USING GRAPHING CALCULATORS TO INTEGRATE MATHEMATICS AND SCIENCE

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The computational, graphing, statistical and programming capabilities of today's graphing calculators make it possible for teachers and students to explore aspects of functions and investigate real-world situations in ways that were previously inaccessible because of computational constraints. Many of the features of graphing calculators can be used to integrate topics from mathematics and science. Here we provide a few illustrations of activities that use the graphing, parametric graphing, regression, and recursion features of graphing calculators to study mathematics in science contexts.

Increasingly, graphing calculators are being used in secondary mathematics teaching. Textbooks, teachers' guides, and even high stakes examinations are being written with expectations that teachers and students in high school mathematics courses (and in some cases middle school courses) are using graphing calculators. Graphing calculator manufacturers are sponsoring workshops and producing supplementary materials to help teachers learn how to use their latest products and incorporate them into mathematics curricula. Furthermore, state departments of education are now revising their curriculum guidelines to include graphing calculators. Indeed, the *Virginia Standards of Learning* (SOL's) for introductory algebra [1] stipulate: "...graphing utilities (graphing calculators or computer graphing simulators) should be used as tools to assist problem solving. Graphing utilities enhance the understanding of functions..."(p. 18). The Virginia SOL's also specify that graphing utilities be used in algebra, trigonometry, and mathematical analysis courses because they "enhance the understanding of realistic applications through mathematical modeling and aid in the investigations of functions and their inverses" (p. 25).

The NCTM *Curriculum and Evaluation Standards* [2], written before the current widespread use of graphing calculators, also advocates the use of graphing utilities in courses in algebra, trigonometry, and functions. Moreover, the Standards for grades 9-12 assume that "Scientific calculators with graphing capabilities will be available to all students at all times." (p. 124).

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The benefits of graphing calculators are numerous. The computational, graphing, statistical and programming capabilities of today's graphing calculators make it possible for teachers and students to explore aspects of functions and investigate real-world situations in ways that were previously not feasible because of computational constraints. Features of graphing calculators also make it possible to study traditional topics in new ways and in more depth and to study new topics that were previously impractical at the secondary level using only paper and pencil methods. Mathematical modeling, simulation, connecting multiple representations, and data analysis are examples of mathematical topics that can be studied more efficiently and effectively using graphing calculators.

Many of the features of graphing calculators can be used to integrate topics from mathematics and science. Here we provide a few illustrations of activities that use the graphing, parametric graphing, regression, and recursion features of the Casio 9850 Plus graphing calculator to study mathematics in science contexts. These activities can be carried out with other graphing calculators as well.

Sample Integrated Calculator Activities

Parametric Graphing

The parametric graphing features of graphing calculators allow students to dynamically simulate the actual paths of projectiles by generating equations for motion in both the x and y directions using *time* as a parameter. These paths can assist students in understanding the components of projectile motion and their associated equations by providing appropriate visual support. Without using parametric equations, students could only graphically represent the trajectory of a projectile with *time* as the x-variable and *height* as the y-variable, and of course graphs of such relationships do not simulate the actual paths of projectiles. Research documents, however, that many students interpret such height-time graphs as true paths of projectiles because they interpret the change in *time* as motion in the x direction [3-5]. This type of misinterpretation of a graph is sometimes referred to as *iconic interpretation* of a graph [6]. Such graphical misconceptions can be avoided, and even analyzed, when students use parametric graphing.

Rocket Simulation. This activity asks students to use parametric equations to simulate the actual path of a rocket launched straight up with an initial velocity of 98 meters/second.

Students are first asked to simulate both the constant upward motion of a rocket unaffected by gravity, and the downward freefall accelerating motion of a rocket due to the force of gravity. The parametric equations that generate dynamic graphs of these two motions are: $x_1=1$, $y_1=98t$, and $x_2=2$, $y_2=-(.5)9.8t^2$. Students are then asked to consider these two components of motion together.

The first screen shot shown in Figure 1 simulates the paths of one rocket fired straight up with a constant velocity of 98 m/s and another rocket freefalling for 10 seconds. The second screen shot shows the paths of the same two rockets after 20 seconds, but also includes the path of a third rocket launched with an upward velocity of 98 m/s *and* under the influence of gravity. The equation generating the path of the third rocket combines those of the first two, namely $x_3=3$, $y_3=98t-(.5)9.8t^2$. The third screenshot zooms in on the graph representing the actual path of the rocket (note: t/320 has been added to the x component to move the downward portion of the graph over 1 pixel from the upward portion).

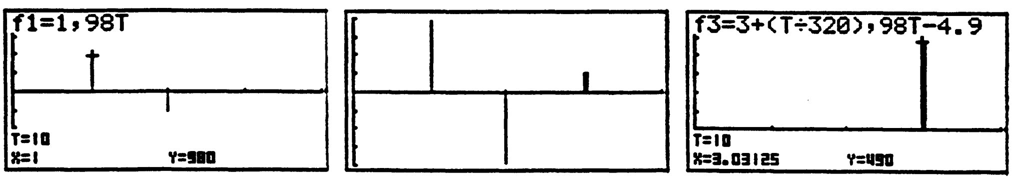

Figure 1: Rocket Simulation Screenshots.

Students are able see that after 10 seconds the distance traveled by the rocket moving upward at a constant velocity is greater than the distance traveled by the freefalling rocket, and can surmise that a rocket moving up and under the influence of gravity will still be above the ground. Also, students will see that at 20 seconds the distances traveled by both rockets are equal, and can predict that a rocket launched up at 98 m/s affected by gravity will hit the ground at 20 seconds. This prediction can then be verified graphically. Observing the three graphs over several time intervals, tracing the simulated paths, and comparing the distances traveled at various times allow students to observe graphically and numerically how constant velocity and acceleration are related.

Activities like this one, besides helping students to better understand velocity and acceleration, also help students connect the different terms of quadratic equations to formulas from Newtonian mechanics describing the different aspects of projectile motion.

Freefall. This activity, adapted from [7], gives students the gravity data shown in Table 1[1] and asks them to simulate the paths of objects freefalling from 500 feet above the surface of each of the planets.

Planet	Period	Distance from Sun	Gravity
Mercury	88	57.900	3.70
Venus	225	108.200	8.87
Earth	365	149.600	9.78
Mars	687	227.900	3.69
Jupiter	4332	778.300	23.12
Saturn	10760	1427.000	8.96
Uranus	30685	2869.328	8.69
Neptune	60189	4496.672	11.00
Pluto	90456	5913.500	0.66

Table 1: Planet Data.

The screen shots in Figure 2 show the parametric equations used to generate three of the graphs and the graphs of the paths of the objects freefalling from 500 feet for 9 seconds (with the names of planets abbreviated above each). This activity allows students to compare the effect of gravity dynamically by seeing the relative motions over time. Tracing various paths and relating the changes in distance over time to the equations helps students get a better feel for the effect of different gravitational constants on freefall motion.

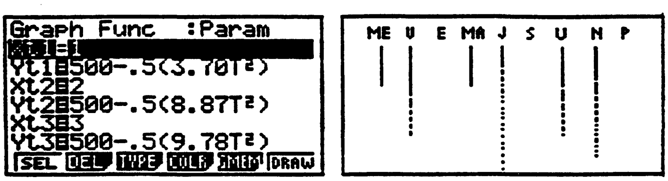

Figure 2: Freefall Screenshots.

[1]This data was found at ***http://nssdc.gsfc.nasa.gov/planetary/planetfact.html***

Projectile Motion. This activity investigates non-vertical projectile motion and incorporates trigonometry (and calculus if desired). Students are asked to simulate the paths of three projectiles launched with an initial velocity of 64 feet/second at 30, 45, and 60 degrees, respectively. The screenshots in Figure 3 show the (truncated) parametric equations used to generate the graphs, the simulated paths, and the simulated paths with the derivative tracing feature activated (for use with calculus students).

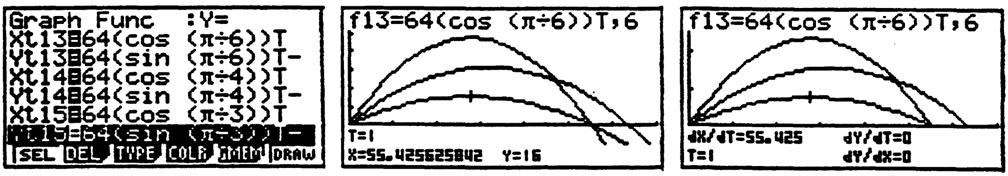

Figure 3: Projectile Motion Screenshots.

Activities such as this, with appropriate questioning and follow-up tasks, help students connect mathematical equations to formulas describing motion, connect coefficients in equations to features of graphs, and understand how derivatives represent slope at a point.

Curve Fitting

Graphing calculators make it possible for students to plot data, visually explore relationships between variables, and determine the equations of best-fitting curves in two ways: by using the graphing features to successively approximate best-fit curves or by using the regression capabilities to calculate least-squares regression equations.

Deriving Kepler's Third Law. This activity, adapted from [8], gives students the data in Table 1 and asks them to plot the approximate average distance from the sun (length of the semi-major axes) versus the period for each planet. From the plots students can easily conjecture that a relationship exists between these variables, and using the least-squares regression capabilities of the calculator, they can calculate the coefficients of the curves of best fit. The screen shots in Figure 4 show the plotted data points, the coefficients of the power function of best fit, and the drawn regression curve.

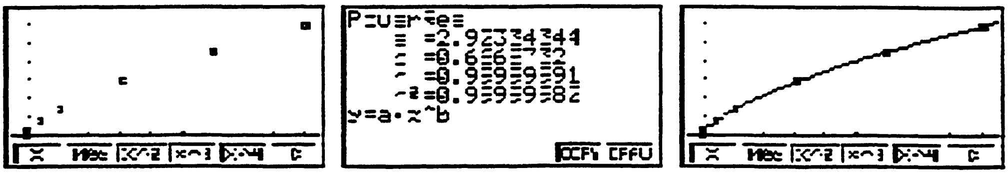

Figure 4: Kepler's Third Law Screenshots.

Students can see that $y = 2.9x^{.66}$, or distance = $2.9(\text{period})^{2/3}$, fits the data almost perfectly, and can relate the square of the period to the cube of the approximate distance from the sun by using basic algebra. Hence, they are deriving Kepler's Third Law. Activities like this one help students better understand that formulas in science are derived from data and that mathematics plays an integral role in such derivations.

Monthly Temperature. This activity gives students the temperature data in Table 2 and asks them to plot the temperature data and examine the variation over a year.[2]

	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr
Wash. DC	54	64	73	77	75	68	57	46	37	34	36	45	54
Verk. Russia	5	32	54	57	48	36	5	-35	-53	-57	-48	-25	5
Buenos Aries	63	55	48	50	52	55	59	66	72	73	73	70	63

Table 2: Average Monthly Temperature Data.

One version of this activity is to ask students to fit a sine curve to the data without using the regression feature of the calculator. (Not all graphing calculators have a sine regression feature). The screen shots in Figure 5 show a plot of the data, a sequence of sine curves approximating the data with one coefficient being adjusted at a time, and the coefficients of a regression equation calculated using the regression feature of the graphing calculator. Here the first sine curve uses an amplitude coefficient derived from examining the maximum and minimum values of the temperature [$y = 21.5 \sin(x)$], the next curve adds a vertical shift coefficient also derived from the maximum and minimum values [$y = 21.5 \sin(x) + 55.5$], the

[2] A fuller version of this activity, by R. Ward, can be found at *http://pen1.pen.k12.va.us:80/Anthology/Pav/MathSci/calc/TEMP*

next adjusts the period ($2\pi/12$) for 12 months [$y = 21.5 \sin(.52x) + 55.5$], and the fourth curve adjusts the horizontal shift [$y= 21.5 \sin(.52x - .59) + 55.5$]. The last coefficient can be approximated either visually, through use of the trace feature, or algebraically using one data point and the previously found coefficients.

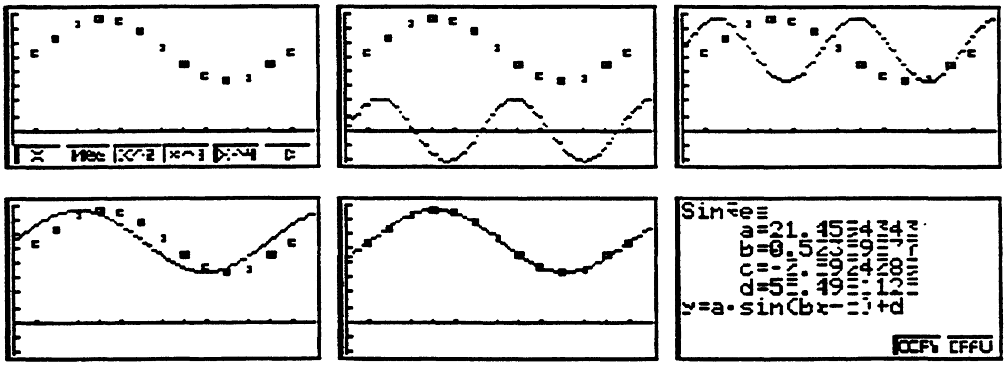

Figure 5: Washington, DC Temperature Screenshots.

Students are then asked to compare the derived equations and discuss their similarities and differences.

Figure 6 shows the plots of the data for all three cities.

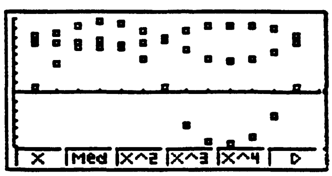

Figure 6: Temperature Plots for Three Cities.

Students can be asked to relate relevant aspects of the geography of the cities to amplitude and phase differences between the graphs and the coefficients of the equations describing them. Activities like this help students develop an understanding of how different coefficients affect the graphs of trigonometric functions.

Recursion

Graphing calculators facilitate the study of many topics from discrete mathematics. Examples of such topics include matrices, combinatorics, and recursion.

Solution Mixture Problem. This activity asks students to first solve the following problem:

> Consider two containers, A and B, containing 100 cc of *solution a* and *solution b,* respectively. Ten cc of *solution a* is taken from container A and placed in container B. The solutions in container B are then mixed up and 10 cc of this blend is placed in container A. Determine if there is then more of *solution a* in container B or more of *solution b* in container A.

Students usually try to solve this problem using a combination of intuition and algebra, and a substantial number of them do not solve it correctly. After this simple case is resolved, we ask students to predict what would happen if this process is continued many times. Students are then asked to calculate the amount of *solution a* in each container after each iteration. Using algebraic equations to answer this question can be complicated and inefficient. It is easier to solve this problem using recursion. At each iteration, the number of cc of *solution a* in container A and container B, respectively, can be represented as:

$$a_{n+1}=(10/11)a_n+(1/11)b_n$$
$$b_{n+1}=(1/11)a_n+(10/11)b_n$$

The graphing calculator screenshots in Figure 7 show these recursive equations (truncated), a table listing the number of cc of *solution a* in containers A and B, and a plot of the amount of *solution a* in each container at each step. Notice the amounts converge to 50 cc from above and below.

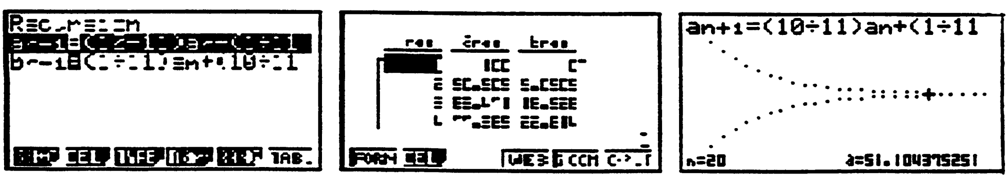

Figure 7: Solution Mixture Problem Screenshots.

Discussion

The graphing calculator activities described above use mathematics to model and analyze problems arising in scientific situations. Such activities provide contexts for school mathematics topics. It is widely acknowledged that such contexts can be helpful to learners, and that traditional mathematics instruction has been woefully inadequate in this regard [2, 9, 10].

Each of the above activities makes use of numerical, algebraic, and graphical representations of mathematical functions. Research has shown that many students have difficulty connecting multiple representations of functions. Activities such as those presented above can facilitate the making of such connections [2, 11-13].

These applications not only help students better understand the mathematics involved, but also help students develop better understanding of aspects of the science involved, namely, scientific concepts, how scientific laws are derived, and how the doing of science is facilitated by mathematics. Teachers and students could further explore science and its connection to mathematics by designing and conducting their own experiments using data collection devices and probes developed for use with graphing calculators. ■

References

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