

# USING MANIPULATIVES IN UNDERGRADUATE MATHEMATICS COURSES

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D. MOORE

*Dept. of Mathematics, University of Puerto Rico, Mayagüez*  
moore@shuttle.uprm.edu

## Abstract

Students in undergraduate mathematics classes not only benefit from the use of manipulatives in the classroom, but also enjoy them. This paper specifically outlines one successful activity that used manipulatives in a large section of a precalculus course and then explores possibilities in other courses. It also addresses the use of mathematics manipulatives as a platform to introduce both active and cooperative learning in a large lecture setting.

## Introduction

Learning requires reflection and occurs best in an atmosphere that provides enjoyable interaction [1]. In fact, student-student, as well as student-faculty, interactions can have a great impact on learning [2]. Leinhardt's work suggests that students must build on prior knowledge, and that the act of learning is a social act. Learning involves construction of knowledge, an active process. As students learn, their knowledge is produced and then transformed by themselves as well as by their interactions with others [3].

Active learning has been called learning in which students are doing things and then thinking about what they are doing [4]. Science professors have had success implementing active learning in their large lecture sections, often in conjunction with cooperative learning [5-7]. Cooperative learning has been examined in numerous studies as a teaching strategy to promote active learning. In fact, cooperative learning has been called the most researched of all instructional methods and its impact has been addressed at multiple levels of student achievement [8-10]. Although it initially influenced K-12 institutions, there is a definite growing interest in cooperative learning at the undergraduate level [11,12].

This article will relay how a specific activity was implemented in a large lecture section of a precalculus course in an effort to incorporate both cooperative and active learning.

## Targeted Course

One professor assigned to teach a section of *Precalculus I* at the University of Puerto Rico, Mayagüez (UPRM) with 84 students, decided to implement some cooperative learning activities and demonstrations, both of which used manipulatives, to improve student interest and

learning in her course. The professor was inspired to do this by Eric Mazur who has been implementing small-group activities in his large physics lectures [13].

The *Precalculus I* course is required of all UPRM students. It is the most basic three-credit mathematics course offered, although students entering with insufficient backgrounds in mathematics are directed to a zero-credit basic mathematics course. Passing rates tend to be low in the *Precalculus I* course, especially in the second semester when most of the students are repeating the course due to their unsuccessful attempts in the first semester.

### **Experimental Activity**

On the first day of class, the professor decided to adapt an exercise taken from Marilyn Burns [14]. The students were each given a sheet of centimeter-squared paper and asked to trace around their hand, then to find its area. The professor then circulated throughout the auditorium-style lecture hall looking at each student's work. Common questions that arose with the professor's responses are given below:

*Student:* Does it matter if I open or close my fingers when I trace around my hand?

*Professor:* (Showing her hand in both positions) What do you think? Is the area affected?

*Student:* How can I find the exact area?

*Professor:* You don't have to have an exact number, just estimate. How would you go about estimating? [Some students had no idea where to begin; in those cases, the professor encouraged them to ask their neighbors how to start.]

*Student:* What formula should I use to find the area?

*Professor:* What formulas do you know for area? How could you use them?

Students used many ways to calculate the area of their hands. Some counted the number of squares on the centimeter-squared paper for an answer. Others divided the hand into a combination of rectangles or rectangles and triangles, and summed the answers from formulas they knew. Others drew a small rectangle inside the tracing of the hand and another outside the tracing of the hand to determine a lower and upper bound, and then estimated an answer using them.

Once students had their answers, the professor asked them to compare their answers with their neighbors. Students often held their hands together palm-to-palm to compare and to see if the bigger palm did indeed have the bigger area. Others just overlaid their tracings and held them to the light to compare. Many realized after talking with other students that  $\text{cm}^2$  were the appropriate units. In more than one instance, a student with a much larger hand had calculated a smaller area than a student with a much smaller hand. In those cases, the students tended to check with others to see which area needed “to be fixed,” recalculated that area, and then compared answers again.

The professor concluded by demonstrating the exercise on an overhead projector with her hand, using many of the ways that students in the class had. For the second part of the activity, she asked the students to take a second sheet of centimeter-squared paper and to draw a square, not a rectangle, on it with the same area as they had found for their hands. She asked them to work on this individually. After two minutes, she told students they could discuss their ideas with their neighbors. During this activity, the professor walked up and down the partitioned aisles to see how students were doing. The student who finished first had an area of  $121 \text{ cm}^2$ . The professor let the activity continue for about five minutes. Then she asked the student with  $121 \text{ cm}^2$  hand area to give his results. She went around the room soliciting responses from other students. Some students had used rectangles even though instructed not to do so because they couldn't find a square with integer side lengths that would work. Another common way to solve the problem, for example with a hand of  $105 \text{ cm}^2$ , was to say that the square had to have sides longer than 10 cm but smaller than 11 cm, so 10.5 cm was used. Some students pulled out calculators and would find even better decimal answers using trial and error. A few even knew that the side of the square should be the square root of the area of the hand.

The above activity was described in detail because it is a very rich activity. First, it was appropriate to use on the first day of class. It involved mathematics but was still an icebreaker. The estimation of the hand area did take some time, but that allowed the professor to see each student individually face-to-face—at least for a few seconds. This is not often done in a large lecture section. Although the numbers were not calculated, the professor felt that a higher percentage of these students came to see her during office hours than usual. More specifically, the activity:

- helped students recognize the difference between area and perimeter;

- showed the power of estimation;
- helped teach the difference between units of measure in one-dimension and square units used to measure two-dimensions;
- introduced the concepts of upper and lower bounds;
- showed that mathematics problems are not always solved with a formula—but that known formulas can be adapted and applied in creative ways;
- showed that there is not one right way to find an answer;
- established the importance of checking to see if an answer is reasonable;
- emphasized that even though a square is a rectangle, a rectangle is not necessarily a square;
- pointed out the difference between an estimation (a little over 7) and an exact number (the square root of 51);
- confirmed that numbers are hardly ever integers in real-life situations;
- allowed students a deeper, more concrete understanding of square roots.

One activity has been outlined above in detail. The professor also used a spring with weights and increasing rows of similar pattern blocks in two different activities designed to teach the definition of a function and multiple representations of a function, respectively. The spring was a Slinky®. It was suspended from the classroom ceiling with a styrofoam cup attached to the bottom. The cup acted as a “basket” for the weights which were coins. An initial measurement was taken to see how far the bottom of the “basket” was from the floor. Additional measurements were then taken to determine the distance from the basket to the floor when one quarter was added to the basket, then two quarters, then three. The students were then asked to determine how many quarters would be needed in order for the basket to touch the floor. As long as the Slinky® is not stretched too far, this develops into a nice decreasing, linear relationship. The professor implemented only the three activities, but chose them to address topics that seemed most crucial in the *Precalculus I* course.

### **Another Course**

The same professor also taught the *College Geometry* course in the same semester. Rather than spending the first few class periods reviewing basic geometric concepts that students should bring from high school and prerequisite college courses, the instructor decided to assign two- and three-dimensional string art projects to cooperative groups of students whose instructions were written using the basic terminology that students should know. Two of the

construction projects included: a three-dimensional figure with concurrent lines on parallel edges of an icosahedron; and, a three-dimensional figure with eight raindrops on an octahedron. The instructor noticed that students enjoyed the cooperative projects, but that they also seemed to benefit from the construction of the models. It was unclear if they benefited more from the application rather than sheer memorization of the terms or more from the actual hands-on involvement of creating the models. Regardless of which was more beneficial, the students did master the terms much more quickly and enjoyably than they would have in the period of two to three lectures and a quiz.

### Observations

Table 1 shows relative grade distributions for the experimental section of *Precalculus I* which had 84 students in the spring of 2001. Even though most students typically enrolled in this course have either failed *Precalculus I* in the fall semester or have had to take the zero-credit basic mathematics course, 65% receiving a passing grade of “D” or better (60% received a grade of “C” or better). This passing rate for the experimental section is higher than passing rates have been in past years in either the fall or spring semester (*Precalculus I* passing rates have ranged between 45% and 63% over the past ten years at UPRM). So, students with weak mathematics backgrounds or students who were unsuccessful in their first attempt at precalculus are now doing just as well in a large lecture section as students in smaller sections who entered the university with much stronger mathematics backgrounds. The author acknowledges that these results are being compared against other precalculus sections taught largely by less-experienced teaching assistants and is careful not to draw strong conclusions; however, departmental regulations at UPRM dictate that 90% of all student grades for *Precalculus I* do come from common departmental examinations.

**Table 1.** Grade distributions for 84 *Precalculus I* students in the spring of 2001.

GRADE	STUDENTS (%)
A	13
B	23
C	24
D	6
F	18
W	17

## Conclusion

Professors, including mathematics professors, should rise to the challenge of better meeting the needs of all their students. Teaching strategies that allow for more active learning on the part of the student should be implemented [15]. The professor in this study tried to incorporate only three new activities. However, these activities were all connected to crucial or prerequisite topics in precalculus that could be used to span multiple ideas.

Besides implementing different kinds of activities to teach mathematical concepts, the professor tried to present material in such a way that it would capture the interest of the students. She also tried to ensure that students understood that student-student and student-professor interactions were encouraged. The curriculum can easily be enhanced with experiences that provide students opportunities to strengthen communication skills, while supplying a more active learning environment. Although it calls for more creativity on the part of the instructor, more active, cooperative work should be implemented and tested for its effectiveness in undergraduate mathematics classrooms. ■

## Bio

Deborah Moore is Associate Professor of Mathematics at the University of Puerto Rico, Mayagüez. Her interests are the integration of mathematics and science, the role of mathematics in the sciences, and the use of manipulatives to promote conceptual understanding of mathematics.

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