THE LEARNING OF MATHEMATICAL CONCEPTS AND PRINCIPLES THROUGH THE INTEGRATION OF TECHNOLOGY IN LABORATORY ACTIVITIES

E. MORERA GONZÁLEZ
University of Puerto Rico, Cayey
edwinmorera@yahoo.com

Introduction

The role of educational technology—computers, calculators (scientific, graphing, programmable, and others) Calculator Based Laboratory (CBL), sensors, videodiscs, CD-ROMs, and telecommunication networks through which real data can be accessed—are instruments that aid the learning process in mathematics, and have given teaching an innovative quality, capable of greatly influencing mathematical knowledge and reasoning. Although it is not the solution to teaching and learning problems in mathematics, there is evidence that technology will slowly become a catalyst agent of change in mathematics education [1].

Thanks to the possibilities offered through the dynamic manipulation of mathematical objects in multiple systems of representation within interactive structures, technology opens spaces that allow students to have new mathematical experiences which are hard to achieve in a traditional medium; in which they can manipulate directly mathematical objects within an exploration setting. In considering solutions to problems, such as the approach to the teaching of mathematics and the construction of knowledge as a learning model, some authors [1,2,3] have established that the pedagogical principles that serve as the foundation for the constructivist paradigm may contribute to the integration of new technologies in education. Through this approach, qualitative changes in the nature of learning and teaching in mathematics may be promoted.

Laboratory Activities as an Option for the Learning of Concepts

The results found in mathematics courses that follow traditional teaching methods, such as the exposition of content as a finished body of knowledge, the theoretical administration of results, and the mechanical solution of problems point toward changes that lead to the consideration of more active methods. Using these methods, students explore, make conjectures and deductions, elaborate justification, test arguments, and understand that the primary responsibility of learning lies within themselves [4]. These ideas are not new, as Polya wrote in

1975, "if learning mathematics is reflected to some degree in the invention of this science, there must be a place in it for intuition, for the plausible inference." [5]

Through well-designed laboratory activities that integrate technology, students participate actively in the process and construct their mathematical knowledge. In this setting, the students' task is not to use the technology to make calculations mechanically (this may be done in other settings), but rather to analyze and reason about the results obtained through this technology. To achieve this reflection and reasoning, we must seize the pedagogical advantages it offers. For example, the graphing calculator promotes: speed in computation, visualization, interaction, and learning from mistakes. Besides properties such as its graphing, numerical, symbolic, and programming capabilities, and its ease of use, it allows students to construct processes and mathematical objects which are complemented by the graph to attain higher levels of learning, compatible with a quasi-experimental mathematical presentation [6].

In these laboratory activities, the important element is the active construction process that links new knowledge with prior knowledge, observation, reflection, analysis, argumentation, proof of results, and others, but not the result. Instead of receiving the information in a passive manner, or simply copying the information from the professor or the textbook, students analyze the information in an active way from the start, trying to make sense of it and to relate it with what they already know about the subject. This constructive process is important because unless students construct representations of the new knowledge, making it their own as they paraphrase it and consider its meanings and implications, the learning will be retained only as mechanical and inert memories relatively void of meaning [7]. In this process, the exposition of recipes that are memorized for a brief period is out of place. Learning will be more meaningful through discoveries that occur during explorations motivated by curiosity [8].

In consequence, these laboratory activities should be designed within the framework of guided discovery, through which students are provided the opportunity to manipulate mathematical objects actively and transform them through direct actions. Also, they are designed in such a way that they stimulate students to seek, explore, analyze or process, in one way or another, the information they receive instead of only responding to it. These laboratory activities will be fruitful whenever the following is taken into consideration:

- The complexity of the mathematical content to be taught;

- The complexity of the cognitive processes involved in the learning of mathematics;
- The fundamental role that curriculum designers and faculty should play in the design and implementation of teaching situations which address students' difficulties and needs, and take advantage of technology to create spaces in which students can construct broader and more powerful mathematical knowledge.

A radical change is proposed, from a passive method based on receiving information toward an active method in which mathematical knowledge is constructed. In a setting such as this, the cooperative participation of students is fundamental [9-12]. This setting allows students to interact regularly with many of their peers, to discuss interesting questions about the course and to learn from each other. This is why it is necessary to provide an adequate physical environment where students can carry out these laboratory activities that will lead toward higher levels of learning mathematical concepts and principles.

Implementation of Laboratory Activities

The need of an adequate physical environment for the implementation of laboratory activities refers to more than a room full of the necessary equipment; it is a place where there is an environment in which students can explore the objects they study. It should be a place where students have the freedom to comment, ask questions, and make conjectures about the course matter. In this environment, the professor is available to serve as a facilitator who offers students the opportunity to verify their analysis, so that they may identify mistakes in their reasoning for themselves and generate feedback on their own knowledge. This is the way that they construct and reconstruct the object of the learning process.

The physical environment must fulfill the conditions that allow students to carry out their work, without unnecessary distractions, and promote the interaction of ideas among peers, the professor, and teaching assistants. As the students set the process of the scientific method in practice through the laboratory activity, they construct high level mathematical knowledge.

Considerations for Writing a Laboratory Activity

The preparation of a laboratory activity requires more elaboration time to achieve the exploration and discovery of a concept. The following are some general considerations that provide guidance for writing a laboratory activity [13].

1. According to the proposed objectives, the laboratory activity should fall into one of the following categories:
   - Developed before the presentation of a topic; the activity should be initiated with a problem that stimulates discussion. The discovery of concepts belongs to this type.
   - Those that include interesting applications with data that have not been manipulated because of the extent of the calculations. Questions about analysis and interpretation of the obtained results are suggested so that the experience is not reduced to a simple numerical calculation. An example of this is the laboratory in which a phenomenon is modeled and the characteristics and properties are explored.
   - Those in which the content presented is broadened or reinforced in class. The professor can provide questions to motivate the analysis of the situation and help students to observe and predict or make conjectures about the results, according to the topic previously explained.
2. It is imperative that professors master the content of the class very well, even more than if it were an expository class, so that they can provide adequate answers to questions that emerge during experimentation.
3. The problems to be studied should be carefully selected so that they are not too easily solved, but require analysis of the situation, besides being interesting and pertinent for students.

Example: Representation systems[1]

The use of technology allows the dynamic handling of multiple representation systems of mathematical objects. This is one of their relevant characteristics from the perspective of learning mathematics. Representation systems are a central aspect of the students' understanding of mathematical objects and their relations, as well as the mathematical activities that they perform when they carry out tasks that have to do with these objects [8,9,10]. External representations allow the student to organize mathematical experiences and to organize the information internally. From this perspective, a representation system is composed of a set of symbols that are manipulated according to rules that identify or create characters, operate within them, and

[1] The complete laboratory is very long, so only a brief description is included.

determine relations among them. The same mathematical object can be represented by different representation systems.

We have developed a laboratory where the student manipulates the symbolic representation, the graph, and the table of values of a quadratic function. The function $f(x)=x^2+5x-6$ is represented by students in the symbolic representation system, in the graphic representation system, and in the value table representation system (see Figure 1), among others.

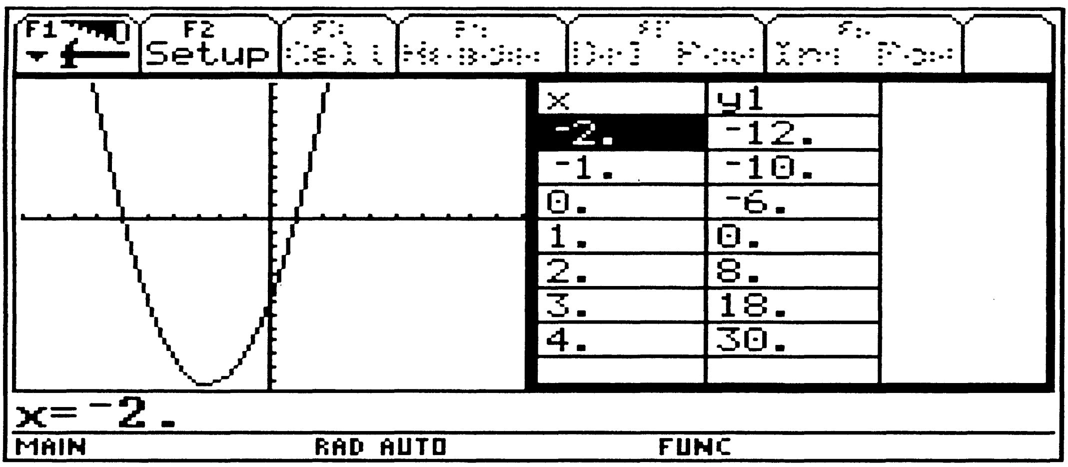

Figure 1.

The idea of representation makes it possible to characterize the students' activities as they carry out the task. The laboratory is designed so that students do syntactic transformations within the same representation system. They transform $f(x)=x^2+5x-6$ into $f(x)=(x-1)(x+6)$ or into $f(x)=(x+2.5)^2-12.25$ in the symbolic representation system; they also transfer the graph horizontally or vertically or when the dilation in the graphic representation system varies. The second type of mathematical activity that students perform in the laboratory is the translation between representation systems. That is, the relation of the function on the graphic as it goes from the base symbolic representation $f(x)=x^2$ to the expression $f(x)=(x+2.5)^2-12.25$ (in which it is possible to identify the localization of the vertex) or to the expression $f(x)=(x-1)(x+6)$ (in which the roots may be located).

In this way, students handle procedurally the representation systems *and this action* serves as a base for evolving into a conceptual understanding of the mathematical object and the mathematical relations. Student understanding evolves along two axes: one axis is horizontal along which the management of the representation systems advances from a mathematical concept; along the second axis advancement is made in the materialization process (from procedural to conceptual) of the same concept [14].

Final Comments

During the past few years, several groups have recommended the use of new educational technologies for the learning of mathematics. Some authors have pointed out that the sustained use of technology in the classroom will convert it into a setting where the student discovers, formulates conjectures, justifies and tests arguments. Our experience has been to use these laboratory activities as an additional experience to the traditional classroom.

The development of these activities takes up more time and effort, since the needs of the students must be considered. From this viewpoint, the textbook becomes one more reference and the professor becomes less dependent on it.

The results with future teachers found in the integration of these laboratory experiences in their mathematics classes are heartening. The level of the type of questions they pose is higher than the traditional ones. The students become familiar with the way in which mathematical knowledge is constructed, promoting the compression of the epistemology of the knowledge area they will teach. The evaluations of these activities by students have been positive. In focus groups carried out with students, they have expressed that: "when I'm in the classroom as a teacher I will follow this methodology"; "I like the laboratory activities because they are more active than in the traditional class"; "the use of the graphing calculator allows us to do the analysis faster and I have more time to understand the material."

Technology is obviously not the solution to teaching and learning problems in mathematics, but it is making us think about it. It is possible that the major contribution of technology to the teaching and learning process of mathematics consists of the interaction between it, the professor, and the student and this is changing the vision that students have of mathematical content and the educational process. ■

Bio

Edwin Morera González is a professor in the Department of Mathematics-Physics at the University of Puerto Rico, Cayey. His topic of interest is the integration of technology to the learning of mathematics, in particular the graphing calculator.

References

[1] P. Gómez, *Tecnologia y Educación Matemática: una empresa docente*, Grupo Editorial Iberoamérica, Mexico, 1998.

[2] R. Oliveros, "El efecto de la calculadora en la enseñanza de matemáticas," *Memorias Décima Reunión Centroamericana y del Caribe*, Puerto Rico, 1996.

[3] M. Solis, "Profesores, matemática, enseñanza y tecnología: estudio sobre la aceptabilidad de la supercalculadora como recurso didáctico," *Memorias Octava Reunión Centroamericana y del Caribe*, Costa Rica, 1994.

[4] E. Morera, "Cambio de Paradigma en el Proceso de Enseñanza-Aprendizaje de las Matemáticas: Del Conductismo al Constructivismo," Conferencia presentada en la Semana de las Matemática del Colegio Universitario de Cayey, Cayey, PR, 1998.

[5] G. Polya, *Matemáticas y razonamiento plausible*, Editorial Tecnos, Madrid, 1975.

[6] I. Lakatos, *Pruebas y Refutaciones. La lógica del descubrimiento matemático*, Alianza Editorial, Madrid, 1982.

[7] T. Good, and J. Brophy, *Psicología Educativa Contemporánea*, Longman Publishing Group, New York, 1996.

[8] F.A. Hitt, "Representaciones Mentales y Visuales en el Aprendizaje de Conceptos Matemáticos," Conferencia dictada en el Colegio Universitario de Cayey, Puerto Rico, 1998.

[9] C. Janvier, *Problems of Representation in the Teaching and Learning of Mathematics*, Lawrence Erlbaum Associates, Hillsdale, 1987.

[10] J.J. Kaput, "Technology and Mathematics Education," in D.A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, New York, 1992.

[11] J. Bruner, *School for Thought: A Science of Learning in the Classroom*, MIT Press, Cambridge, MA, 1993.

[12] E. Cohen, *Designing Group Work: Strategies for a Heterogeneous Classroom*, Teachers College Press, New York, 1986.

[13] T. Good, M. McCastlin, and B. Reys, "Investigating work groups to promote problem solving in mathematics," in J. Brophy (ed.), *Advances in Research on Teaching: Vol. 3, Planning and managing learning tasks and activities*, JAI Press, Greenwich, CT, 1992.

[14] D. Tall, "Computer Environments for the Learning of Mathematics," in R. Biehler, et al. (eds.), *Didactics of Mathematics as a Scientific Discipline*, Kluwer, Dordrecht, 1994.