DEVELOPING MATHEMATICS ENRICHMENT WORKSHOPS FOR MIDDLE SCHOOL STUDENTS: PHILOSOPHY AND SAMPLE WORKSHOPS

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This paper describes our approach to organizing enrichment activities using advanced mathematics topics for diverse audiences of middle school students. We discuss our philosophy and approaches for the structure of these workshops, and then provide sample schedules and resource materials. The workshops cover activities on the following topics:

Graphing Calculators
The Chaos Game
Statistical Sampling
CT Scans--the reconstruction problem
The Platonic and Archimedean solids
The Shape of Space
Symmetry
The Binary Number System and the game of NIM
Graph Theory: Proof by Counterexample

Overview

The University of Minnesota Talented Youth Mathematics Program (UMTYMP) has created programs to capture and maintain the interest of middle school students who enjoy mathematics. The activities discussed in this paper have been developed and implemented for two of our intervention programs: Project YES (Young Emerging Scholars) and Project PRIME (Professions and Recreations: Intermediate Mathematics Enrichment). The overall objectives for both programs are:

1) to provide a comfortable and interesting setting for students to learn and enjoy mathematics and become aware of its applications
2) to provide opportunities for students to do mathematics in a stimulating, small-group environment
3) to allow students to learn mathematics using innovative visualization and graphics
4) to help students to become aware of communication problems in mathematics and improve their communication skills

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5) to enable students to better understand how advanced mathematics and science courses enhance career opportunities in mathematics and related areas
6) to familiarize students with a variety of careers and disciplines that rely upon mathematics.

The enrichment activities may be used as stand-alone activities that do not rely on prior student knowledge, group involvement, or connections to the larger community of mathematicians and scientists. One way, however, to maximize the effectiveness of these enrichment activities is to develop them as a series of events that focus on a central theme and that take place on a regular basis or are part of a larger ongoing program.

The primary goal of Project YES is to provide a rich and diverse program of enrichment activities to support the further study of mathematics and to enhance interest in career opportunities in mathematics and the sciences for 7th and 8th grade students. Project YES especially encourages female students, students of color, and economically disadvantaged students to participate. Since the program began in 1990, 100 to 125 students have participated each academic year. Project YES consists of a comprehensive academic year program of monthly activities and a four-week summer enrichment institute. It is targeted at students identified as motivated to study mathematics by a variety of school and community sources, parents, and other non-traditional resources, as well as those who nearly obtained admission into UMTYMP. Families are invited to attend several of the activities, and parents are involved in helping their children participate in the program. The program helps students learn how to appreciate mathematical ideas gleaned from advanced topics. It does not pursue acceleration in one subject, but instead develops a series of challenging mathematical and enrichment activities covering a broad range of topics. An important aspect of the activities is the student friendships that develop. Students thrive when they are given the opportunity to spend time with students like themselves who are interested in math and like to be challenged. Learning and teaching emphasize hands-on activities. Students build models and actively use graphing calculators and computer software. In addition to mathematics workshops, the program includes presentations by professionals who use math and science in their careers.

Based on the success of Project YES, Project PRIME, which focuses on a younger population with an interest in mathematics enrichment, was started in 1996. Project PRIME was initially developed to reach out to females in the fifth and sixth grades from a diverse group of inner-city schools. An enrollment of fifty students was anticipated. Based on an overwhelming response from students, parents, and teachers, however, the program was opened to a larger audience. Two hundred young females registered for the 1996-97 academic year program; the project was opened to male students in 1997. A week-long institute was developed for the summer of 1997.

The 1996-97 PRIME program included a variety of events throughout the academic year: fall orientation, two hands-on mathematical workshops focusing on geometry through the use of visualization software applications, the annual math fair with exhibits and activities from a wide range of institutions, and a day of science and engineering lab visits and career speakers. Students also received subscriptions to the bi-monthly PRIME Times newsletter, which offered them the chance to write in and share their solutions to a variety of math puzzles.

In the fall of 1997, 355 male and female students enrolled in both Project PRIME (225 5^{th} and 6^{th} graders) and Project YES (130 7^{th} and 8^{th} graders). Based on the current waiting lists for these programs, it is anticipated that 50 to 60 more students will be invited to participate in the spring events. The students will attend three-hour workshops scheduled throughout the year. Project PRIME includes five activities, featuring hands-on workshops centered on the mathematics of knots. Project YES has eight activities, including interactive workshops and computer labs on topics ranging from graphing calculator functions to combinatorics *(http://www.math.umn.edu/itcep/)*. Both of these programs strive to give students a sense of participation in the culture of mathematics and a sense of how their interest in this culture can help them throughout their lives.

All of the workshops provide excellent teacher preparation opportunities for pre-service students. Each workshop is developed and taught by post-doctoral fellows. Secondary school teachers and mathematics and mathematics education graduate students who are enrolled in teacher certification programs also teach the workshops. Each instructor is assisted by an undergraduate student with an interest in mathematics education. The student/instructional team ratio is 25:2.

While many of the students who participated in Project YES and Project PRIME were identified as potentially talented in mathematics, the evaluations showed all of the participants maintained a high level of interest and involvement in the programs. For example, the survey results of the October 1997 Project PRIME workshop indicated that 94% thought it was fun or great and for the October 1997 Project YES workshop, 97% thought it was fun or great. Thus, we feel that these workshop activities are useful and stimulating for a diverse audience of middle school students. All that is required might be some modification in the length and possibly the depth of some of our activities. We strongly encourage any teacher to use these activities with students who are interested in new approaches to mathematics, and to use them to help students better understand the variety of mathematical applications.

Developing Workshops

Our workshops for middle school students always include multiple activities built around one main topic. As the workshop progresses, its segments build on knowledge or discoveries from earlier segments. As experience within a topic grows, more student questions develop and are addressed.

By the end of the session, we try to achieve some level of mathematical closure. This does not require that every mathematical question be resolved by the time that the students leave. In fact, we often give students open problems or additional materials to take home. At the end of each activity, students should feel that some interesting conclusions were reached as a result of their investigations, and some will wish to explore the topic further. (They should find enough gold to convince them that there is more gold to be found.)

While part of each workshop (typically the introductory material) is led in a lecture/discussion style, a significant portion of the workshop consists of hands-on activities that students accomplish in pairs or groups of 3-4 students. By pushing students to probe for answers in their own minds and from their own experiences, the hands-on activities strongly reinforce the learning process. Within a cooperative group, students share materials, make conjectures, and draw conclusions together through discussion.

In addition to being enjoyable and helping to build social relationships between students, we want the students to appreciate some of the educational benefits of cooperative learning. To this end, activities promoting a pooling of intellectual resources are created, allowing students to progress through new material more quickly than they might on their own. Opportunities for processing their thinking aloud through discussion seems to make the material more tractable for students. Verbal discussion forces students to make their reasoning precise enough that others can understand it. The small group discussions encourage active participation from students who would not feel as comfortable sharing their thoughts with the entire classroom of students.

Generally, we begin each workshop with all participants in a common session before breaking into separate classrooms and then further into groups. In addition to introducing the main topic, we often initiate a brief physical or verbal exercise (e.g., a resounding cheer for prime numbers) to get everyone in motion. There is a palpable energy and excitement when a large number of students gets together to discover the motivation for the mathematical journey upon which they are about to embark. This format also means that only one teacher needs to prepare remarks for the opening session! At the end of the workshop, we reconvene as a large group for a wrap-up discussion. During the closing time we have participants answer a short (4 to 5 question) evaluation of the event. The feedback from these evaluations is used when planning future events.

The components that we weave together to form a workshop are often activities that were designed by other teachers and curriculum developers in a format that is nearly ready for our purposes. If we are lucky, these pieces may already have been tested with prior groups of students, and are both realistic and effective. Some student activity sheets are used in their original form. Otherwise, we might cut and paste prepared activity sheets to make them fit our workshop vision. Sometimes we find articles from teachers that sketch an activity that can be done, and we fill in our own details. There is simply no need to start to build a program from sand and water when it can be built from bricks and sound scaffolding! This consideration is especially relevant during the school year when time is scarce.

We have also found that readily available software and manipulatives often enhance our examination of a topic. Many of the software applications we have used were obtained for free via the internet, most often from the NSF Geometry Center *(http://www.geom.umn.edu)* and from Rick Parris' Peanut Software *(http://www.exeter.edu/~rparris)*. Other excellent web sources are the Mathematics Forum, Swarthmore College *(http://www.forum.swarthmore.edu)*, and MEGAMATH *(http://www.c3.lanl.gov/mega-math/welcome.html)*. We use Polydron manipulatives (800-452-9978) for a variety of geometry topics. A one-time purchase of a classroom set of Polydrons has been a worthwhile investment. We frequently utilize lending programs of graphing calculator companies, which may include delivery of measurement devices for collecting "real world" data and activity booklets, if needed.

In our opinion, at some point in a teacher's development, the experience of combining basic mathematical concepts with a goal of creating some enrichment curricula is important. This is most realistically done during a summer period when teachers can focus on curriculum development as a project. At the NSF Geometry Center's 1994, 1995, and 1996 summer workshops for secondary teachers, for example, teachers from several states nationwide were involved in the development of curricula surrounding the "Shape of Space" video and the "KaleidoTile" software. The deeper understanding that the teachers gained while developing the curricula enabled them to use the materials more effectively in their classrooms. Many of them have gone on to develop their own enrichment programs.

In the remainder of this paper we describe in detail some of the workshops that we have developed for Projects YES and PRIME. For each example, we provide an event schedule and a description of activities and/or resource list. In cases where we created original material, copies of the activity sheets are listed on our web site, making these active documents available to teachers planning similar activities *(http://www.math. umn.edu/itcep/)*.

Graphing Calculator Workshop

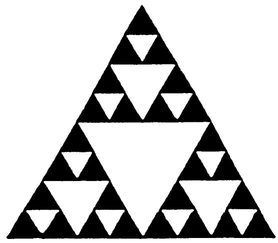
One of our most successful activities has been the Graphing Calculator Workshop, which was developed for a four-hour format. The data that is used for graphing is collected from sounds, motions, and temperatures. These investigations can be explored using the human voice or musical instruments, bodily movements or any source of motion such as bouncing balls, and any substance that responds to changes in temperature such as water. This enables the teacher to vary the lesson each time it is presented by using a variety of sources for the experiments.

Graphing Calculator Curriculum	
1.	Interactive demonstrations to small groups (15 to 10 students) on the basic use and functions of the graphing calculator, led by an instructor and teaching assistant.
2.	Rotation of teams through three stations using the graphing calculators with Calculator-Based Laboratory systems (CBLs) to graph data from experiments that explore motion, sound, and temperature. (CBLs are portable hand-held data collection devices with sensor probes for collecting 'real world' data that can be retrieved by graphing calculators for analysis.) A refreshment break is provided between two of the rotations.
3.	Reconvening of the groups to discuss their experiences and the implications of their findings.

The introductory demonstrations include instructions on using the calculator to perform basic arithmetic calculations, using a coordinate system, definition of a function, using the calculator to construct a function table and graph, and tracing and zooming on a graph. The ideas for the CBL activities are from Texas Instruments' CBL System Experiment Workbook [1]. More information can be obtained from 1 (800) TICARES. Student comments are very supportive of the positive effects of this workshop—the relevance of using graphs when solving problems in the classroom and the real world.

The Chaos Game

We spent several days during the summer YES program using The Chaos Game to explore dynamical systems. The Chaos Game, based on an idea of Kevin Lee, College of St. Catherine, St. Paul, MN, is an interactive game that develops a fractal on its game board. It can be played with pencil, paper, and a die. The game starts with only the vertices of an equilateral triangle each assigned 2 numbers between 1 and 6, and a random point inside the triangle. On successive moves the die rolls determine where to place the new dot by locating the new point halfway between the previous point and the triangle vertex chosen by that die roll. The compellingly beautiful results of applying the game's algorithm can best be viewed on a computer. The process forms the Sierpinski Triangle fractal as its end result or basis of attraction:



This topic and its mathematical justification can be spread over several days, with other fractal topics interspersed, as listed below:

The Chaos Game Curriculum	
Day 1	
1.	Explanation of the game and use of half-rulers.
2.	Students play the game on transparencies, with a partner, and then the class witnesses and analyzes the results.
3.	Collaboration to create a giant paper Sierpinski triangle for the classroom wall; Computation of the number of triangles at each stage.
4.	Collaboration to create a giant Sierpinski tetrahedron.
5.	Discussion of self-similarity.
6.	Worksheets for thinking about why The Chaos Game works.
7.	Coloring Pascal's triangle to make the Sierpinski Triangle.
8.	Introduction of contractions and rotations into The Chaos Game, and the use of fractals in storing and generating pictures.
Day 2	
1.	Play Fractalina (computerized version of The Chaos Game) on the computer.
2.	Play The Chaos Game (target version) as a class, on the giant Sierpinski Triangle.
Day 3	
1.	Play The Chaos Game (target version) in pairs on the computer.

Sources for material are the NCTM's Fractals for the Classroom publications [2] and Bob Devaney's home page at Boston University *(http://math.bu.edu/DYSYS/dysys.html)*, which has Java computer applet versions of the games.

Statistical Sampling

Using materials from the estimation section of the Challenge of the Unknown video and activity book (Norton & Company), this workshop introduces a variety of strategies for estimation. The meatiest of these is statistical sampling, which is introduced via a shark population example. Students emulate the approach indicated in the video using a pail of goldfish crackers. We structured this curriculum for a two-hour workshop.

Statistical Sampling Curriculum	
in auditorium	
1.	Introduction using envelope/winner experiment where the group tries to estimate how many people in the room received "winner" cards in their envelopes; discussion of uses of estimation in students' lives and in the larger world.
in individual classrooms	
2.	Numerical calculation warm-up (from book).
3.	Grocery bill totaling example.
4.	Discussion of how to estimate the number of M&Ms in a jar, and submission of estimates.
5.	Using a grid to estimate the crowd size at the Million Man March.
	(snack break--estimate the cost of the food for the event)
6.	Tag and Recapture statistical sampling worksheets and goldfish activity (from book).
7.	Gathering data for eye color sampling experiment; use the classroom proportions to estimate the proportions among all participants.
in auditorium	
8.	Collect totals for eye color experiment and compare classroom predictions to the actual value.

CT Scans

This topic was spread over several days in our summer YES program, and concluded with a visit to the radiology department of a hospital to see the CT scanners in use. It combines materials from the New Tools for New Technologies video and activity book set [3] published by COMAP (800 77C-OMAP), a May 1996 article in the Mathematics Teacher on "Medical Applications of Systems of Linear Equations"[4], and a Newton's Apple episode on 3D MRI brain scans.

CT Scans Curriculum	
1.	New Tools... video introduces the problem of reconstruction of information.
2.	Graph theory problem: Students try to reconstruct a graph from subgraphs (from New Tools... activity book).
3.	Discussion: interpretation of solutions to a set of linear equations; using linear equations to model the CT scan reconstruction problem (from article).
4.	Students work on activity sheets from article.
5.	Visit local hospital for lecture by a physicist and tour of the radiology department.
6.	View video on 3D MRI brain scans.

Platonic and Archimedean Solids

This is one of our longest-running workshops, based on The Platonic Solids video and The Platonic Solids Activity Book [5] from Key Curriculum Press (800 995 MATH). The KaleidoTile software may be downloaded from the NSF Geometry Center's web page at *http://www.geom.umn.edu/software/download/*. Materials for use with the software were prepared by teachers at the Center, Summer 1995.

Platonic and Archimedean Solids Curriculum	
in auditorium	
1.	As students arrive, they begin to cut out two-dimensional nets that they will use later to build a cube and an octahedron that fit together, demonstrating duality.
2.	Using activities P1 and P3 in the activity book, students deduce a method for computing interior angle measure for regular polygons.
in individual classrooms	
3.	Using Polydron manipulative pieces or heavy paper, students work in groups to build regular polyhedra, i.e., polyhedra that satisfy the properties that (a) all faces are regular polygons, (b) all faces are the same, and (c) each vertex has the same number of faces coming together; each group might be assigned a particular number and type of face for its vertices, e.g., restricting its solid to have exactly 3 pentagons at each vertex.
4.	Based on the angle sizes determined in the introductory session, the class discusses why there are exactly five Platonic solids.
5.	Discover duality by filling in a chart of the number of faces and vertices for the Platonic solids.
6.	Optional, if time allows: Build other symmetric polyhedra, as described in activity 3 of the activity book.
7.	Watch The Platonic Solids video.
8.	With KaleidoTile software, examine relationships and transformations between some of the Platonic and the Archimedean solids. Sample activities are described in the KaleidoTile materials.

Shape of Space

In this workshop, we begin to construct a view of four-dimensionality and *of spaces* that are not Euclidean. Of course, we are restricted to looking at these from a three-dimensional perspective, so we do this by analogy with the way that a three-dimensional world would appear to a two-dimensional creature. We use soap bubbles on plastic straw and paper clip models of tetrahedra and cubes to try to visualize four-dimensional extensions of these objects. Other activities are based around Jeff Weeks' video, "The Shape of Space." Review copies of this video are available on loan by contacting the NSF Geometry Center at the University of Minnesota, (612-626-0888) or admin@geom.umn.edu. The activities, such as playing the game of Dots on a torus, are collected in a set of materials written by teachers in Summer 1995 at the Center. The Shape of Space software created by Jeff Weeks is available on the internet via *http://www.geom.umn.edu/docs/forum/weeks_software*.

Symmetry

One special workshop that combines a significant amount of existing materials with a substantial component of our own ideas is our four-hour workshop on symmetry. The new ideas, offering additional approaches to the deep content of the existing materials, have developed naturally from using the existing materials and from doing additional reading.

Symmetry Curriculum	
1.	Introductory lecture/slide show on the basic types of mathematical symmetry and the 7 possible band patterns.
2.	Using Kali software (freeware) to see how students' initials look in the 7 band patterns.
3.	Making a chart of symmetries for regular polygons.
4.	Constructing paper snowflakes that exhibit 4-, 5-, and 6-fold rotational symmetry: What folding schemes are required?
Snack Break	
5.	Choreographing movements to show transformations between symmetry types.
6.	Reflecting and translating in xy-coordinate systems.
7.	Cutting pictures out of magazines that exhibit symmetry and identifying the types of symmetry.
8.	Discussing and summarizing *Why is this mathematics?*
9.	Viewing video[1] about reflection in 2 and 3 dimensions.

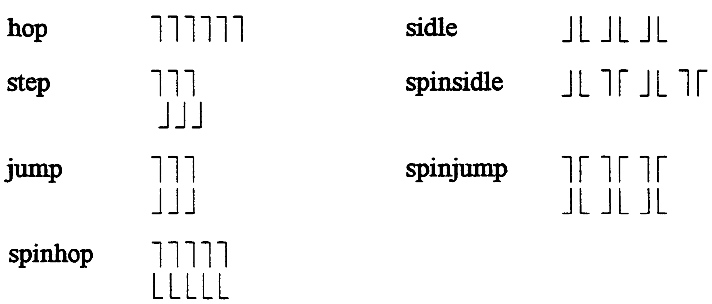
[1] The video shown is one produced by Anna Gardberg at the NSF Geometry Center, called "Mad About Mirrors." Unfortunately, it is not currently available to the public.

These activities were assembled for Project PRIME, and thus include some movement exercises that may be more popular with certain age groups than others.

Students who attended the symmetry workshop brought home their snowflakes and all of the worksheets that were distributed. Most of the motivation for these activities came from two sources: (1) a book called *Symmetry: A Unifying Concept* (Hargittai and Hargittai, Shelter Publications) [6], and (2) curricular materials developed by the NSF Geometry Center to go with its Kali software *(http://www.geom.umn.edu/locate/lori/kali/)*. The software is downloadable for free from the Center's web site *(http://www.geom.umn.edu/software/download/)*.

The movement choreography addressed two main themes. The first was experimentation with reflection and rotation symmetry, in pairs and circles of students. Following oral instructions, students formed reflective poses that assumed symmetry planes in locations specified by the leader. They made designs with k-fold rotational symmetry and stepped their designs around from equivalent position to equivalent position, until returning (after k changes) to the original position.

The second theme was that of the seven symmetry band patterns possible for a horizontal pattern of a repeated figure accomplished only by a combination of translation, relection, glide reflection, and half-turn transformations. These are translation ("hop"), translation and horizontal reflection ("step"), horizontal reflection and translation ("jump"), translation and two-fold rotation ("spinhop"), translation and vertical reflection ("sidle"), two-fold reflection and vertical reflection ("spinsidle"), and vertical and horizontal reflection ("spinjump"). See the figure below.



Each student in a group of five made a colored arrow on an index card. The group then chose a starting and ending symmetry band pattern for its arrows and planned a sequence of movements to change from one to the other using the transformations that were discussed. The groups performed these for one another, holding their arrow cards up for all to see.

The Binary Number System and the game of NIM

This material was spread over several days of our YES summer program, concluding with a presentation by a University of Minnesota professor about strategies for the game of NIM.

The Binary Number Curriculum	
in individual classrooms	
1.	To introduce the binary representation for the numbers 0-16, Doug Shaw conceived the idea of using student volunteers to form a human demonstration of a ripple counter. (Details of this activity are provided at the end of this section.)
2.	For each number 1-13, student groups receive a strip of cardstock; the students use the strip to make four equal-sized cards for that number, two labeled with the decimal representation, and two with the binary representation; decimal representations should be written in one color, binary ones in another, and the numbers should be underlined to indicate the bottom.
3.	Students play the familiar card game of "concentration," in which they try to locate pairs of cards bearing the same number; a number will be considered the same whether its decimal representation or its binary representation is shown.
4.	(Optional: binary arithmetic activities)
5.	Introduction to the game of NIM via playing the computerized version on the University of Toronto Mathematics Network's web page at *http://www.math.toronto.edu/mathnet/games*
in auditorium	
6.	Presentation of strategies for winning NIM, using binary number representations.

Human Binary Number Ripple Counter: A binary counting exercise developed by Assistant Professor Doug Shaw, School of Mathematics, University of Minnesota.

Human Binary Number Ripple Counter Exercise	
1.	Choose four student volunteers who are good at following directions, and invite them to the front of the class. They should sit in a row of chairs, facing the other students.
2.	There are two possible states for each student: sitting or standing. Define a "change of state" to be the action of moving from sitting to standing or from standing to sitting, depending on a person's starting state. (For example, if she/he is sitting, a change of state will be accomplished by standing up.) All students should start out in the sitting state.
3.	Students will need to change states in a prescribed way when the class claps hands: • The student on the right (from the class' point of view) will change state on every clap. • The one next to her/him will change state whenever the first one <u>sits down.</u> • The third will change state whenever the second <u>sits down</u>. • The fourth will change state whenever the third <u>sits down</u>. • Who will be changing most often? Who least often?
4.	Have the class clap together at a very slow rate (especially at first!), and have the first two people practice changing state. Start over and try it with the third. Start over, including the fourth, and continue until all four of them are seated again. This will take 16 claps. Try it again, a little faster, to make sure they have it. On the next time, return to a slow speed, but instead of clapping have the class count aloud. At "one" the first stands. At "two" the first sits and the second stands. At "three" the first stands and the second remains standing. At "four" the first sits, the second sits, the third stands. (If the students catch on quickly to the movements, another four can switch in for this counting section. Middle school students may find this difficult to coordinate, though.)

5.	Suggest that the name of the starting state, when they were all seated, will be 0. • Ask when the first person stood for the first time (1). • Ask when the second stood for the first time (2). • Ask when the third stood for the first time (4). • Ask when the fourth stood for the first time (8). • Ask when they all sat. (It was on the count of 16, but this was really the 0 state again.)
6.	After the discussion, have each one hold a card with a number on it. (The numbers, left to right, will be 8, 4, 2, and 1.) Now start counting with state changes again, but on "six" say "STOP". Note that the 4 and the 2 are standing, and 4+2=6. Let them start again, and stop them on 13. Note that 8+4+1=13, and 8, 4, and 1 are standing. • What is the biggest number that can be represented this way? (8+4+2+1=15). • What would be a natural way to represent 16? (An extra person on the left side!)

Graph Theory: Proof by Counterexample

This workshop was inspired by problems on the University of Victoria's Mathmania web page (*http://www.csc.uvic.ca/~mmania*). The progression of activities allows students to learn some vocabulary of graph theory representations, while exploring the notions of conjectures and counterexamples. With the workshop as preparation, they can then move to working independently to create new counterexamples that can expand the understanding of the as yet unsolved degree/diameter problem described on the web site.

The logic to be digested in these problems is in the statement of the conjecture. Once students understand what the conjecture says, the method employed here is simply to find an example of a graph that contradicts the proposed conjecture, thereby disproving it directly.

Graph Theory Curriculum	
1.	Introduction to graph theory vocabulary; after introducing edges, vertices, and the definition of a *k*-regular graph, this sheet poses the first opportunity for students to try to disprove a conjecture by finding a counterexample.
2.	Introduction to cliques and independent sets for Ramsey Theory; this provides two more opportunities for students to devise a counterexample to a conjecture.
3.	Disproving a conjecture in matching theory.
4.	Definition of degree and diameter, and statement of the degree-diameter problem; in class or on their own, students can work on finding a planar graph with diameter 3 and degree 3 larger than those previously discovered.

Visit our web site (*http://www.math.umn.edu/itcep/*) for additional examples and activities from our programs.

Evaluation and Assessment Process

In order to assess the impact of the Project PRIME and Project YES enrichment activities, to analyze if we are meeting our primary objectives, and to gain useful data for refinements and future directions, student and parent surveys are used regularly to collect both qualitative and quantitative data.

Student attitudes about the concepts introduced at each workshop and their interest in the topic/s are measured by two to three open-ended questions on the survey. The questions are written by the curriculum developers and cite specific items from the lessons. For example, the Statistical Sampling Workshop survey asks the following two questions: "What part of the estimation problem did you find most interesting?" and "What did you learn from the Tag and Recapture Methods activity?"

On each survey, students are asked to rate their overall reaction to the workshop on a scale of 1 to 5: 1 no enjoyment, 2 little enjoyment, 3 neutral, 4 fun, and 5 great event. Demographic questions asking the students to list their gender, grade level, and reason for attending the event, (i.e., I wanted to attend, my parents wanted me to attend, and other) are also included on each survey.

Parent surveys are given at the end of each family activity and at the final event of the academic year. The questions probe whether or not the parents' expectations of the activities were met, to what extent their child shared his/her experiences with the family following the workshops, and what they felt their son/daughter gained from the experience.

The identical survey questions (except those that relate to a specific lesson) have been used for the past three years in order to enable us to do a longitudinal study. Those that are related to a specific lesson use the same format and are placed in similar context. After the results are collected and entered into our database, statistical reports are generated and descriptive analyses are written. These reports help us to assess the impact of the workshops and programs and to plan for the future.

Conclusion

The University of Minnesota Talented Youth Mathematics Program, which offers a nationally recognized rich and challenging curriculum, continues to serve 175 to 200 of Minnesota's most mathematically talented middle school students each academic year. However, a wider range of middle school students and their parents, teachers, and school coordinators has consistently expressed a strong interest in similar mathematics enrichment opportunities. To serve this audience, and to capture and maintain the interest of middle school students who enjoy mathematics at a variety of levels, we have developed and implemented several successful intervention programs. Over 350 middle school students participated in either Project YES or Project PRIME activities during the summer of 1996 and the 1996-97 academic year. Over 400 of these students, their family members, and teachers participated in our 1997 annual math fair, a one-day event of mathematical enrichment activities.

The activities that we developed for these programs sought 1) to broaden middle school students' understanding of mathematics by introducing topics that go beyond the standard curriculum, 2) to encourage meaningful insights, and 3) to provide opportunities for the students to become a part of the mathematics community. Several student evaluations have confirmed that our goals are being reached. For example, in response to the survey statement: *List the most interesting thing you learned today*, student comments ranged from "I learned the difference between Archimedean and Platonic solids" to "I learned how estimation is a very useful tool and works much faster than counting everything" to "I learned about how mathematics is not just numbers, and a lot about its daily uses."

In summary, we encourage all teachers to consider using our curricula as a model for developing engaging activities to build on middle school students' natural interest in the exploration of mathematics and its applications. The results can be very rewarding. To quote a parent's response to a Project YES survey, "Activities like these offer the opportunity to brainstorm, to experiment, and to create. The program was a fertile environment where ideas were exchanged and younger students got to experience a *creative* environment without stress or pressure. It was an ideal opportunity for youngsters to follow their mathematical ideas and see that doing so was worthwhile." ■

References

[1] *CBL™ System Experiment Workbook*, Texas Instruments, 1994.

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