

## USE OF THE WORLD WIDE WEB IN MATHEMATICS INSTRUCTION

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Many mathematics instructors may find they can use the World Wide Web to distribute information and facilitate discussion and interaction in their classrooms, while actually reducing their administrative workload. Here is a discussion of some of the benefits (including better student understanding!) which an instructor might enjoy from taking the plunge.

There is a lot of discussion in the literature about the ability of new technology to improve the teaching of mathematics, primarily by facilitating better and more exciting presentations by the instructor and better participation by the student in the learning and doing of mathematics. There seems to be little attention paid to the question of how the use of technology, and the World Wide Web in particular, can enable a reduction of the instructor's administrative burdens and improve the real core of the teaching experience—the direct interaction between student and teacher.

In most colleges and in many high schools, access to the World Wide Web is nearly universal. If course documents are already produced on a computer (or easily could be), then significant benefits can accrue by using the web in mathematics courses – including a lightened administrative load, the ability to reach students more individually and efficiently, and more involvement and success for the students.

Future teachers are likely to be among those responding most enthusiastically, since they will be aware of the rapid spread of technology throughout all levels of education. They will expect to be required to use technological resources when they themselves are teaching, and they will welcome the opportunity to become familiar with the web and its various uses.

Using the web is neither necessary nor sufficient for a contemporary college or high school math course. A perceived need to be involved with the newest and hottest thing is not motivation enough. An instructor should be able to see clear and obvious benefits for his or her classes and teaching style.

The web has been used for a number of years in both college and high school math classes, with varying degrees of sophistication and with differing objectives for the role of the web, depending on the teaching style of the instructor and the capabilities and resources of the students. It is not difficult to acquire the minimal specific knowledge required to post documents to the web, and there are many different approaches to mitigate the difficulty of inputting math symbols into a computer. A good first step is to take a minimalist approach. Whatever the structure of the class and teaching method, the instructor can simply replace, whenever possible, documents printed and distributed on paper with the same documents made available on the web.

With that change only, and despite the difficulty of writing mathematics in text editors and word processors, an instructor can use the internet to increase his or her efficiency and improve students' understanding. At lower levels of math, the effectiveness of homework exercises which drill basic computational skills can be improved. In upper level courses, students can receive better coaching on the writing, logic, and organizational skills necessary for writing good proofs. Savings in the time required to perform basic administrative tasks augment this improvement by freeing up time to teach.

Ten reasons to use the web in math classes include:

1. The instructor is probably already producing the basic documents needed for the web.

Nearly any document created on a computer can now be saved to HTML format for posting on the web, retaining formatting, graphics, and so forth. The program MathType—which facilitates the creation of formulas and equations by pointing and clicking—is one convenient way to upgrade the mathematical symbol set built into Word and similar programs, and ease the problem of document creation. A syllabus becomes a home page for the course, with links, to homework assignments and solutions, problem discussion, test results, grades, and other interesting web sites that may interest students.

There is, of course, no shortage of commercially available software to help do all of this in a more professional and automated manner, and there are web sites which will “host” (for free) a home page for an instructor, without requiring any additional software [1]. Taking advantage of such services may improve the look and smoothness of web pages. However, it may require more effort than the instructor is currently expending, since

existing documents often cannot be used, and new ones must be custom made. However, for large courses, with many instructors or teaching assistants, it may be that what would be a large amount of work for a single section, can efficiently be spread over many sections. At any rate, an instructor certainly ought to examine the possibilities of such sites, including those often offered by publishers.

## 2. Paper work tasks will consume less time.

Posting the syllabus on the web makes it the home page for the course. Links are easy to add to homework assignments, any take-home tests, and the solutions to homework assignments, tests, and exams. The same computer programs—especially Microsoft Word—and hardware (a scanner is useful) which created documents to be printed will create the documents to post on the web. So while creation of the documents is identical to the pre-web days, there is no longer the distribution of hard copies, which saves the time of making copies, distributing them, and replacing them when students lose them. When the need arises to update one of the documents, it is particularly easy to do so and then e-mail an announcement to the entire class with one click. Since a student can access e-mail and the web totally at the student's convenience, there is no longer any reason not to be up-to-date on class requirements.

The more documents an instructor can place on the web, the more time will be saved from the administrative tasks of copying and distributing them. On the other hand, posting and maintaining a web site will be non-trivial, also. However, there will be a net gain in time that can then be used for teaching instead of paperwork.

## 3. Web pages do not need to be flashy to be useful and effective.

Instructors should not waste time trying to create exciting web-based material. Even the burden of making a web-available copy of lectures would increase the workload significantly, overwhelming the benefits cited above. There is plenty of interactive material out there (often from textbook publishers), with more arriving daily. As an instructor finds such material which can be helpful to students, it is easy to add links from the class home page to that material. But, nothing should be done to encourage students to believe they needn't come to class. Few of us would have become teachers if we believed direct, in-person contact with our students was of no value.

#### 4. Students will appreciate the additional access to course material.

The fact is well promoted that students have far more diverse backgrounds and current lifestyles than was the case thirty years ago. More students are working, more students are married with family obligations, and more students are commuting greater distances. Since it will be a requirement that students have computers, students will see only an upside to the use of the web.

It is amazing how often a student will send e-mail to apologize for missing a class, and to find out if there are any additional or changed material or assignments. So, students are able to access information when it is convenient for them. They will also be able to work from more convenient locations. It is not at all unusual today to have several students commuting fifty miles or more. Such students particularly appreciate the ability to submit work without having to come to school. The use of the web thus allows the additional flexibility needed by the changed circumstances of many students, without requiring lower expectations and standards for the quality and amount of work the students must perform.

#### 5. Instructors will have another effective avenue of communication with students.

Because of their more complicated schedules, students often find it hard to take advantage of office hours. The expectation that work will be submitted via e-mail, and conversation and discussion encouraged, enables a student to engage in discussions with teachers and fellow students that might not otherwise occur. For instance, it is not hard to employ a system by which students can post comments and questions to a web page. In addition, students in lower level classes, who may be there simply to satisfy the requirements of their major, often feel they are not “math people,” and seem reluctant to reveal their difficulties in face-to-face meetings with their teachers. Some of this reluctance is sometimes reduced when questions can be sent via e-mail (so they can be carefully thought through) and responses can also be read in private (eliminating, perhaps, embarrassment at an “obvious” answer).

The experiences of a number of instructors suggest that this greater discussion occurs both with small sections of ten or fewer students, as well as in larger, lower level courses with section sizes of thirty to forty. This would probably be even more true in the larger classes of several hundred students which occur in some schools, since students are likely to feel even more removed from direct interaction with their instructors.

In fact, in one case, an *Introduction to Analysis* section with six or eight good students was taught with heavy reliance on the web and e-mail for communication between students and instructor. The course was taught during the second semester by an instructor who did not use the web for the course, and who was not even particularly comfortable using e-mail for mathematical discussions. Nonetheless, he encouraged the students to continue e-mailing him with questions. Even though he seldom responded to the e-mail, he found it helped him prepare better for class, since he had a better inkling of the areas of confusion for his students.

6. The difficulties of writing mathematics on a computer can be reduced.

The difficulties of writing mathematics on a computer has been reduced, but probably not completely overcome. It is not yet feasible to require students to use any particular word processor, let alone augment it with a program like MathType. One approach is to develop a Convention on Denotations for each course, to enable students to write mathematics in a text file (the simplest e-mail files). Of course, students who have the higher-powered programs should be encouraged to use their features. Here's an excerpt from such denotation conventions, which typically develop and change over the life of the course, for an *Introduction to Abstract Algebra*. It is rare to encounter a mathematical expression for which a satisfactory "text-only" substitute cannot be found.

**Fractions**

For simple fractions:

$$\frac{1}{2} = 1 / 2$$

$$\frac{x}{x^2 - 1} = x / (x^2 - 1)$$

For more complex expressions:

$$\frac{x+3}{x^2+3} = \text{frac}(x+3, x^2+3)$$

**Functions**

→: Use ->

Write f(g) for  $f \circ g$

**Greek alphabet**

Write alpha for  $\alpha$

Write beta for  $\beta$

Write gamma for  $\gamma$

Write phi for  $\varphi$

**Infinity**

Use infinity for  $\infty$

**Matrices, Determinants, and Tables**

Write  $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$  as

matrix ( a b c

d e f

g h i )

7. At the lower levels, more drill can be encouraged.

In our lower level classes, such as precalculus and calculus, we normally have thirty to forty students and no administrative help for correcting homework and tests. As a consequence, most students are assigned very little, if any, homework that is corrected and returned. Most of the student homework exercises require calculating an answer, not writing a theoretical proof; and, most of our grading does not give partial credit for an answer that is "close," or that used the right method, but did the calculations incorrectly.

Anecdotal evidence supports the belief that the average grade and, of course, understanding, of a lower level class increases directly with the total amount of corrected homework the typical student submits. As use of the web increases, allowing more corrected drill by the students, it is reasonable to expect an increase in the percentage of B's and C's in a class. On the other hand, it is probably rare that homework of this sort would make a significant difference in the number of A's in a class. Depending upon the size and level of the class, several different homework options/approaches are available.

In order to create a reasonable work load for the instructor, minimize the difficulties for students of writing math on a computer, and yet not deprive the students of consistent feedback on the quality of their work, one can choose to simply require submission of the answers to the homework assignment. However, with no supporting work shown, one also sacrifices the ability to comment on the details of the students' efforts.

Multiple choice exercises make the work particularly easy for the student to submit and for the instructor to correct, but multiple choice exercises allow a student to work towards an answer, and guides the student's thinking in too limited a manner. Another approach is to require that the student find the answer, rather than simply choosing one. This approach requires greater reliance on a Convention on Denotations, and requires greater effort and time for correction. In addition, the instructor should write out on a computer all the answers, to assure that each answer can easily be written using the class Convention on Denotations.

A third possibility is now being offered by some web sites (such as that developed by Brooks/Cole, the textbook publisher) which allows the entry (and automatic correction)

of free form answers according to “calculator syntax.” When a phrase such as “ $x^2$ ” is entered for an answer, the phrase “ $x^2$ ” appears on the computer screen, so the student has a chance to review and correct his work before submitting it. None of these approaches is perfect, but each offers significant advantages, particularly compared to the approach of leaving all homework exercises to the students’ discretion.

Textbook publishers and others have programs which allow automatic generation of groups of exercises (from an exercise “bank”) satisfying certain criteria, and sometimes automatic grading of those exercises. While it is possible that the use of such programs would actually take more work than simply continuing present methods, each instructor should make an individual judgment. That judgment would depend on whether a quiz is taken by a single section of thirty students or, for instance, 450 students in fifteen sections.

8. At the upper levels, student logical and writing skills can be improved.

In our upper level courses, such as *Introduction to Abstract Algebra* or *Introduction to Topology*, writing proofs is heavily emphasized. So the advantages to the student of writing up homework on a computer (with text editing abilities) are significant. Because rewriting is so easy on the computer, in some small classes of ten or fewer students I establish a deadline for draft submissions, which encourages students to begin their work early and to seek help when they encounter problems. For instance, here’s how one student’s ideas evolved as I commented on her efforts to express the concept that one set was the Cartesian product of two others.

Her first solution was:

$$iii) x = y \times z \text{ iff } \forall (a,b) ((a,b) \in x \leftrightarrow a \in y \wedge b \in z)$$

I noted to her:

This is a good beginning idea, but is it possible that some set  $w$  could be an element of  $x$  if  $w$  is not itself an ordered pair?



She next came up with:

$$\text{iii) } x = y \cdot x z \text{ iff } \forall z \exists (a,b) (z \in x \leftrightarrow z = (a, b) \mid (a \in y \wedge b \in z) )$$

My comment to her was:

Ok – here, the use of the vertical line isn't right, and you seem to have used the variable  $z$  for two different purposes. Additionally, the statement would be much clearer if you moved the existential quantifier inside the major set of parenthesis. I think you are trying to say something like, “iff for all  $z$  ( $z$  is in  $x$  iff there is some. . . )”

And I don't think you really want to posit the existence of an ordered pair, rather than the two elements ordered in the pair, since those are the ones you subsequently go on to describe further.

Her final response was:

$$\forall w (w \in x \leftrightarrow (\exists a \exists b) (w = (a, b) \wedge a \in y \wedge b \in z) )$$

Furthermore, in some cases the text editing capabilities of the computer can aid the student in writing better proofs. In a class such as *Introduction to Set Theory*, using these word processing capabilities might help students gain more confidence about how to start or continue a proof because it is so easy to reuse an existing proof! For instance, here's how one student responded to the challenge of deriving a proof of one of DeMorgan's Laws from a proof of the other, by changing “intersection” to “union” and vice versa, and by substituting “and” for “either .. or.”

To see that  $\overline{A \cup B} = \overline{A} \cap \overline{B}$ , just note that

$$\overline{x \in A \cup B} \text{ iff } x \notin A \cup B \text{ iff it is not the case that (either } x \in A \text{ or } x \in B)$$

$$\text{iff } x \notin A \text{ and } x \notin B \text{ iff } x \in \overline{A} \text{ and } x \in \overline{B} \text{ iff } x \in \overline{A} \cap \overline{B}$$

To see that  $\overline{A \cap B} = \overline{A} \cup \overline{B}$ , just note that

$$\overline{x \in A \cap B} \text{ iff } x \notin A \cap B \text{ iff it is not the case that (} x \text{ is in } A \text{ and } x \text{ is in } B)$$

$$\text{iff either } x \notin A \text{ or } x \notin B \text{ iff } x \in \overline{A} \text{ or } x \in \overline{B} \text{ iff } x \in \overline{A} \cup \overline{B}$$

Students also get a better sense of what it is to do mathematics, since when they engage in some back and forth discussion with the instructor or other students, they begin intuitively to understand that a proof often does not emerge full-blown in an instant, but may consist of iterations of some good guesses and insights leading to roadblocks, leading again to some more guessing and insights. And, the easy ability to share different proofs of the same exercise on the web can often stimulate students to a better appreciation that there is more than one approach. It is particularly fun to be able to e-mail a student to say that she submitted a proof better than the one I had posted.

## 9. Correcting homework and tests by e-mail will be more effective and efficient.

When homework is sent by e-mail, it tends to be on time, allowing the instructor to correct it when and where he or she wants. And it is legible! There are no words struck out, no ugly erasure marks, no arrows pointing the instructor to answers scribbled in corners or on the back of a page. This can be particularly valuable in the lower level courses, where a student's lack of confidence and interest in the subject may be subconsciously (or consciously!) affecting the quality of the student's work. The job of correcting the submissions thus goes more quickly and pleasantly, allowing the assignment of more work, or the correction of a greater percentage of the exercises assigned.

As remarked earlier, because lower level courses are frequently so large and the prospect of correcting homework so daunting, instructors often rely on the fact that a college student should be responsible for understanding how to work correctly all assigned exercises, as well as for follow-up with questions to the teacher or to other students until

the understanding is complete. But the fact is that many of the lower level students need to learn the skills of studying and self-discipline as much as they need the content of the course. At the lower level, students are seldom skilled or enthusiastic about math, else they would already have taken the courses in high school. Most enormously underestimate the benefits of practice and drill at this level. In particular, if mathematics is the study of pattern, then skill at mathematics is closely related to the ability to recognize patterns. It is, of course, impossible to have a chance of discovering those patterns if not enough work is done, and not enough examples are seen. So a way is needed to give these students a chance to learn the good studying habits that are so essential, but without allowing the burden of correcting homework to take all the joy out of teaching. Submission of simple answers by e-mail seems one good approach to this conflict.

Homework submitted by e-mail allows both the student and the teacher to have copies of the work, and it allows a "marking up" of the work which doesn't destroy the original. And the ease of writing and editing clearly on the computer may enable the instructor to make more extensive comments than might be the case with handwritten student submissions and handwritten corrections.

At the upper level, in particular, where the writing and logical skills needed for proofs (and for life) are stressed, the fact that the work is submitted in a form which allows the instructor to copy parts of it easily is a terrific benefit. What the student has written can be compared to what the instructor would have written; pointing out ways to eliminate ambiguity, to be more rigorous, to be more succinct, and so forth. And, because students frequently have similar difficulties, it is easy to recycle comments originally meant for one student to other students.

All of this greater ease at making comments, suggestions, and corrections can help improve teacher/student relations in ways that somewhat compensate for the students' busier lives and their consequent inability to see the teacher face to face as often as in the "good old days." For instance, a student whom I never saw outside of the classroom had asked me a question in class about ordinals; subsequently, she found it was explained in a homework exercise which developed the idea of the form and uniqueness of the  $n^{\text{th}}$  ordinal. Her submission was:

10. 1st member:  $\emptyset$ , since  $X_{x_0} = \{x \in X \mid x < x_0\}$  and  $x_0$  is the minimal element.

2nd member:  $\{\emptyset\}$ .

The  $n$ th member is given by  $\{x_{n-1}, x_{n-2}, \dots, \emptyset\}$ .

This implies that there is only one  $n$ th ordinal, uniquely determined by the  $(n-1)$ th ordinal.

Ah ... should have done the homework before I asked that question in class.

My response was:

10) is good, too. And while the more homework you do the earlier, the more you get out of class:

You can't always be so far ahead that the text holds no surprises!

Furthermore, the web enables the instructor to encourage students to learn from each other. Good student proofs can be posted with (or instead of) the instructor's solutions, and one student's solutions can be easily quoted to another student. In some upper level courses, a spirited culture of both competition and cooperation has been established; students compete to be the first to post correct answers to certain challenging problems, and cooperate in solving problems by posting suggestions, proposed solutions, and corrections in a "thread" relating to a particular topic.

10. More time for teaching.

I would be interested in your experiences relative to and expanding upon my ten reasons, and I welcome your e-mail. ■

## Reference

[1] VQuad com, Internet: <http://www.vquad.com>