LEARNING TO UNDERSTAND: THE MATHEMATICAL PREPARATION OF PROSPECTIVE TEACHERS

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This paper describes the development of a two-course sequence in mathematics content for prospective elementary teachers. Community college and university personnel collaborated to develop a course sequence that would prepare prospective elementary teachers to teach mathematics with an understanding of concepts to support their abstract mathematical knowledge. The strategy was to begin with a broad vision and then focus on the smaller pieces which would achieve that vision. The course changes are validated by documents published by various educational and mathematical groups advocating an increased emphasis on teaching for understanding rather than rote learning. Significant change is difficult without support from colleagues and sufficient time, both necessary to the change process. The noteworthy components of Austin Community College's revised course are a safe environment in which students become independent learners and written communication as an integral part of the course resulting in students who have increased their conceptual understanding. As a result of taking the course, students accept responsibility for their own learning, have increased self-confidence, and show enthusiasm for mathematics. While requiring a major commitment from faculty, the results are well worth the effort.

"Any fool can know. The point is to understand."
- Albert Einstein

History of the Program

Austin Community College has a long history of involvement with the mathematics preparation of future elementary school teachers dating back to at least 1977, four years after the college was established. The college's six-semester-hour, two-course sequence has always transferred to the University of Texas at Austin because instructors at both institutions are aware of each other's activities. For a community college, our current and cumulative experience in this area is very large.

This association with the university was strengthened when, in 1994, the current vision, philosophy, and structure of our two-course sequence was developed in collaboration with university personnel. Mary Hannigan and Tracy Rusch received funding to develop a content course that would better serve prospective elementary teachers. This curriculum effort was shared with other interested instructors from both institutions during all stages of development. Vera Preston was one of the earliest advocates of this new structure, as it

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complemented her personal vision, and added her expertise to subsequent revisions. Support for this new vision of preparing teachers continued to increase among ACC faculty and is now accepted college-wide.

Educational Philosophy

A democracy can survive only with an educated populace. Any culture is only a generation from anarchy. The education of our children is essential; therefore, the education of the teachers of our children is critically important to all of us. It is in the selfish best interests of all citizens to educate our children. American Indian leaders consider the next seven generations when they make decisions for their people. The rest of the United States could benefit as a people if our leaders considered how their decisions would affect the people one hundred forty years from now.

> The process of teaching mathematics and learning mathematics is iterative: the way preservice elementary teachers are taught influences their understanding of and beliefs about mathematics; their understanding of and beliefs about mathematics influence the way they teach; and the way they teach influences their students' understanding of and beliefs about mathematics [1].

If college professors are sensitive to the variety of learning styles of their students and help students become aware of their own learning styles, it is likely that the students will carry that sensitivity with them into their own classrooms. Providing the students with a foundation of concepts to support their abstract mathematical knowledge will also go with them and will benefit future generations. Creating a classroom environment that addresses these issues is a college professor's contribution to breaking the cycle of mathematically ill-prepared elementary teachers.

Vera Preston's interest in preparing prospective elementary teachers began when asked by the child of a migrant worker to explain division. Her secondary mathematics preparation had only prepared her to teach the algorithm, rather than the concept. This prompted her to enroll in courses that prepared her to teach elementary mathematics, which she could later share with her college students. Her teaching strategies have evolved over the years to adapt to the needs of her students.

Mary Hannigan's experiences began in a similar way with a class of rural seventh-graders who needed a conceptual foundation that Mary's secondary mathematics program had not prepared her to provide them. She took graduate courses in elementary methods and began attending conferences to help her determine how to better serve her college students so they would not end up in the same predicament she had. Serving as a mentor to a new fourth-grade mathematics teacher, her mother, helped define for her the needs of beginning elementary teachers.

Course Development

As part of her graduate studies at the University of Texas, Mary, with colleagues Tracy Rusch and Susan Hull, was given the opportunity to rethink the needs of prospective elementary teachers. In the summer of 1994, Mary and Tracy received funding to rewrite the curriculum for this course which both would teach during the fall term, Mary at ACC and Tracy at the university. Soon after, these newly developed curriculum materials were shared with Vera. Working in isolation, Vera had made some changes to her course, but it is challenging to make such drastic changes on your own. With a support network in place, ACC's course for prospective elementary teachers began a coordinated evolution.

Revisions to the curriculum are ongoing. While Mary and Vera share ideas and materials, based on the same philosophy, each has her own "style", of course. These materials were shared at a workshop with other faculty who had expressed an interest in their work. As a result of this workshop, the vision of this course shared by Mary, Vera, and Tracy has been adopted college-wide.

The changes in the course are influenced by personal experiences and are validated by documents published by the National Council of Teachers of Mathematics (NCTM), the Mathematical Association of America (MAA), the American Mathematical Association of Two-Year Colleges (AMATYC), and the Texas Statewide Systemic Initiative (Texas SSI) [1-5]. Vera was involved, as president of Women and Mathematics Education, in developing the *Curriculum and Evaluation Standards for School Mathematics* [2]. As a member of the Action Team on Strengthening the Mathematical Preparation of Elementary Teachers of the Texas SSI, Mary co-authored a survey sent to college of education deans and mathematics department chairs which showed a lack of community college participation in the preparation

of prospective elementary teachers [6]. Austin Community College's long involvement with this course as well as the current efforts in curriculum revision indicate that ACC is a state leader in preparing future elementary teachers.

As part of her involvement with the Texas SSI, Mary visited universities in the state which were funded by the Texas SSI to improve their courses for prospective elementary teachers. Observing other faculty making changes to their courses which were similar to the ones made at ACC, confirmed for her that ACC's vision was shared by others across the state.

Observations on the Process of Change

It is important to understand that the significant changes that occurred in Austin Community College's course for prospective elementary teachers took time, energy, and thought and was a collective effort on the part of many people. It was by no means a "quick fix"; it was an incredibly challenging endeavor.

ACC's course has been evolving since 1994 and continues to evolve. Every instructor who teaches the course is always searching for new projects and activities that are mathematically powerful and will develop and/or deepen students' understanding of mathematics. Quality resources for this college population are not readily accessible, so this is not an easy task.

The strategy of beginning with a broad vision and then focusing on the smaller pieces worked well for ACC. Within the scope of the philosophy, there is much room for individual creativity on the part of the instructor. While the vision remains relatively constant, the pieces are adjusted to fit the interests and needs of a specific class.

Working together to make changes is essential. Vera, working alone, had many frustrations with the course she was developing; there just weren't enough hours in the day to make drastic changes. Conversely, Mary and Tracy, while still frustrated implementing change, had each other to share the creative load, the triumphs and failures. Having a support group, even if small, is essential to successful implementation of a new course.

Effects of Change on the Students

Prospective elementary teachers are often highly anxious about mathematics and, to varying degrees, believe that they already know all the mathematics necessary to teach elementary school. ACC's course generally changes those ideas. Most students complete the course with more confidence in their ability to do (50% of students completing an end-of-course survey agreed that elements of the course helped them develop confidence in their problem solving skills, 41% strongly agreed) and understand mathematics (83% agree or strongly agreed) and realize that there is more to mathematics than regurgitating information.

Students' increased confidence in their mathematical ability (83% of students reported a very low to moderate level of confidence at the beginning of the course, 78% reported a high or very high level of confidence upon completing the course), hopefully increases their confidence in their ability to explain mathematics to their future students. At the beginning of the semester many of these students express concern that they will perpetuate the math anxiety they have experienced. By the end of the course, they are convinced that this will not happen, and they haven't even had a course in the methods of teaching elementary mathematics! The students' have realized that they know more mathematics than they believed they knew and can explain it to others.

The excitement at seeing the growth these students display--improving conceptual understanding, showing enthusiasm for mathematics, accepting responsibility for their own learning, in addition to increasing self-confidence--more than makes up for the difficulties involved in making changes and managing this labor-intensive course. It is the most gratifying and important course the authors teach.

A Vision for the Future

Our efforts are driven by the commitment to provide future generations with elementary teachers

- who are confident in their ability to do and to learn mathematics
- who have well-developed mathematical analysis skills which they use readily and routinely
- who have a deep understanding of the mathematical ideas they share with their students

- who are able to thoughtfully select mathematical learning experiences and clearly explain mathematical ideas
- who embrace the opportunity to share the magic, beauty, and fun of mathematics with children [7].

This vision guides the curriculum and environment of the mathematics content course for prospective elementary teachers at Austin Community College. The remainder of this article describes the underlying structure of the course sequence by highlighting the strengths of the course.

"Those who can, do. Those who understand, teach."
- Lee Shulman [8]

Students develop an understanding of concepts, not just an ability to use algorithms. Projects are an integral part of the course intended to build students' conceptual understanding of mathematics. While students are encouraged to work together on the projects, the projects can be completed individually. Many of the projects utilize manipulatives to develop a conceptual foundation on which to build a bridge to abstraction.

Another component of the course, challenge problems, requires the students to justify to a peer the solution of the problem. In writing this justification, the students develop an explicit understanding of the mathematics [9]. They are instructed that this paper is not to be the steps of their solution process.

A challenge problem is a problem that requires more than simply applying a process in order to solve it. Challenge problems can generally be solved by a variety of problem solving strategies–pictorially, numerically, or using more sophisticated computational procedures. As long as the solution is justified, any method of solution is acceptable. The classic handshake problem (How many handshakes are made between eight people if everyone shakes hands with everyone else exactly once?) is an often used challenge problem in the patterns unit.

"Tell me, and I forget;
Show me, and I remember;
Involve me, and I understand."
- Ancient Chinese saying

Students participate in a mathematical community which requires that they clearly communicate mathematical ideas and work together to model concepts. By working on the projects cooperatively, students are provided the opportunity to develop their skills in mathematical communication. Using tools (manipulatives) to model concepts requires that the student share a common language.

Participation in a mathematical community develops skills in the practice of mathematics. Mathematical inquiry or the practice of mathematics has long been the exclusive domain of mathematicians; in the desire to impart mathematical information, they have often slighted many students by not providing them the opportunity to discover the joys of mathematical inquiry. ACC's course has attempted to facilitate this experience for prospective teachers so that they will one day provide it for their own students.

An additional benefit that results from participating in a mathematical community is that students recognize the different learning styles that exist. In discussing problems they are confronted with the various ways that people think about mathematics. The end result of that thinking is the same, but the path to that end can be very different. Recognizing and accepting these differences will benefit them when considering how to present a lesson to their own future students.

"They know enough who know how to learn."
- Henry Adams (1838 - 1918)

Instructors establish a safe environment in which students become independent learners. To develop the skills of communication and mathematical inquiry, math anxious students must feel safe enough to take risks or make mistakes or "contribute ideas for investigation or discussion" [1]. Once they are secure in the knowledge they will not be denigrated for "not knowing", they will be willing to take greater risks to achieve greater understanding. This safe environment doesn't just happen, instructors must make a conscious effort to foster this environment. This is not an easy task, but one well worth the effort required.

Teachers must be flexible and able to adapt to ever-changing curriculum. New, unfamiliar content may be added to the curriculum or a new course must be prepared that hasn't been reviewed recently. To adequately prepare to teach, instructors must be able to "teach" themselves this material. New teachers spend an inordinate amount of their preparation time trying, sometimes unsuccessfully, to teach themselves the mathematics rather than developing engaging, mathematically rich lessons [10]. As a result, they resort to telling students how "to do" mathematics rather than teaching concepts [11]. Prospective teachers who have experience "teaching themselves" and have developed confidence that they can adequately prepare themselves to teach mathematics are in a better position when they begin their career. ACC's course attempts to develop independent learners, students who have both the ability and confidence to teach themselves.

"How do I know what I think until I see what I say."
- E. M. Forster

Written communication is an integral part of the course. Many of the course components require the students to communicate mathematical ideas in writing. Questions on projects require a justification for the solution; challenge problems are entirely a justification of the solution to a problem. Focus questions, another written component of the course, require students to describe connections among mathematical ideas and relate those ideas to other disciplines and to life; the focus question for the sorting and classifying unit asks "All of the projects in this unit focus on characteristics (or "attributes") and on the development of organizational strategies. These two concepts are essential, fundamental elements in the study of mathematics. Why?". To support the argument presented, the students present examples drawn from mathematics as well as other areas. While not a technical paper, a good focus question will show evidence of significant reflection upon mathematical ideas.

In addition, students also write essays which reflect on their feelings, beliefs and emotions regarding the course and their mathematical growth. These types of essays are found as summaries of the various units, as part of a portfolio assignment, or exist independently as a cumulative end-of-semester assignment. Students are provided with guidelines for the assignment and then allowed the freedom to sculpt the paper to reflect their mathematical knowledge.

Final exams are also an extensive writing assignment. The goal of the final exam, generally a take-home exam, is to allow students to demonstrate their ability to learn independently. The questions posed are mini-challenge problems or challenge problem/focus question hybrids, but not ideas specifically covered during the course of the term. Thus students are required to utilize resources other than the instructor to justify a problem solution or make connections among mathematical ideas.

Written communication is an important aspect of the course. This emphasis is due to the lack of time available to evaluate oral communication, as in student presentations. If the students can effectively communicate through written language, they surely can communicate effectively through the spoken word.

Conclusion

For colleges considering a revision of an existing course or developing a new course for prospective teachers there are many issues that must be considered. There is no one blueprint for this endeavor. Faculty needs ample time to consider the objectives of the program, the needs of the students, and the needs of the faculty.

The success–based on the comments of students, methods faculty at the local universities, and local public-school administrators–of Austin Community College's program for prospective teachers lies, in part, with the quality time spent discussing the goals and objectives of the course and the means to achieve those goals. Included in the discussions were a description of the typical student exiting the program, the mathematical skills and understanding they possess, and their level of preparation for subsequent courses, in this case the university methods course. The opportunity to have meaningful conversations on these topics was made possible by funding and the support of the institution. Developing a course of this nature is not a "spare time" task.

Once the underlying structure of the course is established (a commitment to a strong writing component, the vision of the environment, etc.), the pieces that pull the whole course together must be located; in ACC's case, the problems to use as challenge problems, the focus questions, and the individual projects. At this point, there may be minor differences among instructors as to the best vehicles to achieve the objectives, but that is what gives all

participants ownership in the course.

The actual curriculum materials that evolve or are located need to be compatible with an institution's vision for this course. The key consideration is that the activities must facilitate the mathematical growth of the students. The selected materials must be appropriate for the students, at their current level of mathematical sophistication, and that then take them beyond that level, striving for mastery of appropriate mathematical content.

> Assuming that it would be desirable for U.S. teachers to have better mastery of their subject matter, it not obvious that the simple step of requiring more mathematics courses for future teachers will achieve this goal. Standard undergraduate courses do not dwell heavily on the relevance of their subject matter to the...curriculum, and teacher candidates may not be able to make the connections on their own [12].

The advantage of ACC's course for prospective elementary teachers is that it DOES provide the students with opportunities to make the connections between the mathematics they have studied and elementary school mathematics. The faculty believe that the current course is NOT just more mathematics, but better, more appropriate mathematics which prepares them to teach the next generation of students.

This course requires a major commitment from the faculty who teach it. But despite the additional workload, we are impressed by the changes we see in our students. Their mathematical sophistication and their self-confidence increase dramatically. Because of the preparation we provide, we believe we significantly impact future generations of elementary schoolchildren which makes the effort worthwhile. ■

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