CLASSROOM TEACHERS' USE OF RESEARCH TO EFFECT CHANGE IN PRE-SERVICE ELEMENTARY TEACHERS

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Introduction

Activities based on mathematics education research are incorporated into a preservice content course designed to reflect the course objectives of (i) broadening and deepening preservice teachers' understanding of the complexities of teaching and learning, (ii) encouraging them to develop reflective practices, and (iii) exposing them to the scholarship of teaching. Research articles and videos are used throughout the semester to generate cognitive dissonance that facilitates the reconstruction of inappropriately formed concept images and as sources of classroom investigations and assessment questions. The activities provide pre-service elementary teachers with experiences which result in changes in their attitudes about mathematics and engage them in professional practices that inform their instructional, pedagogical, and theoretical perspectives.

Research-based activities include a seminar experience which serves to inform students about some of the issues involved in the teaching and learning of mathematics. Assigned problems such as the Towers Problem, generate discussions that reveal students' thinking, their understanding of proof, their beliefs about the elementary school mathematics content they need to know, and their assumptions about grade school children's mathematical abilities. Ann's Fraction Cookie is an activity which models the use of questions as a constructivist instructional practice and introduces the student to problems involving part/whole and part/part relationships in the absence of rote symbolic manipulations, with the goal of developing conceptual understanding of these relationships. These activities and their impact on students are described in this report.

The Seminar

The seminar introduces students to mathematics education research and the scholarship of teaching. In preparation for the seminar, students are given a handout describing a seminar:

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> A seminar brings together an interested group of learners who have done some preparation outside of class, including having read, thought about and written about various research reports and articles. Seminar is a time to "mine" the research, to work it over as a group, to think aloud about it, and to test your ideas against those of other members of the group. It is a special time for a unique intellectual activity – the exchange of ideas focused on a source (research articles, a book, a video, etc.)

A second handout with suggestions for writing an effective and useful seminar paper, together with a list of focus questions directed at salient features of the assigned readings is also distributed. Attendance at seminar is mandatory.

Students are given copies of three or four research articles, which they are assigned to read in preparation for the seminar. Sources of these seminar readings include various mathematics education research journals, papers presented at conferences, and/or books which include various researchers' writings on specific topics/issues of mathematics education. A short list of questions designed to focus students' attention on some of the salient features of the assigned readings is distributed along with the readings. Students are requested to note text passages they find most interesting and to review the articles in light of the focus questions with which they have been supplied. Each student writes a short paper (3-5 pages typewritten) in which the research articles are analyzed and the relevance of the research to the student's own previous mathematics experiences is described. Students report to seminar with their written question(s) about some feature of the readings they would like clarified and at least one notion they found particularly interesting/relevant that they are prepared to discuss. The questions and the particular ideas or topics of interest, together with the students' seminar papers, form the basis for discussion during the seminar.

Prior to the seminar, students review the individual and group responsibilities described in the seminar handout. They are responsible for conducting the seminar and directing the discussion. The instructor's role in seminar is that of an observer and occasional participant as s/he moves from group to group and room to room, neither the focus of attention nor the authority who tells students what they should learn. In order to provide greater opportunity for all students to actively participate in the discussions, participants form small groups (5-6 students). Seminar discussions take place in two adjacent classrooms, three groups per classroom. A follow-up discussion at the next class meeting provides opportunities for students from the various groups to share their seminar experiences with other members of the class and students submit their written papers for evaluation.

Reactions to the seminar experience, based on the seminar discussions and their written papers, suggest that most preservice teachers re-examine their own mathematics learning as a result of their engagement in this reflective activity. The following excerpt from a student's seminar paper is typical of the comments:

> When I read these articles, I began to think about my own math education. I thought back to when I was in first grade and in high school. I would have to say I used instrumental understanding all the way through. That is the way I was taught. In first grade, I was taught to count the points on the numbers, but when I got to 9 + 9, there were no points, so I couldn't solve the problem. In high school, when we were using algebraic equations, I never knew why. I never knew why, if you used a certain equation, you would come up with this answer. We were always given an equation, showed how to apply it to a number and that was it. No explanations of why it worked the way it did.

This student's written description and her subsequent comments during the seminar and in the follow-up session, suggest that she has a conceptual orientation, i.e., she feels a need to know *why* something works as well as *how* it works. It seems that her prior remembered experiences occurred in a classroom in which the instructor provided only a calculational orientation [1]. These prior experiences left her with feelings of frustration and a distaste for mathematics. The assigned readings and seminar activities provided a validation of her expectations that mathematics should make sense and gave the student a renewed confidence which was evident throughout the semester.

Classroom and Journal Investigations

Research articles are a rich source of problems which provide documentation of students' thinking and beliefs. Two such problems featured in research reports, *Building Towers* and *Ann's Fraction Cookie*, are described, along with their use in the preservice course.

Building Towers [2]: Pre-video Experiences

Early in the semester, preservice teachers are assigned the problem, *Building Towers*, as homework:

> Given plastic cubes of two different colors (red and blue):
> a) Build as many different towers as possible four cubes high without omitting or

duplicating any.

b) Convince other members of your group that you have built all possible different towers and that you do not have any duplicates.

During the next class meeting, students discuss their work and solution(s) to the problem with other members of their group, justifying their results. Members of different groups then share their discussions, solutions, methods of investigation, and justifications. The groups then explore *Building Towers II*:

Given plastic cubes in two different colors (red and blue):

a) Build as many different towers as possible five cubes high without omitting or duplicating any.

b) Convince other members of your group that you have built all possible different towers and that you do not have any duplicates.

Preservice teachers' initial explorations of this problem generate classroom discussions that reveal their thinking about the problem. It is a problem characterized as easily accessible–that is, every student is able to get started on the problem and work out a solution, though the solutions of many preservice teachers are frequently incorrect. After the class investigations and further discussion, students are assigned a reflection journal as homework and asked to complete the following:

I first thought....

I first attempted the problem by....

After talking with my group I realized....

I know I have all the towers because....

These written reflections reflect the range of abilities and ways of thinking of different students. A typical response was:

> When I first looked at this problem I thought it was going to be very complex and difficult. I don't know if I have all the answers...Every time I think I have all the towers I find a couple more. It's driving me mad!!! I know there is a pattern but I'm not sure what it is....However, I just realized that some towers repeated, so this pattern really doesn't work. Augh!!!

Occasionally, some students indicate they have made connections with previous problems which they recognize as having a similar structure. A student recently wrote:

I first thought that this problem might be like the ice cream problem that we did from the book. We were dealing with 4 elements (cubes) in the tower problem and 4 toppings in the ice cream problem. I first approached the problem by reviewing the ice cream problem. I remembered that there were 16 different combinations of toppings in the ice cream problem....I also remembered the following table from our text:

Number of Elements	Number of Subsets
1	2
2	4
3	8
4	16
5	32

This student went on to complete the statement: *I know I have all the towers because...*

This helped me generalize about a tower with n numbers of blocks. For any set with n elements, the number of subsets (combinations in this case) of that set is 2^n. This helped me verify that there would indeed be 16 different towers of 4 cubes and with 5 cubes we would get 2^5 which is 32 combinations.

In general, preservice teachers' understandings of the nature of proof provide little cause for rejoicing by their instructors. Few students are able to provide a valid proof. They generally fail to recognize that the fourth- graders use various methods of proof to justify their work, including proof by exhaustion and proof by induction. More representative of the responses to the question of whether they have found all the cubes is the statement:

I know I have all the towers because I have talked with the people in my group and we all had the same answer.

Some students, perhaps not quite convinced that their group members are correct, look to other groups for verification:

We have also talked to other classmates and have all come to an agreement. Therefore we have all possible towers.

Unfortunately, they don't. In a recent class, twelve of the twenty-three students did *not* find all possible four- and five-cube towers. The approximate percentage of 50% of students who find a correct solution has remained fairly consistent during the four years we have used the problem.

Building Towers I and II: Post-video Reflections

Following the preservice students' investigations of the *Towers* problem, a twelve-minute video of a group of four fourth-graders working on the same problem is viewed by the class. (The videotape is part of a longitudinal research study on the nature of elementary grade children's experiences with proof.) The videotape presentation generates discussion in which students' beliefs are revealed about elementary school mathematics content they need to know and their assumptions about grade school children's mathematical abilities. Students are assigned to write a second journal in which they are asked describe their observations about the four fourth graders on the video clip, together with reflections on their own investigations of the *Towers* problem. They were asked to answer four questions:

> As a result of class discussion, I now realize about the Tower Problem....
> Before seeing the video of the 4th graders doing the Tower Problem, I thought that....
> After seeing the video, I now....
> I have the following additional comments and/or reflections....

Several students were surprised to discover the variety of ways the problem could be approached, an observation typical of a majority of students after viewing the video. Many of the preservice teachers commented that, prior to watching the video, they believed "the fourth graders would have a harder time dealing with the problem than they did." Several preservice teachers acknowledged that fourth grade students did better than they (the preservice teachers) had done and that the elementary grade students were more capable of doing a higher level of mathematics than previously thought. Typical comments were:

> After seeing the video, I realize just how intelligent 4th graders are. They seem to be so willing to explore different ways about finding solutions.

> Children are doing more difficult tasks at a younger age than I was.

> I realized that 4th graders can handle this concept and with ease! I felt a bit 'challenged' by the intelligence and confidence the 4th graders displayed.

A few students, after viewing the video, saw connections to other problems they had missed in their previous investigations:

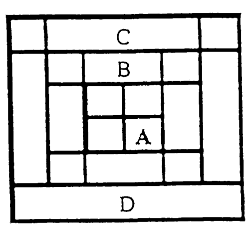
> The pattern was Pascal's triangle, and those kids were a heck of a lot smarter than I thought. Their ideas and thoughts were very advanced.

The reflections and subsequent class discussions form the basis for later investigations into the nature and role of proofs, including an analysis of the proofs given by the fourth graders in the video clip.

Ann's Fraction Cookie [3]

Preservice teachers generally have little conceptual understanding of fractions and have learned operational algorithms by rote. Less than fifteen percent of preservice teachers each semester have been able to demonstrate mastery (85% correct) on a thirty question competency test of basic arithmetic skills given during the first week of class. The areas of greatest difficulty for these students are the questions which test basic skills with fractions and those dealing with proportional reasoning. A common strategy for finding the sum $9 + 7\ 3/8$ is to change both addends to improper fractions, combine the terms, and attempt to convert the answer to a mixed number. More than seventy percent of the students who have enrolled in the preservice content course during the past six years used this strategy.

Experiencing cognitive dissonance can be effective in the restructuring of previously-acquired inappropriate concept images. The following activity has been effective in generating cognitive dissonance which resulted in students' restructuring of their existing schemas. Ann's fraction cookie activity is designed to provide students with experiences of part/whole and part/part relationships. Students are given the "cookie" and asked the following series of questions:



The Questions

1. A is what part of the whole cookie?
2. B is what part of the whole cookie?
3. How did you figure out your answer?
4. C is what part of the whole cookie? Why?
5. Can you show me half of the cookie?
6. Can you see any more halves?
7. How many ways can you make one-half?
8. What is more important, the shape or the number of squares?
9. How many ninths would make half of the cookie?
10. How many ninths would make the whole figure?
11. How did you figure that out?
12. If I give you forty-five ninths of this cookie, how many cookies would you be able to make?
13. Suppose I give you one thirty-sixth and one eighteenth of a cookie. What part of the cookie have I given you? Why?
14. Suppose I give you two thirty-sixths and three eighteenths, what part of the cookie would you have? Why?
15. What part of the cookie would you have if I give you three ninths and two eighteenths of it?
16. Instead, suppose I give you one sixth and one ninth of the cookie, what part of the cookie would you have?
17. If I give you two sixths of the cookie and then three ninths of the cookie, what part of the cookie would you have? Why?
18. Suppose I have a certain amount of money. I would like to give you one fourth and one eighth of that money. What part of my money would I have given you?

The responses to these questions are based on understanding the relationships among the various components of the fraction cookie manipulative. Students are not permitted to use pencil and paper to calculate their responses, only to record their answers to this series of questions. For homework, they are to read the research article that reports Ann's investigations based on the same questions. Additionally, the preservice teachers are to identify the fraction operations in the questions used during the initial class investigation. They are to note the sequence in which the operations were introduced. Students are given a second copy of the cookie, which they use as manipulatives to continue their explorations of the various relationships.

The investigations utilizing the Fraction Cookie manipulatives introduce a sequence of activities leading to discussion of "What's my unit?" and a more conceptual understanding of the operational algorithms students have learned previously. This problem introduces students to the use of manipulatives. Students' final course interview comments typically mention their reaction to the introduction and use of manipulatives:

> Instead of thinking I have a math disability of some kind, I am starting to think of math as a journey into more exciting learning experiences. I had so much fun with the fraction manipulatives, I went and bought some; my husband laughs at me because I like to do problems with them during the commercials on tv.

Assessment and Evaluation Activities

Research articles are also a source of problems used as evaluation items on small group take home exams and on small group oral exams. Preparation for the small group oral exam offers students opportunities to demonstrate the creativity inherent in mathematical thinking in ways most of these students have not experienced previously. Two problems that have been extremely effective in revealing how creatively students think about a problem are *Sam's Cookies* and *The Bowl and Measuring Cup* [4]. Both problems afford students opportunities to demonstrate their ability to use manipulatives effectively and appropriately–opportunities which allow students to use their imagination in wonderfully creative ways.

Sam's Cookies

> Sam has 35 cups of flour. He makes cookies that require 3/8 of a cup each. If he makes as many such cookies as he has flour for, how much flour will be left over?

The manipulatives that students use to demonstrate their solutions to these problems have included hand-made manipulatives as well as "found" manipulatives. One group used Legos to demonstrate their solution to *Sam's Cookies*. They arranged thirty-five 8-cell units, forming a base layer representing the thirty-five cups of flour on a flat Lego platform. On top of the thirty-five 8-cell units, they placed 3-cell units, arranged to cover as much of the 35 8-cell units as possible.

The Bowl and the Measuring Cup

Perhaps the problem that students have the most fun with is *The Bowl and the Measuring Cup*:

> You are given a large bowl and a small measuring cup. As quantities to measure, you have some rather large pebbles and some fine-grained sand. You repeatedly fill the measuring cup with the pebbles, transferring them to the large bowl until it is filled. By counting you have determined that it required 27 cups of pebbles to fill the bowl. You then empty the bowl.
>
> a) Calculate how many cups of sand will be needed to fill the bowl to the top.
>
> b) Put 20 cups of pebbles into the empty bowl and calculate how many cups of sand will be needed to fill the bowl to the top. Consider what arithmetic operation is appropriate for this situation, if any. Once you have made your choice, carry out the appropriate calculation (you may use a hand-held calculator), then verify your answer.
>
> c) Describe the essential mathematical tasks contained in this problem.

Each semester, one or more groups attempt to replicate the problem conditions as precisely as they can. One group, working at a member's home, crept out during a rainstorm and "borrowed" pebbles from the neighbor's driveway across the street and sand from the next-door neighbor child's sandbox. Another group, not certain a cup was a cup was a cup, used every bowl in the house, then borrowed additional bowls from the neighbors as they investigated the problem, using cereal and sugar; hard candy and salt; and other combinations of materials–they couldn't find any pebbles or sand, nor could they find a bowl. Another group spent a week locating "proper equipment" before attempting to solve it–they measured how much liquid every bowl in each of their four houses could hold, trying to find a bowl that held exactly twenty-seven cups.

The adventures that preservice teachers have investigating this problem are described by the student who wrote on her final portfolio evaluation:

> We solved many word problems over the semester (and I think that math learning should be mostly word problems), but my favorite had to be the problem on the oral exam concerning the large bowl, the measuring cup, the sand, and the pebbles. The sand, pebbles and bowl problem asked us to make a prediction about the ratio of the sand to the pebbles. This led to a long discussion about whether "a cup is a cup" no matter what is inside. This lively debate caused our group to rethink preconceptions about this idea, and to perform tests to prove or disprove our various hypotheses. What made it most enjoyable was all the time that our orals group spent arguing about it, and the fact that we were pouring fish rocks in the Denny's Restaurant for hours. We argued, we laughed, and we griped, but we all learned from sharing one another's ideas and methods. It was a rich experience that I'll not soon forget.

Her change in attitudes and beliefs during the semester is clearly demonstrated if one compares the portfolio evaluation comments with comments written during the first week of the semester in her autobiography. At the beginning of the semester, this student wrote:

> If you have the words 'Beth', 'math', and 'highlights' in one sentence, there must be an oxymoron in there somewhere. Although I don't feel that I'm afraid of math as much as I'm just frustrated with it, I must be one of those students who suffers from math anxiety.

Summary

Students' lack of conceptual understanding of foundational mathematical ideas–their inability to interpret and use ambiguous mathematical notation effectively and appropriately; the plethora of misconceptions and inappropriate concept images they enter class with; their innate and learned over-reliance on rote procedures and inflexible schemas; together with their negative attitudes and beliefs about mathematics are some of the issues that confront the instructor of preservice students. The responses cited in this paper illustrate the lack of prerequisite mathematical knowledge and skills preservice students bring to our courses. College students who cannot add two mixed numbers correctly; who are convinced they need to change a whole number to an improper fraction before adding to a mixed number; who believe that fourth graders know more mathematics and can solve problems more easily than they can–these are the students who hope to teach mathematics to the future generations of students.

How do we narrow the gap between the under-preparedness of preservice students and the level of conceptual understanding and competence necessary to teach future generations of students? Given their lack of mathematical competence and conceptual understanding, the accomplishment of many of our students in the short period of a sixteen week semester are quite remarkable. But is it enough? A growing body of research [5] suggests that instruction designed with a reconceptualized view of mathematics and learning can improve students' competency and understanding of what mathematics is, what it means to know mathematics, and how to go about learning mathematics. The literature provides a framework within which to interpret our observations and student work, and is a rich source of problems which reveal students' thinking. A student's self-assessment upon completing the preservice content course summarizes the impact research-based activities had on her beliefs and attitudes:

> Upon entering this class, I was really stuck in a rut–if it's math, I can't do it...Mathematics almost seems a philosophy to me at this point. I am taking an astronomy course in conjunction with this class, and the discovery in both classes of the appearance of patterns as a problem-solving tool has really solidified this technique for me. I see math as a tool that is there to work for me rather than something to make things more difficult.

Students' responses such as this, together with the improvements in mathematical competence and understandings they have demonstrated, suggest that activities and evaluations grounded in research can play a vital role in the development of future teachers. More work on effective uses of the research literature is needed. We invite you, the reader, to join with us as we continue to explore ways to incorporate research findings into our instructional practices. ■

References

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[2] C. A. Maher and A. M. Martino, "The Development of the Idea of Mathematical Proof: A 5-Year Case Study," *Journal for Research in Mathematics Education*, 27 (2) 194-214 (1996).

[3] A. Sáenz-Ludlow, "Ann's Fraction Schemes," paper presented at the Illinois Council of Teachers of Mathematics Annual Meeting, Peoria, IL, (1991). *Note:* A more extensive write-up of a similar teaching experiment is also available. (See: A. Sáenz-Ludlow, "Michael's Fraction Schemes," *Journal for Research in Mathematics Education*, 25 (1) 50-85 (1994).

[4] R. B. Davis, "Reflections on Where Mathematics Education Now Stands and on Where It May Be Going," *Handbook on Research in Mathematics Teaching and Learning*, Macmillan Publishing Company, New York, 165-196, 1992.

[5] *Issues of Curriculum Reform in Science, Mathematics and Higher Order Thinking Across Disciplines*, U.S. Department of Education, 16-35, 1994.