

THE WRITE APPROACH TO MATHEMATICS OR HOW I FOUND THE MIDDLE WAY

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Revising a course is a multifaceted process. Often, reform efforts are focused on a particular aspect, that of inquiry-based collaborative learning. This article discusses the implementation of another aspect of the reform of a course for pre-service elementary teachers: the use of journals and writing exercises for evaluation and assessment. The evolution of this particular reform is traced, with emphasis on the reactions of students and faculty, the issues raised by these reactions, and the solution and resolution attained by the author is outlined.

Redesigning a course in one department that is primarily to serve the students of another department, and in fact a department housed in a separate School, is a delicate process. Sometimes, it seems the two departments in question, Education and Mathematics, do not even speak the same language. As a twenty-five year veteran of the Department of Mathematics, I thus felt some apprehension when the New York Collaborative for Excellence in Teacher Preparation (NYCETP) asked me to revise *Math 185*, the single college-level, mathematics course for pre-service elementary school teachers. Historically, *Math 185* had been used not only to provide pre-service teachers with a deeper understanding of mathematics, but also to satisfy part of the science core requirement for non-science students. Thus, some of the mathematics in the course was neither geared toward the needs of the education students, nor structured to conform to the Standards of the National Council of Teachers of Mathematics (NCTM) or the New York State Department of Education Standards. I began the planning process by reviewing these standards. The NCTM [1] quite reasonably listed problem solving as the very first standard and very few in the mathematics community would argue with that viewpoint. It was the next two standards that really resonated in my mind: mathematics as reasoning and mathematics as communication. How often have mathematics teachers heard, “I know the answer,” [maybe!] “I just can’t explain how I got it.” This is indeed a serious shortcoming in a soon-to-be professional facilitator of explanations.

I began the revision cautiously, introducing student journals and some group problem solving activities and games. At first, I gave very little direction about the journals, asking students to comment on the lesson and their understanding of it. The journals were to be done at home and handed in once or twice a week, since I was ever jealous of classroom time. I told the students not to censor their comments, but I needn't have worried — they were candid, often painfully (my pain!) so. However, I found that the journals were too unfocused and too often, time-stressed students made a brief entry that showed very little thought. I started to use more direct questions, asking them to describe specifically what they had learned, what had confused them, how had they overcome the confusion, and what they had liked or disliked about the topic. I wrote comments in the journals in response to their entries, sometimes briefly, and sometimes more at length, always trying to keep my responses positive and encouraging. When faced with an egregious error or a particularly painful comment, my response would be noncommittal, a gentle correction or perhaps an 'OUCH!'

The issue of when and what corrections to make in a journal is an important one. I agree wholeheartedly with Countryman [2] that one must primarily respond to content rather than mechanics, since the act of writing is primarily a communication between student and teacher. However, I did not take the 'freewriting' approach to journals that she suggests. She recommends having students write rapidly for a short time in class. Since I wanted more crafted answers, I asked the students to write their journals outside of class and present a polished, finished product. I did tell the students I would be happy to read anything they wrote and that they would not be graded on their spelling and grammar. However, I corrected these types of mistakes, with particularly glaring errors receiving an 'OUCH.' In the same vein, when they wandered too far afield in their journals, I tried not to judge them, but to gently bring their minds back to the task at hand. Or when necessary, I told them, "You need to put more thought into your journal entries." However, I believe it is very important in *this kind* of personal writing not to make the student feel judged or defensive. Conversely, any time student work is collected and corrected, there is another issue that rears its unsightly head: the question of grading. Because of the considerations mentioned above, I decided not to grade the journals conventionally, but to use a check mark, \checkmark , to indicate that the journal was 'acceptable.' If the student had done outstanding work in some way, either insightful ideas or excellent writing, I gave a grade of $\checkmark+$. Very rarely, if the student seemed to have given the

material almost no thought, and had been warned by “You need to put more thought...” before, I awarded a $\sqrt{-}$. I believe, for this type of personal, written work, that maintaining a supportive climate is crucial if you wish the students to give their full effort. In a reversal of the usual idea (the harder they think the test will be, the harder they will work for it), instead students will sometimes freeze up if they think they cannot perform at a satisfactory level. Tobias [3] notes that students’ passivity often results from their fears of making mistakes. In her seminal work on math anxiety, she catalogs the nature of the terrain, describing a number of issues that clearly emerged as I read the journals.

The first problem that I observed was an issue of language itself. Mathematics prides itself on its precision. Mathematical words and symbols are to have but one meaning. However we, as professionals, often do not realize that for the students, there is an ambiguity inherent in the fact that we use common English words to describe precise mathematical concepts. Is zero nothing, a place-holder, or just another ‘regular’ number; and if so, why can’t we divide by it? Pimm [4] explores the linguistic and conceptual difficulties that students encounter during the translation between the languages of mathematics and those of the classroom. Suffice it to say that the journals helped me in my exploration of this important problem. I began to realize that for some of my students solving the problem might not have been the main difficulty; the trouble may have lain in the decoding of the words of the mathematics. Thus, not only did my running dialogues with students help me provide clarifications for them, but it helped me to learn what misconceptions these, and certainly past generations of my students, held. The journals furnished me, the instructor, with enormous insight into my students’ mathematical mindscape.

There is yet another issue that the journals helped me to confront. The difficulty is by no means limited to our school, but is more severe because of our particular student population. City College is part of an urban university in New York City. The College’s original mandate was to provide an education to the children of the poor and working class people and to open to all new immigrants the opportunities of America. The campus is located in a section of Manhattan with large minority and new immigrant populations. Many of our students do not have English as their native language. Of course, there are programs within the College that address these deficiencies, but it became clear to me, as I read the student journals, that even students who had passed ESL

and writing courses had not yet achieved what I felt to be an adequate mastery of the language. I had *not* been mistaken in feeling the necessity of a strong writing component in my reformation. Once again, it was clear to me that the understanding of a procedure without the ability to clearly explain it was close to useless for a prospective teacher.

However, as I mentioned at the start of the article, I was ever aware that the course that I was modifying was a *math* course. Journals might work well in an education course, but would surely seem alien to my colleagues who would be teaching the course. One of my students approvingly characterized the journals as ‘math therapy,’ a concept that I did not think would fly with the rest of my department. In addition, I was becoming dissatisfied with the lack of focus in many of the journals. I needed to know how well students understood concepts, but too often, well-meaning students did not discuss their misapprehensions. Instead, as to be expected in a population with such a high level of math anxiety, students often merely restated what we had done in class or what they had read in the textbook. I came to the realization that there were two, truly useful types of journal entries: those that revealed students’ mathematical background/outlook, and those that offered suggestions for changes in, or reactions to, the course.

The time was ripe for another revision. I created a series of fifty-three writing exercises. The first exercise was a ‘math autobiography’ where I asked students for their history of and feelings about mathematics. The journals had not prepared me for the flood of terror and horror this series of questions elicited. As Countryman suggests [2], it certainly “brought the issues of self-esteem and confidence ...into the open, where they can be confronted.” However, *how* to confront them was not so obvious. I offered encouragement as best I could and tried to keep the lines of communication open. In the felicitous cases, this communication offered both of us an opportunity to observe their growth. One eloquent student, who entitled her autobiography “Math Virgin,” began the course by saying, “I have only a few memories of my early math experiences and they’re all bad.” Through the semester, I watched her abilities and confidence grow, until by the end she was using words like ‘fun,’ ‘exciting,’ and ‘stimulating’ as modifiers for mathematics. As I implied earlier, the journals were not only useful as a window into the students’ minds, but as sources of comments and critiques on the course itself. They helped me clarify material and activities that had appeared muddled or unfocused to the

student. So the last writing exercise also maintained the journal format for what I called a ‘final journal entry.’ In it, the students were to discuss their reactions to the course. They were to tell what they had gained from the writing exercises and activities, what they had enjoyed, and how they would change the course, with a strong admonition to BE HONEST. THEY WERE! Before I discuss the reactions to the other revisions I made in the course, this new series of writing exercises, and a set of activities I created, it seems appropriate to describe the final revisions themselves.

In the eventual course description prepared for NYCETP, *Math 185* remains fairly conventional in terms of topics taught, and indeed no particular approach is mandated in many of the lessons. However, there is a lot of opportunity for extra problem solving (which is the *first* NCTM Standard [1]) and collaborative work built into the course. I created four, full-period activities, one on each of the four major topics in the course. They are to be done by students using manipulative materials in a math resource room. (My difficult, but successful battle for the creation of such a collaborative learning center, ‘owned’ by Mathematics, is yet another story.) Additionally, I created/compiled twelve, brief collaborative learning exercises, consisting of more challenging problems or ‘mini-activities,’ which required no manipulatives and could be done in a regular classroom on a weekly basis. In the new, improved writing component, besides the autobiography and course summation, students were required, every week, to submit relatively brief answers to four or five ‘thought’ questions: either, challenging problems, explanations, or analysis of fictitious students’ work; or, comments on material taken from various ‘real-life’ sources.

Now we come full circle to my opening remark. Importation of educational reform into a course, which has been ‘conventionally’ taught since time immemorial by a species of department resistant to what it views as flighty fads, is tough. Now, before a posse of mathematicians saddles up to bring me to justice, let me clarify my remarks. I am not a whole-hearted constructivist. I believe there is a place for lecture in the classroom. I admit to a hankering for a tee shirt that says, “...because I’m the teacher.” However, I do believe that, unless we get our students to be more actively involved in and to take more responsibility for, their own learning, ...well, we can all finish that sentence. So how was I to effectively preach my middle way?

The activities were the easy part. After piloting them in my own classes, I offered to run them with colleagues also teaching the course. I invited faculty to attend these jointly run sessions via notes, fax, email, and unashamed begging. Clever notices piqued curiosity. (“Don’t miss a chance to obtain tickets hotter than those for the new Rolling Stones’ concerts. And Dave and I are younger [not by much!] than Mick.”) About fifteen members of the Mathematics Department came to observe the activities and the responses were almost universally positive. One senior member of the Department commented, “What I saw deserved videotaping...terrific interaction, exploration.” The chairman mandated that the activities were to be an official part of the course. The inclusion was especially easy, since by then we had the resource room with a file drawer containing the materials for each activity. I met with instructors each term to go over the activities and am in the process of writing up a cover sheet for each investigation for the faculty members teaching the course. I think I’ve clinched the sale.

The students were even easier to convince. They fairly uniformly adored the activities from the start. An “A” student who began the course saying (in answer to a question from the autobiography) that she was too busy to work with other students and found group work to be a waste of time, ended by noting that she found the activities “challenging and intriguing...[they] made the concepts much clearer.” Even a failing student, who felt the course was unfair and demanded too much work, relented by saying, “I really enjoyed the activities.” The universal cry, especially from the weaker students, was, “More activities!” Now some of this may be ascribed to the fact that there was a strong ‘game’ flavor to the activities. Furthermore, students enjoyed the interaction with classmates, often finding the act of explanation to be empowering. To wit, “Today I suggested to Geraldo that there was more than one answer and *I proved it to him!*” Many of them echoed the student who, in referring to using colored chips to represent signed numbers said, “I wish my teacher in junior high school had known about this.” Students that were already in classrooms with children brought some of the material I gave them to those classrooms. One student, thrilled by the reaction of her class to a challenging problem, said, “I just love it when the children get into a heated discussion about math.” Amen!

One full-period activity is to be done as an introduction to number bases and a review of positional notation. The students earn and spend ‘money’ in fictional countries

with base six and four currencies. Of course, the words “number base” are not used. They use the ‘money’ in various situations and then answer a series of questions designed to lead them to a deeper understanding of arithmetic operations in our positional numeration system. Student responses to this activity were almost invariably positive. One woman said that she finally truly understood what ‘carrying’ in addition meant. Another rejoiced that, “I [now] view math as part of life.” A mother remarked that she had begun playing the game at home with her children. My favorite comment was, “[the activity was] a game in which you never lose because you are learning and having fun at the same time.”

To be fair, the response was not 100% positive. One foreign student felt uncomfortable with the interactions within his group, feeling that his classmates were not serious or respectful enough. Another thoughtful student felt that the activities provided some interesting facts, but did not allow enough time to “learn to apply the formulas to various types of problems that might arise.” She felt that they sometimes “took time from the lesson which could have been spent preparing for the topic.” I’m sure many math faculty would agree with her, as perhaps even I would, but the activities did not absorb the bulk of the semester. I mostly taught by ‘imparting knowledge,’ keeping the students active by having them present problems and calling on them for answers during class.

What of the new writing exercises? Of course, *I* felt that they were a great success. I was able to continue the dialogue with the students and I tried to keep my comments as encouraging as I could. I was able to become even clearer about their misunderstandings and direct them to the kind of material they needed to study and urge students to see me to talk about the ideas. The applications drawn from newspapers or classroom scenarios made the fourth NCTM Standard, making connections, come alive. All the difficulties in using mathematical language and the confusion of mathematical terms with those same words in natural language, as discussed above, were even more clear than they had been with the journals. I also could see the clarification process, as when students, who could not distinguish between the concepts of factor and multiple, were able to do so after an activity that focused on these concepts. It certainly was work to correct and comment on the writing exercises, to give hints and encouragement on the problems, but I felt that I was amply repaid by the learning that I saw occurring. Also,

some of my best students were lavish in their praise of the use of the writing exercises, saying, “I would feel much more comfortable [at the end of the semester] if I ever were to have to explain [math] in words.” Another rhapsodized about “how wonderful it is to speak in numbers and words.” Many asserted that the writing exercises had helped them to get a better understanding of the topics. However, only a minority unreservedly approved of the required writing, and they were the people in the class who felt comfortable with the process of expressing themselves. The majority of the students did not really like this type of assignment. They found it “hard,” “vague,” “mind-wrenching,” “way too much,” and “too demanding,” to quote a few negative comments. Even when they admitted it was ‘good’ for them, they likened it to a dose of castor oil. They told me it was too time-consuming and they resented it being given in addition to conventional homework. Suddenly the writing had gone, for some, from being ‘math therapy’ to ‘math punishment.’

It was clear to me that there were two major components in this unfortunate transformation. The first hinges on the answer students gave to the question, “Are you busy?” in their autobiography. Most said numerous obligations and responsibilities filled their lives. The vast majority had jobs, some full-time, as well as families that included one or more children. Many of the women headed single-parent households. Others said that they were taking a large number of credits that semester because they needed to finish their degrees quickly. Their final summations revealed that they found that the course demanded too much of their time. The new writing required much more time and effort than the journal entries I had required in the past. The second, related issue is a fact just glancingly mentioned in the first paragraph of this paper. *Math 185* was, until very recently, the *single* college-level course required of pre-service teachers, with a prerequisite of only about 1½ years of high school mathematics. The reformed course, with its increased emphasis on the importance of explanations and higher order thinking, demanded a stronger background than this requirement. Through my unrelenting nagging and the intercession of the New York State Education Department, the College has now strongly recommended the adoption of a new prerequisite for *Math 185*: a NYCETP-developed Quantitative Reasoning course. I am confident that as the students enter *Math 185* strengthened by this new requirement, they will find the demands of the course to be more reasonable.

There remains one other, dirty little reason that the students do not extol the virtues of the course writing requirement. I grade it. I use a scale of 1 to 10, rarely giving a grade below 6 to a serious effort. I tell them I will drop the lowest three grades and count the average of the remaining exercises as one test. Generally, this score helps most students' averages, but they don't like being graded. How did such an initially empowering activity come to such a sorry pass? I started grading for two reasons. I felt the need for alternative assessments and the writing provided an excellent vehicle for evaluating creative or reflective approaches to addressing problems. Open-ended problems, questions on each of the four activities (horrifying for those students who enjoyed the activities, but not these questions), discussions of communication issues, and analyses of complex situations are all forms of alternative assessment. I use these along with quizzes and class participation to augment the grades from class tests and the final. (See Kulm [5] for an excellent discussion of assessment options.) But it is amazing how appealing rote recall of algorithms can appear when compared to being asked to provide cogent, well-phrased answers and explanations.

Finally, what about my colleagues? Well, they hate the idea of the writing exercises almost as much as my students. Reading such material in the context of a math class seems to feel no more natural to most people than writing it. However, I have acquired a powerful ally. The New York State Education Department has strongly encouraged 'writing across the curriculum' in liberal arts classes for education students. The new state licensure exams, including the parts that test mathematics, require that students exhibit proficiency in reading and writing. I will continue to refine the writing exercises, hone the activities, and lecture when I see fit. My strongest students offer me support in my own middle path between the needs and demands of mathematics and those of education. A student, who began the course fearful of mathematics and achieved a grade of "A," comments on a strongly constructivist methods course in which she was enrolled concurrently with my course. "I like the hands-on approach to teaching, but it moves too slowly for me. [Also,] I have a yearning to conquer my fear of 'traditional' math. I need to be challenged and I want to learn some of the 'deeper' concepts that I could never grasp. I find the mixture [of approaches] to be the most stimulating." Viva language and the 'middle way!' ■

Bio

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References

- [1] *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 1989.
- [2] J. Countryman, *Writing to Learn Mathematics*, Heinemann Education Books, Portsmouth, NH, 1992.
- [3] S. Tobias, *Overcoming Math Anxiety*, W.W. Norton & Co., New York, 1993.
- [4] D. Pimm, *Speaking Mathematically*, Routledge & Kegan Paul, London, 1987.
- [5] G. Kulm, *Mathematics Assessments*, Jossey-Bass Publisher, San Francisco, CA, 1994.