MATHEMATICS FOR GENERAL EDUCATION: ANOTHER RULE OF THREE

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1. General Education Students

A commonly held view of mathematics faculty is that general education (GE) students, those compelled to study mathematics as a degree requirement, comprise, by far, the most difficult audience to please. Indeed, many departments of mathematics are happy if these students can go elsewhere, philosophy, computer science or business, for example, to fulfill such a requirement.

Our union stewards, if we had them, would grieve the resulting employment loss to the mathematical community, not a trivial issue these days. But the availability of alternatives to mathematics for GE requirements for this large population may, in many instances, protect the mathematics department. We know these students dislike mathematics, so if we can shift the burden, or blame, to other disciplines, the English majors and History majors and all those other majors who later acquire influence over science research budgets won't have such negative feelings about mathematics. In turn, many of these students are only too happy to join the conspiracy to export jobs to mathematically underdeveloped departments. Perhaps most disturbing is the possibility that a potential K-8 teacher would do 'mathematics' in this way.

There are students, however, frequently uninterested in mathematics as mathematics, who nevertheless have very positive feelings toward the subject. Engineers are quite aware that they cannot survive without the tools provided in calculus, differential equations, linear

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algebra, probability and so on. As students, these people are avid consumers of our mathematical wares. When an engineering student asks, "What's this stuff good for?" it is a gesture of friendship.

We would all like to teach mathematics to students who are intrigued by the structural aspects of the subject. Indeed, some GE courses in mathematics are designed to introduce students to axiomatic methods; the instructors presumably aiming to show students how mathematics is done by those in the trade. Such an inward view of mathematics for GE imposes a very great burden on both the instructor and the student; the former to avoid frustration and the latter to avoid boredom. But engineering students *voluntarily* take mathematics courses. This being so, we need to ask: assuming there were actually an explicit choice in the matter, would we be better off if we taught non-science students as if they were engineers or as if they were mathematicians?

2. Engineering Students

Mathematicians know that mathematics, besides being a very entertaining way to while away the time, is an indispensable tool for analysis of the real world. It is the latter characteristic, not the former, that will convince lay people that mathematics and mathematicians deserve support. And how shall this claim of indispensability be verified? We can just make it repeatedly and hope that it sticks. This style of argument is used regularly in public venues. It requires the aid of lobbyists. A better way might be to get the students themselves to do some work and see that without mathematics, they cannot understand why particular pieces of real data are important or what those data have to say about design decisions.

It is fair to assume that this understanding is precisely what attracts engineering students to mathematics. But the desire for genuine external application often makes engineering students an imperfect fit for many mathematics classes. On one hand, these are talented, hard-working, quantitatively oriented students who are often among the top performers in our classes. On the other, engineering students get impatient if confronted with too much 'theory'. In fact, if they and their engineering faculty get really annoyed, one finds mathematics courses taught in engineering departments. *This* migration to mathematics courses outside the mathematics department is cause for rather more concern, as we know from frequent panel sessions on the subject at mathematics meetings. In addition to the too-much-theory problem, lack of deep familiarity with the engineering application on the part of mathematicians surely plays an important role in this phenomenon. From the point of view of abstract mathematics, this is, therefore, not a completely positive situation.

Perhaps a better way to grasp what engineers like in their mathematics courses is to examine the most highly mathematical texts that are used in engineering courses. These can already be found at the sophomore-junior level. Courses in linear circuit analysis, for example, cover a lot of ground that might otherwise be found in linear algebra, complex function theory and differential equations courses. A mathematician would complain that too much is left out of that coverage. However, the entire context of the treatment is connected with lumped circuit elements, a physical setting that is simultaneously crucial to the electrical engineer and forbidding to someone not introduced to Coulomb, Faraday and Ohm, the (hard) freshman physics parents of the hardware. The EE student who does not think of a transfer function in physical terms will suffer the consequences.

We all know that those EE students who take a standard course in linear algebra will have the benefit of mathematical preparation useful for many things besides circuit analysis. In fact, because there are different kinds of engineers, that standard course is precursor to courses in mechanical, civil and chemical engineering as well. The real point here is that until the day the EE student works with the actual circuits, that student's faith in much of linear algebra relies mainly on a promise. Similar situations occur in chemical thermodynamics, mechanics of materials, and numerous other engineering regimes.

3. The Rule of Three

In an attempt to abstract from these examples, we identify the characteristics of a pleasurable mathematical experience for an engineering student.[2]

1. Students should encounter real data, reliably gathered from a familiar setting, as

[2] No one will mistake this 'Rule of Three' with its predecessor, made famous by the Harvard Calculus Consortium [1].

the raw material for mathematical problems;

2. Students should perform some analysis on that data, typically some calculation; the methods of analysis should conform to the practices of people who work in the setting;
3. The results of the analysis should cast a genuinely informative light on the setting of the problem.

The typical engineer maintains an interest in mathematics for just so long as it continues to provide these services. When these things are not happening, the engineering students complain. And if those complaints are legitimate, what does this say about the way we treat GE students?

Take some examples that are imposed on GE students:

a) Problems in high-school algebra that begin: "I was three years old when my uncle's first wife ..." and end, "How much does my dog weigh?" fail on all three counts;
b) the techniques of symbolic logic fail on the second count as they are rarely employed by lawyers or others in the arguing business;
c) the fitting of simple functions to statistical data, with no supporting model, fails on the third count;
d) mathematical modelling with no verifying reference to external data fails on the first count.

Notwithstanding lapses relative to our three ground rules, many mathematicians have found these kinds of problems a convenient, and sometimes effective way of approaching general education students. Our intent here is to capture what engineering students find attractive about mathematics and to describe what we might do to induce a similar reaction in others.

It is more-or-less obvious why engineers want their mathematics to conform to this 'Rule of Three'. They live their professional lives in a sea of real data. Furthermore, the standard methods of analysis are standard because they have been found useful; engineers who use arcane mathematical techniques are bound to be poorly understood by their colleagues. Lastly, there is little reason for an engineer to do a computation if it doesn't give any information about the problem at hand.

For only slightly different reasons, this Rule of Three is also pertinent to non-science students. Let us assume that we want these students to escape the 'mathematics as a textbook exercise' view.

1. To omit Rule 1 is to confine the students' work to toy problems. Trying to find a good garbage pickup route in a town with six streets will convince few students that graph theory is a useful analytical tool.
2. To omit Rule 2 is to have students doing mathematics that is irrelevant to the practice in the external discipline. This kind of work isolates the students from practitioners of that discipline just as we are trying to convince those students that mathematics is useful. No one in penology has much interest in the rate at which the searchlight scans the prison wall, even though this is a favorite problem in the calculus text.
3. To omit Rule 3 is to use the setting as an excuse to do mathematics. Say we fit a quadratic function to the incidence of hepatitis B from 1984 to 1996. This may be a good exercise in polynomial algebra. But what do we now know about the etiology of hepatitis?

Now, general education students are not aspiring engineers. Besides being quantitatively less literate than their engineering sisters and brothers, they have no 'subject' that requires mathematics in any crucial fashion. In particular, prospective K-8 teachers want to teach, they do not want to engineer. This means that an effective approach to engineering-style mathematics for non-science students will require a simultaneous time-efficient introduction to some subject of common acquaintance, if not common interest. That subject will have to entail some identifiable mathematical considerations. This raises a disturbing prospect. An instructor who wants students to employ our Rule of Three will have to know something about the setting. To put things bluntly, the instructor will have to know something besides mathematics. This kind of familiarity with the setting of an applied problem is atypical in a mathematics course.

Take a simple example. In Chapter 1 of the Harvard calculus book [1], there is a discussion of the near exponential behavior of the population of Mexico during a certain time period. In treating this problem, most instructors will utterly ignore the fact that Mexico is under consideration. The tabular data and its management are the issue. There are at least three valid reasons why the instructor does this:

a) This course is labeled mathematics, not sociology;
b) The instructor doesn't know anything about the factors that influence Mexican population growth;
c) Time taken to talk about Mexico will be lost to important mathematics.

While these reasons are quite legitimate in a calculus class where students and instructor have a common mathematical agenda, they are not particularly germane to a GE course. In fact, the reverse is true: If an instructor wants to get students to employ mathematics as a tool in understanding some external setting, the ambience of that setting is going to be virtually as important as the mathematics itself. The fact that both mathematics and real context are desirable almost surely demands that we confine that context as narrowly as possible.

4. Implementing the Rule: An example

With very considerable advice from engineers and scientists at the NASA-Langley Research Center in Hampton, Virginia, material has been developed for a GE mathematics course which is designed to conform to the standards described above. The development of the course, "The Mathematics of Powered Flight", was largely prompted by the recent establishment of an 'everyone takes mathematics' requirement at William and Mary. The physical setting is airplane flight. Hence, we try to prod students to consider what aspects of flying can best be understood by doing elementary mathematics. 'Elementary' is very much an operative word here. A beginning course in aerodynamics is not the agenda. In fact, unless there is some elementary analysis available for a problem, we simply ignore it. The advertised prerequisites are high school algebra (algebra I is sufficient) and geometry. Some students have taken physics or chemistry in high school, others have not.

Those looking for ways to improve a GE mathematics course are often determined to make the course 'interesting' for students. This is obviously a desirable goal. It is for us, however, something of a fringe benefit. Rather than hoping that students be intrigued by the subject of airplane flight, we merely ask that they recognize its familiarity. Do we not all have ears that pop with changing altitude? Do we not all see air-resistance-driven violations of Galileo's law of falling bodies, $v = gt$? Do we not all see mysterious 'road' signs near the runway at an airport? Ultimately, the idea is not, "Do you find this interesting?" Instead, we ask, "Do you see that your computations help you to understand what is going on here?"

This is, we think, not a trivial issue for prospective K-8 teachers. It is one thing for a teacher to tell ten year olds that some *other* people use mathematics to help understand the world. It is quite another for that teacher to say to those children, "I used some algebra (or some geometry or some trigonometry) to work out certain engineering problems." No one need confess an interest or lack thereof in the problem itself.

As it happens, many students do get interested, precisely because of familiarity. One student in a class has made parachute jumps. Students are impressed when we calculate a terminal velocity (about 20 feet per second) that matches what jumpers are told in training. Moreover, a student may use a map of the main airport at his or her hometown to find sources of data. The high-level navigation maps that students use present a picture of the continental United States that has no political boundaries, but instead is full of geometric data, much of which can be verified by measurement or calculation.

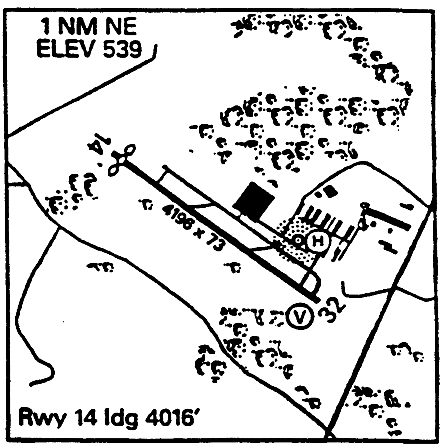
It is safe to say that *every* student is attentive when the Microsoft Flight Simulator is used to illustrate some piece of mathematics. Perhaps because it looks like (and is sometimes advertised as) a game, one may think of managing all the gauges on the panel to keep from crashing. In some sense this is true, for the Simulator as well as for a real airplane. For our purposes, the computational and geometric content of the gauges are the things that bear exploitation.

Students who do a group taco project can directly manipulate a three dimensional geometric figure to see how it can be analyzed with the tools of *plane* trigonometry. Some purchase blowup globes of the earth to help with work on latitude and longitude. Some cut oranges to study the definitions of those angles. Some even acquire a feel for geography (where is Topeka ?) that they missed in school.

A brief account of several topics will give the reader an idea of the course content.

We can begin with a map of a runway, for instance, the airport at *Gaithersburg*, a Maryland suburb of the District of Columbia. This map is taken from [2]. It identifies the single bi-directional runway at Gaithersburg as 14-32. The student must learn that Runway 14 specifies a runway that heads in a direction 140° east of north; that is, in a southeasterly direction. It is, therefore, a fact of geometry that the opposite heading must be 320°. For this purpose, runway direction are rounded to the nearest 10°. Among other things, this means that a student in an airplane who looks out the window during the ground taxi will be perpetually reminded of a fact of geometric/arithmetic invariance: The identifiers on the red runway signs must always differ by 18. For some students, it is fair to assume that they know a little something about trigonometry. If this be the case, the material can proceed somewhat faster. Otherwise, one must take an hour to show students some similiar right triangles and get them to look up sine and cosine function values on a hand calculator.

One may continue, then, with more real data by calling (301) 977-2971, a telephone number that will yield a current account of the flying weather near Gaithersburg. Included in that weather report is a vector, namely the wind direction and speed. Students must now calculate the component of that wind in the direction of the Gaithersburg runway. This is an exercise carried out by all working pilots. As a matter of life and limb, they are always concerned about the relationship between the wind and the direction along which their wheels are rolling on concrete. Incidentally, as with much of mathematics that is applied in the world, terminology is adapted to the setting rather than to ideas of textbook mathematics. This is the case with wind data, which is *not* provided in the standard Cartesian mode.



GAITHERSBURG AIRPORT

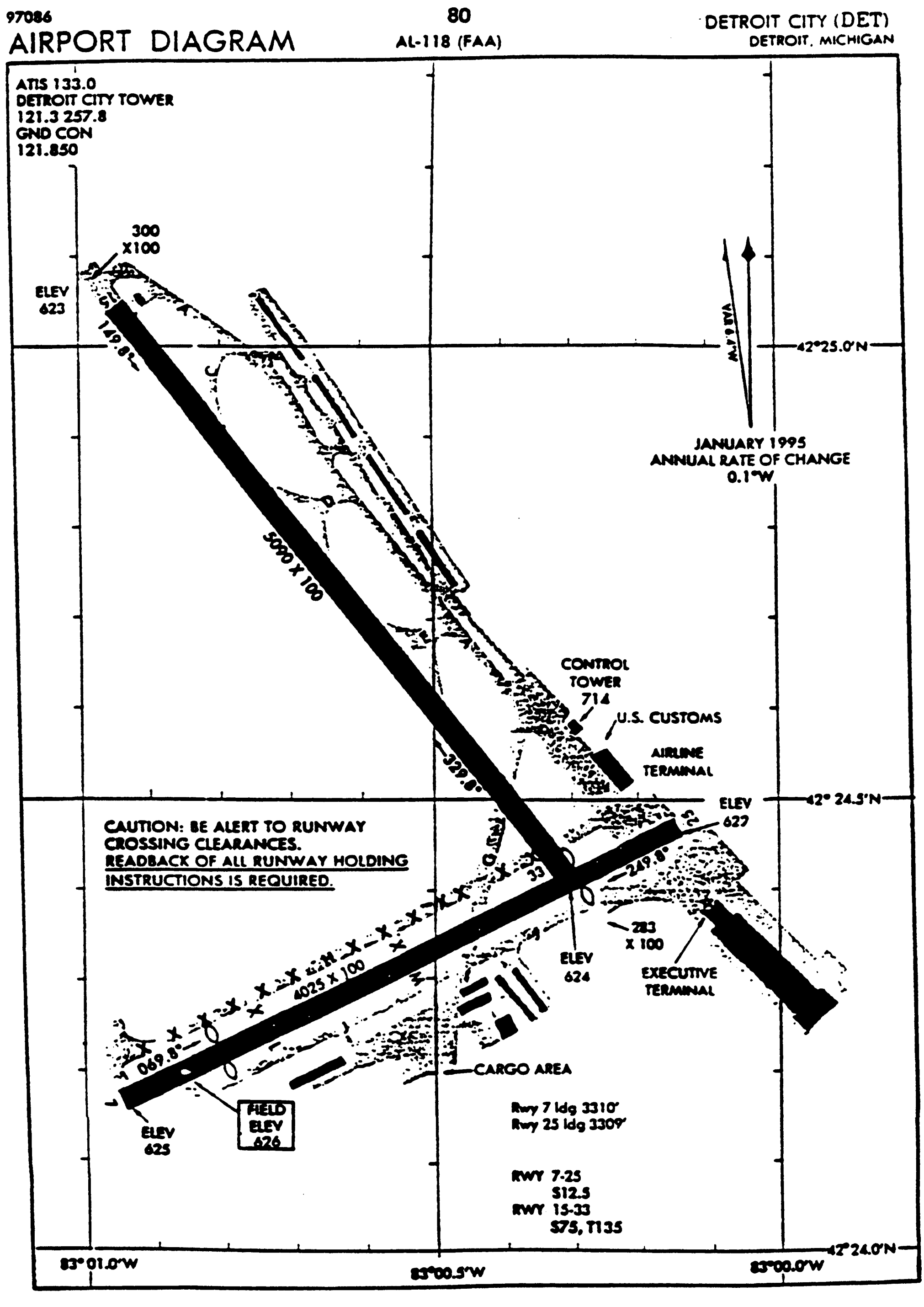
Next, a related issue. Why was the 14-32 direction chosen for Gaithersburg in the first place? It may well be that in particular cases, the runway layout must conform to the geometry of the land available for construction. More typically, the pattern of prevailing winds plays a critical role. It is no accident that runways 5-23 are found at Norfolk International, Oceana Naval Air Station and the Chesapeake Muni airport, all on the southside of the James River in Tidewater, Virginia. For our purposes, we can have students examine wind histories provided in [3]. These histories have been reformatted in [4], specifically for use in an FAA runway layout program [5]. Each individual student is given the airport diagram for a major airport in the continental United States. With the aid of the FAA program, the student may study the connection between the direction of the runway(s) and the wind history provided.

The diagrams for large airports are sources of other useful data. For example, at the Detroit City airport, one sees that runway 7 actually has compass direction 69.8°. Now it is easy for students to measure the geometric heading of that runway by using a protractor set against the 83°00.5' meridian. That geometric heading may be combined with the magnetic variation displayed at the top right of the airport diagram in the shape of a wedge. A harpoon arrow points to compass north, a spade head arrow points true (= geometric) north. The actual measured variation in 1995 was 6.4° west of north. All this information should fit together into a neat addition.

Notice as well that the Detroit City airport has an elevation of 626 feet above sea level. This may be verified with the aid of the Microsoft Flight Simulator. The simulator will permit one to place an aircraft at the Detroit airport and read the altimeter. Better yet, the user may change the weather, in particular, the ambient pressure, to see what *quantitative* effect this has on the altimeter reading. In order to do this, we have to work out a solution to the hydrostatic equation for a compressible fluid. Using this model, the pressure, P_h, at altitude h, is given by

$$P_h = P_0 \exp(-\kappa h),$$

where P_0 is sea-level pressure measured on a particular day (and announced in an airport weather service), h is in feet and $\kappa = 8.05 \times 10^{-2}/(14.7 \cdot 144)$. In fact, the notation used in the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CAUTION: BE ALERT TO RUNWAY
CROSSING CLEARANCES.
READBACK OF ALL RUNWAY HOLDING
INSTRUCTIONS IS REQUIRED.
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formula, eminently readable for a mathematician, is replaced for student consumption by

$$P_h = P_0 \times (.999962)^h, \tag{1}$$

a formula derived from a discrete version of the hydrostatic equation.

Students are required to examine the structure of this calculation: All the numbers have an important physical meaning: 8.05×10^{-2} is the weight density of air in pounds per cubic foot; 14.7 is standard air pressure in pounds per square inch; 144 is the number of square inches in a square foot. In observing the instrument panel, the student will see that the pilot must take account of weather changes in order to get correct altimeter readings. Dangerous altimeter errors resulting from failure to attend to the weather may be calculated using (1).

Equation (1) is also important when one studies the pressurization schedule of an aircraft. Maintaining ground level pressure in the passenger cabin of an aircraft at altitude is not feasible because repeated cycling through such large pressure differentials would quickly destroy the airframe. In order to simultaneously protect the aircraft and keep passengers comfortable, certain standard constraints are enforced. For fear of oxygen deficit, passengers should not 'feel' as if they are any higher than 8000 feet. Equation (1) determines the corresponding minimum pressure. But the maximum pressure differential for a certain aircraft might be, say, 8 pounds per square inch. Now it is time for students to compute: How high can this aircraft fly?

The solution to the hydrostatic equation is itself copied from that of an earlier problem: How rapidly does an unpowered missile lose horizontal velocity as it passes through a resisting medium? Arguing from conservation of momentum, we show that the velocity, as a function of *distance*, falls off exponentially. By further exploiting this result, we can establish a rule for the calculation of terminal velocities for bodies falling in air:

$$g = C_D \frac{A}{W} \frac{\rho V^2_{terminal}}{2}. \tag{2}$$

In this equation, g is the acceleration due to gravity, C_D the drag coefficient for the body, A its cross sectional area, W its weight, ρ the weight density of air, and $V_{terminal}$ the terminal velocity of the body.

Equation (2) appears to require a heavy dose of physics. But, in fact, the word 'acceleration' is never used in the development and (2) is itself obtained mainly from a proportionality argument together with the intuitively reasonable conservation of momentum. Indeed, the treatment of physical laws is sufficiently remote from beginning physics that the idea of mass is never used either. For that matter, we consistently employ British Engineering Units, first because they are familiar to students and second because much published aeronautical data is given in that system. Students may employ (2) to work out terminal velocities for baseballs and parachutes. Results are a good match with published data.

At least as important as the availability of the terminal velocity formula is its structure. In general, C_D depends on the Reynolds number of the corresponding flow, and therefore on the velocity itself. But, for speeds up through 300 knots, C_D is nearly independent of velocity. Otherwise, C_D depends almost entirely on the character of the surface of the body (rough or smooth), and on its geometry (streamlined or blunt). Hence, we may pose for the student:

> Two smooth balls are dropped at the same time from the Tower of Pisa. One is made of iron and is one inch in diameter. The other is made of aluminum and is three inches in diameter. Which hits the ground first?

Owing to the similarity in surface texture and geometry, one may assume the two drag coefficients are the same. Thus, this iron-aluminum problem requires students to calculate the weight to cross-sectional area ratio of the two bodies. The students themselves must look up the weight density of the two metals.

Working with drag coefficients leads us to a study of the conditions for smooth level flight of an aircraft. Students may demonstrate for themselves the significance of the weight to (wing) area ratio of the aircraft.

Returning to the magnetic variation indicator on the Detroit City map, we may take up a large family of navigation problems. First of all, a vast array of radio navigation aids is scattered across the country. Among other things, those navigation aids (they are commonly called VOR's, the letters individually articulated) broadcast a nominal magnetic north that is established by the FAA. The fact that magnetic variation slowly changes over time causes some curious 'disagreements' among nearby VOR's. These disagreements can be resolved with some elementary mathematics.

As far as navigation itself, we may begin with short distance approximations that make the earth flat near a particular site. Calculations of distances and flight headings may be verified with the Flight Simulator. For large distances, a somewhat deeper foray into trigonometry is required. But we do just enough to work out long distances on the curved earth and corresponding headings for great circle flights. We erect none of the standard structure of spherical trigonometry.

Central to this geometric analysis is a canonical form for a great circle. Let H be the most northerly point on the great circle, where H has latitude and longitude $(\lambda_H\ \tau_H)$. It is intuitively clear that the choice of H will fix the great circle. Then, a generic point P, with latitude and longitude (λ, τ) falls on that great circle if and only if

$$\tan\lambda_H = \frac{\tan\lambda}{\cos(\tau - \tau_H)}. \tag{3}$$

This result may be derived trigonometrically. But it may also be demonstrated with the aid of a cardboard and glue construction that is appropriate as a team project. Among other things, the student is asked:

A pilot flies a great circle path whose most northerly point is Reykjavik, Iceland. Does the airplane fly north or south of Cleveland?

The further usefulness of (3) may be seen from a simple formula for calculating distances and headings. Let an aircraft lie at $P = (\lambda, \tau)$ en-route to the 'high-point' $H = (\lambda_H, \tau_H)$ of a certain great circle. The angular separation, β, along that great circle between P and H satisfies

$$\sin\beta = \cos\lambda\,\sin(\tau - \tau_H). \tag{4}$$

The value of β, obtained from (4), determines the surface distance between P and H according to

$$distance = 2\pi \cdot 3440 \cdot \frac{\beta}{360},$$

where β is measured in degrees, the way pilots do. 3440 is the radius of the earth in nautical miles; we treat the earth as a sphere in these calculations. Furthermore, the (geometric) heading, γ must satisfy the equation

$$\cos\gamma = \tan\lambda \tan\beta, \tag{5}$$

an equation whose similarity to (3) is shown to be no accident. Calculations in every specific case may be verified against navigation maps published by the National Ocean Service.

It might be argued that an entire semester devoted to airplane flight demands of students an interest they may not possess. But, as we have indicated above, 'interest', as such, is not really on our agenda. We do insist that the students' work be genuinely edifying in a context. It is essentially a matter of conserving time that, in our view, precludes one from jumping from airplanes to horticulture to bowling and elsewhere, hoping to hit a topic that will seize the imagination of almost every student. Finding even one topic in which mathematics actually contributes to GE students' understanding, as prescribed by our Rule of Three, is not so easy.

5. Conclusion

The course described above has found substantial popularity at William and Mary. A single instructor of record has complete responsibility for an individual section of the course. We are able to manage a class size of 55, meeting three hours per week in lecture-dialogue sessions. Approximately 15% of the typical graduating class will take this course. The FAA program [5], and the wind data base [4] are available on the student accessible server.

The wide ranging use of highly focused external data described in §4 is, for many mathematicians, a daunting matter. It might be suggested that team-teaching is an appropriate way to manage the foreign territory of flight applications. One great benefit is that the mathematics instructor would not be burdened by extraneous information. Moreover, a pilot, say, would surely be in a better position to explain much of the nuance of the physical application, presumably to the edification of the students.

But there are also some good reasons to avoid team teaching. First, it is expensive to pay two people for one job. Second, although there is indeed a real sense in which this is an 'interdisciplinary' course, the honest objective is to get students doing meaningful mathematics. In the hands of a pilot, for example, there is danger of turning this into a flying course; interesting perhaps, but not mathematics.

In this regard, another consideration is important. It should be clear from the description in §4 that there is little time spent on the development of mathematics for its own sake. The absence of any such development is driven, not by a disdain for mathematics, but rather by a desire to have GE students use their limited time to make a genuine connection between mathematical practice and the external world. However, it is one thing to avoid making mathematical developments and quite another to be unaware of them.

We are certain that serious mathematical training of the instructor is a valuable asset in the use of these course materials. In particular, preparation of those materials had to be carried out as a persistent search for mathematical problems whose solutions are elementary. That judgement could only be made on mathematical grounds. In particular, it is easy for a mathematician to omit an investigation that might be irresistible to a practitioner of some particular aspect of flight.

There is a mathematical quid pro quo. Some colleagues have suggested that flight scheduling problems might be an interesting way to get students to do some graph theory. There is, however, no obvious way in which GE students could construct, say, the flight timetable of Delta Airlines. Only if we could find a way for them to do such a real problem could we permit graph theory to enter the conversation without violating our *Rule of Three*.

All of these are issues that will confront any institution that tries to establish some real connections between intellectual disciplines. Real connections, if they are to be real, have to amount to something more than a narrative describing what other people can do in such a regime. The more prominent successful efforts along legitimate interdisciplinary lines are found at the advanced undergraduate or graduate level. We hope we have demonstrated that such things are also possible for general education students. ■

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