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A Theoretical Simulation of the Settling of Proppants in a Hydraulic Fracturing Process

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A THEORETICAL SIMULATION OF THE SETTLING OF PROPPANTS IN A HYDRAULIC FRACTURING PROCESS

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science

In Chemical Engineering at Virginia Commonwealth University

by

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In the name of ALLAH, most compassionate, most merciful.

Blessings of ALLAH on the prophet Mohammad and the family of Mohammad.

Alhamdulillah, praises be to ALLAH for the blessing for the completion of this thesis.

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Dedication

I would like to dedicate this work to the person for whom all the words in the world cannot express her virtues. She passed away, but her soul is still with me to light up my days; to my beloved Mother.

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Abstract

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By Nisreen Alseamr, M.Sc.

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Hydraulic fracturing is a process for the extraction of hydrocarbons from underground formations. It involves pumping a specialized fluid into the wellbore under high pressures to form and support fractures in the rock. Fracturing stimulates the well to increase the production of oil and the natural gas which are the pillars of the energy economy. Key to this process is the use of proppants, which are solid materials used to keep the fractures open. Understanding the transport of proppant particles through a fluid is important to improve the efficiency and reduce environmental impact of fracturing. An increase of the settling velocity for instance, will impede the hydraulic fracturing process by reducing well productivity, or necessitate use of chemical additives. This thesis presents a theoretical investigation of the settling velocity of proppant particles. The effect of different parameters on the settling velocity were studied by manipulating the main factors that can influence particle transport. These include size of the particle (300 µm-

2000 μm), sphericity, density (1200 kg/m^3-3500 kg/m^3) and concentration. These typical values were obtained from commercially available proppants currently used in industry.

Various correlations were investigated, assuming the carrier (fracturing) fluid to be an ideal Newtonian and as a power law (non-Newtonian) fluid. This will help predict the settling velocity for proppant particles in order to increase well productivity, and improve hydraulic fracturing efficiency. The models show that changing the carrier fluid viscosity and particle properties such as diameter, density, sphericity, and concentration leads to a significant change in the proppant settling velocity. For instance, reduction in particle size, density, and sphericity tend to reduce the settling velocity, while increasing the concentration of the particles and the fluid viscosity reduce the settling velocity.

Chapter 1

Introduction

1.1 Background

The oil and natural gas (O&N) industry plays a significant role in the industrial sector of the world because of its great importance in different aspects of life. The industry is based on hydrocarbon materials, which in turn are extracted from underground reservoirs or deposits, and represent an important source of energy. Because of the urgent need for energy, many engineering techniques are employed in extracting and processing the raw hydrocarbon feedstocks (1). One of these techniques that has gained recent and increasing prominence is hydraulic fracturing.

Hydraulic fracturing, sometimes simply called fracturing (1), is a process used to stimulate O&N wells, increase efficiency, and increase the extraction of crude oil and natural gas. Fracturing works by injecting a liquid at high pressure to create fractures in rocks below the surface of the earth. Fracturing techniques can typically raise the general production rate of a well from 5 to 15 (%), and increase the production of hydrocarbons from 1.5 to 30 times more than other traditional methods such as the vertical drilling of the wells (2). However, one of the biggest challenges associated with hydraulic fracturing is that this process can have a negative impact on the environment since it requires and creates materials with potentially dangerous effects on soil, water resources, and often, nearby populations (1).

The process of hydraulic fracturing involves pumping a fracturing fluid, which is a fluid used to create permeable channels in the rock formations, under high pressure. This causes fractures in the wellbore to release the crude oil and natural gas to the top of the surface (**Figure 1.1**). Initially, we outline the steps of the process. Fracturing fluid consists commonly of water, chemicals, and a solid material known as a proppant. Proppants are solid materials that are used to support the fractures that the fracturing fluid creates and keep them open to release the desired hydrocarbon materials. Each well is different in the amount of these additives (water, chemicals, and proppant) (3), as shown further below.

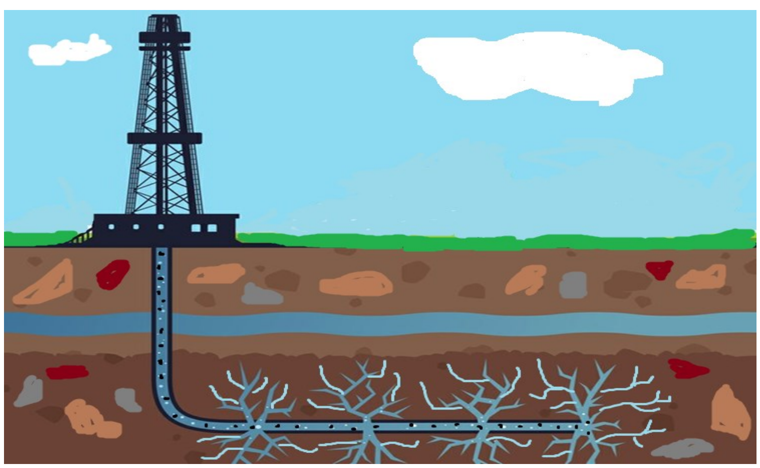

Figure 1.1: Hydraulic fracturing process.[1]

[1] Image adapted from (http://www.businessinsider.com/well-regulated-fracking-wont-contaminate-our-drinking-water-2015-6).

1.2 Process overview

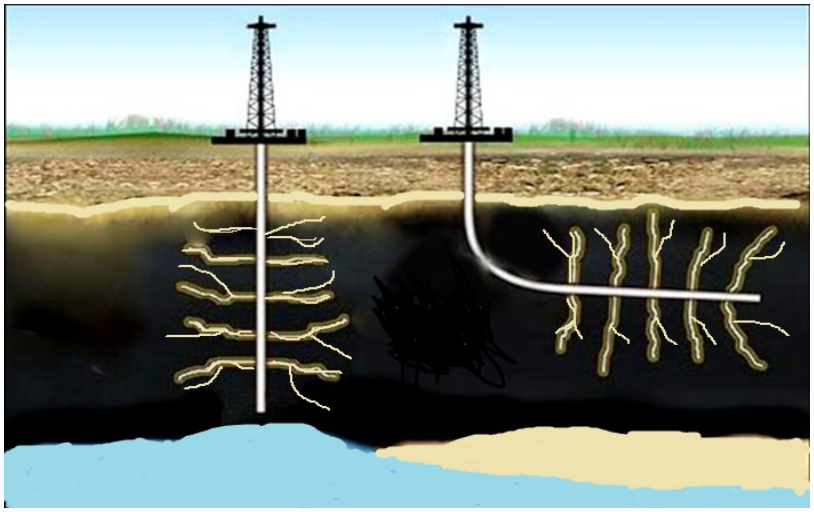

Figure 1.2: Vertical drilling of hydraulic fracturing in the past vs. a combination of vertical and horizontal drilling of hydraulic fracturing now.[2]

The process of hydraulic fracturing has been around for a number of decades. It was first developed in 1947. It was first applied on a vertical wells in 1950. The first horizontal drilling was used in 1930s. In 1974 (4), hydraulic fracturing and the horizontal drilling were combined, and it is now commonly used for extraction (**Figure 1.2**). However, it has only been for the past decade or so that the process of hydraulic fracturing has dramatically changed the energy landscape in the world. As mentioned above, the process of hydraulic fracturing involves pumping a specific fluid into the wellbore under high pressure in order to create fractures in the rock formations. Then, the crude oil and the natural gas are released to the top of the surface. In this process the wells are drilled horizontally. The fracturing fluid carries chemical additives and solid grains called proppants. Proppants are solid materials which are used to keep the fractures open after the end of the pumping process (4). These fractures will serve as permeable channels

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[2] Image adapted from (https://blog.zintro.com/2011/05/24/hydraulic-fracturing-or-fracking-raising-health-and-safety-concerns/).

for the hydrocarbons materials that are confined in the ground to be free. These channels, or fractures, reach about 10 mm in width and several hundred of meters in length. Each fracture can be almost 50-100 m in height (5).

1.3 How hydraulic fracturing works

There are several mechanisms used to complete the hydraulic fracturing process. One method includes extending a tube to the bottom of the ground. This tube has many moving packers that have sleeves which in turn clog the perforation initially in the tube. There is a hole between the sleeves in each packer. This hole varies in its size from large to small and the packers divide the horizontal tube into sections. The horizontal tube is located half a mile or more from the vertical tube (**Figure 1.3**). The process starts in the last section of the horizontal tube to be fractured. This will be after proppant is pumping. After that a ball is placed in the last packer which will close the hole because the ball is larger in size than the hole. High pressure is applied on the packer in order to push it and move the sleeves to open the perforation before the sleeves and so on until the last packer beside the vertical tube to be fractured (1).

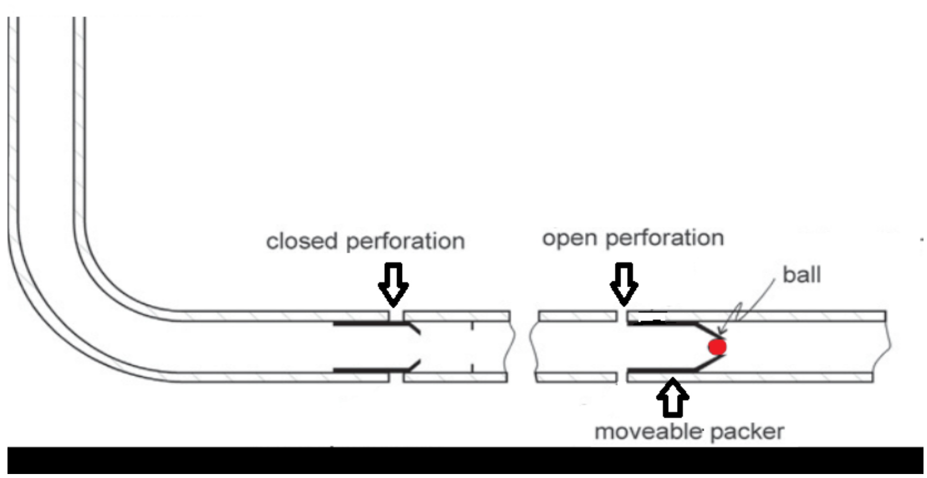


Figure 1.3: One mechanism of hydraulic fracturing (1).

1.4 Fracturing fluid

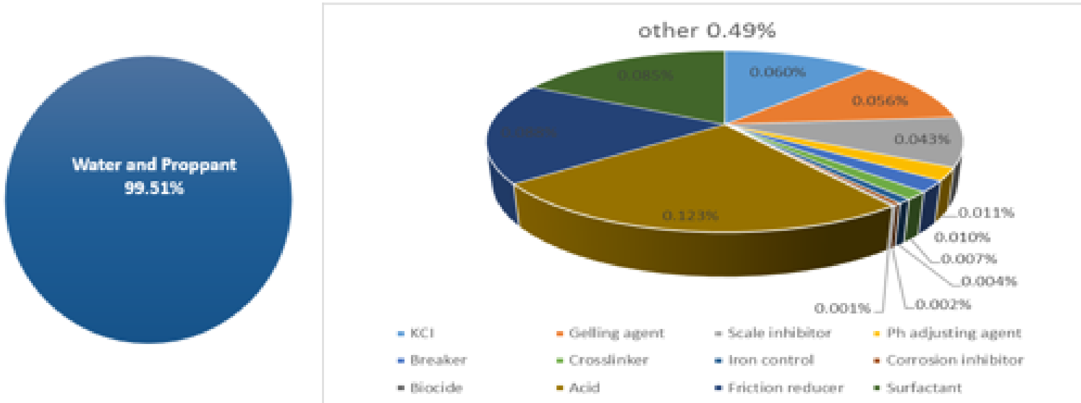


Figure 1.4: The compositions of the fracturing fluid and their percentages (3).

The fracturing fluid is one of the most important elements of the hydraulic fracturing process. Fracturing fluid contains water, various chemicals, and solid grains called proppants (**Figure 1.4**). The proppants play a vital role in fracturing because they work as "propped agents" to keep the fractures open and prevent them from closing after the pumping stops. This helps to release the hydrocarbons to the wellbore through the perforations. There are different types of fracturing fluids, including (1):

- Oil based fracturing fluid.
- Water based fracturing fluid.
- Alcohol based fluid.
- Acid frac [3] fluid.
- Foams.
- Slick water.

[3] (frac is a commonly used abbreviation of fracturing)

In addition to these types, other types include:

- Emulsion fluid.
- Noncomplex gelled water fracture.
- Nitrogen-foam fracture.
- Complexed gelled water fracture.
- Premixed gel concentrates.

It should be noted here that 90% of the used frac fluids is water (6). The remainder is made of proppant material (~9%), and various chemical additives added to the fracturing fluid (~0.5%). Each element of these additives has a specific role in the hydraulic fracturing process. **Table 1.1** shows the types of the additives and their usage in the fracturing, and **Figure 1.4** indicates the percentages of these additives. Typically, successful hydraulic fracturing processes tend to use 3 to 12 of these additives in various proportions (3).

Table 1.1: Typical composition of fracturing fluid and their usage (3).

Additives	Usage
Fluid (typically water)	Create the fractures in the formation and carry a propping agent.
Hydrochloric acid (HCl)	Help dissolve minerals and remove damage near the well bore by clean; also helps initiate fissures in the rock matrix.
Corrosion inhibitor (typically ammonium bisulfate)	Used only when acid is used to prevent pipe corrosion.
Biocides (typically sodium hypochlorite or chlorine dioxide)	Control bacterial growth in the water injected into the well and prevent pipe corrosion.
Friction reducers (typically polyacrylamide based compounds)	Reduce pipe friction and pressure in the piping required to pump fluids.
Gelling agents (guar gum and cellulose)	Thicken water-based solutions and help in suspension and transport of proppants into formation.
Crosslinking agent (boric acid, titanate and zirconium)	Enhance abilities of the gelling agent to even further aid in transport of proppant material.

Breaker solution	Cause the enhanced gelling agent to break down into a simpler fluid so it can be readily removed from the wellbore without carrying back the proppant material.
Oxygen scavenger (ammonium bisulfate)	Prevent corrosion of pipe by oxygen.
Iron control and stabilizing agents (citric acid and acetic acid)	Keep iron compounds in soluble form to prevent precipitation.
Surfactant	Promote flow of the fluids used in the fracturing process
Scale Inhibitor (ethylene glycol)	Control the precipitation of specific carbonate and/or sulfate minerals.
Proppants	Hold fissures open so gas and oil can be extracted.

Fracturing includes pumping the fracturing liquid to the reservoir rocks under extreme pressure in order to stimulate the rocks and create fractures. The fractures remain open during the pumping process because of the pressure of the driven fluid. However, after the pumping stops, the fractures tend to close because of the decrease in the applied pressure, and reduce the permeability of the fracture. To solve this problem a propping agent - proppant, is used. The proppant is a grain material placed in the frac fluid. Because of the presence of the proppant, fractures will hold open after the pumping has stopped and lose pressure. This therefore results in an increase the conductivity of the well, and increase in the production of the crude oil and natural gas through the wellbore (7). In order for the proppant to do its mission in the hydraulic fracturing process, the material should have certain characteristics to qualify it to withstand the high pressures inside the fracture. For example, it has to be strong enough to resist the applied pressure and thus keep the fracture open to obtain the required productivity. Otherwise, the particle will be crushed and generate fines, which will clog the fracture and reduce its permeability. Though large proppant particles create porous media that helps to increase the well permeability, small particles have a significant role of reducing the settling velocity of the proppant particles. Low density of the proppant particles has a great effect of controlling and reducing the settling velocity of the proppant particle. Finally, spherical shape of the proppant is a good factor to increase the porosity. However, increasing the shape of the proppant toward the sphere shape tends to increase the settling velocity which is the main interest in this research.

1.5 Proppants in the past and the present

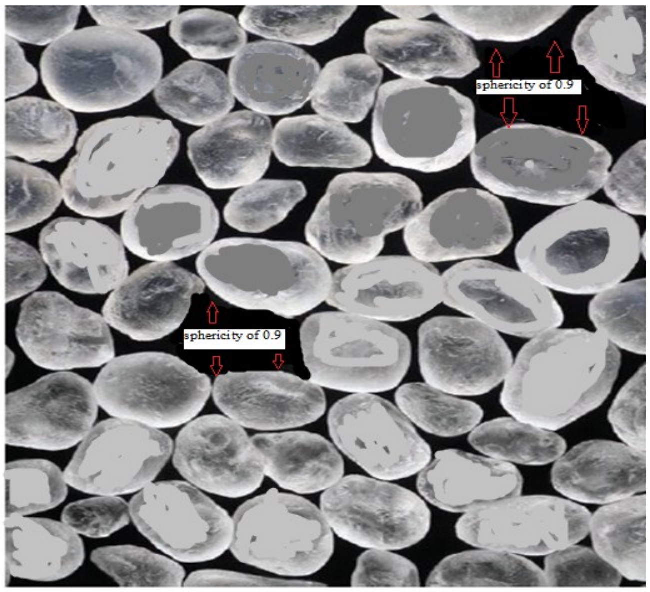


Figure 1.5: Northern White Sand (Ottawa sand).[4]

When the work in hydraulic fracturing processes started in the end of the 1940s, the used proppant was a natural sand from sandstone. This kind of sand is called Ottawa sand because it mined near Ottawa, Illinois. The range of the size for the sand grains is 841 to 400 microns. Ottawa sand is also called white sand with a sphericity of 0.9 (8) (**Figures 1.5 and 1.6**).Currently proppants from Hickory sandstone formation, mined near Brady, Texas and called Brady sand

[4] Image adapted from (http://www.intechopen.com/books/effective-and-sustainable-hydraulic-fracturing/modeling-of-proppant-permeability-and-inertial-factor-for-fluid-flow-through-packed-columns).

(7, 8) are also commonly used. The grains of the Brady sand tend to be angular with a sphericity from 0.6 to 0.7 where the sphericity is how close the particle of the proppant from the shape of the sphere.

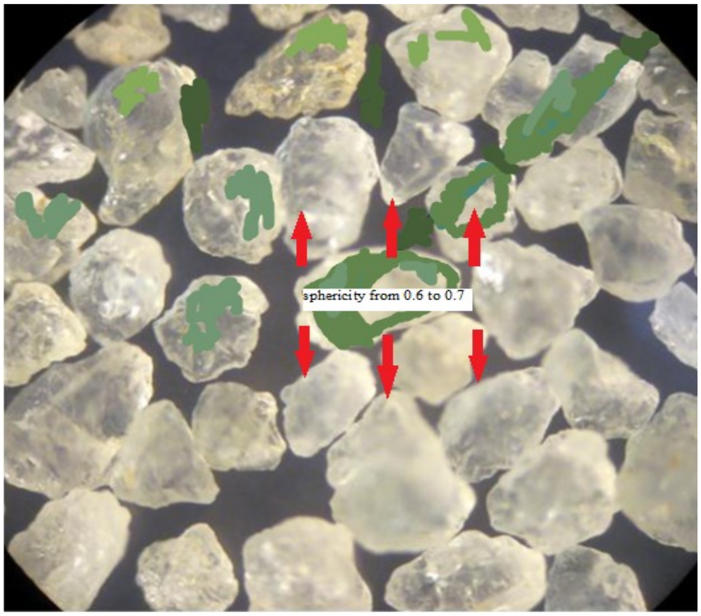


Figure 1.6: Brown Sand (Brady sand).[5]

In the 1970s the drilling process for the wells in the hydraulic fracturing became deeper wherein the sand grains became unable to resist the high pressure in the fractures under the ground. Thus the need arose to produce proppant grains with higher strength. This resulted in proppants from sintered bauxite, resin-coated sand in its both type curable sand and procured, ceramics, and zirconia (no longer used) (7). See **Tables 1.2 and 1.3** for more information on proppant properties (8). **Table 1.4** contains information about commercial sources of proppants available today, and their properties.

[5] Image adapted from (http://www.intechopen.com/books/effective-and-sustainable-hydraulic-fracturing/modeling-of-proppant-permeability-and-inertial-factor-for-fluid-flow-through-packed-columns).

Table 1.2 General information on Proppants (8).

Provenance	Strength	Type
Natural	Low strength	Quartz sand
Synthetic	Intermediate strength	Low density alumina silicate proppants
		High density alumina oxide and silicate proppants
	High strength	Alumina oxide proppants

Table 1.3 Summary of proppant properties (8).

Proppant Type	Size and Shape	Strength	Conductivity
Sand	Irregular size and shape , dependent on source	Low	Low
Resin-coated sand	Irregular size and shape, smooth, rounded	Medium	Medium
Ceramic	Uniform size and shape, round	High	High
Resin-coated ceramic	Uniform size and shape	Very high	High

Table 1.4: Types and properties of commercially available proppants

Composition	Wt. %	Specific gravity	Size	Roundness	Sphericity
Alumina silicate: -aluminum oxide -aluminum silicate	 15-85% 15-85%	2.85-3.5	105 µm-1680 µm	0.9	0.9
Silica: -quartz (crystalline Silica – SiO_2. -aluminum oxide -iron oxide -titanium oxide	 98-99% <0.8% <0.1% <0.1%	2.65	210 µm -1190 µm	0.6-0.9	0.7-0.9
-quartz -resin: -phenol-Formaldehyde resin -hexamine (hexamethyle-netetramine) resin	95-98% 2-5% <0.4	2.65-2.85	210 µm -841 µm	0.6-0.8	0.6-0.8
-aluminum oxide(alumina) -resin: phenol-formaldehyde resin	95-100 % 0-5 %	2.65-3.40	297 µm -1680 µm	0.9	0.9
-aluminum oxide(alumina) -resin: phenol-formaldehyde resin	95-100 % 0-5 %	2.65-3.40	297 µm -1680 µm	0.9	0.9
-Aluminum oxide (alumina), bauxite	100%	2.65-3.34	177 µm -1680 µm	0.8-0.9	0.8-0.9

Based on the above Tables, it may be observed that the main and important factors that affect the selection of the proppant grains can be summarized as follows:

- The size of the proppant.
- The density of the proppant.
- The shape of the proppant.
- The concentration of the proppant.
- The viscosity of the carrier fluid.

Improper selection for these factors can results not only in failure of the material but also settling problem which is our primary target in this research.

1.6 Statement of the problem and the objective of the study

1.6.1 Statement of the problem

Once the proppant properties as well as the factors that affect the settling of proppants determined, it may be noted that there are other issues faced by proppant particles during the transport to the fractures such as flow-back and leak-off phenomena. However, the problem of proppant settling is the primary focus of this study, since it has the greatest influence on the success and efficiency of the fracturing process. The aim of this research to investigate the settling of the proppant particle during its transport in the carrier fluid and understand the relative significance of the different factors that can affect it. Thus, various models were used to predict the settling velocity of the particles theoretically and its relationship with the size, the density, the concentration, and the shape of the proppant particles. Finally, the properties of the carrier fluid was also studied. **We are interested in a theoretical understanding of the effect of various properties on proppant behavior so that we can propose strategies to reduce the settling of the proppant particles during their transport inside the fracture.**

1.6.2 The objective of the study

Accordingly, the goal of this research is to theoretically investigate the settling velocity of proppant particles and to reduce it to achieve a better conductivity from the well. This is done by identifying the main factors that influence the settling velocity, and understanding how these factors can affect the settling velocity. Several correlations were used to calculate and predict the settling and reducing it to the minimum value. These calculations were performed based on assuming a "theoretical fracturing fluid" that was assumed to be both of the fluid types - Newtonian and non-Newtonian.

Overall, these calculations can enable us to predict the properties of the proppant and how we can control them to reduce the settling. For example, decreasing both the size and density of the proppant can lead to reduction in settling velocity. However, as discussed above, reduced size and density are accompanied by issues with porosity and mechanical integrity. Similarly, considering the shape of the proppant - when the particles are spherical in shape, the settling is greater despite the importance of spherical shape in the increasing of the permeability of the fractures more than the angular shape (8). The concentration of the proppant particle has an inverse relationship with the settling velocity. Whenever the concentration of the particles increases, the settling decrease. This is because the increase in the concentration of the proppant leads to increase in the viscosity of the slurry (5). On the other hand, increasing concentration would imply greater proppant and pumping loads that may be more expensive. It was also found that increase in the viscosity of the proppant carrier fluid causes a decrease in the settling velocity. It is clear that the problem of proppant transport is a complex one and changing any one of these factors alone is not feasible without it making an impact on other properties. Understanding the factors that affect the settling velocity and their relationship with it enable us to estimate how we can reduce the settling to achieve better transport for the proppant particles during hydraulic fracturing. Details on the calculations and results are presented in the subsequent chapters.

1.7 Structure of the thesis

The research was organized as the following:

Chapter 1 provides overview, background, and the aim of the research.

Chapter 2 provides the method and research methodology.

Chapter 3 shows the calculations performed and the results.

Chapter 4 gives the discussion about the results.

Chapter 5 summarizes the conclusions and recommendations.

Chapter 2

Settling velocity in Newtonian and non-Newtonian fluids

2.1 Introduction

Settling is one of the most important problems that can impede the success of the hydraulic fracturing process because of its effect on the proppant - one of the basic elements in the fracturing process. Proppants keep the fractures open after the pumping of the fracturing fluid stops, and release the hydrocarbons stored underground to the surface. When these proppants settle under the influence of gravity, the production of hydrocarbons is reduced because of the closing of the fractures due to deposits and loss of the particles. There is therefore a need to calculate the settling velocity of proppants in order to estimate it for certain systems before the fracturing process is performed. This is the focus of the research work in this thesis and is aimed at helping to design strategies to minimize this velocity (lower settling velocity = lower settling) and ensure higher efficiency of fracturing. Understanding the factors that affect the settling velocity therefore has a vital role in its prediction. This chapter presents a few correlations that were studied to determine the settling velocity. These correlations were used to estimate the settling velocity of proppants under different conditions. Further, the validity of these correlations are presented here.

2.2 Assumptions

In Chapter 1, the transport of the proppant in a wellbore in the hydraulic fracturing was discussed. It is evident that the proppant should not settle down in the wellbore, because this will prevent the proppant from doing its job *prior* to reaching the fractures. Ideally, the proppant particles should be placed in the fracture to keep it open after the pumping process stops. This will allow the extracted hydrocarbons to be released to the surface. This is why it is important to avoid settling, or at the very least, to reduce the settling velocity as much as possible.

Initially, research was conducted by reviewing the literature to identify the factors that can affect the settling velocity and how they can be controlled. Most typically, four of these factors are related to the particle itself - the size, the density, the shape factor, and the concentration of the proppant particles. Another important factor is the viscosity of the carrier fluid i.e. the fracturing fluid that carries the proppant particles. In order to investigate the effect of these factors on settling velocity, initially, we make a few simplifying assumptions (9):

- The pressure varies only in the direction of the flow.
- The fluid is incompressible (i.e. it has a constant density).
- The flow is steady, there is no change with time.
- The fluid may be Newtonian or non-Newtonian (power law rheology).
- Edge effect is neglected.
- No leak off to the formation.
- Constant width of the fracture.
- Particles and fluid have the same velocity.

- Particles are assumed to be spherical.
- Fully developed flow meaning that there is a balance between the pressure and the only one shear stress τ_{yx} (Pressure Driven Flow effective in the direction of the flow).

Once these assumptions were set, correlations related to both Newtonian and non-Newtonian fluids were identified to be used for settling velocity calculations. A theoretical investigation from the literature on these parameters for the proppant and type of fluid was used. It should be mentioned here that the assumption “no leak-off to the formation” is not typically applicable in real situations as there is fluid loss due to the formation permeability (7).

2.3 Method

Based on commercially available proppant particles and typical carrier fluids, certain key parameters were selected for the analysis. These refer to the size, density, sphericity, concentration of the proppant particles, and the fluid that carries the proppant particle (carrier fluid). The values of the properties of the proppant particles chosen are close to the representative values for sand proppant particles. The data were tested on correlations related to both Newtonian and non-Newtonian fluids in order to make reliable predictions about our goal, which was to understand the effect on, (and ultimately, reduce) the settling velocity. The parameters of size (particle diameter), density, sphericity, and the concentration of the particles were varied. Finally, the viscosity of the fluid which carries the proppant was also varied.

The size, the density, and the sphericity (also known as shape factor) - which is the ratio of the surface area of a sphere which has the same volume to the surface area of the particle, have a positive relationship with the settling velocity. In contrast, the concentration of the proppant particle and the viscosity of the carrier fluid are expected to have an inverse relationship with the settling velocity. The various correlations and data used are initially discussed and in the next chapter, the results and the discussion are discussed.

2.3.1 Newtonian fluid

Initially, the carrier (fracturing) fluid is assumed to be Newtonian. Three different correlations were studied.

2.3.1.1 Correlation N1

Equation 2.1 obtained from the work by Nguyen et al. (10), is valid for spherical particle flows in Newtonian and with a range of Archimedes number from 0 to 512,000 where the Archimedes number (*Ar*) is a dimensionless number, representing the ratio between the gravitational force and the viscous force of the motion of the fluids due to density differences.

$$\frac{v_{stokes}}{v_s} = 1 + \frac{Ar}{96}(1 + 0.079\ Ar^{0.749})^{-0.755} \qquad (2.1)$$

The above correlation is used to determine the settling velocity of a single particle, which moves in an unbounded, Newtonian fluid (10). Here v_{stokes} is the Stokes' terminal velocity of a single particle moving in the Stokes regime, and is valid for Newtonian, dilute, laminar, and creeping flow with Reynolds number less than 0.1, determined as:

$$v_{stokes} = \frac{d^2\ g\ (\rho_p - \rho_f)}{18\ \mu_f} \qquad (2.2)$$

Stokes velocity is the result of the balance of the three forces - gravitation force (F_g), buoyancy force (F_b), and drag force (F_d).

v_s is the settling velocity for a single particle, and *Ar* is the dimensionless Archimedes number.

The Archimedes number for Newtonian fluid is given by (11):

$$Ar = \frac{g\, d^3\, (\rho_p - \rho_f)\rho_f}{\mu^2} \qquad (2.3)$$

d is the diameter of the proppant particle, g is the acceleration due to gravity m/s^2, ρ_f is the density of the carrier fluid, kg/m^3, ρ_p is the density of the proppant particle, kg/m^3, μ_f is the fluid viscosity, $Pa.s$. Archimedes number is used to determine the motion of fluids due to difference in their densities, i.e. it is used to establish the theoretical settling range for the particles moves in the fluids. If $Ar >> 1$ then gravitational forces dominate, i.e. less dense bodies rise and denser bodies sink, if $Ar << 1$ then viscous forces dominate.

2.3.1.2 Correlation N2

The correlation of Chhabra and Peri (12) is considered:

$$Re = a\, Ar^{\,b} \qquad (2.4)$$

This correlation is valid when:

1. $10 < Ar < 10^6$
2. $1 < Re < 10^4$
3. $0.38 \leq n \leq 1$
4. Newtonian and non-Newtonian fluids are used

Where n is the power index, a and b are the following:

$$a = 0.1 \times \exp\left(\frac{0.51}{n} - 0.73\, n\right) \qquad (2.5)$$

$$b = \frac{0.954}{n} - 0.16 \qquad (2.6)$$

For a Newtonian fluid, the Archimedes number Ar is given by Equation 2.3, and the particle Reynolds number is given by (13):

$$Re = \frac{\rho_f \ v_s \ d}{\mu_f} \tag{2.7}$$

The Reynolds number is the ratio of inertial forces to viscous forces and is a dimensionless quantity that uses to predict the flow regime. If *Re* is less than 2100, the flow is laminar and if the *Re* is 4000 and higher, the flow is turbulent. The transition flow is the area between the laminar flow and the turbulent flow (9). The Reynolds number in Equation 2.7 is related to the object, the spherical particle in our case, in a fluid and is used to determine the falling velocity of a particle. When the viscous force is high, the flow is laminar and the *Re* is small.

2.3.1.3 Correlation N3

The correlation of Turton and Clark (12) was used.

$$Re = Ar^{1/3} \{ 1/(\frac{18}{Ar^{2/3}})^{0.824} + (\frac{3}{4} \frac{0.428}{Ar^{1/3}})^{0.412} \}^{1.214} \tag{2.8}$$

The above correlation is valid for Newtonian fluids with a range of Archimedes number of (1-10^8) and Reynolds number from 0.1 to 2 x 10^4.

2.3.2 Non-Newtonian fluid

Assuming the carrier fluid to be non-Newtonian, two correlations were used as the following:

2.3.2.1 Correlation NN1

Initially correlation N2, Equation 2.4 was used. As mentioned above, this correlation is valid for both Newtonian and non-Newtonian fluids. Different formulas for Reynolds number and Archimedes number related to the non-Newtonian fluid were used. The Reynolds number for non-Newtonian fluid also called “generalized Reynolds number” is given by (12):

$$Re' = \frac{\rho_f \ {v_s}^{2-n} \ d^n}{k} \tag{2.9}$$

Archimedes number for non-Newtonian fluid is given by:

$$Ar' = \frac{4 \ g \ d^{(2+n)/(2-n)}}{3 \ k^{2/(2-n)} \ \rho^{2/(n-2)}} \left(\frac{\Delta\rho}{\rho}\right) \tag{2.10}$$

Where:
$$\Delta\rho = \rho_p - \rho_f \tag{2.11}$$

2.3.2.2 Correlation NN2

The second correlation used to determine the properties of the proppant in the non-Newtonian fluid was developed by Chein (14).

$$ {v_s}^2 + 4.458 \ e^{(5.03 \ \phi)} \left(\frac{\mu_f}{d \ \rho_f}\right) v_s - 19.45 \ e^{(5.03 \ \phi)} \ d \left(\frac{\rho_p}{\rho_f} - 1\right) = 0 \tag{2.12}$$

Where: Φ is the sphericity (shape factor of the particle), *d* is the mean diameter of the particle. The sphericity ranges from 0.3 to 1 where the latter is a perfect sphere and 0.3 indicates a completely irregular shape. In reality, there are no perfectly spherical particles usable as commercial proppants. For the usage of this correlation for determination of the properties of a non-Newtonian fluid, sphericity of 0.9 was used which reflected the assumption of using near-spherical particles and close to the sphericity of commercial sand proppants (0.7-0.9). The above correlation is valid for both Newtonian fluids and non-Newtonian (power law) fluids. Also, it is valid for particle Reynolds number in the range of 0.001 to 10,000. The viscosity of a non-Newtonian fluid is called the apparent viscosity. It is not constant and is a function of shear rate, given by: ($\eta \left(\dot{\gamma}_{yx}\right) = \mathrm{k} \left(\left|\dot{\gamma}_{yx}\right|\right)^{n-1}$) (15).

For the motion of the spherical particle in a liquid over the entire surface of the particle, the average share rate will be ($\dot{\gamma} = v_s/d$) (16, 17).

So, the apparent viscosity for non-Newtonian fluid will be equation (2.13) given by (14, 16):

$$\mu_f = k\left(\frac{v_s}{d}\right)^{n-1} \tag{2.13}$$

Where k is the consistency index, $Pa.s^n$. k and n are the parameters that describe the rheology, deformation and the flow, of the non-Newtonian fluid. k is the shear stress at shear rate of 1 (7).

2.4 Objective parameters for Newtonian fluids and non-Newtonian fluids

2.4.1 Particle size (*d*)

Ten different sizes of proppant particles (300, 500, 700, 900, 1000, 1500, 1600, 1700, 1800, and 2000 *μm)* were tested using the three correlations for the Newtonian fluid (water), and five different sizes (1500, 1600, 1700, 1800, and 2000 *μm)* were tested using the two correlations for non-Newtonian fluid. These correlations were tested as follows - with high, medium, and low particle density; (1500, 2000, and 2650 kg/m^3) were chosen, which are related to the density of sand particles, because sand is the common material used in hydraulic fracturing. Then, settling velocities were obtained corresponding to each size for all correlations. The assumptions and the validity of each correlation were checked to ensure the valid range of each correlation. The percentage increase and the percentage decrease of the settling velocity were obtained for each section in the mentioned correlations. It should be mentioned that the data (n = 0.8610*,* k = 0.0865 $Pa.s^n$*,* and ρ_f = 1000.211 kg/m^3) for non-Newtonian fluid were taken from reference (18) in order to perform the calculations.

2.4.2 Particle density (ρ_p)

In this section five values of proppant density were chosen - from 2500 kg/m^3 to 3500 kg/m^3 for the non-Newtonian fluid. Whilst for the Newtonian fluid, eleven values of density (1000 – 3500 kg/m^3) were chosen. The selection of these values was done because they were close to the density of the sand particle which is the common type of the proppant. Testing were conducted for these values of the density using the Newtonian correlations in three parts, similar to what was done for the particle size- large, medium, and small particle sizes. For the non-Newtonian carrier fluid, similar steps were performed. Testing was made for the five values of the densities on the two correlations for non-Newtonian fluid in three parts. First part with a large particle size, which was equal to 2000 μm. The second one and the third one were 1500 μm and 1000 μm respectively. Substituting the data and calculating the reduction in the settling velocity for each part in the two correlations were done. The density of the carrier fluid was 998.2 kg/m^3 for Newtonian fluid which is the typical density of water at 20 ^oC and 1000.211 kg/m^3 for the non-Newtonian fluid which was taken along with k and n from the data of Kelessidis and Mpandelis (2004) in the reference (18).

2.4.3 Particle shape factor (ϕ)

The influence of the shape factor (also called sphericity), is studied here because of its practical and industrial importance in the hydraulic fracturing process, wherein sand or other natural materials are used that are often not spherical. Increasing the sphericity tends to increase the porosity between the particles, resulting in an increase in the conductivity of the well. The conductivity is an important measure used in fracturing operations and refers to how effectively fluids are transported through permeable media, (*length*^{2}*/time*). The higher in the shape factor, the more the settling, which is not desirable. Estimation of the sphericity was done by using Chein's correlation (NN2) that was used for the non-Newtonian fluid. Assuming the shape factor and then calculating the settling velocity for the proppant were done by using Equation 2.12. Testing was performed in four types of shape factor 0.3 (irregular shape), 0.6, 0.9 (nearly spherical particle), and 1 (perfect sphere) with large particle size, 2000 μm and then with the medium and small sizes of particles of 1500 μm and 1000 μm respectively.

2.4.4 Particle volume fraction (C)

In a dense suspension, the settling velocity is called "hindered velocity" due to the high concentration of particles. To test the effect of the volume fraction on the settling velocity, the correlation of Richardson and Zaki (R&Z) was used (19, 20):

$$\frac{v_h}{v_s} = \varepsilon^z = (1-c)^z \qquad (2.14)$$

Where: ε is the porosity; the voids between the proppant particles (21):

$$\varepsilon = 1 - c \qquad (2.15)$$

For concentrated suspension, $\varepsilon < 0.85$ (20).

For the dilute solution, $\varepsilon > 0.92$ (20).

C is the volume fraction of the particle. v_h is the hindered settling velocity for the suspension. The above correlation, Equation 2.14, is valid for:

1. $0 < C < 0.37$ ($0.37 \cong 0.4$), (22).
2. Newtonian fluid and creeping flow, (19).

3. Uniform size of particles (monodisperse).
4. Spherical particle.
5. Inelastic fluid (non-Newtonian fluid), (15, 23)
6. $0.6 \leq n \leq 1$ (23, 24).
7. $d/D \leq 0.16$ (23, 24), where *d/D* is the ratio of the particle diameter to the tube diameter (19).

For Newtonian fluid, *z* is a modified form (15) of *z* for the Newtonian fluid in the R&Z correlation (19):

$$z = 4.65 + 20\left(\frac{d}{D}\right) \; ; \quad Re \leq 0.2 \qquad (2.16)$$

$$z = \left(4.40 + 18\left(\frac{d}{D}\right)\right)(Re)^{-0.3} \; ; \quad 0.2 \leq Re \leq 1 \qquad (2.17)$$

$$z = \left(4.40 + 18\left(\frac{d}{D}\right)\right)(Re)^{-0.1} \; ; \quad 1 \leq Re \leq 200 \qquad (2.18)$$

$$z = 2.40 \; ; \quad Re > 200 \qquad (2.19)$$

For the non-Newtonian fluid *z* is (15):

$$z = 4.65 + 20\left(\frac{d}{D}\right) + \frac{(1-n)}{n} \; ; \; Re' \leq 0.2 \qquad (2.20)$$

$$z = \left(4.7 + 8.8\left(\frac{d}{D}\right)\right)(Re')^{-0.1} \; ; \; 0.3 \leq Re' \leq 165 \qquad (2.21)$$

2.4.4.1 Steps to determine the effect of the volume fraction on the settling velocity

By using the three correlations for Newtonian fluids, and the two correlations for non-Newtonian fluids, the settling velocities were estimated. These settling velocities were used in the R&Z

correlation (19) to determine the hindered settling velocities after assuming certain values of the volume fraction close to what were found in the literature. The outline of the steps for each correlation in both fluids (Newtonian and non-Newtonian), is provided below:

2.4.4.1.1 Newtonian fluid

2.4.4.1.1.A Correlation N1

1. Using correlations N1 (Equation 2.1) for Newtonian fluid to get v_s (settling velocity for a single spherical particle).
2. Determining *Re* by using v_s in Equation *Re* 2.7.
3. Selecting the appropriate equation to determine z based on the value of *Re*.
4. Using the R&Z correlation, Equation 2.14, to determine v_h (hindered settling velocity) by assuming large, medium, and small values for volume fraction in the range (0.01-0.35). These values were chosen under the restrictions of the validity of the R&Z correlation and the range of the porosity.

2.4.4.1.1.B Correlation N2

1. Using correlations 2 for Newtonian fluid (N2) to determine *Re*.
2. Determining v_s from the formula of *Re* for the Newtonian fluid.
3. Selecting the appropriate equation to determine z based on the value of *Re*.
4. Using the R&Z correlation and assuming same values for the volume fraction (0.01-0.35) to determine v_h (hindered settling velocity).

2.4.4.1.1.C Correlation N3

Repeating same steps in correlation (N2) for Newtonian fluid to determine the hindered settling velocity in correlation 3, Equation 2.8, for the Newtonian fluid.

2.4.4.1.2 Non-Newtonian fluid

2.4.4.1.2.A Correlation NN1

Following the same procedure in correlation (N2) for the Newtonian fluid but replacing Re by Re' , Equation 2.9, and Ar by Ar', Equation 2.10, to get the hindered settling velocity for non-Newtonian fluid.

2.4.4.1.2.B Correlation NN2

1. Using correlations (NN2) for non-Newtonian fluid, Equation 2.12, to determine v_s the settling velocity for a single spherical particle.
2. Determining Re' by using v_s .
3. Determining z from Re' (selecting the appropriate equation to determine z based on the value of Re').
4. Using R&Z correlation and assuming four values for the volume fraction in the range (0.01-0.35) to get v_h (hindered settling velocity).

2.4.5 Fluid viscosity

Three different values of the fluid viscosity were chosen - large, medium, and small for the Newtonian fluid. These values are (0.001, 0.005, and 0.02) *Pa.s*. As mentioned earlier, the viscosity of a water frac fluid at 68 ^oF is ~0.001 *Pa.s* and the other values were considered when more additives were added to the water like Guar gum, to increase the viscosity to 50 *cP* or 100 *cP* or more (depending on additives that were used to form linear gels or cross-linked gels frac fluids) (7). The variation between the values was under the restriction of the validity of the correlation. As done for the size and the density of the particle, the test was done for the three values of the viscosity in three sections on each correlation. Each section had a different value of size. The sizes were 2000, 1500, and 1000 μm in each correlation. For a non-Newtonian fluid, the value of k, consistency index, was changed to see the effect of the viscosity on the settling velocity because the viscosity of non-Newtonian fluid depends on k and has a positive relationship with it, Equation 2.13.

Chapter 3

The effect of proppant carrier fluid properties on the settling velocity

3.1 Introduction

The settling of proppants is one of the factors that has a significant influence on the success of hydraulic fracturing. It has an effect on the efficiency and conductivity of the oil and natural gas wells. After outlining the correlations that were used to calculate the settling velocity in Chapter 2, here we demonstrate the results that were obtained from those calculations. The primary target of this study v_s represents the settling velocity for a single particle (*m/s*).

Briefly, the parameters that were used in all the correlations are mentioned here:

Ar and *Ar′* are the Archimedes numbers, and *Re* and *Re′* are the Reynolds numbers for Newtonian and non-Newtonian fluids respectively. *d* is the diameter of the proppant particle, *g* is the acceleration due to gravity, 9.81 m/s^2, ρ_f is the density of the carrier fluid, 998.2 kg/m^3 for Newtonian fluid (water) and 1000.211 kg/m^3 for non-Newtonian fluid, ρ_p is the density of the proppant particle, kg/m^3, μ_f is the fluid viscosity, *Pa.s*, Φ is the sphericity. *k* and *n* are the non-Newtonian fluid rheology, the consistency index and the power index, 0.8610, respectively. Finally, *C* is the volume fraction of the particle, and v_h is the hindered settling velocity for the suspension, *m/s*.

3.2 Particle Size

3.2.1 Newtonian fluid correlations

Ten sizes of the particles were tested on the three correlations of the Newtonian fluid where $\mu_f =$ 0.001 $Pa.s.$

3.2.1.A Correlation N1

Recall Equation 2.1:

$$\frac{v_{stokes}}{v_s} = 1 + \frac{Ar}{96}\left(1 + 0.079\ Ar^{0.749}\right)^{-0.755}$$

The ten sizes of the particle were tested in three parts on Correlation N1:

a- With high particle density: ρ_p = 2650 kg/m^3

Table 3.1: Settling velocity with high particle density.

d (μm)	d (m)	v_{stokes} (m/s)	Ar	v_s (m/s)
300	0.0003	0.081	436.725	0.043
500	0.0005	0.225	2021.87	0.078
700	0.0007	0.441	5548.02	0.112
900	0.0009	0.729	11791.6	0.142
1000	0.001	0.900	16175	0.157
1500	0.0015	2.026	54590.6	0.223
1600	0.0016	2.304	66252.8	0.235
1700	0.0017	2.602	79467.7	0.248
1800	0.0018	2.917	94332.5	0.259
2000	0.002	3.601	129400	0.282

b- With medium particle density: $\rho_p = 2000\ kg/m^3$

Table 3.2: Settling velocity with medium particle density.

d (μm)	*d (m)*	v_{stokes} *(m/s)*	*Ar*	v_s *(m/s)*
300	0.0003	0.049	264.869	0.029
500	0.0005	0.136	1226.25	0.055
700	0.0007	0.268	3364.82	0.080
900	0.0009	0.442	7151.47	0.103
1000	0.001	0.546	9809.97	0.114
1500	0.0015	1.228	33108.6	0.164
1600	0.0016	1.398	40181.6	0.173
1700	0.0017	1.578	48196.4	0.182
1800	0.0018	1.769	57211.7	0.191
2000	0.002	2.184	78479.7	0.209

c- With low particle density: $\rho_p = 1500\ kg/m^3$

Table 3.3: Settling velocity with low particle density.

d (μm)	*d (m)*	v_{stokes} *(m/s)*	*Ar*	v_s *(m/s)*
300	0.0003	0.025	132.673	0.017
500	0.0005	0.068	614.225	0.033
700	0.0007	0.134	1685.43	0.049
900	0.0009	0.222	3582.16	0.065
1000	0.001	0.273	4913.8	0.072
1500	0.0015	0.615	16584.1	0.106

1600	0.0016	0.7000	20126.9	0.113
1700	0.0017	0.790	24141.5	0.119
1800	0.0018	0.886	28657.3	0.125
2000	0.002	1.094	39310.4	0.137

The settling velocity compared to the particle size is summarized below:

Table 3.4: Settling velocity vs. particle size (Correlation N1).

d (μm)	High density 2650 kg/m^3 v_s *(m/s)*	Medium density 2000 kg/m^3 v_s *(m/s)*	Low density 1500 kg/m^3 v_s *(m/s)*
300	0.043	0.029	0.017
500	0.078	0.055	0.033
700	0.112	0.080	0.049
900	0.142	0.103	0.065
1000	0.157	0.114	0.072
1500	0.223	0.164	0.106
1600	0.235	0.173	0.113
1700	0.248	0.182	0.119
1800	0.259	0.191	0.125
2000	0.282	0.209	0.137

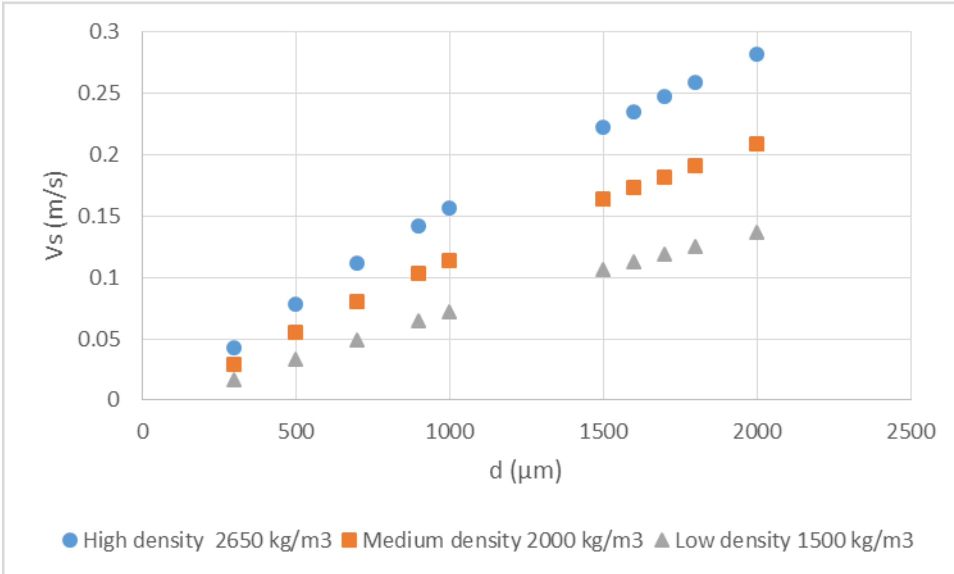


Figure 3.1: Settling velocity vs. particle size (Correlation N1).

There is a direct relationship between the particle size and the settling velocity - as the particle size increases, the settling velocity increases.

3.2.1.B Correlation N2

Recall Equation 2.4:

$$Re = a\, Ar^{\,b}$$

The same procedure was followed here - ten sizes of the particles were tested in three parts on Correlation 2, Equation 2.4, and organized as the following (where: $n = 1$ for Newtonian fluids).

a- With high particle density: $\rho_p = 2650\ kg/m^3$

Table 3.5: Settling velocity with high particle density.

d (µm)	*d (m)*	*Ar*	*Re*	v_s *(m/s)*
300	0.0003	436.725	9.987	0.033
500	0.0005	2021.87	33.718	0.068

700	0.0007	5548.02	75.151	0.108
900	0.0009	11791.6	136.747	0.152
1000	0.001	16175	175.757	0.176
1500	0.0015	54590.6	461.702	0.308
1600	0.0016	66252.8	538.426	0.337
1700	0.0017	79467.7	622.073	0.367
1800	0.0018	94332.5	712.806	0.397
2000	0.002	129400	916.148	0.459

b- With medium particle density: $\rho_p = 2000\ kg/m^3$

Table 3.6: Settling velocity with medium particle density.

d (µm)	*d (m)*	*Ar*	*Re*	v_s *(m/s)*
300	0.0003	264.869	6.714	0.022
500	0.0005	1226.25	22.668	0.045
700	0.0007	3364.82	50.524	0.072
900	0.0009	7151.47	91.935	0.102
1000	0.001	9809.97	118.161	0.118
1500	0.0015	33108.6	310.401	0.207
1600	0.0016	40181.6	361.983	0.227
1700	0.0017	48196.4	418.219	0.246
1800	0.0018	57211.7	479.218	0.267
2000	0.002	78479.7	615.924	0.309

c- With low particle density: $\rho_p = 1500$ kg/m^3

Table 3.7: Settling velocity with low particle density.

d (μm)	*d (m)*	*Ar*	*Re*	v_s *(m/s)*
300	0.0003	132.673	3.878	0.013
500	0.0005	614.225	13.093	0.026
700	0.0007	1685.43	29.181	0.042
900	0.0009	3582.16	53.098	0.059
1000	0.001	4913.8	68.246	0.063
1500	0.0015	16584.1	179.277	0.120
1600	0.0016	20126.9	209.069	0.131
1700	0.0017	24141.5	241.549	0.142
1800	0.0018	28657.3	276.78	0.154
2000	0.002	39310.4	355.737	0.178

The settling velocity compared to the particle size is summarized below:

Table 3.8: Settling velocity vs. particle size (Correlation N2).

d (μm)	High density 2650 kg/m^3 v_s *(m/s)*	Medium density 2000 kg/m^3 v_s *(m/s)*	Low density 1500 kg/m^3 v_s *(m/s)*
300	0.033	0.022	0.013
500	0.068	0.045	0.026
700	0.108	0.072	0.042

900	0.152	0.102	0.059
1000	0.176	0.118	0.063
1500	0.308	0.207	0.120
1600	0.337	0.227	0.131
1700	0.367	0.246	0.142
1800	0.397	0.267	0.154
2000	0.459	0.309	0.178

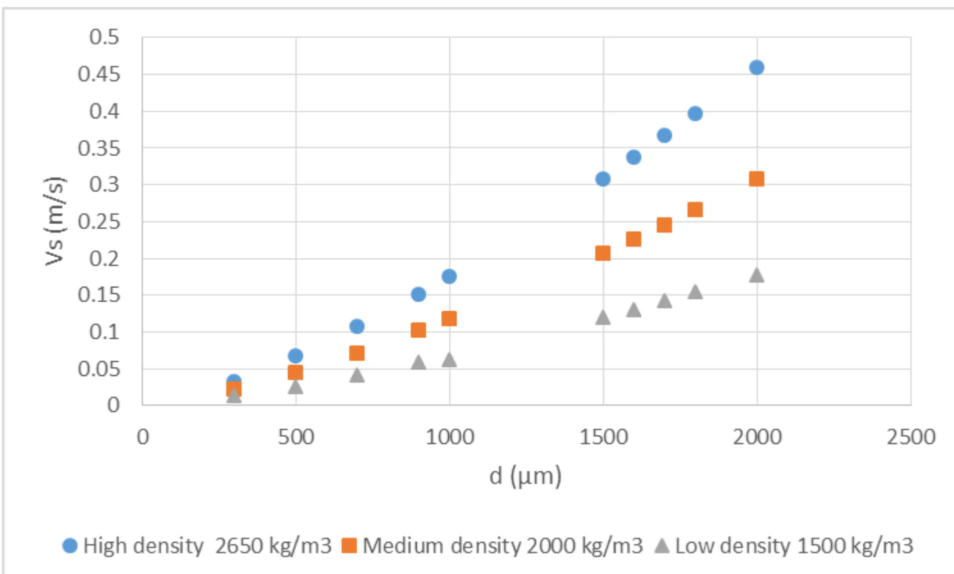


Figure 3.2: Settling velocity vs. particle size (Correlation N2).

It is observed that the increase in the particle size corresponded to an increase in the settling velocity.

3.2.1.C Correlation N3

Recall Equation 2.8: $Re = Ar^{1/3}\ \{\ 1/(\frac{18}{Ar^{2/3}})^{0.824} + (\frac{3}{4}\ \frac{0.428}{Ar^{1/3}}\ )^{0.412}\ \}^{1.214}$

The particle sizes were tested using the above correlation as the following:

a- With high particle density: $\rho_p = 2650\ kg/m^3$

Table 3.9: Settling velocity with high particle density.

d (μm)	*d (m)*	*Ar*	*Re*	v_s *(m/s)*
300	0.0003	436.725	27.377	0.091
500	0.0005	2021.87	117.395	0.235
700	0.0007	5548.02	315.349	0.451
900	0.0009	11791.6	664.473	0.740
1000	0.001	16175	909.232	0.911
1500	0.0015	54590.6	3051.23	2.038
1600	0.0016	66252.8	3701.13	2.317
1700	0.0017	79467.7	4437.46	2.615
1800	0.0018	94332.5	5265.64	2.931
2000	0.002	129400	7219.18	3.616

b- With medium particle density: $\rho_p = 2000\ kg/m^3$

Table 3.10: Settling velocity with medium particle density.

d (μm)	*d (m)*	*Ar*	*Re*	v_s *(m/s)*
300	0.0003	264.869	17.385	0.058
500	0.0005	1226.25	72.437	0.145
700	0.0007	3364.82	192.936	0.276
900	0.0009	7151.47	405.101	0.451
1000	0.001	9809.97	553.752	0.555

1500	0.0015	33108.6	1853.81	1.238
1600	0.0016	40181.6	2248.13	1.408
1700	0.0017	48196.4	2694.88	1.588
1800	0.0018	57211.7	3197.32	1.779
2000	0.002	78479.7	4382.41	2.195

c- With low particle density: $\rho_p = 1500$ kg/m^3

Table 3.11: Settling velocity with low particle density.

d (μm)	*d (m)*	*Ar*	*Re*	v_s *(m/s)*
300	0.0003	132.673	9.536	0.032
500	0.0005	614.225	37.588	0.075
700	0.0007	1685.43	98.412	0.141
900	0.0009	3582.16	205.139	0.228
1000	0.001	4913.8	279.819	0.280
1500	0.0015	16584.1	932.068	0.622
1600	0.0016	20126.9	1129.78	0.707
1700	0.0017	24141.5	1353.74	0.798
1800	0.0018	28657.3	1605.6	0.894
2000	0.002	39310.4	2199.57	1.102

The settling velocity compared to the particle size is summarized below:

Table 3.12: Settling velocity vs. particle size (Correlation N3).

d (μm)	High density 2650 kg/m^3 v_s (m/s)	Medium density 2000 kg/m^3 v_s (m/s)	Low density 1500 kg/m^3 v_s (m/s)
300	0.091	0.058	0.032
500	0.235	0.145	0.075
700	0.451	0.276	0.141
900	0.740	0.451	0.228
1000	0.911	0.555	0.280
1500	2.038	1.238	0.622
1600	2.317	1.408	0.707
1700	2.615	1.588	0.798
1800	2.931	1.779	0.894
2000	3.616	2.195	1.102

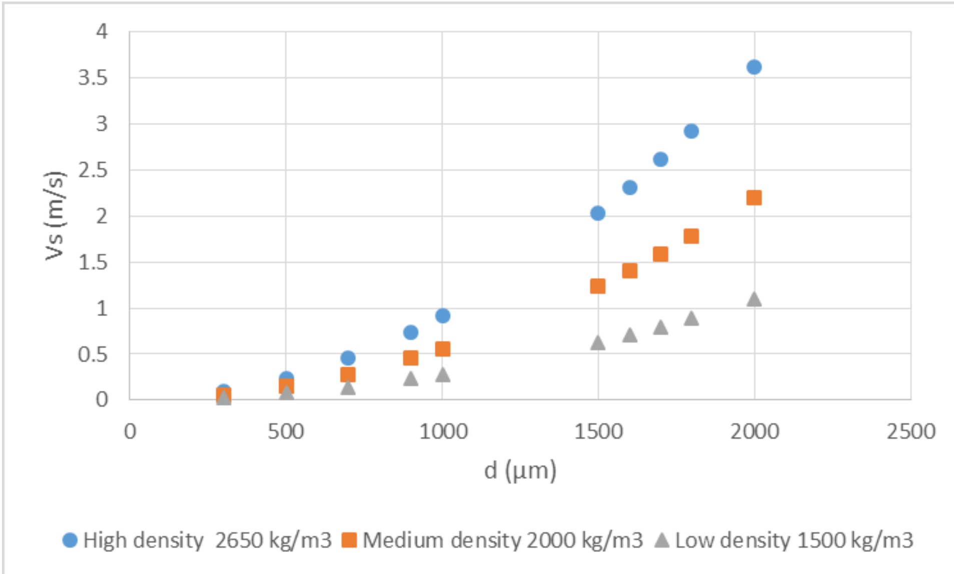


Figure 3.3: Settling velocity vs. particle size (Correlation N3).

There is observed to be a positive relationship between the particle size and the settling velocity.

3.2.2 Non-Newtonian fluid correlations

3.2.2.A Correlation NN1

Recall Equation 2.4:

$$Re = a\,Ar^{\,b}$$

Five particle sizes on the above correlation were tested in three sections as the following:

Where: $k = 0.0865\ Pa.\ s^n$.

a- With high particle density: $\rho_p = 2650\ kg/m^3$

Table 3.13: Settling velocity with high particle density.

$d\ (\mu m)$	$d\ (m)$	Ar	Re	$v_s\ (m/s)$
1500	0.0015	52.494	4.014	0.125
1600	0.0016	61.729	4.675	0.136
1700	0.0017	71.879	5.394	0.147
1800	0.0018	82.973	6.174	0.159
2000	0.002	108.102	7.917	0.182

b- With medium particle density: $\rho_p = 2000\ kg/m^3$

Table 3.14: Settling velocity with medium particle density.

$d\ (\mu m)$	$d\ (m)$	Ar	Re	$v_s\ (m/s)$
1500	0.0015	31.802	2.506	0.082
1600	0.0016	37.397	2.919	0.089
1700	0.0017	43.549	3.368	0.097
1800	0.0018	50.267	3.854	0.104
2000	0.002	65.490	4.942	0.119

c- With low particle density: $\rho_p = 1500\ kg/m^3$

Table 3.15: Settling velocity with low particle density.

$d\ (\mu m)$	$d\ (m)$	Ar	Re	$v_s\ (m/s)$
1500	0.0015	15.885	1.305	0.046
1600	0.0016	18.679	1.520	0.050

1700	0.0017	21.751	1.753	0.054
1800	0.0018	25.108	2.007	0.058
2000	0.002	32.712	2.573	0.067

Change in the settling velocity compared to the particle size is summarized below:

Table 3.16: Settling velocity vs. particle size (Correlation NN1).

d (μm)	High density 2650 kg/m^3 v_s *(m/s)*	Medium density 2000 kg/m^3 v_s *(m/s)*	Low density 1500 kg/m^3 v_s *(m/s)*
1500	0.125	0.082	0.046
1600	0.136	0.089	0.050
1700	0.147	0.097	0.054
1800	0.159	0.104	0.058
2000	0.182	0.119	0.067

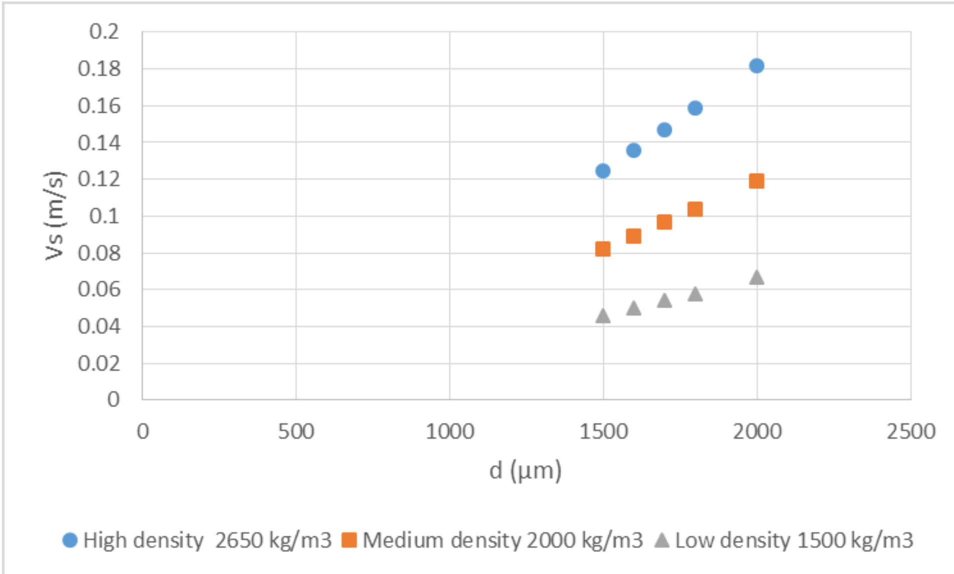


Figure 3.4: Settling velocity vs. particle size (Correlation NN1).

There is a positive relationship between the particle size and the settling velocity.

3.2.2.B Correlation NN2

Recall Equation 2.12:

$$v_s^{\,2} + 4.458\, e^{(5.03\,\phi)} \left(\frac{\mu_f}{d\,\rho_f}\right) v_s - 19.45\, e^{(5.03\,\phi)}\, d \left(\frac{\rho_p}{\rho_f} - 1\right) = 0$$

Testing was done using five particle sizes on the above correlation in three sections as the following: (ϕ = 0.9 (near-spherical particle)).

a- With high particle density: $\rho_p = 2650\ kg/m^3$

Table 3.17: Settling velocity with high particle density.

d (μm)	*d (m)*	v_s *(m/s)*
1500	0.0015	0.392
1600	0.0016	0.446
1700	0.0017	0.504
1800	0.0018	0.564
2000	0.002	0.690

b- With medium particle density: $\rho_p = 2000\ kg/m^3$

Table 3.18: Settling velocity with medium particle density.

d (μm)	*d (m)*	v_s *(m/s)*
1500	0.0015	0.227
1600	0.0016	0.260
1700	0.0017	0.295
1800	0.0018	0.331
2000	0.002	0.410

c- With low particle density: $\rho_p = 1500\ kg/m^3$

Table 3.19: Settling velocity with low particle density.

d (μm)	*d (m)*	v_s *(m/s)*
1500	0.0015	0.103
1600	0.0016	0.118

1700	0.0017	0.134
1800	0.0018	0.151
2000	0.002	0.189

Change in the settling velocity compared to the particle size is summarized below:

Table 3.20: Settling velocity vs. particle size (Correlation NN2).

d (μm)	High density 2650 kg/m^3 v_s (m/s)	Medium density 2000 kg/m^3 v_s (m/s)	Low density 1500 kg/m^3 v_s (m/s)
1500	0.392	0.227	0.103
1600	0.455	0.260	0.118
1700	0.513	0.295	0.134
1800	0.574	0.331	0.151
2000	0.690	0.410	0.189

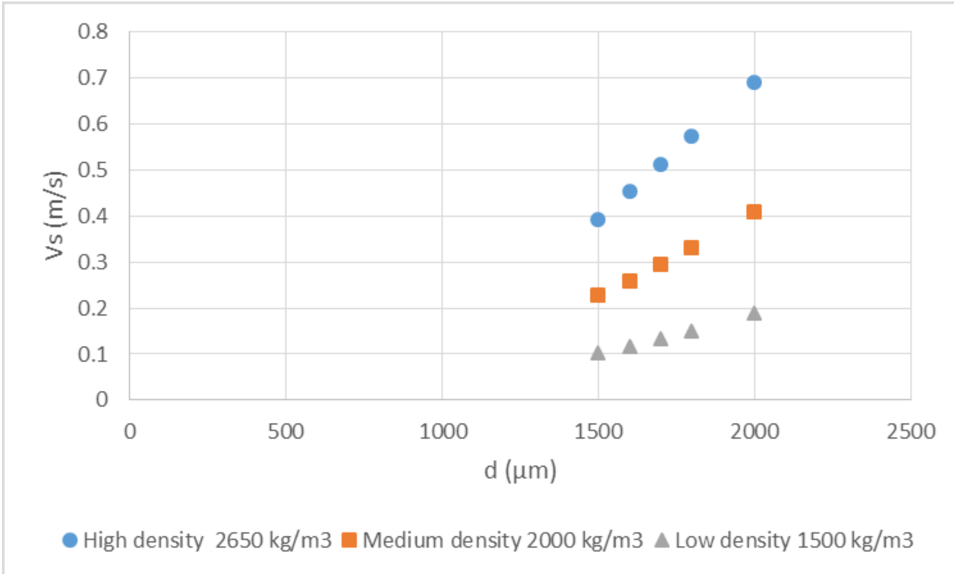


Figure 3.5: Settling velocity vs. particle size (Correlation NN2).

As expected, an increase in the particle size results in an increase in the settling velocity.

3.3 Particle density

3.3.1 Newtonian fluid correlations

3.3.1.A Correlation N1

Eleven different particle densities were tested using correlation N1 as the following:

a- With large particle size: $d = 2000\mu m$

Table 3.21: Settling velocity with large particle size.

ρ_p (kg/m^3)	$v_{stokes} (m/s)$	Ar	v_s (m/s)
1200	0.440	15808.8	0.077
1500	1.094	39310.4	0.137
1700	1.530	54978.1	0.168

2000	2.184	78479.7	0.209
2200	2.620	94147.5	0.233
2500	3.274	117649	0.266
2650	3.601	129400	0.282
2800	3.928	141151	0.297
3000	4.364	156818	0.316
3200	4.800	172486	0.335
3500	5.454	195988	0.361

b- With medium particle size: $d = 1500\mu m$

Table 3.22: Settling velocity with medium particle size.

ρ_p (kg/m^3)	v_{stokes} (m/s)	Ar	v_s (m/s)
1200	0.247	6669.32	0.059
1500	0.615	16584.1	0.106
1700	0.861	23193.9	0.131
2000	1.228	33108.6	0.164
2200	1.474	39718.5	0.184
2500	1.842	49633.2	0.211
2650	2.026	54590.6	0.223
2800	2.209	59548	0.235
3000	2.455	66157.8	0.251
3200	2.700	72767.6	0.266
3500	3.068	82682.4	0.287

c- With small particle size: $d = 1000\mu m$

Table 3.23: Settling velocity with small particle size.

ρ_p (kg/m^3)	v_{stokes} (m/s)	Ar	v_s (m/s)
1200	0.110	1976.09	0.039
1500	0.273	4913.8	0.072
1700	0.382	6872.27	0.090
2000	0.546	9809.97	0.114
2200	0.655	11768.4	0.128
2500	0.818	14706.1	0.147
2650	0.900	16175	0.157
2800	0.982	17643.8	0.166
3000	1.091	19602.3	0.177
3200	1.200	21560.8	0.188
3500	1.363	24498.5	0.204

The change in the settling velocity compared to the particle density is summarized below:

Table 3.24: Settling velocity vs. particle density (Correlation N1).

ρ_p (kg/m^3)	Large size $2000\mu m$ v_s (m/s)	Medium size $1500\mu m$ v_s (m/s)	Small size $1000\mu m$ v_s (m/s)
1200	0.077	0.059	0.039
1500	0.137	0.106	0.072

1700	0.168	0.131	0.090
2000	0.209	0.164	0.114
2200	0.233	0.184	0.128
2500	0.266	0.211	0.147
2650	0.282	0.223	0.157
2800	0.297	0.235	0.166
3000	0.316	0.251	0.177
3200	0.335	0.266	0.188
3500	0.361	0.287	0.204

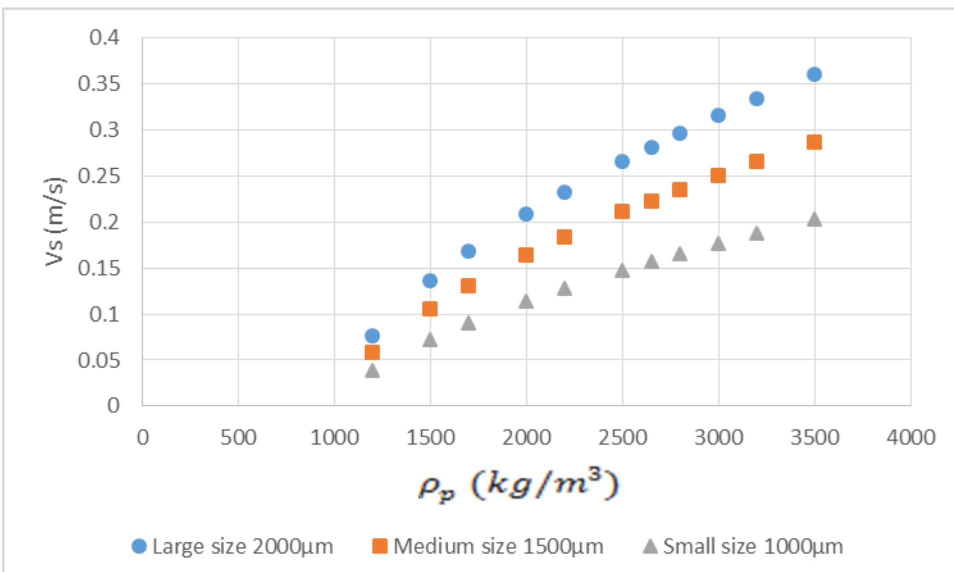


Figure 3.6: Settling velocity vs. particle density (Correlation N1).

There is a positive relationship between the particle density and the settling velocity as expected.

3.3.1.B Correlation N2

Testing was done for eleven values of density on Correlation N2 as the following:

a- With large particle size: $d = 2000\ \mu m$

Table 3.25: Settling velocity with large particle size.

ρ_p (kg/m^3)	Ar	Re	v_s (m/s)
1200	15808.8	172.59	0.086
1500	39310.4	355.737	0.178
1700	54978.1	464.302	0.233
2000	78479.7	615.924	0.309
2200	94147.5	711.695	0.356
2500	117649	849.448	0.425
2650	129400	916.148	0.459
2800	141151	981.61	0.492
3000	156818	1067.17	0.535
3200	172486	1150.99	0.577
3500	195988	1273.86	0.638

b- With medium particle size: $d = 1500\ \mu m$

Table 3.26: Settling velocity with medium particle size.

ρ_p (kg/m^3)	Ar	Re	v_s (m/s)
1200	6669.32	86.978	0.058
1500	16584.1	179.277	0.120
1700	23193.9	233.99	0.156

2000	33108.6	310.401	0.207
2200	39718.5	358.666	0.240
2500	49633.2	428.088	0.286
2650	54590.6	461.702	0.308
2800	59548	494.692	0.330
3000	66157.8	537.813	0.359
3200	72767.6	580.054	0.387
3500	82682.4	641.972	0.429

c- With small particle size: $d = 1000\ \mu m$

Table 3.27: Settling velocity with small particle size.

ρ_p (kg/m^3)	*Ar*	*Re*	v_s (m/s)
1200	1976.09	33.110	0.033
1500	4913.8	68.246	0.068
1700	6872.27	89.073	0.089
2000	9809.97	118.161	0.118
2200	11768.4	136.534	0.137
2500	14706.1	162.961	0.163
2650	16175	175.757	0.176
2800	17643.8	188.315	0.189
3000	19602.3	204.73	0.205
3200	21560.8	220.811	0.221
3500	24498.5	244.381	0.245

Change in settling velocity compared to the particle density is summarized below:

Table 3.28: Settling velocity vs. particle density (Correlation N2).

ρ_p (kg/m^3)	Large size *2000μm* v_s (m/s)	Medium size *1500μm* v_s (m/s)	Small size *1000μm* v_s (m/s)
1200	0.086	0.058	0.033
1500	0.178	0.120	0.068
1700	0.233	0.156	0.089
2000	0.309	0.207	0.118
2200	0.356	0.240	0.137
2500	0.425	0.286	0.163
2650	0.459	0.308	0.176
2800	0.492	0.330	0.189
3000	0.535	0.359	0.205
3200	0.577	0.387	0.221
3500	0.638	0.429	0.245

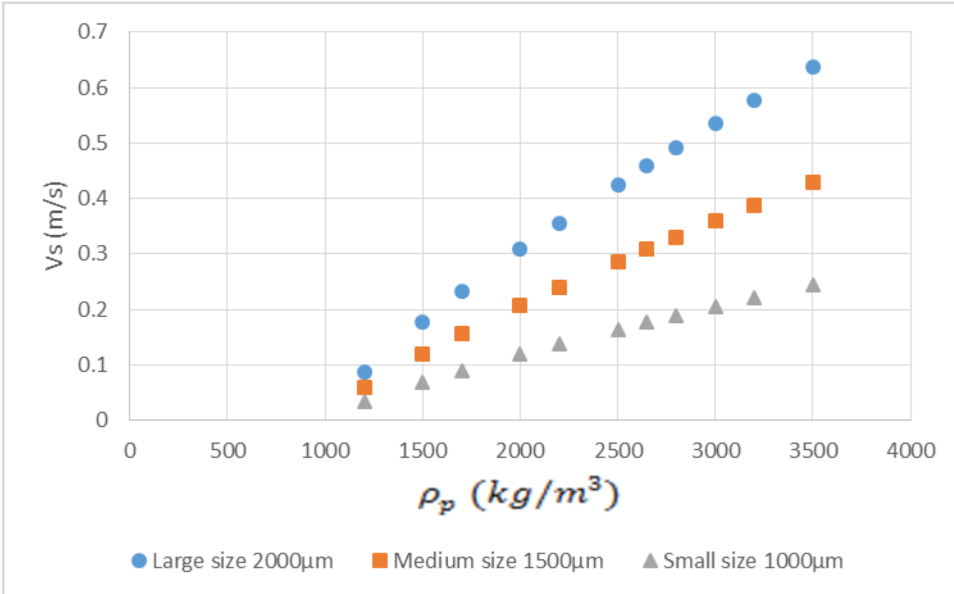


Figure 3.7: Settling velocity vs. particle density (Correlation N2).

There is a direct relationship between the particle density and the settling velocity.

3.3.1.C Correlation N3

The density of the particle was changed using Correlation N3 as the following:

a- With large particle size: $d = 2000\ \mu m$

Table 3.29: Settling velocity with large particle size.

$\rho_p\ (kg/m^3)$	*Ar*	*Re*	$v_s\ (m/s)$
1200	15808.8	888.791	0.445
1500	39310.4	2199.57	1.102
1700	54978.1	3072.84	1.539
2000	78479.7	4382.41	2.195
2200	94147.5	5255.33	2.632
2500	117649	6564.58	3.288

2650	129400	7219.18	3.616
2800	141151	7873.75	3.944
3000	156818	8746.45	4.381
3200	172486	9619.17	4.818
3500	195988	10928.2	5.474

b- With medium particle size: $d = 1500\ \mu m$

Table 3.30: Settling velocity with medium particle size.

ρ_p (kg/m^3)	*Ar*	*Re*	v_s *(m/s)*
1200	6669.32	378.123	0.253
1500	16584.1	932.068	0.622
1700	23193.9	1300.88	0.869
2000	33108.6	1853.81	1.238
2200	39718.5	2222.32	1.484
2500	49633.2	2774.96	1.853
2650	54590.6	3051.25	2.038
2800	59548	3327.51	2.222
3000	66157.8	3695.84	2.468
3200	72767.6	4064.14	2.714
3500	82682.4	4616.57	3.083

c- With small particle size: $d = 1000\ \mu m$

Table 3.31: Settling velocity with small particle size.

$\rho_p\ (kg/m^3)$	Ar	Re	$v_s\ (m/s)$
1200	1976.09	114.814	0.115
1500	4913.8	279.819	0.280
1700	6872.27	389.48	0.390
2000	9809.97	553.752	0.555
2200	11768.4	663.177	0.664
2500	14706.1	827.231	0.829
2650	16175	909.232	0.911
2800	17643.8	991.213	0.993
3000	19602.3	1100.51	1.102
3200	21560.8	1209.78	1.212
3500	24498.5	1273.66	1.376

The change in the settling velocity compared to the particle density is summarized below:

Table 3.32: Settling velocity vs. particle density (Correlation N3).

$\rho_p\ (kg/m^3)$	Large size $2000\mu m$ $v_s\ (m/s)$	Medium size $1500\mu m$ $v_s\ (m/s)$	Small size $1000\mu m$ $v_s\ (m/s)$
1200	0.445	0.253	0.115
1500	1.102	0.622	0.280

1700	1.539	0.869	0.390
2000	2.195	1.238	0.555
2200	2.632	1.484	0.664
2500	3.288	1.853	0.829
2650	3.616	2.038	0.911
2800	3.944	2.222	0.993
3000	4.381	2.468	1.102
3200	4.818	2.714	1.212
3500	5.474	3.083	1.376

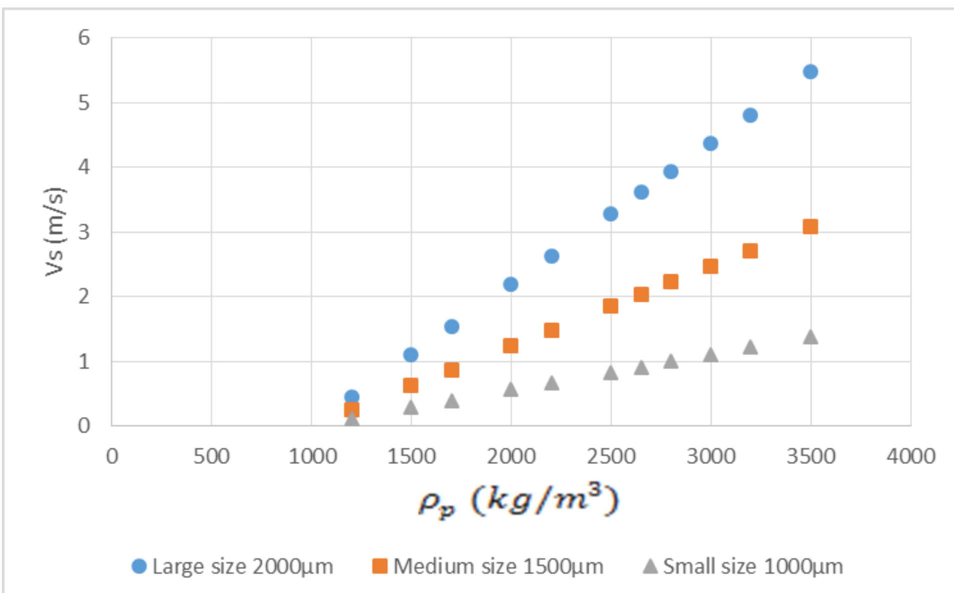


Figure 3.8: Settling velocity vs. particle density (Correlation N3).

The relationship between the particle density and the settling velocity was positive.

3.3.2 Non-Newtonian fluid correlations

3.3.2.A Correlation NN1

Testing was done for five values of the density on Correlation NN1 as the following:

a- With large particle size: $d = 2000\ \mu m$

Table 3.33: Settling velocity with large particle size.

ρ_p (kg/m^3)	Ar	Re	v_s (m/s)
2500	98.299	7.240	0.163
2650	108.130	7.919	0.182
3000	131.070	9.489	0.214
3200	144.178	10.378	0.231
3500	163.841	11.703	0.257

b- With medium particle size: $d = 1500\ \mu m$

Table 3.34: Settling velocity with medium particle size.

ρ_p (kg/m^3)	Ar	Re	v_s (m/s)
2500	47.734	3.671	0.115
2650	52.508	4.015	0.125
3000	63.647	4.811	0.146
3200	70.013	5.263	0.158
3500	79.561	5.935	0.176

c- With small particle size: $d = 1000\ \mu m$

Table 3.35: Settling velocity with small particle size.

ρ_p (kg/m^3)	Ar	Re	v_s (m/s)
2500	17.243	1.409	0.067
2650	18.968	1.542	0.072
3000	22.992	1.847	0.084
3200	25.292	2.021	0.092
3500	28.741	2.279	0.103

Change in settling velocity compared to the particle density is summarized below:

Table 3.36: Settling velocity vs. particle density (Correlation NN1).

ρ_p (kg/m^3)	Large size $2000\mu m$ v_s (m/s)	Medium size $1500\mu m$ v_s (m/s)	Small size $1000\mu m$ v_s (m/s)
2500	0.163	0.115	0.067
2650	0.182	0.125	0.072
3000	0.214	0.146	0.084
3200	0.231	0.158	0.092
3500	0.257	0.176	0.103

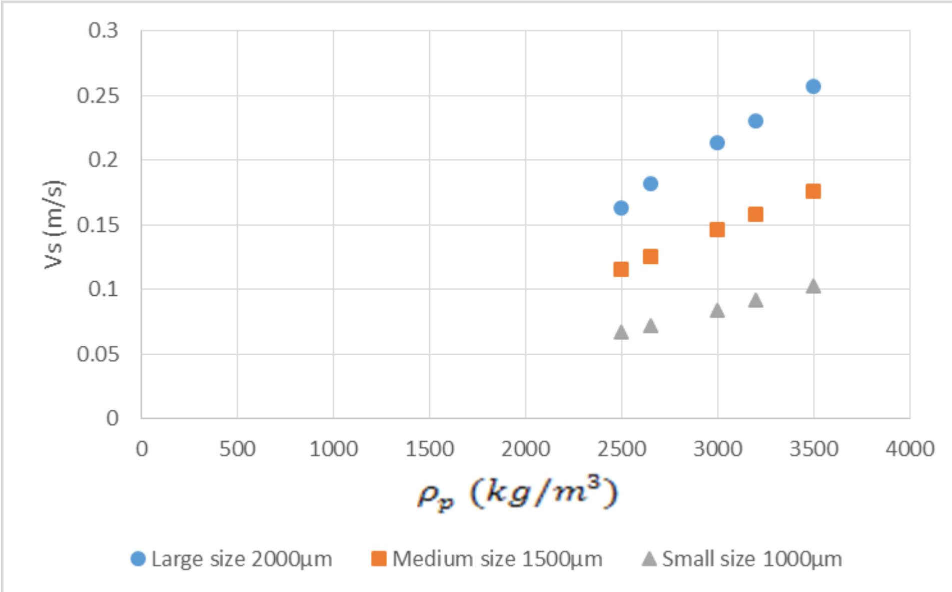


Figure 3.9: Settling velocity vs. particle density (Correlation NN1).

The relationship between the particle density and the settling velocity was positive.

3.3.2.B Correlation NN2

The density of the particle was changed using Correlation NN2 as the following:

a- With large particle size: $d = 2000\ \mu m$

Table 3.37: Settling velocity with large particle size.

$\rho_p\ (kg/m^3)$	$v_s\ (m/s)$
2500	0.623
2650	0.690
3000	0.842
3200	0.928
3500	1.055

b- With medium particle size: $d = 1500\ \mu m$

Table 3.38: Settling velocity with medium particle size.

$\rho_p\ (kg/m^3)$	$v_s\ (m/s)$
2500	0.352
2650	0.392
3000	0.484
3200	0.537
3500	0.618

c- With small particle size: $d = 1000\ \mu m$

Table 3.39: Settling velocity with small particle size.

$\rho_p\ (kg/m^3)$	$v_s\ (m/s)$
2500	0.150
2650	0.167
3000	0.209
3200	0.233
3500	0.270

Change in the settling velocity compared to the particle density is summarized below:

Table 3.40: Settling velocity vs. particle density (Correlation NN2).

ρ_p (kg/m^3)	Large size *2000µm* v_s *(m/s)*	Medium size *1500µm* v_s *(m/s)*	Small size *1000µm* v_s *(m/s)*
2500	0.623	0.352	0.150
2650	0.690	0.392	0.167
3000	0.842	0.484	0.209
3200	0.928	0.537	0.233
3500	1.055	0.618	0.270

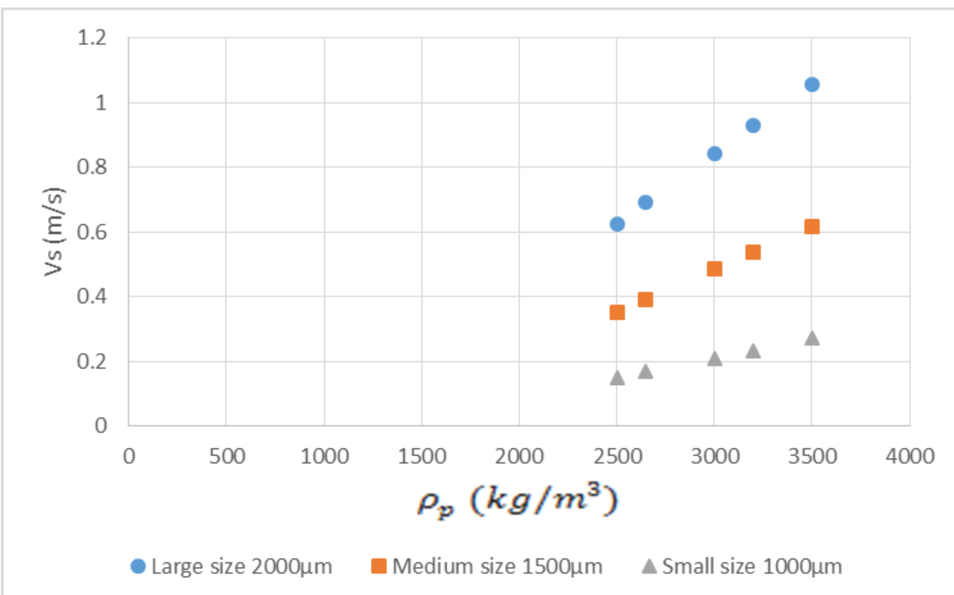


Figure 3.10: Settling velocity vs. particle density (Correlation NN2).

The relationship between the particle density and the settling velocity was positive.

3.4 The shape factor of the particle

For the shape factor, Chein's correlation (14) was used, Equation 2.12, for Newtonian and non-Newtonian fluid. ($\rho_p = 2650\ kg/m^3$, $\mu_f = 0.001\ Pa.s$, $k = 0.0865\ Pa.s^n$)

3.4.1 Newtonian fluid

For Newtonian fluid, Chein's correlation was used as the following:

a- With large particle size: $d = 2000\ \mu m$

Table 3.41: Settling velocity with large particle size.

ϕ	v_s *(m/s)*
1	2.971
0.9	2.339
0.6	1.125
0.3	0.535

b- With medium particle size: $d = 1500\ \mu m$

Table 3.42: Settling velocity with medium particle size.

ϕ	v_s *(m/s)*
1	2.499
0.9	1.980
0.6	0.964
0.3	0.461

c- With small particle size: *d = 1000 µm*

Table 3.43: Settling velocity with small particle size.

ϕ	v_s *(m/s)*
1	1.903
0.9	1.531
0.6	0.767
0.3	0.372

The change in the settling velocity compared to the particle shape factor is summarized below:

Table 3.44: Settling velocity vs. particle sphericity in Newtonian fluid.

ϕ	Large size *2000µm* v_s *(m/s)*	Medium size *1500µm* v_s *(m/s)*	Small size *1000µm* v_s *(m/s)*
1	2.971	2.499	1.903
0.9	2.339	1.980	1.531
0.6	1.125	0.964	0.767
0.3	0.535	0.461	0.372

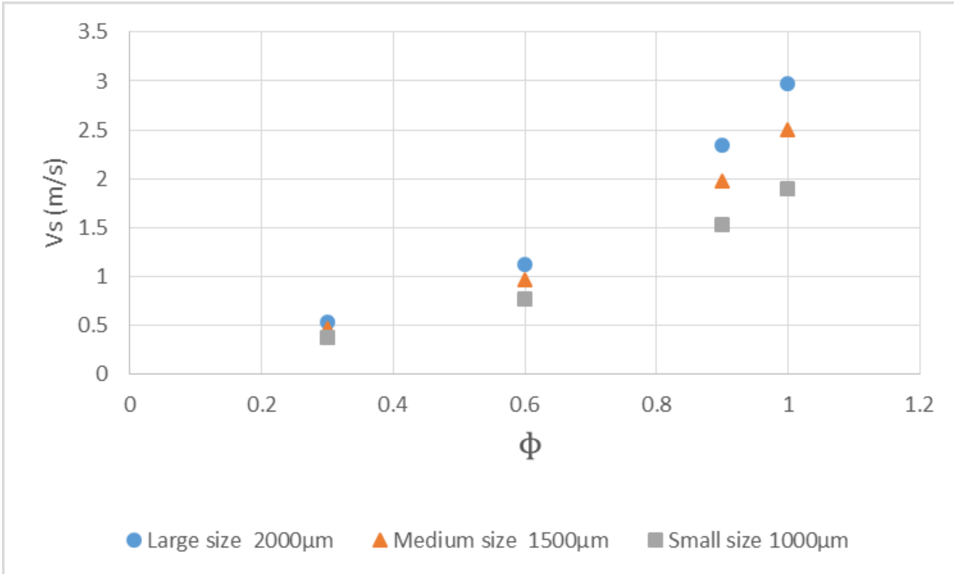


Figure 3.11: Settling velocity vs. particle sphericity in Newtonian fluid.

There is a positive relationship between the sphericity of the particle and the settling velocity.

3.4.2 Non-Newtonian fluid

For non-Newtonian fluid, Chein's correlation was used as the following:

a- With large particle size: $d = 2000\ \mu m$

Table 3.45: Settling velocity with large particle size.

ϕ	v_s *(m/s)*
1	0.650
0.9	0.690
0.6	0.513
0.3	0.343

b- With medium particle size: $d = 1500\ \mu m$

Table 3.46: Settling velocity with medium particle size.

ϕ	v_s (m/s)
1	0.312
0.9	0.392
0.6	0.280
0.3	0.218

c- With small particle size: $d = 1000\ \mu m$

Table 3.47: Settling velocity with small particle size.

ϕ	v_s (m/s)
1	0.169
0.9	0.167
0.6	0.162
0.3	0.142

The change in the settling velocity compared to the particle shape factor is summarized below:

Table 3.48: Settling velocity vs. particle sphericity in non-Newtonian fluid.

ϕ	Large size 2000μm v_s (m/s)	Medium size 1500μm v_s (m/s)	Small size 1000μm v_s (m/s)
1	0.650	0.312	0.169

0.9	0.690	0.392	0.167
0.6	0.513	0.280	0.162
0.3	0.343	0.218	0.142

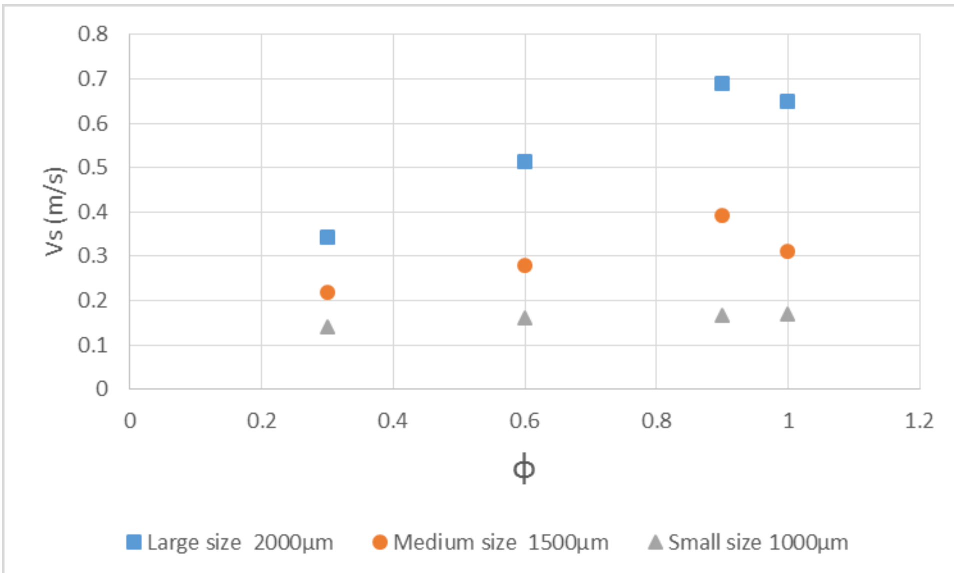


Figure 3.12: Settling velocity vs. particle sphericity in non-Newtonian fluid.

There is a positive relationship between the particle shape factor and the settling velocity.

3.5 Particle volume fraction

3.5.1 Newtonian fluid correlations

Applying the following data on the Newtonian correlations: $\rho_p = 2650\ kg/m^3$, $d = 2000\ \mu m$, and $\mu_f = 0.001\ Pa.s$

3.5.1.A Correlation N1

First, from Correlation 1 for Newtonian fluid, the settling velocity was $v_s = 0.282$ m/s. Second, determining Reynolds number, $Re = 562.985$, and Equation 2.19 is used to determine z which is ($z = 2.40$). Third, Equation 2.14; Richardson and Zaki (16, 17):

$$\frac{v_h}{v_s} = \varepsilon^z = (1-c)^z$$

is used to determine the hindered settling velocity v_h *(m/s)* as the following:

Table 3.49: Settling velocity vs. particle volume fraction (Correlation N1).

c	v_h *(m/s)*
0.35	0.100
0.25	0.141
0.1	0.219
0.01	0.275

3.5.1.B Correlation N2

First, from Correlation 2 for Newtonian fluid we determined the settling velocity, $v_s = 0.459$ m/s. Second, determining $Re = 916.148$ and then determining $z = 2.40$ by using Equation 2.19. Third, determining the hindered settling velocity by using R&Z correlation as the following:

Table 3.50: Settling velocity vs. particle volume fraction (Correlation N2).

c	v_h *(m/s)*
0.35	0.163
0.25	0.230
0.1	0.356

0.01	0.448

3.5.1.C Correlation N3

First, from Correlation 3 for Newtonian fluid we determined the settling velocity, v_s = 3.616 m/s.

Second, determining Re = 7219.18 and determining z = 2.40 by using Equation 2.19.

Third, determining the hindered settling velocity by using R&Z correlation as the following:

Table 3.51: Settling velocity vs. particle volume fraction (Correlation N3).

c	v_h *(m/s)*
0.35	1.286
0.25	1.813
0.1	2.808
0.01	3.530

Particle volume fraction compared to the change in the settling velocity is summarized below:

Table 3.52: Settling velocity vs. particle volume fraction in Newtonian fluid.

c	Correlation N1 v_h *(m/s)*	Correlation N2 v_h *(m/s)*	Correlation N3 v_h *(m/s)*
0.35	0.100	0.163	1.286
0.25	0.141	0.230	1.813
0.1	0.219	0.356	2.808
0.01	0.275	0.448	3.530

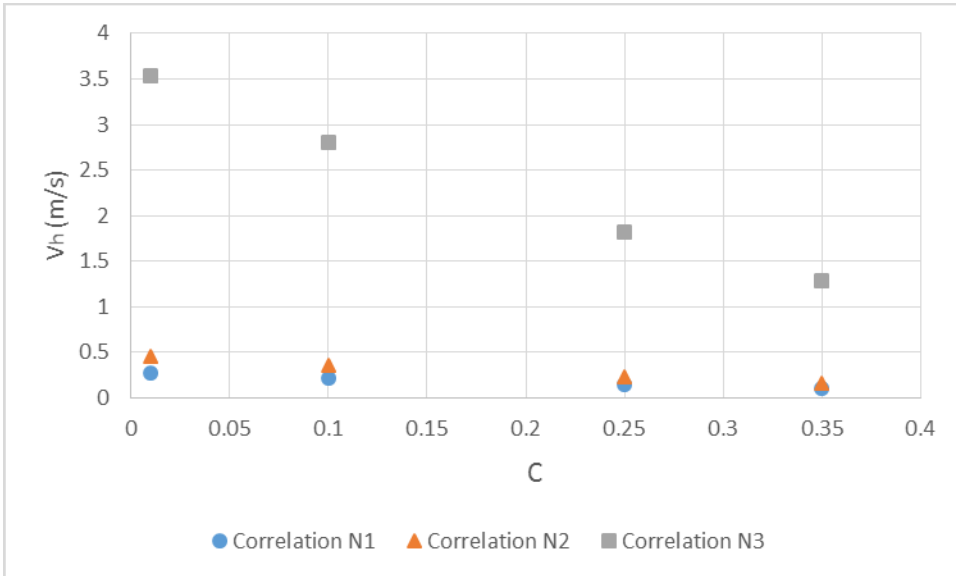


Figure 3.13: Settling velocity vs. particle volume fraction in Newtonian fluid.

The relationship between the particle concentration and the settling velocity was inverse.

3.5.2 Non-Newtonian fluid correlations

Applying the following parameters on the non-Newtonian correlations as the following:

$\rho_p = 2650\ kg/m^3$, $d = 0.002\ m$, $k = 0.0865\ Pa.s^n$, the diameter of the cylindrical container $D = 0.1\ m$ and $d/D = 0.02$.

3.5.2.A Correlation NN1

First, by using Correlation 1 of non-Newtonian fluid, the Reynolds number was $Re' = 7.917$ and then the settling velocity was obtained as 0.182 m/s. Second, by using Equation 2.21, $z=3.964$. Third, by using R&Z, the hindered settling velocities as the following:

Table 3.53: Settling velocity vs. particle volume fraction (Correlation NN1).

c	v_h *(m/s)*
0.35	0.032
0.25	0.058
0.1	0.119
0.01	0.174

3.5.2.B Correlation NN2

First, by using Correlation 2 for a non-Newtonian fluid, the settling velocity was 0.690 m/s and then $Re' = 35.951$.

Second, by using Equation 2.21, $z = 3.408$.

Third, by using Richardson and Zaki correlation, the hindered settling velocity was obtained as:

Table 3.54: Settling velocity vs. particle volume fraction (Correlation NN2).

c	v_h *(m/s)*
0.35	0.159
0.25	0.259
0.1	0.482
0.01	0.667

The change in the settling velocity compared to the particle volume fraction is summarized below:

Table 3.55: Settling velocity vs. particle volume fraction in non-Newtonian fluid.

c	Correlation NN1 v_h *(m/s)*	Correlation NN2 v_h *(m/s)*
0.35	0.032	0.159
0.25	0.058	0.259
0.1	0.119	0.482
0.01	0.174	0.667

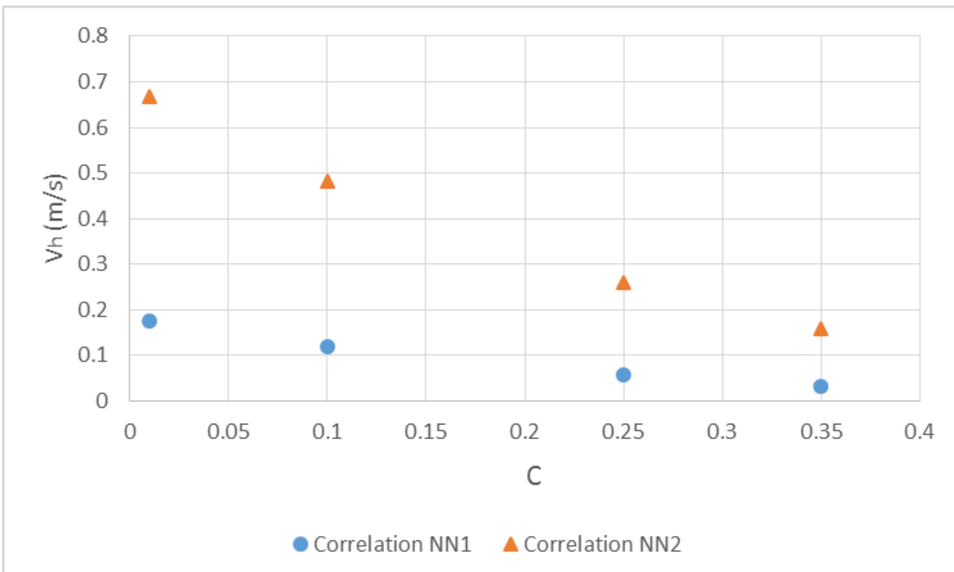


Figure 3.14: Settling velocity vs. particle volume fraction in non-Newtonian fluid.

The relationship between the particle volume fraction and the settling velocity was negative.

3.6 Effect of Carrier Fluid viscosity

3.6.1 Newtonian fluid correlations

3.6.1.A Correlation N1

Equation 2.1 for Newtonian fluid was used as the following:

Where: $\rho_p = 2650\ kg/m^3$.

a- With large particle size: $d = 2000\ \mu m$

Table 3.56: Settling velocity with large particle size.

μ_f (Pa. s)	v_{stokes} (m/s)	Ar	v_s (m/s)
0.001	3.601	129400	0.282
0.005	0.720	5176	0.186
0.02	0.180	323.5	0.101

b- With medium particle size: $d = 1500\ \mu m$

Table 3.57: Settling velocity with medium particle size.

μ_f (Pa. s)	v_{stokes} (m/s)	Ar	v_s (m/s)
0.001	2.026	54590.6	0.223
0.005	0.405	2183.62	0.138
0.02	0.101	136.476	0.068

c- With small particle size: $d = 1000\ \mu m$

Table 3.58: Settling velocity with small particle size.

μ_f (Pa. s)	v_{stokes} (m/s)	Ar	v_s (m/s)
0.001	0.900	16175	0.157
0.005	0.180	647	0.086
0.02	0.045	40.438	0.037

Change in settling velocity compared to the change in the fluid viscosity is summarized below:

Table 3.59: Settling velocity vs. fluid viscosity (Correlation N1).

μ_f *(Pa. s)*	Large size $2000\mu m$ v_s *(m/s)*	Medium size $1500\mu m$ v_s *(m/s)*	Small size $1000\mu m$ v_s *(m/s)*
0.001	0.282	0.223	0.157
0.005	0.186	0.138	0.086
0.02	0.101	0.068	0.037

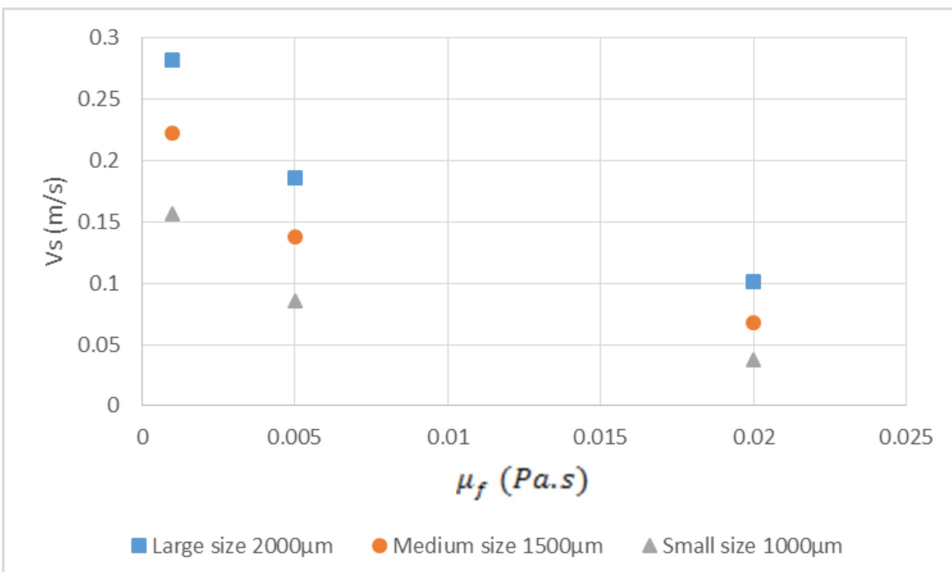


Figure 3.15: Settling velocity vs. fluid viscosity (Correlation N1).

There is an inverse relationship between the settling velocity and the fluid viscosity.

3.6.1.B Correlation N2

Recall Equation 2.4, this equation was used as the following:

a- With large particle size: *d = 2000 μm*

Table 3.60: Settling velocity with large particle size.

μ_f *(Pa. s)*	*Ar*	*Re*	v_s *(m/s)*
0.001	129400	916.148	0.459
0.005	5176	71.122	0.036
0.02	323.5	7.869	0.004

b- With medium particle size: *d = 1500 μm*

Table 3.61: Settling velocity with medium particle size.

μ_f *(Pa. s)*	*Ar*	*Re*	v_s *(m/s)*
0.001	54590.6	461.702	0.308
0.005	2183.62	35.842	0.024
0.02	136.476	3.966	0.003

c- With small particle size: *d = 1000 μm*

Table 3.62: Settling velocity with small particle size.

μ_f *(Pa. s)*	*Ar*	*Re*	v_s *(m/s)*
0.001	16175	175.757	0.176
0.005	647	13.644	0.014
0.02	40.438	1.510	0.002

Change in settling velocity compared to the change in the fluid viscosity is summarized below:

Table 3.63: Settling velocity vs. fluid viscosity (Correlation N2).

μ_f (Pa. s)	Large size 2000μm v_s (m/s)	Medium size 1500μm v_s (m/s)	Small size 1000μm v_s (m/s)
0.001	0.459	0.308	0.176
0.005	0.036	0.024	0.014
0.02	0.004	0.003	0.002

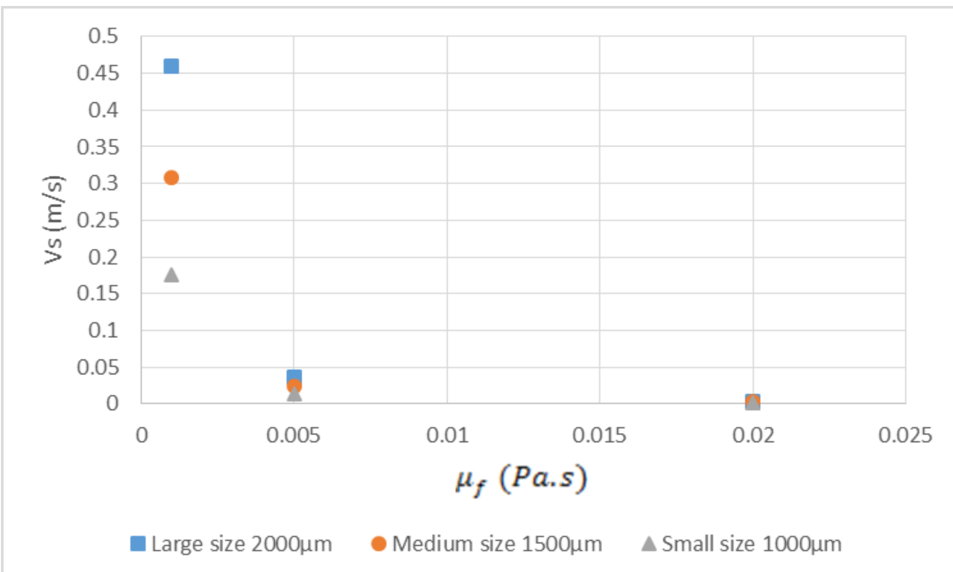


Figure 3.16: Settling velocity vs. fluid viscosity (Correlation N2).

The relationship between the fluid viscosity and the settling velocity was negative.

3.6.1.C Correlation N3

For Newtonian fluid, Equation 2.8 was used as the following:

a- With large particle size: *d = 2000 μm*

Table 3.64: Settling velocity with large particle size.

μ_f *(Pa. s)*	*Ar*	*Re*	v_s *(m/s)*
0.001	129400	7219.18	3.616
0.005	5176	294.51	0.148
0.02	323.5	20.811	0.010

b- With medium particle size: *d = 1500 μm*

Table 3.65: Settling velocity with medium particle size.

μ_f *(Pa. s)*	*Ar*	*Re*	v_s *(m/s)*
0.001	54590.6	3051.25	2.038
0.005	2183.62	126.51	0.084
0.02	136.476	9.766	0.007

c- With small particle size: *d = 1000 μm*

Table 3.66: Settling velocity with small particle size.

μ_f *(Pa. s)*	*Ar*	*Re*	v_s *(m/s)*
0.001	16175	909.232	0.911
0.005	647	39.465	0.040
0.02	40.438	3.777	0.004

Change in fluid viscosity compared to the change in the settling velocity is summarized below:

Table 3.67: Settling velocity vs. fluid viscosity (Correlation N3).

μ_f (Pa. s)	Large size $2000\mu m$ v_s (m/s)	Medium size $1500\mu m$ v_s (m/s)	Small size $1000\mu m$ v_s (m/s)
0.001	3.616	2.038	0.911
0.005	0.148	0.084	0.040
0.02	0.010	0.007	0.004

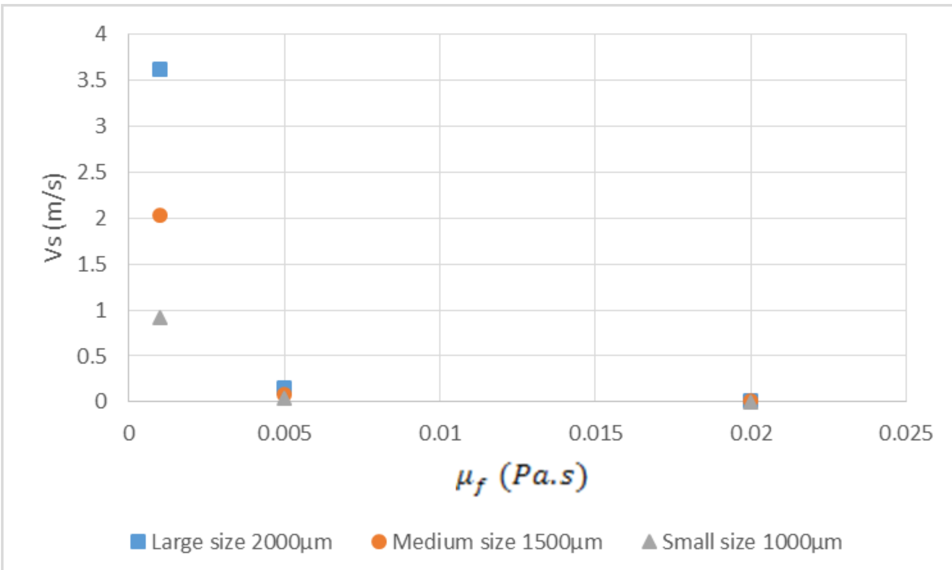


Figure 3.17: Settling velocity vs. fluid viscosity (Correlation N3).

There is an inverse relationship between the fluid viscosity and the settling velocity.

3.6.2 Non-Newtonian fluid correlations

Applying the following data on NN1 and NN2: $\rho_p = 2650\ kg/m^3$, $d = 2000\ \mu m$.

3.6.2.A Correlation NN1

Recall Equation 2.4 which was used as the following:

Table 3.68: Settling velocity vs. fluid consistency index (Correlation NN1).

k $(Pa.s^n)$	*Ar*	*Re*	v_s *(m/s)*
0.1868	12.644	1.053	0.061
0.0865	108.120	7.918	0.182
0.0165	891.643	57.537	0.243

3.6.2.B Correlation NN2

Recall Equation 2.12 which was used as the following: ($\Phi = 0.9$)

Table 3.69: Settling velocity vs. fluid consistency index (Correlation NN2).

k $(Pa.s^n)$	v_s *(m/s)*
0.1865	0.305
0.0865	0.690
0.0165	1.865

Change in fluid viscosity compared to the change in the settling velocity is summarized below:

Table 3.70: Settling velocity vs. the fluid consistency index non-Newtonian fluid.

k $(Pa.s^n)$	Correlation NN1 v_s (m/s)	Correlation NN2 v_s (m/s)
0.1865	0.061	0.305
0.0865	0.182	0.690
0.0165	0.243	1.865

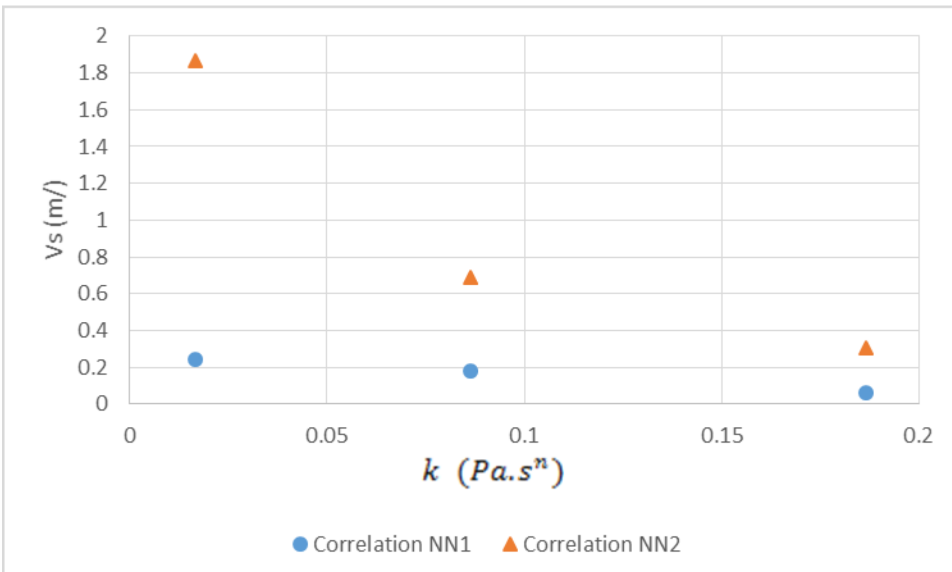


Figure 3.18: Settling velocity vs. the fluid consistency index in non-Newtonian fluid.

The relationship between the fluid viscosity and the settling velocity was negative.

3.7 Conclusions:

Summarizing the results across different correlations, we can present Table 3.72. The percentage change was used to describe the trend in the properties of the proppant particle and the carrier fluid in both Newtonian and non-Newtonian fluids along with the effect of these properties on the settling velocity. This investigation is useful to predict the settling velocity for a specific particle with certain characteristics as can be seen in the table below; Table 3.71.

Table 3.71: Summarizing the results that were obtained for one representative particle.

d (μm)	ρ_p (kg/m^3)	N1 v_s (m/s)	N2 v_s (m/s)	N3 v_s (m/s)	NN1 v_s (m/s)	NN2 v_s (m/s)
		$\mu_f = 0.001$ Pa.s			$k = 0.0865$ Pa.s^n $\phi = 0.9$	
2000	2650	0.282	0.459	3.616	0.182	0.690

Table 3.72: Summarizing the results that were obtained after these calculations.

The change in the properties of the particle and carrier fluid of N	The change in the settling velocity in N	The change in the properties of the particle and carrier fluid of NN	The change in the settling velocity in NN
85%↓ Particle size	92%↓	25%↓ Particle size	38%↓
66% ↓ Particle density	86% ↓	29% ↓ Particle density	39% ↓
70% ↓ Particle sphericity	81% ↓	70% ↓ Particle sphericity	31% ↓
97% ↓ Particle concentration	175% ↑	97% ↓ Particle concentration	382% ↑
1900% ↑ Fluid viscosity	90% ↓	91% ↓ Fluid consistency index	405% ↑

- ↑ Increase.
- ↓ Decrease.
- N: Newtonian fluid.
- NN: Non-Newtonian fluid.

Chapter 4

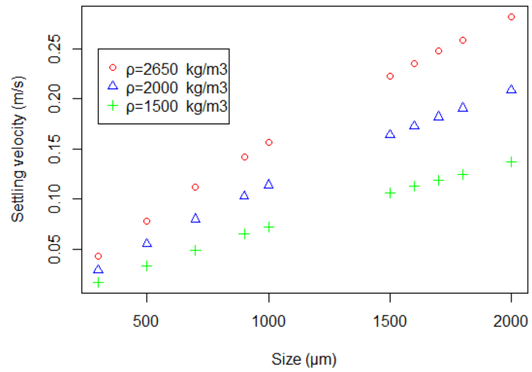
Analysis of the settling velocity models

4.1 Introduction

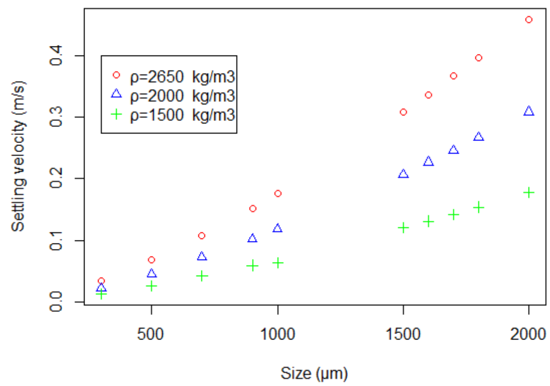
In this chapter the results obtained through the calculations discussed above are summarized. The trends from the figures are presented and discussed here. The comparison between the correlations assuming a Newtonian fluid or a non-Newtonian fluid, as well as within the correlations of Newtonian fluid and non-Newtonian fluid are considered.

4.2 Comparison of the particle size effect on the settling velocity between the correlations for Newtonian fluid

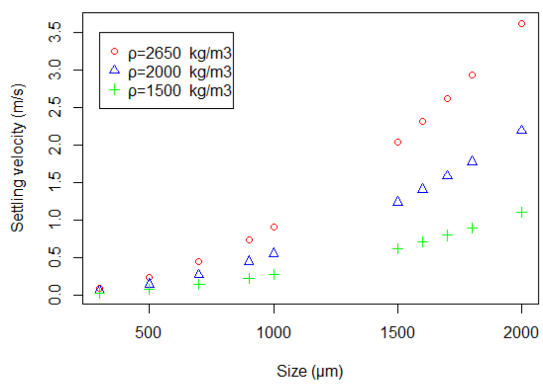
Figure 4.1 shows the settling velocity obtained using all the three correlations (N1 – N3) assuming that the carrier fluid is a Newtonian fluid. The settling velocity is plotted against the particle size for the three different particle densities considered (Figure 4.1(a-c)). It is clear that the lowest values of the settling velocities are obtained for the lowest value of the particle density in all the correlations. This is an expected result that also is a good indicator to show the positive relationship between the settling velocity and the particle density. However, the percentages of the reduction in the settling velocity obtained from the calculations are interesting. For instance, in the calculations, when the particle size was reduced to 85% by using high density (2650 *kg/m*^{3}) in correlation N1, the reduction in the settling velocity was 85%. When medium and small particle densities (2000 and 1500 *kg/m*^{3}) were used in correlation N1 with the same reduction in the particle size, the reductions in the settling velocity were similarly 86% and 88% respectively. This shows that the reduction in the velocity increases slightly at lower densities and the relationship may be assumed to be linear. Correlations N2 and N3 predict a nonlinear relationship between the particle size and the settling velocity and indicate that there is no effect of reducing the particle density on the reduction in the settling velocities with respect to the reduction in the particle size. The reductions in the settling velocity are directly in proportion to the reduction in density (93% and 97% respectively) with the same reduction in the particle size. The reduction in the velocity is the same for all the three values of densities. It may be noted that all the percentages of the reductions in the settling velocity were close to each other for all the three correlations of Newtonian fluid in general.



(a) Correlation N1



(b) Correlation N2



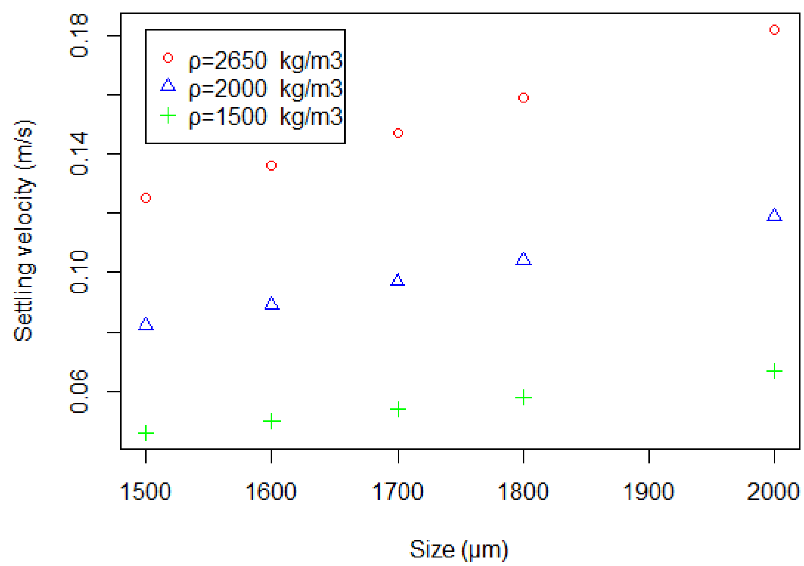
(c) Correlation N3

Figure 4.1: Settling velocity vs. particle size in the Newtonian fluid correlations.[6]

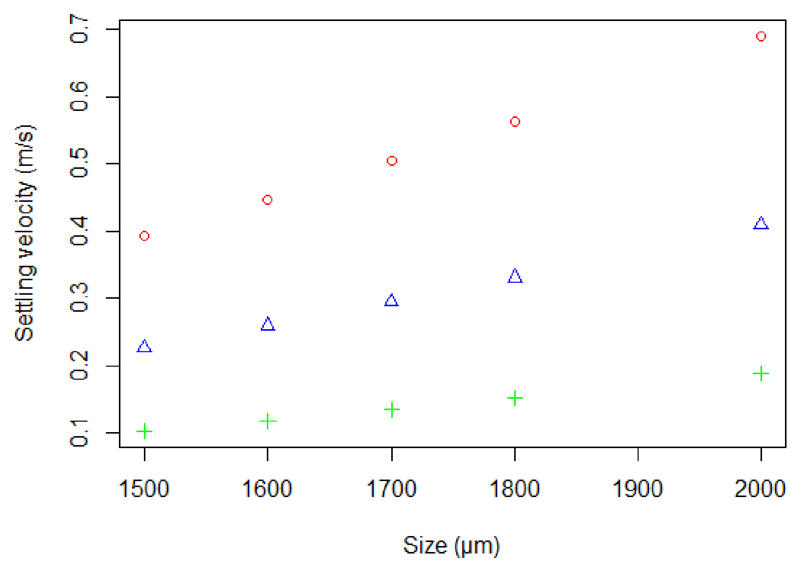
[6] (a) Correlation N1 of Newtonian fluid.
(b) Correlation N2 of Newtonian fluid.
(c) Correlation N3 of Newtonian fluid.

4.3 Comparison of the particle size effect on the settling velocity between the correlations for non-Newtonian fluid

Particle size was plotted against the settling velocity in non-Newtonian fluid in Figure 4.2 using the two correlations NN1 and NN2 assuming the carrier fluid to be non-Newtonian. Again, it is clear that the lowest values of the velocities arise from using low particle density in both correlations. Also, it appears from the figure that the lowest reduction in the settling comes from using low particle density in (a) and (b). Correlation NN1 predicts a strictly linear relationship showing an identical reduction in the settling velocity of 31%, for all the three values of densities (high, medium, and low). In general, this implies that there is no effect of the reduction in the density on the reduction in the settling velocity On the other hand, correlation NN2 shows a similar reduction but not strictly linear as the reductions in the settling velocity were 43%, 45%, and 46% using the three values of density respectively. All these reductions in the velocities corresponded to 25% reduction in the particle size. Thus correlation NN2 appears to predict that at low densities the effect of reducing particle size has a greater effect on reducing the settling velocity. Further, the highest values of settling velocities in correlation NN2 came from the highest sphericity was used compared to the results of the velocities from correlation NN1.



(a) Correlation NN1



(b) Correlation NN2

Figure 4.2 Settling velocity vs. particle size using the correlations for non-Newtonian fluid.[7]

[7] (a) Correlation NN1 of non-Newtonian fluid.
(b) Correlation NN2 of non-Newtonian fluid.

4.4 Comparison between the correlations for Newtonian and non-Newtonian fluids

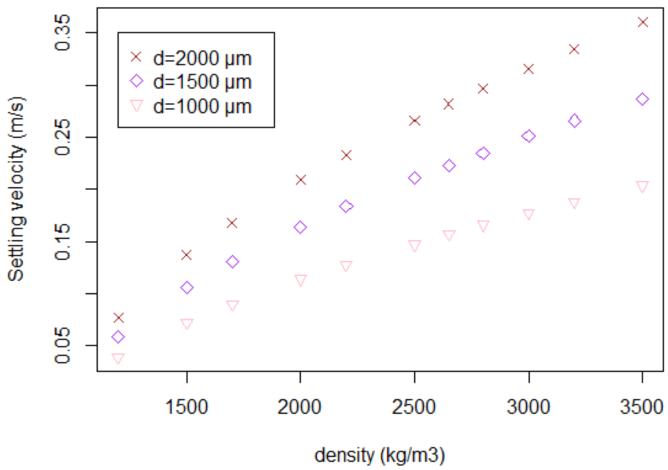
Based on the above discussion, it is observed that as expected, the settling velocity decreases with a decrease in particle size regardless of the density of the proppant particle and whether the carrier fluid is Newtonian or non-Newtonian fluid. Typically, this relationship appears to be mostly linear, although some of the predictions appear to indicate that at higher particle sizes, the settling velocity increases more than the increase in the particle size. Thus smaller particles may be preferred for improving the efficiency of the fracturing process.

4.5 Comparison of the effect of particle density on the settling velocity between the correlations for Newtonian fluid

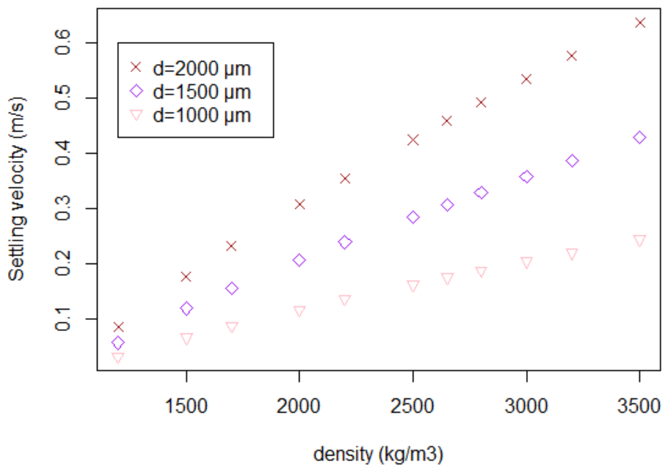
Figure 4.3 shows the relationship between the particle density and the settling velocity using the correlations N1, N2, and N3 assuming that the carrier fluid is Newtonian (Figure 4.3 (a, b, c)). In each correlation, three different particle sizes (2000, 1500, and 1000 μm) were used. The calculations using correlation N1 with large particles (2000 μm) led to a 79% reduction in the settling velocity when the particle density is reduced by 66%. Also, using medium and small particle sizes (1500 and 1000 μm) in correlation N1 with the same reduction in the particle density led to 79% and 81% reductions in the settling velocity, respectively. Therefore, there is a greater reduction in the settling velocity when small particles were used. Thus the effect of particle density seems to be more pronounced at smaller particle sizes in comparison to larger particles.

Also, the reductions in the settling velocity obtained using correlation N2 were 87%, 86%, and

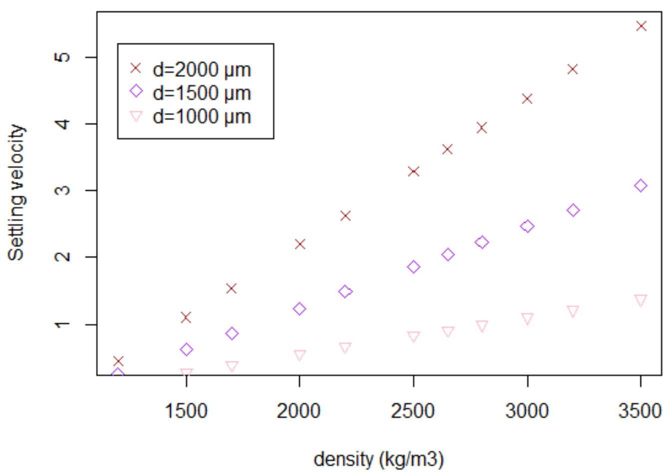
87% which corresponded to the same reduction in the density with using the particle sizes (2000, 1500, and 1000 μm), respectively. Similarly, the reductions in the settling by using correlation N3 were 92%, 92%, and 92% corresponding to 2000, 1500, 1000 μm respectively for the same reduction in the density (66%). Therefore these results were at odds with correlation N1 as they indicate that the size of the particle does not affect the settling.



(a) Correlation N1



(b) Correlation N2



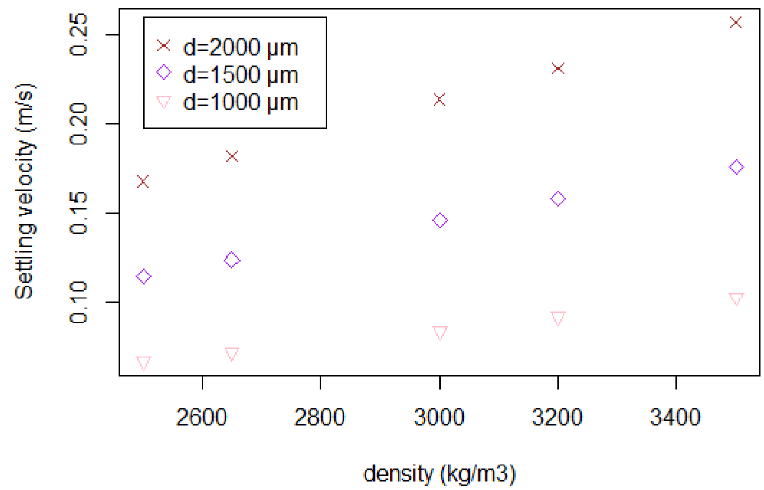
(c) Correlation N3

Figure 4.3: Settling velocity vs. particle density for all Newtonian fluid correlations.[8]

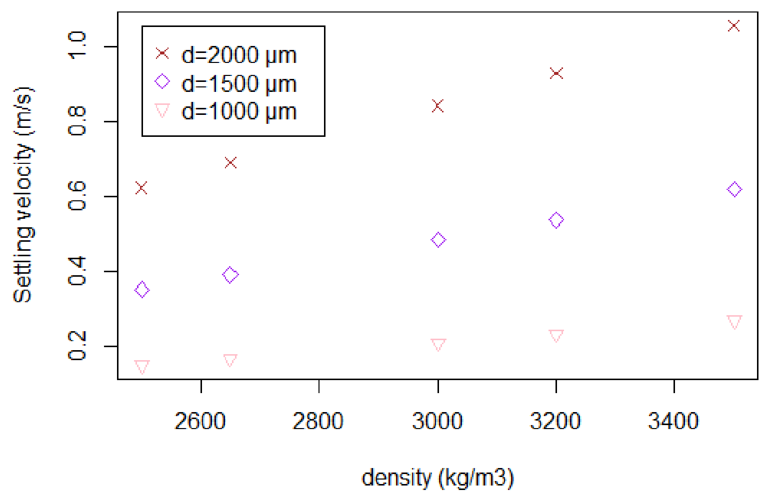
[8] (a) Correlation N1 of Newtonian fluid.
(b) Correlation N2 of Newtonian fluid.
(c) Correlation N3 of Newtonian fluid.

4.6 Comparison of the effect of particle density on the settling velocity between the correlations for non-Newtonian fluid

The settling velocity was plotted as a function of particle density using the correlations NN1 and NN2 for a non-Newtonian carrier fluid with three particle sizes; large, medium, and small (Figure 4.4 (a) and (b)). The percentages of the reductions in the velocity obtained through the calculations from using correlation NN1 were 35%, 35%, and 35%, and for correlation NN2 were 41%, 43%, and 44%, corresponding to the 29% reduction in the density with the particle size. Both correlations show the expected result of an increased settling velocity with an increased particle density. However, the correlations predict an almost linear relationship between the particle density and the settling velocity that is essentially independent of the particle size. (correlation NN2 appears to show that smaller particles have a slightly higher dependency, which is in agreement with that observed with correlation N1 earlier). This implies that choosing different sizes of particles would not make a difference in the settling as long as the density is low.



(a) Correlation NN1



(b) Correlation NN2

Figure 4.4: Settling velocity vs. particle density for all non-Newtonian fluid correlations.[9]

[9] (a) Correlation NN1 of non-Newtonian fluid.
(b) Correlation NN2 of non-Newtonian fluid.

4.7 Comparison of the effect of particle density on the settling velocity between the Newtonian and non-Newtonian fluid correlations

These results can be summarized for all the correlations of Newtonian and non-Newtonian fluids as below. At the large particle size (2000 µm), it is observed that the predicted velocities from all the correlations are also close to each other except for the correlation N3, which showed outlier behavior in comparison to the other correlations.

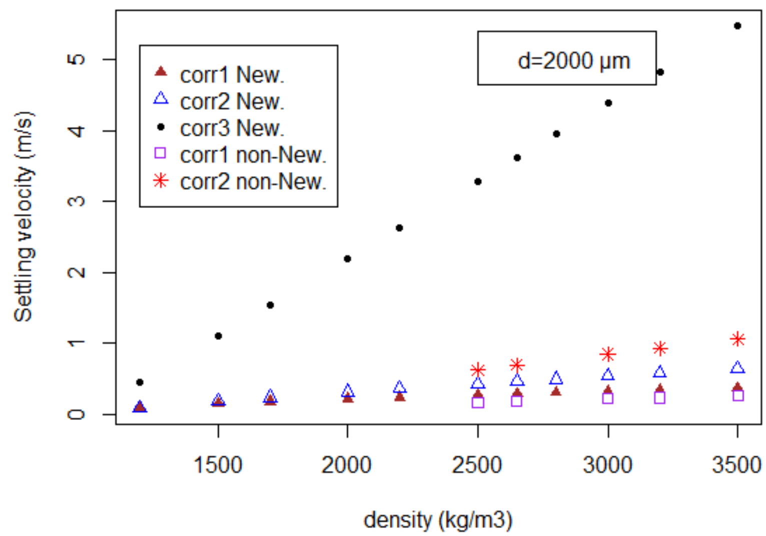


Figure 4.5: All correlations with 2000 µm particle size.

Figures 4.6 and 4.7 with medium and small particle sizes showed the same trend that was observed in Figure 4.5. In Figure 4.7 correlations N1, N2, and NN2 were in a good agreement with each other and there was a little difference between them and correlation NN1.

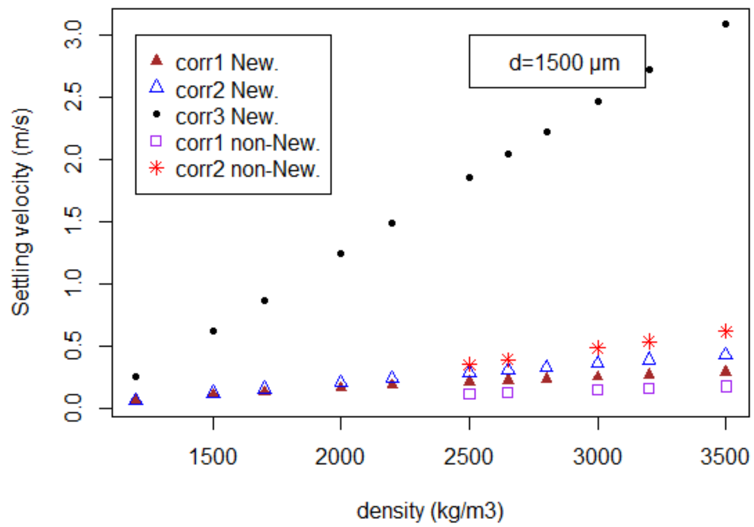


Figure 4.6: All correlations with 1500 µm particle size.

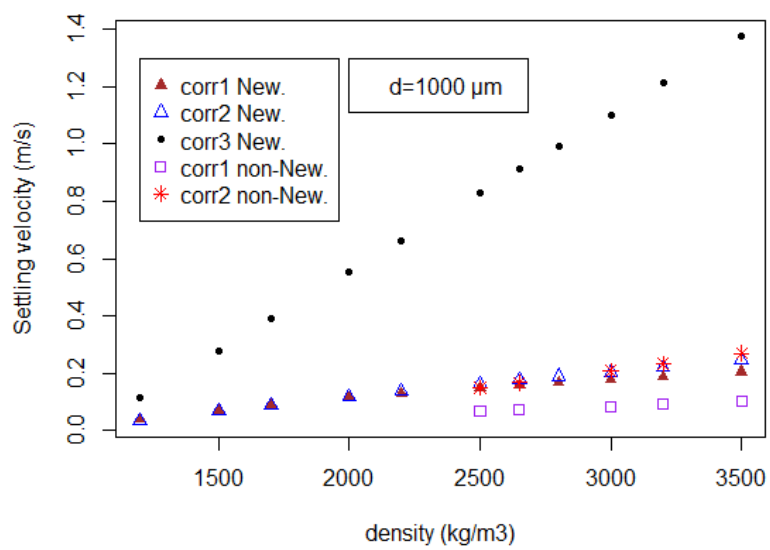


Figure 4.7: All correlations with 1000 µm particle size.

4.8 Comparison of the effect of the particle shape factor on the settling velocity

The effect of shape factor (sphericity) on the settling velocity was plotted using Chein's correlation for Newtonian fluids considering large, medium, and small particle sizes (Figure 4.8). The correlation suggests a linear relationship between the sphericity and the settling velocity for a Newtonian fluid. It may be observed from the figure that the largest reduction in the settling velocity was attained when large particle size was used (reductions in the velocity were 82%, 82%, and 80% corresponding to 70% reduction in the sphericity using particle sizes 2000, 1500, 1000 μm respectively). Interestingly, for non-Newtonian fluids, the reductions in the settling were 47%, 30%, and 16% for a 70% reduction in sphericity (Figure 4.9). This shows that when the particles are small, the effect of sphericity is much less in comparison to when the particles are large. Expectedly, in both fluids the lowest values of the velocities were obtained when small particle sizes were used.

Thus, both correlations indicate that the sphericity has an important role to play in modulating the settling velocity of the proppant. Higher sphericity particles tend to have a higher settling in comparison to a more irregularly shaped particle under the same conditions. Thus, the use of irregular particles in fracturing operations may reduce the settling. It may be noted however, that as discussed earlier, this may come at a cost of reduced porosity and increased clogging of the wells. An optimum is therefore needed to reduce settling while maintaining proppant packing.

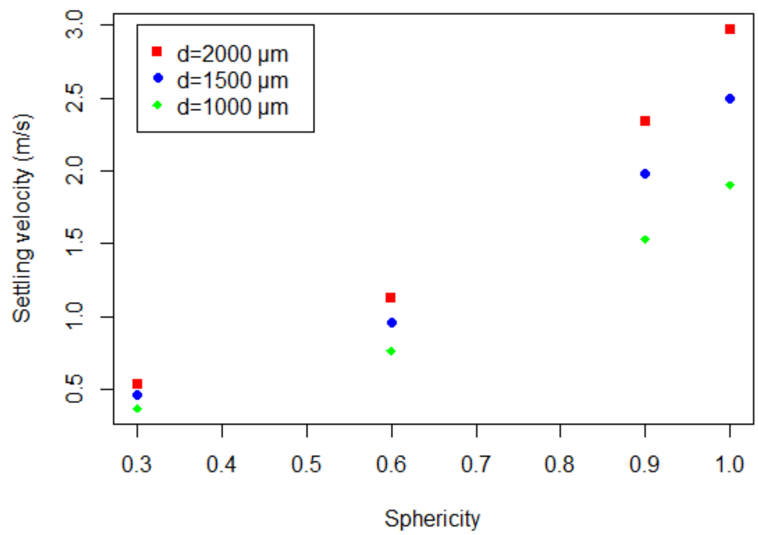


Figure 4.8: The effect of the sphericity on the settling velocity in Newtonian fluid.

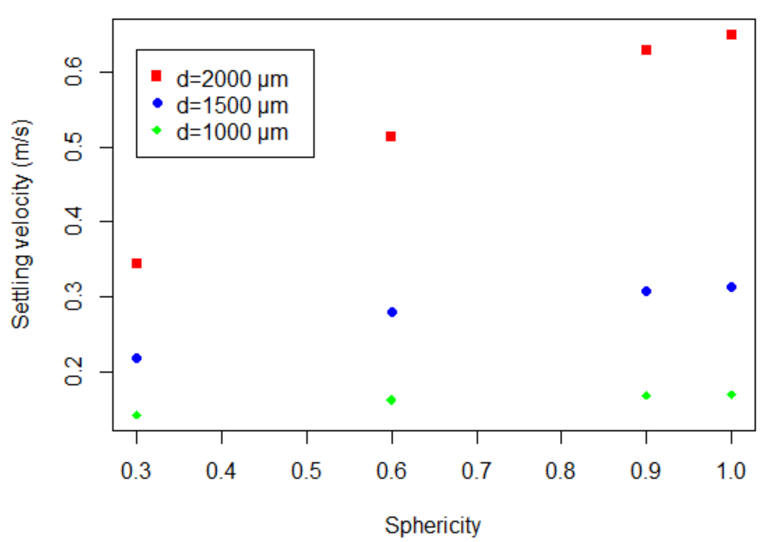
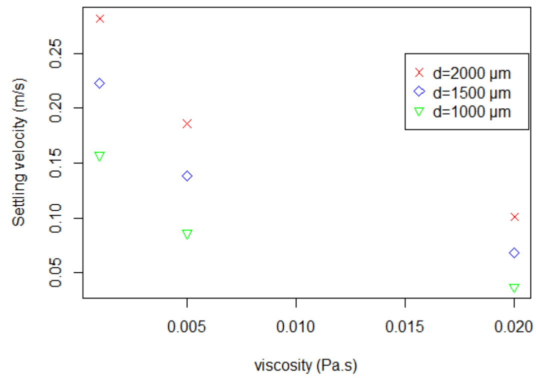


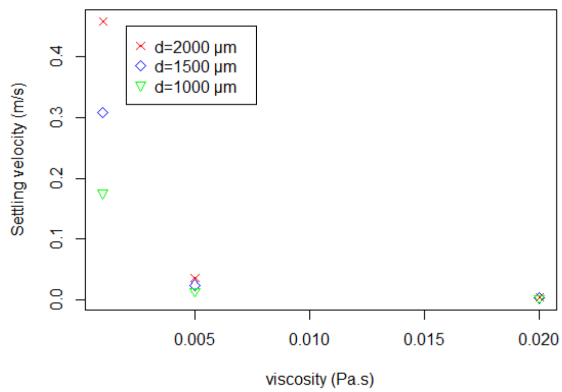
Figure 4.9: The effect of the sphericity on the settling velocity in non-Newtonian fluid.

4.9 Comparison of the effect of the fluid viscosity on the settling velocity between the correlations of Newtonian fluid

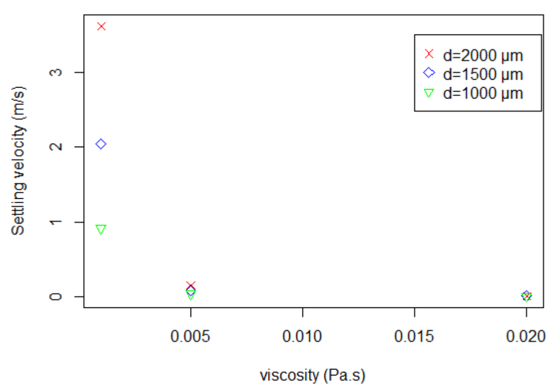
In Figure 4.10, the particle setting was plotted as a function of fracturing/carrier fluid viscosity for all Newtonian correlations N1-N3. Most importantly, it is observed that as the carrier fluid viscosity increases, the settling velocity drops. Initially, the viscosity was assumed to be close to water (*0.001 Pa.s*), as the carrier fluid is ~90% water. However, as the fluid viscosity is increased significantly (*0.02 Pa.s*, which corresponds to a liquid similar to oil), the settling velocity drops significantly. It is observed from Figure 4.10a that the reductions in the settling velocity using large, medium, and small particle sizes were close to each other and the greatest reduction in the velocities were observed using large particle size in correlation N1. In general, there is a steady (almost linear) decrease as the fluid viscosity increases even with a 20x increase in viscosity. However, correlations N2 and N3 showed a dramatic decrease in settling velocity as a function of fluid viscosity, with the predicted velocity dropping by ~2 orders of magnitude for this 20x increase in viscosity. Both correlations N2 and N3 predicted similar results and the velocities did not appear to depend on the particle size either. It is obvious that the results that obtained from correlations N2 and N3 are anomalous in comparison with the results from correlation N1, indicating a lower reliability of these correlations. While the assumption of a Newtonian fluid is a simplification of a real scenario, the analysis shows that as the viscosity increases, the settling velocity drops. This is recognized in industrial applications and operations may use thickening agents as additives to increase the viscosity of the fracturing fluid (See Table 1.3). Conversely, it may be expected that higher viscosity fluids may incur a greater pumping load for a fracturing operation.



(a) Correlation N1



(b) Correlation N2



(c) Correlation N3

Figure 4.10: The effect of the fluid viscosity on the settling velocity in a Newtonian fluid.[10]

[10] (a) Correlation 1 of Newtonian fluid.
(b) Correlation 2 of Newtonian fluid.
(c) Correlation 3 of Newtonian fluid.

4.10 Comparison of the effect of the fluid viscosity on the settling velocity between the correlations of non-Newtonian fluid

The consistency index (k) was plotted against the settling velocity using both correlations of non-Newtonian fluid (Figure 4.11). It is seen from the figure that the values of the velocities were higher in correlation NN2 than in correlation NN1 and the increase in the settling obtained from correlation NN2 was very large and satisfied the results showed through the calculations that the increase in the settling was 512% in correlation NN2, and 298% in correlation NN1 which corresponded to 91% reduction in the consistency index in both correlations.

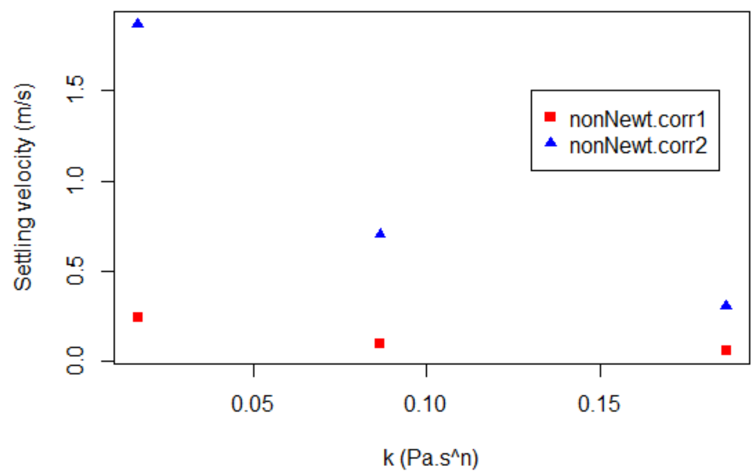


Figure 4.11: The effect of the consistency index on the settling velocity in a non-Newtonian fluid.

4.11 Comparison of the effect of the particle volume fraction on the settling velocity between Newtonian and non-Newtonian fluids

The particle volume fraction was plotted against the settling velocity by using the three Newtonian correlations (Figure 4.12). The settling velocities predicted using correlations N1 and N2 were close to each other. The largest values of the velocities came from correlation N3 as noted in Figure 4.12, which also showed the largest decrease in settling velocity as a function of concentration.

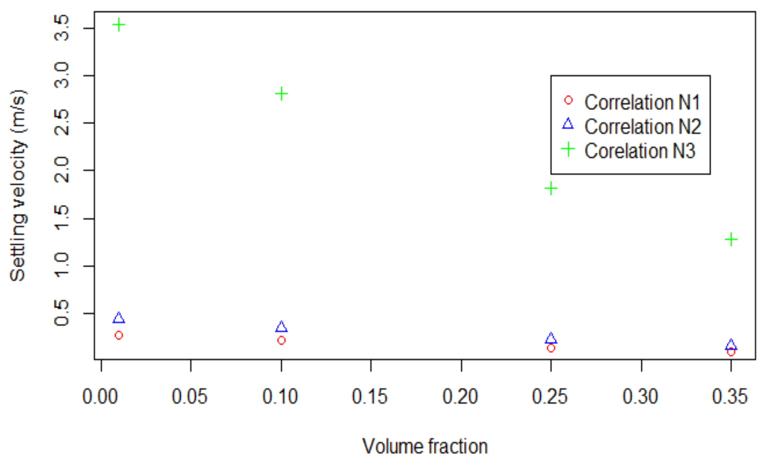


Figure 4.12: Settling velocity vs. particle volume fraction using the correlations of Newtonian fluid.

Similarly, the effect of volume fraction on the settling velocity using both correlations of non-Newtonian fluid (Figure 4.13) showed slight differences between the predicted behavior. Correlation NN1 showed a smaller effect (319%) vs. correlation NN2 (444%) corresponding to an increase in particle volume fraction. Also, there was a large difference between the predicted velocities obtained from both correlations as was clear in Figure 4.13. In general, both correlations predict that as the volume fraction of particles increases, the settling velocity of an individual particle is likely to decrease. Therefore, higher volume fractions are favored in order to improve the transport of proppants. Once again, it must be noted that increasing particle volume fraction corresponds to a higher proppant and pumping load. Therefore the desire to reduce settling must be balanced against the economics of the fracturing process.

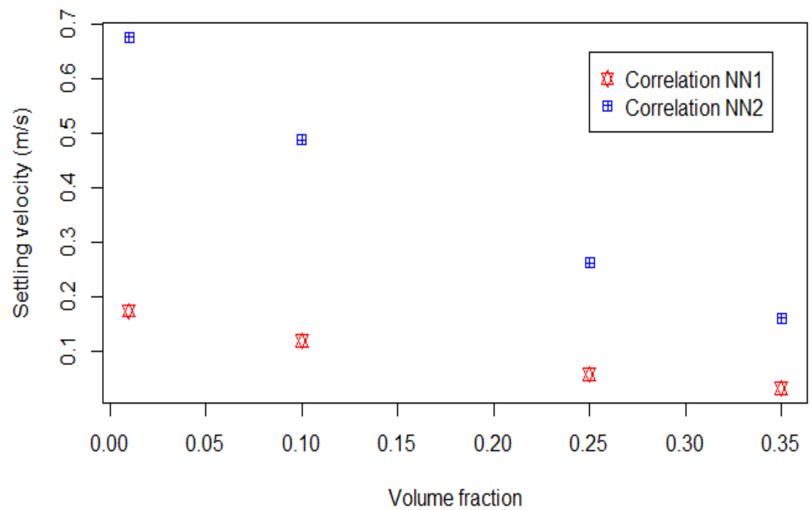


Figure 4.13: Settling velocity vs. particle volume fraction using non-Newtonian fluid correlations.

All the correlations were plotted on the same figure to show the relationship between the particle volume fraction and the settling velocity in Figure 4.14. Interestingly, the results of the velocities in correlation N1 and N2 of Newtonian fluid and correlation NN1 and NN2 of non-Newtonian fluid were in good agreement with each other, showing that these volume fraction dependencies were not affected by the nature of the fluid. The outliers were the results of correlation N3 of Newtonian fluid which were different from others.

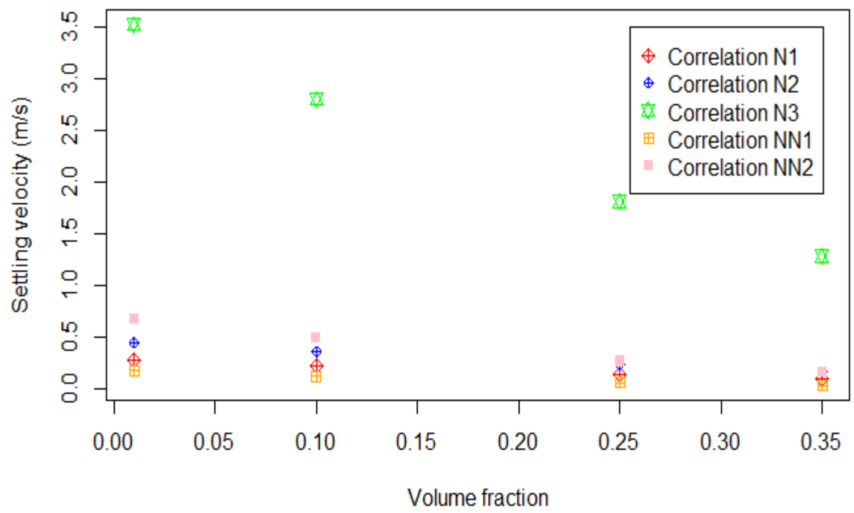


Figure 4.14: Settling velocity vs. particle volume fraction across all correlations.

4.12 Comparison of the settling velocity of this model to previous data

Comparison was done between the settling velocity from this work and the settling velocity from the previous researchers (12) on the same correlations that were used to validate the presented results.

Case study was taken for correlation N1 and correlation N2 with an actual data from the former researchers and it was found that the ranges of the settling velocity were (0.031-0.305) for correlation N1 and (0.026-0.309) for correlation N2. These ranges were so close to the ranges that were obtained in this work which were (0.043-0.282) for correlation N1 and (0.033-0.459) for correlation N2.

Also, the Root Mean Square Error on velocity (RMS-V) was calculated for correlations N1 and N2 to quantify the difference between the values of the settling velocity that were obtained in this work and the values of the settling velocity from the previous researchers to evaluate our work. RMS-V was 0.0586 for correlation N1 and 0.0283 for correlation N2. The RMS-V for correlation N1 was small and was very small for correlation N2. This indicates that the work that was done in this research was accurate and reliable. Plotting was made between the results of the settling velocity from the actual data in correlation N1 and the results from correlations N1, N2, and N2 with actual data in figure (4.15). Figure (4.15) confirmed the results that were obtained in RMS-V that there is no big difference between the actual data and the data of this work.

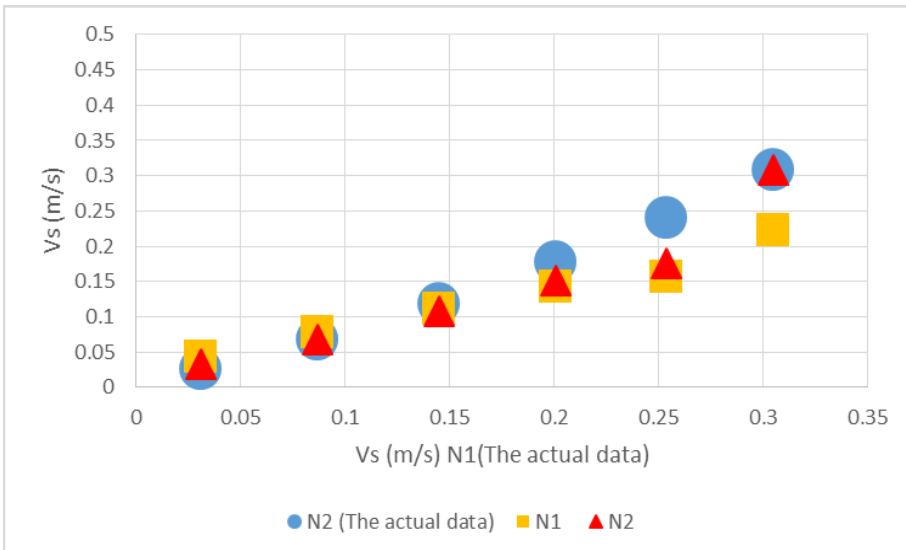


Figure 4.15: Comparison of the settling velocity of this model to previous data.

4.13 Conclusions

It is clear that the particle properties and the carrier fluid have a significant effect on the settling velocity. Particle size has a great effect on the settling velocity and has a positive relationship with it but changing other factors as the same time beside the particle size (such as density) gives a better prediction on what the settling velocity would be. The similar procedure with the particle density is performed as well using other factors like the particle sphericity and concentration, and the fluid viscosity. This makes the prediction of the settling velocity is more accurate and allows us to predict the settling velocity under any conditions in terms of the particle properties and the fluid. Finally, for a specific particle with certain size, density, and sphericity moving in either Newtonian or non-Newtonian fluids, it is easier to identify the particle settling velocity.

The finding of this research is important economically because it reduces the settling, one of the major problems in hydraulic fracturing, and then increases the conductivity of the well thereby improving the efficiency of the fracturing process.

Chapter 5

Conclusions

5.1 Conclusions and recommendations

Hydraulic fracturing process is an advanced technique used to stimulate wells and extract more oil and natural gas, especially for the wells that have reached limits of production of these materials using horizontal drilling. Hydraulic fracturing involves injecting a special fluid consisting of water, additives and granular material into the wells at high pressure. As discussed in the previous chapters, the transport of the granular material (proppant) is critical to the success of the fracturing process. The objective of this research involved understanding the factors that affect the proppant transport in the fluid, specifically with the objective of controlling the settling velocity of the proppants and improving the efficiency of the fracturing process.

This research presented an investigation of various correlations to estimate the settling velocity of particles in Newtonian and non-Newtonian fluids. Various parameters that influence the settling of a proppant particle were studied including particle size, density, volume fraction, and the shape factor as well as the fluid viscosity. It is understood that there are several other factors that could potentially affect the transport. However, in this research, we focused on some of the key parameters that can be externally controlled and thereby control the settling velocity of the proppants. Using several simplifying assumptions, theoretical calculations were made with some typical values of proppants currently available, such as particle sizes and densities.

Various conclusions could be drawn from the research – for instance, reducing the particle size in the range of 1500-2650 kg/m^3 of particle density led to reducing the settling velocity by ~92% in Newtonian fluid. Similar reduction in the velocity occurred when the particle size was reduced 85% using either 2650 kg/m^3 or 2000 kg/m^3 particle density. However, reducing the particle size by ~85% at 1500 kg/m^3 particle density led to ~93% reduction in the velocity. This implies that the effect of particle size is more pronounced for lighter particles than it is for heavier particles. Assuming the carrier fluid to be non-Newtonian, the reduction in the settling velocity was ~38% when the particle size reduced by 25% in the range of 1500-2650 kg/m^3. The reduction in the settling was the same when the same reduction occurred in the size using 2000 kg/m^3 particle density while the same reduction in the particle size with 2650 and 1500 kg/m^3 particle density caused 37% and 39% reduction in the settling. This implied that the effect of particle density was minimal in comparison to reductions in size for a non-Newtonian fracturing fluid.

Similar effects were noted for particle density (increased density leads to increased settling with low effect of particle size), concentration of the particles (reducing the particle concentration led to increase the settling velocity in both Newtonian and non-Newtonian fluid, with a higher change in the latter). In addition, reducing the sphericity by 70% resulted in lowering the velocity of the settling in both Newtonian and non-Newtonian fluids but with a much higher reduction in the former. Increasing the fluid viscosity caused a reduction in the velocity. While some of these trends are expected, the importance of this work is to show the quantitative and qualitative dependence of the velocity on these parameters under various conditions.

We evaluated the results that were obtained from this work to check the validity of the work by comparing the results of the settling velocity with the results of the former researchers. It was found there was only a small difference between the results of the settling velocity of this work and the previous work. As a result, the data from this research can be very useful for hydraulic fracturing and other applications. In general, the need to optimize the settling velocity of the particle and reduce it by knowing the behavior of the particle during its transport inside a fluid is very valuable for industrial applications. Understanding particle transportation inside a fluid such as during hydraulic fracturing is important since it gives the optimal properties for particles with a reduced risk of settling during flow.

Considering the data that is obtained by this work gives insight about how the process in the industry will look like regarding to the settling velocity of the particles. For instance, we have discussed the percentage of the success of the particles transportation with regard to the settling velocity. This in turn will lead to the success of the work. One of the several aspects that were addressed by this thesis is that if certain particles with certain properties are used in a specific application, the settling velocity of these particles can be estimated within a few reasonable assumptions. For example, if particles with 2000 μm size and 2650 kg/m^3 density move in Newtonian fluid, the settling velocity of these particles will be expected to be in the range of (0.282-0.459) *m/s*. Knowing the settling velocity of the particles gives an idea about the results that will be achieved. This is one of the contributions of this work toward the industries that deal with particles.

At this point, it is also important to discuss the following implications of this work – changing a single property is typically not feasible in a field situation because of its unintended effects. For instance, while lower density particles are good for reduced settling velocities, they are also less mechanically robust and may not be able to withstand the high pressures required in such operations. Similarly, reducing the particle size is good for reducing settling. However, smaller particles are typically not preferred because they can lead to clogging of the well bore. Lower sphericity is accompanied by issues with porosity. While low viscosity Newtonian fluid which is a water frac is predicted by our results as leading to a low settling, this leads to lack in the transportation of the proppant particles, thereby requiring high pumping rates. The high pump rate will increase the friction factor and then reduce the pumping pressure. This, in turn, will make the fractures narrower and affects the conductivity of the wells negatively. In other words, changing a parameter must be carefully considered for its effect on other process parameters. Thus, a proper estimation of combinations of these factors or optimization of properties by varying multiple parameters is needed for a truly improved fracturing operation. This research presents some of the initial steps towards this.

5.2 Future Work

This research study used various models to predict the settling velocity that was influenced by certain factors. However, further investigation is needed to study other factors that affect the settling velocity such as the effects of the walls, high pressure flow and presence of different additives that can change the fluid properties. It is also clear that better models are needed to accurately simulate the flow. Several simplifying assumptions were made in our calculations.

The assumption “no leak-off to the formation” is not typically applicable in real situations as there may be fluid loss due to the formation permeability. “Constant width of the fracture” is also challenging because it depends on the pumping rate. Similarly, assumptions such as the use of a spherical particle will have to reflect realistic conditions such that calculations are performed on irregularly shape particles. Experimental work is needed for a more complete understanding and a precise knowledge of the parameters that affect settling velocity and to validate the theoretical values obtained. It is suggested that future work focus on this and combination with theory and experiment to provide a complete understanding of proppant transport and control. Such investigations can have great applied and practical importance to not only better model complex slurry flows but also to improve well conductivity in industrial operations.

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