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Metastable state in a shape-anisotropic single-domain nanomagnet subjected to spin-transfer-torque

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We predict the existence of a metastable magnetization state in a single-domain nanomagnet with uniaxial shape anisotropy. It emerges when a spin-polarized current, which delivers a spin-transfer-torque possessing a field-like component, is injected into the nanomagnet. At a metastable state, the internal torque due to nanomagnet's shape anisotropy cancels the externally applied spin-transfer-torque and hence the *net* torque acting on the magnetization becomes zero. Therefore, it prevents spin-transfer-torque from switching the magnetization from one stable state along the easy axis to the other, even in the presence of room-temperature thermal fluctuations. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4761250]

Spin-transfer-torque (STT) is an electric current-induced magnetization switching mechanism that is widely used to switch the magnetization of a nanomagnet with uniaxial shape anisotropy from one stable state to the other.[1,2] A spin-polarized current is injected into the magnet to deliver a torque on the magnetization vector and makes it switch. This has now become the staple of nonvolatile magnetic random access memory (STT-RAM) technology.[3] Recent experimental measurements of STT[4,5] following its theoretical prediction[6] in magnetic tunnel junctions (MTJs), which is of primary interest for technological applications, showed a significant amount of out-of-plane (or field-like) torque in addition to the traditional in-plane torque.[1,2]

In this letter, we show analytically that the spin polarized current can spawn a metastable state in the presence of field-like torque, which can trap the magnetization vector and impede it from switching. This can be prevented if the spin-polarized current is higher than a critical value or the ratio of field-like torque and in-plane torque is lowered to a small value. Thus, careful design methodologies must be incorporated for feasible implementation of devices employing spin-transfer-torque mechanism in magnetic tunnel junctions.

Consider a single-domain nanomagnet shaped like an elliptical cylinder with elliptical cross section in the *y*-*z* plane (see Fig. 1). The major (easy) and the minor (in-plane hard) axes of the ellipse are aligned along the *z*-direction and *y*-direction, respectively. Let $\theta(t)$ be the polar angle and $\phi(t)$ the azimuthal angle of the magnetization vector in spherical coordinate system. At any instant of time *t*, the energy of the unperturbed nanomagnet is the uniaxial shape anisotropy energy which can be expressed as[7]

$$E(\theta(t),\phi(t)) = B(\phi(t))\sin^2\theta(t) + constant\ term, \qquad (1)$$

where

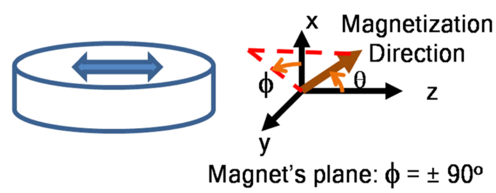

FIG. 1. A nanomagnet shaped like an elliptical cylinder. The stable states are along the $\pm z$ axes. The magnetization direction can be rotated with a spin polarized current.

$$B(\phi(t)) = \frac{\mu_0}{2} M_s^2 \Omega [N_{d-xx}\cos^2\phi(t) + N_{d-yy}\sin^2\phi(t) - N_{d-zz}]. \qquad (2)$$

Here, M_s is the saturation magnetization, N_{d-mm} is the demagnetization factor in the *m* direction (m = *x*, *y*, *z*),[8] and Ω is the nanomagnet's volume.

The magnetization $\mathbf{M}(t)$ of the single-domain nanomagnet has a constant magnitude but a variable orientation, so that we can represent it by the vector of unit norm $\mathbf{n_m}(t) = \mathbf{M}(t)/|\mathbf{M}| = \hat{\mathbf{e}}_\mathbf{r}$, where $\hat{\mathbf{e}}_\mathbf{r}$ is the unit vector in the radial direction. The other two unit vectors are denoted by $\hat{\mathbf{e}}_\theta$ and $\hat{\mathbf{e}}_\phi$ for θ and ϕ rotations, respectively.

The torque acting on the magnetization within unit volume due to shape anisotropy is

$$\begin{aligned}\mathbf{T_E}(t) &= -\mathbf{n_m}(t) \times \nabla E(\theta(t),\phi(t)) \\ &= -\{2B(\phi(t))\sin\theta(t)\cos\theta(t)\}\hat{\mathbf{e}}_\phi \\ &\quad - \{B_{0e}(\phi(t))\sin\theta(t)\}\hat{\mathbf{e}}_\theta,\end{aligned} \qquad (3)$$

where

$$B_{0e}(\phi(t)) = \frac{\mu_0}{2} M_s^2 \Omega (N_{d-xx} - N_{d-yy})\sin(2\phi(t)). \qquad (4)$$

Passage of a constant spin-polarized current *I* perpendicular to the plane of the nanomagnet generates a spin-transfer-torque that is given by

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$$\mathbf{T_{STT}}(t) = s\,[c_s(V)\sin\theta(t)\,\hat{\mathbf{e}}_\theta - b_s(V)\sin\theta(t)\,\hat{\mathbf{e}}_\phi], \quad (5)$$

where $s = (\hbar/2e)\eta I$ is the spin angular momentum deposition per unit time and η is the degree of spin-polarization in the current I. The coefficients $b_s(V)$ and $c_s(V)$ are voltage-dependent dimensionless terms that arise when the nanomagnet is coupled with an insulating layer as in an MTJ.[4–6,9] The strength of field-like torque, i.e., $b_s(V)$ is significant in MTJs, whereas it is small in spin-valve devices with metallic spacer.[4,5,9] We will use constant values of $b_s(V)$ and $c_s(V)$ for simplicity.[10] Furthermore, we will assume $b_s(V) = 0.3\,|c_s(V)|$ and $|c_s(V)| = 1$ to be in agreement with the theoretical prediction[6] and subsequent experimental results.[4,5] For $\theta = 180°$ to $0°$ switching, $c_s(V) = +1$, and for $\theta = 0°$ to $180°$ switching, $c_s(V) = -1$, while $b_s(V) = +0.3$ for both cases.

We can immediately conjecture from Eqs. (3) and (5) that the existence of a field-like torque can render a state, at which these two torques [$\mathbf{T_E}$(t) and $\mathbf{T_{STT}}$(t)] balance each other, which we would discuss next. The magnetization dynamics of the single-domain nanomagnet under the action of various torques is described by the Landau-Lifshitz-Gilbert (LLG) equation as

$$\frac{d\mathbf{n_m}(t)}{dt} - \alpha\left(\mathbf{n_m}(t) \times \frac{d\mathbf{n_m}(t)}{dt}\right) = -\frac{|\gamma|}{M_V}\,\mathbf{T_{eff}}(t), \quad (6)$$

where $\mathbf{T_{eff}}(t) = \mathbf{T_E}(t) + \mathbf{T_{STT}}(t)$, α is the dimensionless phenomenological Gilbert damping parameter, $\gamma = 2\mu_B\mu_0/\hbar$ is the gyromagnetic ratio for electrons, and $M_V = \mu_0 M_s \Omega$. Solving the aforesaid equation, we get the following coupled equations for the dynamics of $\theta(t)$ and $\phi(t)$:[7]

$$(1+\alpha^2)\frac{d\theta(t)}{dt} = \frac{|\gamma|}{M_V}[\{-s(c_s(V) + \alpha\, b_s(V)) + B_{0e}(\phi(t))\}\sin\theta(t) - 2\alpha B(\phi(t))\sin\theta(t)\cos\theta(t)], \quad (7)$$

$$(1+\alpha^2)\frac{d\phi(t)}{dt} = \frac{|\gamma|}{M_V}[\{s(b_s(V) - \alpha\, c_s(V)) + \alpha B_{0e}(\phi(t))\} + 2B(\phi(t))\cos\theta(t)]\ (\sin\theta \neq 0). \quad (8)$$

Note that when the magnetization vector is aligned along the easy axis (i.e., $\theta = 0°, 180°$), the torque due to shape anisotropy, $\mathbf{T_E}(t)$ and the torque due to spin-transfer-torque, $\mathbf{T_{STT}}(t)$ both vanish [see Eqs. (3) and (5)], which makes $d\theta(t)/dt$ as well as $d\phi(t)/dt$ equal to zero. Hence, the two mutually anti-parallel orientations along the easy axis become "stable;" however, thermal fluctuations can dislodge the magnetization from a "stable" state and enable switching.[7] We will now show that there can be a third set of values (θ_3, ϕ_3) for $\theta(t)$ and $\phi(t)$ for which both $d\theta(t)/dt$ and $d\phi(t)/dt$ will vanish.

We determine the values of (θ_3, ϕ_3) as follows. From Eqs. (7) and (8), by making both $d\theta(t)/dt$ and $d\phi(t)/dt$ equal to zero, we get

$$2\alpha B(\phi_3)\cos\theta_3 = B_{0e}(\phi_3) - s\,c_s(V) - \alpha s\,b_s(V), \quad (9)$$

$$2B(\phi_3)\cos\theta_3 = -\alpha B_{0e}(\phi_3) + \alpha s\,c_s(V) - s\,b_s(V). \quad (10)$$

From the above two equations, we get $B_{0e}(\phi_3) = sc_s(V)$. If we put $B_{0e}(\phi_3) = sc_s(V)$ in Eq. (9) or in Eq. (10), we get $2B(\phi_3)\cos\theta_3 = -sb_s(V)$. Accordingly, we can determine the values of (θ_3, ϕ_3) as

$$\phi_3 = \frac{1}{2}\sin^{-1}\left(\frac{(\hbar/2e)\eta I c_s(V)}{(\mu_0/2)\,M_s^2\Omega(N_{d-xx} - N_{d-yy})}\right), \quad (11)$$

$$\theta_3 = \cos^{-1}\left(-\frac{(\hbar/2e)\eta I b_s(V)[\mu_0 M_s^2\Omega]^{-1}}{N_{d-xx}\cos^2\phi_3 + N_{d-yy}\sin^2\phi_3 - N_{d-zz}}\right). \quad (12)$$

Note that ϕ_3 depends on $c_s(V)$ while θ_3 depends on both $c_s(V)$ and $b_s(V)$. Neither depends on the Gilbert damping parameter α.

In order to understand the physical origin of the state (θ_3, ϕ_3), consider the fact that the total torque $\mathbf{T_{eff}}(t)$ can be deduced from Eqs. (3) and (5) as

$$\mathbf{T_{eff}}(t) = \{-2B(\phi(t))\cos\theta(t) - sb_s(V)\}\sin\theta(t)\,\hat{\mathbf{e}}_\phi + \{-B_{0e}(\phi(t)) + sc_s(V)\}\sin\theta(t)\,\hat{\mathbf{e}}_\theta. \quad (13)$$

We immediately see that $\mathbf{T_{eff}}(t)$ vanishes when $\theta(t) = \theta_3$ and $\phi(t) = \phi_3$. Hence, there is no *net* torque acting on the magnetization vector if it reaches the state $\theta(t) = \theta_3$ and $\phi(t) = \phi_3$ at the same instant of time t. Unlike in the case of the other two stable states, where both shape-anisotropy torque and spin-transfer-torque *individually* vanish, here neither vanishes, but they are equal and opposite so that they cancel to make the net torque zero.

If the magnetization ends up in the orientation (θ_3, ϕ_3), then it will be stuck and not rotate further unless we change the switching current I to change the spin-transfer-torque. Since changing I can dislodge the magnetization from this state, it is *not* a "stable" state like the ones when $\theta = 0°, 180°$. Hence, we call it a "metastable" state.

For numerical simulations, we consider a nanomagnet of elliptical cross-section made of CoFeB alloy which has saturation magnetization $M_s = 8 \times 10^5$ A/m (Ref. 11) and a Gilbert damping parameter $\alpha = 0.01$. We assume the lengths of major axis (a), minor axis (b), and thickness (l) to be 150 nm, 100 nm, and 2 nm, respectively. These dimensions (a, b, and l) ensure that the nanomagnet will consist of a single ferromagnetic domain.[12,13] The combination of the parameters a, b, l, and M_s makes the in-plane shape anisotropy energy barrier height ~32 kT at room temperature. The spin polarization of the switching current is always assumed to be 80%.

We assume that the magnetization is initially along the $+z$-axis, which is a stable state. At room temperature, the thermal fluctuations will deflect the magnetization vector by ~4.5° from the easy axis when averaged over time,[7] so that we will assume the initial value of the polar angle to be $\theta_{init} = 4.5°$. We choose the initial azimuthal angle ϕ_{init} as $+90°$ because it is the most likely value (along with $\phi_{init} = -90°$) in the absence of spin transfer torque.[7] Similar assumptions are made by others.[10] We then solve Eqs. (7) and (8) simultaneously to find $\theta(t)$ and $\phi(t)$ as a function of

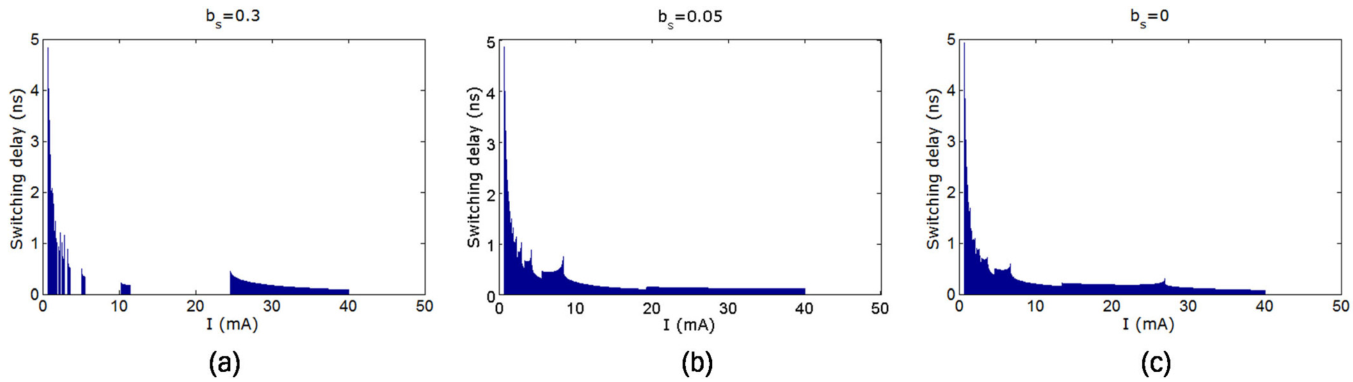

FIG. 2. Switching delays for a range of switching current 700 μA – 40 mA producing spin-transfer-torque for $c_s(V) = -1$ and different values of $b_s(V)$. The switching current is varied in steps of 10 μA. (a) $b_s(V) = 0.3$. No switching occurs for the following switching current ranges: 2-2.05 mA, 2.33-2.49 mA, 2.83–3.26 mA, 3.69–5.09 mA, 5.6–10.24 mA, and 11.41–24.51 mA. Magnetization may start up showing oscillations but ends up at a metastable state (see supplementary Fig. S11). (b) $b_s(V) = 0.05$. Switching succeeds for the entire range of switching current. Metastable states appear for $b_s(V) > 0.05$. (c) $b_s(V) = 0$. Switching succeeds for the entire range of switching current.

time. Once $\theta(t)$ reaches 175.5°, regardless of the value of $\phi(t)$, we consider the switching to have completed. The time taken for this to happen is the switching delay.

Fig. 2 shows the switching delays versus switching current for different values of $b_s(V)$. The switching delay is "infinity" in *some current ranges* when $b_s(V) = 0.3$ because switching failed [see Fig. 2(a)]. However, beyond the current 24.51 mA, switching always occurs within a finite time, meaning that the magnetization never ends up at the metastable state. Simulation results show that if the value of b_s is small enough (≤ 0.05), the metastable state does not show up [see Figs. 2(b) and 2(c)].

The important question is why switching fails only for certain ranges of the current I, i.e., why does the magnetization vector land at the metastable state for certain values of I and not others? The answer is that starting from some initial condition $(\theta_{init}, \phi_{init})$, the angles $\theta(t)$ and $\phi(t)$ must reach the values θ_3 and ϕ_3 at the *same* instant of time t. This may not happen for any arbitrary I. Hence, only certain ranges of I will spawn the metastable state. It is also clear from Eq. (11) that above a certain value of I, there will be no real solution for ϕ_3 since the argument of the arcsin function will exceed unity. This value will be $I_{threshold} = [e\mu_0 M_s^2 \Omega(N_{d-xx} - N_{d-yy})]/[\hbar\eta c_s(V)]$. By maintaining the magnitude of the switching current above $I_{threshold}$, we can ensure that the magnetization vector will never get stuck at the metastable state. For the nanomagnet considered, $I_{threshold} = 32.7$ mA, but switching becomes feasible at even lower current of 24.52 mA since in the range [24.52 mA, 32.7 mA], the coupled θ and ϕ-dynamics expressed by Eqs. (7) and (8) do not allow $\theta(t)$ and $\phi(t)$ to reach θ_3 and ϕ_3 *simultaneously* starting from $(\theta_{init}, \phi_{init})$.

Another important question is whether thermal fluctuations can untrap the magnetization from this state. To probe this, we solved the stochastic LLG equation[7,14] in the presence of a random thermal torque. Fig. 3 shows the magnetization dynamics for a switching current of 24.51 mA at room temperature (300 K). We observe that the magnetization gets stuck at a metastable state with $\theta_3 = 97.58°$ and $\phi_3 = 335.87°$ (and fluctuates around the metastable state due to thermal agitations) ~50% of the time, which means that roughly one-half of the switching trajectories intersect the metastable state and terminate there. The values of θ_3, ϕ_3 are also the angles predicted by Eqs. (11) and (12), thereby confirming that the metastable state indeed has the origin described here. Increasing the temperature to 400 K helps

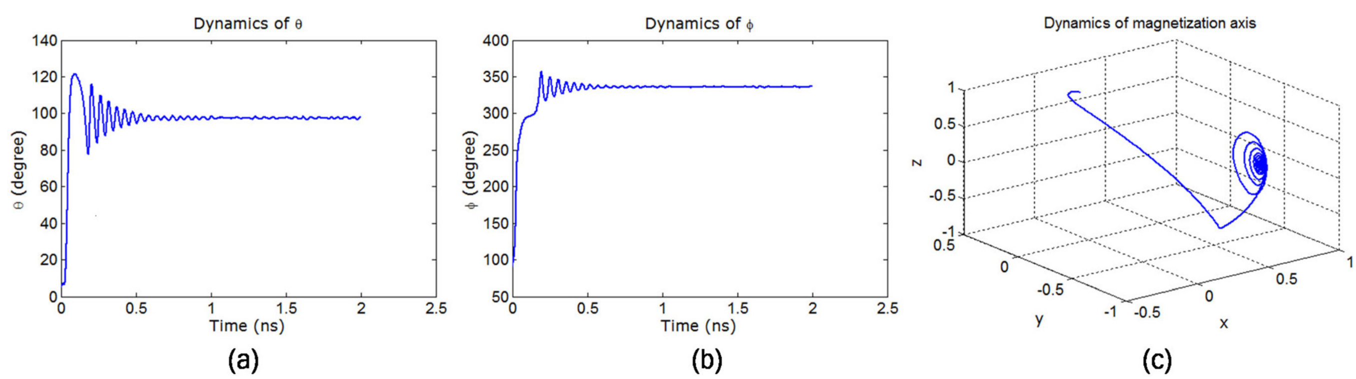

FIG. 3. Room-temperature (300 K) magnetization dynamics when the switching current is 24.51 mA ($c_s(V) = -1$, $b_s(V) = 0.3$). (a) Dynamics of $\theta(t)$. (b) Dynamics of $\phi(t)$. (c) The trajectory traced out by the tip of the magnetization vector in three-dimensional space. The magnetization gets stuck at a metastable state with $\theta_3 = 97.58°$ and $\phi_3 = 335.87°$. This plot was obtained by the solution of the stochastic LLG equation in the presence of a random thermal torque to simulate the effect of thermal fluctuations. This is one specific run from 10 000 simulations performed in the presence of thermal fluctuations which shows that the latter cannot untrap the magnetization from this state at room temperature. This happens for all the 10 000 simulations, if thermal fluctuations are brought into play after the metastable state is reached. This shows that the state is stable against room-temperature thermal perturbations. Random thermal fluctuations occasionally perturb the magnetization around the metastable state but the extent of these fluctuations is less than 3° (see supplementary Fig. S13).

only a little by decreasing the probability that a switching trajectory will intersect the metastable state.[7] What is important, however, is that *if* the magnetization vector gets stuck at the metastable state and the current remains on, then thermal fluctuations cannot dislodge it. In other words, this state is *stable* against thermal perturbations.

It should be emphasized that if $b_s(V) = 0$, then $\theta_3 = 90^\circ$ (x-y plane), however, magnetization cannot remain stuck to that metastable state since no field-like torque is there to balance the out-of-plane shape-anisotropy torque when thermal fluctuations would dislodge the magnetization from $\theta_3 = 90^\circ$. Also, if we consider $\theta = 180^\circ$ to 0° switching ($c_s(V) = +1$), field-like torque aids the rotation of magnetization towards its destination, hence metastable states do not crop up.[7]

We also notice oscillations before magnetization settles into the metastable state (see Fig. 3). This is due to coupled θ- and ϕ-dynamics governing the rotation of the magnetization vector, which causes some ringing. Such ringing signifies that the magnetization is attracted to the metastable state as it comes inside the range of the attractor. Thus, it can be intuitively conceived that if the initial conditions $(\theta_{init}, \phi_{init})$ are changed, the range of currents for which metastable states would emerge can change as well.[7] In general, choosing different parameters for the nanomagnet (e.g., damping constant, saturation magnetization, shape anisotropy[7]) can change the occurrence of metastable states in different current ranges.

Finally, one issue that merits discussion is what happens if the spin polarized current is turned off after the magnetization gets stuck. In that case, the torque due to shape anisotropy will take over and drive the magnetization to the easy axis. One expects that if $\theta_3 > 90^\circ$, then switching should succeed because the nearer easy axis ($\theta_3 = 180^\circ$) is the desired orientation. Equation (12) dictates that $\theta_3 > 90^\circ$ since $b_s(V)$ is always positive. Unfortunately, these simple expectations are belied by the complex dynamics of magnetization. The out-of-plane excursion of the magnetization vector, i.e., deviating from magnet's plane $\phi = \pm 90^\circ$ causes an additional motion that depends on θ_3, ϕ_3 [see the $\hat{\mathbf{e}}_\theta$ component of torque in Eq. (3)]. This motion can oppose the in-plane motion due to damping [see the last term in Eq. (7)].[7] As a result, even when $\theta_3 > 90^\circ$, switching can fail since the magnetization reaches the wrong orientation ($\theta \simeq 0^\circ$) along the easy axis (see Fig. 4).

In conclusion, we have predicted the existence of a metastable magnetization state in spin-transfer-torque switching of a shape-anisotropic single-domain nanomagnet in the presence of field-like torque, which is significant in magnetic tunnel junctions. Since the occurrence of metastable states must be avoided for feasible implementation of devices based on spin-transfer-torque mechanism, we hope that our studies would stimulate experimental research and further theoretical studies onwards.

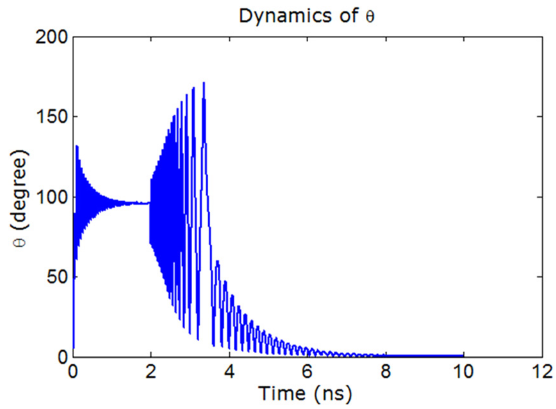

FIG. 4. Time evolution of $\theta(t)$ when the switching current is 20 mA ($c_s(V) = -1$, $b_s(V) = 0.3$). Thermal fluctuations were ignored but we assumed $\theta_{init} = 4.5^\circ$ and $\phi_{init} = 90^\circ$, respectively. The switching current is turned off at 2 ns after the magnetization vector gets stuck at the metastable state with $\theta_3 = 95.74^\circ$ and $\phi_3 = 341.25^\circ$. The dynamics shows that the magnetization vector relaxes to the easy axis because of shape anisotropy, but the final orientation is the *wrong* orientation along the $+z$-axis rather than the desired final orientation along the $-z$-axis. Therefore, switching fails.

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