

ADDRESSING PROSPECTIVE ELEMENTARY TEACHERS' BELIEFS ABOUT THE NATURE OF MATHEMATICS: A CASE FOR EXPLORING STUDENTS' CONCEPTIONS OF MATHEMATICS IN A MATH EDUCATION COURSE

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Our challenge in preparing Virginia's K-8 teachers in mathematics is a complex one. Additional requirements, including more challenging math content courses or more innovative pedagogical courses, may provide part of the solution. It is the premise of this paper, however, that additional knowledge and skills are not enough. In order to prepare teachers who will engage in current best practice pedagogy, we must also address teachers' attitudes and dispositions, their beliefs and conceptions of mathematics itself. This paper will attempt to justify this position. It will also describe strategies used in the *Inquiry in Mathematics* course in Mary Baldwin's MAT program intended to elicit and address graduate students' current, sometimes limited (if not erroneous), notions about what mathematics is really about.

One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it...The issue, then, is not What is the best way to teach? But, What is mathematics really about? [1]

Our challenge in preparing Virginia's K-8 teachers in mathematics is a complex one. Additional requirements, including more challenging mathematics courses or reconceptualized education courses, may provide one piece of the solution. It is the premise of this paper, however, that additional knowledge and skills are not enough. In order to prepare teachers who will engage in "current best practice pedagogy", we must also address teachers' attitudes and dispositions, their beliefs and conceptions of mathematics itself. This paper will attempt to justify this position and to describe an innovative mathematics course that is taught as part of Mary Baldwin College's Master of Arts in Teaching program.

Thompson has stated that for many people "knowing mathematics is equivalent to being skillful in performing procedures and being able to identify the basic concepts of the discipline" [2]. I would propose that many of our prospective elementary teachers hold this

belief, having formed it inductively based on repeated school mathematics experiences in which the focus was mimetic teaching of procedural knowledge, followed by repetitive practice of a skill. In addition, the elementary mathematics curriculum experienced by these students -- pre-1989 and thus pre-NCTM Standards -- was likely to be one dominated by arithmetic.

My research on teachers' conceptions of mathematics had results somewhat contrary to Thompson's comment, however: 68% of participants agreed most strongly that "mathematics is a process by which people attempt to solve personally-meaningful problems", while only 25% defined mathematics instrumentally as "a set of rules, facts, and skills which we should teach to children because of their usefulness for later adult lives" [3]. This suggested that teachers have heard the message of the National Council of Teachers of Mathematics, perhaps through staff development sessions. My observations in these teachers' classrooms, however, revealed the difficulty of enacting a classroom math program that truly reflected this philosophical stance: only one participant seemed to be able actually to change her classroom practice toward real problem-based instruction. The teachers I studied seemed to "talk the talk", but not "walk the walk" when it came to mathematics reform.

So what to do about this? One possible obstacle to enactment of truly reformed math programs is suggested in the science education research dealing with the role of *misconceptions* in learning new concepts. I would suggest that teachers' strong, internal conceptions of mathematics as a rule-driven manipulation of symbols, formed over their years as students in traditional mathematics classrooms, might be viewed as misconceptions of the nature of mathematics. For us, as teacher educators, to simply tell them a "more correct" definition is surely an ineffective approach to addressing the problem.

The science education research also suggests a possible solution. Just as science lessons must confront learners with their misconceptions and present them with tasks which challenge these notions, so may we follow this approach with our prospective elementary mathematics teachers. I would propose that we must, as teacher educators, plan instruction which brings teachers' conceptions of the nature of mathematics out in the open, to be examined, compared with others', and discussed. A one-shot discussion is, of course, not likely to create permanent change, but it is a start. Prospective teachers must put their current beliefs "on the table" and

must confront contradictions between these beliefs and current research in both mathematics and psychology.

In an innovative course in Mary Baldwin's Master of Arts in Teaching program, I attempt to address teachers' misconceptions in a conscious way.

A brief overview of the course content and goals should be helpful. The graduate level course, *Inquiry in Mathematics*, is one of a series of six "inquiry" courses intended to strengthen students' backgrounds in the liberal arts, while also challenging them to explore the nature of each discipline. In *Inquiry in Mathematics*, for example, students learn new mathematics content, but also consider how mathematicians do their work and how the discipline as a whole builds its body of knowledge. By understanding what it means to do mathematics, as opposed to know mathematics, teachers can plan lessons that put students in the role of mathematicians, using the techniques and processes inherent in the discipline.

The course is organized into various strands, such as "What is mathematics?", "How do children learn mathematics?", and so forth, with the content strands (number theory, geometry, probability) interwoven. Students spend time during the initial weeks exploring the nature of mathematics through activities which are intended to probe their current conceptions of the nature of mathematics. This leads logically into a second strand on the history of mathematics, which reinforces the idea that mathematics is a human endeavor; students' group research projects introduce the class to some of the great (sometimes quirky, sometimes inspiring) characters who have worked in the field of mathematics. One activity within this strand is the viewing of the *Nova* episode "The Proof" which follows Andrew Wiles' experiences as he solved Fermat's Last Theorem; class discussion on this often revolves around the surprisingly passionate way this mathematician talks about his work. It also highlights the interdependence of those in the field as they build research on the proofs of those who went before.

This glimpse of a modern mathematician at work leads students into the next strand in which they do library research on some topic in mathematics which is currently being studied; the results are presented as oral reports to the class. This assignment is one which is difficult for students, but its successful completion sends several messages: (a) mathematics goes well

beyond the arithmetic with which they are all so familiar; (b) mathematics is not a finished product; and (c) the students are themselves capable, to some extent, of understanding these new topics— a real confidence-builder! In past semesters, the class has been treated to some wonderful presentations on such topics as fractals, topology, and chaos theory.

I will close with one example of the type of activity that seems to have successfully elicited students' conceptions of mathematics, bringing them out for examination, discussion, and sometimes revision. Students are given cards on which are written quotations about the definition or nature of mathematics, and they are asked to put them into three piles: "I agree", "I disagree", and "I don't understand this". With a partner, they compare and discuss the sorting. Cards in the "I don't understand this" pile are brought to the whole class, to pool students' interpretations. Finally, in a "whip-around", students choose one quotation that seemed particularly meaningful or helpful to them and explain why it seemed so.

You may at this point be asking "Where's the math content"? We are all aware that a number of our prospective elementary teachers do arrive in our classes with gaps and weaknesses in their knowledge of mathematics content, and as teachers of these students we have an obligation to strengthen that knowledge. The content strands of this course, such as number theory, are intended to do just that. In addition, however, they also serve as starting points for discussing pedagogy (as I attempt to model a problem-based approach to teaching mathematics); they give students experience in group problem-solving; and they provide a context in which students improve their communication skills through presentation of solutions, oral reports, and demonstration of models.

In summary, this mathematics education course is intended to prepare prospective teachers for "best practice" in their classrooms by modeling instructional strategies which may be quite different from what they experienced themselves as elementary students. The course builds their understanding of important concepts and skills in mathematics, but it also takes into account that other important facet of curriculum: beliefs, dispositions and attitudes. By consciously addressing teachers' conceptions of what mathematics is, we can help them build richer images of the nature of mathematics, for the benefit of their future students. ■

References

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