

MATHEMATICS FOR A NON-SCIENCE MAJORS CHEMISTRY COURSE

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Abstract

A chemistry course developed for non-science majors has been taught at Virginia Commonwealth University for the past five years. *CHEM 112* uses current event articles from science magazines to make use of a verbal channel of learning in non-science majors, but some mathematics is necessary. Examples are given of successful presentation of nuclear chemistry and data needed for a balanced discussion of global warming. Manipulation of symbols in balancing chemical and nuclear reactions, simple algebra, and logarithms for pH and unit analysis of simple stoichiometric conversions are fundamental to basic chemistry. The population of a voting democracy could benefit from basic education in the concepts of logarithms and algebra in one variable in order to function in a society of increasing dependence on technology.

Introduction

Through the National Science Foundation (NSF) grant that formed the Virginia Collaborative for Excellence in the Preparation of Teachers (VCEPT), several new courses were developed as general education requirements, although students were free to select three from a list of seven or eight such courses in chemistry, physics, and biology. After teaching calculus-based courses in physical chemistry and molecular physics for almost thirty years, it was a new experience to find students preparing for studies in environmental law who were adverse to algebra in one variable. However, we take the position that it is essential that the general population have a working understanding of general principles of science and technology. The course, *CHEM 112: Chemistry in the News*, has attempted to use current events in science to motivate realization of the role of chemistry in our technological society.

It seems obvious that to a large extent students make broad decisions regarding science before choosing an academic major in a college or university. Thus, students in a course for non-science majors may have little interest in science; and yet, our society is increasingly dependent on technology so that science literacy is a component of citizenship. Some students of high ability simply choose majors in non-science fields due to other interests, but a course specifically for non-science majors will likely include students with “math-phobia.” *CHEM 112* is designed to use the strengths of students in writing and reading comprehension, but at the same time increase

understanding of chemical issues with carefully chosen applications of mathematics.

The structure of *CHEM 112* involves student discussion, computer simulations and traditional lecturing twice a week with a writing component. Each week, a new article is introduced from a magazine, such as *Discover* or *Chemical & Engineering News*, and frequently daily newspaper articles are provided to show current applications. Class discussion is stimulated by the professor posing at least two sides of a question and the students are required to write a one-page summary of the article, including a lead-in copy of the first paragraph of the article with a correct citation of the author and source. The lead paragraph is to stimulate good writing style and in essence the students only need to write two paragraphs of their own. Class discussion is supplemented by demonstrations and computer modeling simulations for certain interesting molecules. The reports are graded for content and grammar on a ten-point scale. Studies are underway to compare the improvement of the assigned papers over the course of a semester in *CHEM 112* both as a reinforcement of writing skills and as a “verbal learning channel”; but at this time, it can only be said that the improvement in the quality of the reports improves rapidly within the first three or four weeks of the semester.

At the midpoint of the course, a double (two page) assignment is given to report on the chemistry and environmental quality of one of the rivers in Virginia that empties into the Chesapeake Bay. Groups of four to six students are given this assignment about six weeks before the end of the class to provide time for research (mostly via the Internet) of a given river. All members of a group are given the same grade for the group report and there is evident enthusiasm for this mode of learning. This study fosters awareness of geography on the regional level and it reinforces the importance of aquatic ecology in terms of drinking water and food sources. Maps of the rivers and the Chesapeake Bay are provided to the groups and this is a lively activity with students sharing and consolidating the information they find. The delicate balance of waste treatment and the need for potable water is made quite evident from this study of specific rivers in Virginia. This study also shows that pH is an essential descriptor of water and brings base ten logarithms into discussion of water quality.

Supplemental assignments are offered to all students outside of class in the use of the *SIM-EARTH* [1] computer game by Maxis. As a 3-credit course without a laboratory component, *CHEM 112* has offered scheduled introductions into the use of *SIM-EARTH* by a talented assistant. Future use without an assistant will use out-of-class assignments with the introduction in several lectures.

This game can be quite sophisticated in showing environmental trends on the planets of Venus, Earth, and Mars. After simulating environmental trends in the atmosphere and oceans of planet Earth, the next game assignment is to try to “terraform” Mars by means available to the program, such as using nuclear heat to release carbon dioxide to warm Mars or somehow bring an ice meteor to crash on Mars to provide water. These assignments are available for extra credit in a laboratory with sixteen personal computers. A whole course could be developed using *SIM-EARTH*, but we choose to spend time on other topics as well. This simulation of planetary geospheres puts emphasis on global warming due to burning fossil fuels which increases atmospheric carbon dioxide. This justifies a demonstration of unit analysis showing how much carbon dioxide is formed for each gallon of gasoline (octane) burned. Even a simple example of chemical stoichiometry is very complex for a non-science major, but the need to obtain the result relative to global warming renders this lecture acceptable to non-science students.

Another use of PC technology has been found in the use of the modeling program *CHEMSITE* [2] available from CHEMSW. It has been found that if we draw molecules with the program, we can teach chemistry by asking the students to observe simple valence rules. For instance after drawing about ten molecules, a table can be formed in lecture showing that carbon always has four bonds, oxygen has two, nitrogen has three, and hydrogen has one bond. In this way, students visually “discover” common rules of chemical valence. *CHEMSITE* can also be used to show students that molecules are constantly vibrating at room temperature and also that molecules have complex 3-dimensional shapes. Computer programs accessible during the lecture can be used spontaneously to show a point about a chemical compound if the professor has developed skill in the use of the program outside of class.

Minimum Mathematics

A chemistry course without any mathematics restricts the chemistry a great deal. It should be stated that a number of chemistry texts for non-science majors assume that mathematics should be avoided to the greatest possible extent. Some educators openly profess that there should be no mathematics at all in a non-science majors course in chemistry! In a large class, there is a very broad range of ability and skill in the use of basic mathematics so that some students resist the use of any mathematics, while other students may be bored by trivial lecture material. The strength of non-science majors is their reading comprehension and writing ability so the assignment of weekly, one-page reports is a good way to convey chemistry concepts to them via a “verbal channel.” However, over the last five years, we have explored ways to introduce some mathematics into non-science

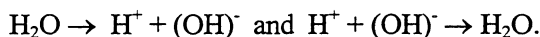
majors chemistry and we have considered what are the minimal levels of mathematics that add the most additional meaning to verbal concepts. Some of our findings have been surprising and will be shared here.

pH

First, a key concept in environmental chemistry wherever water is involved is pH, a logarithmic measure of acidity in water. In a study of the Potomac River, it was found that a certain part of the Potomac has a pH as low as 5, acidity comparable to vinegar. Class preparation in the mathematics of pH helped students appreciate the acidity of river water.

$$\text{pH} = -\log_{10} [\text{H}^+] \quad (1)$$

The basic experimental fact is that water dissociates to a slight degree into H^+ and $(\text{OH})^-$ as



This set of rapid reactions cause an important relationship as an equilibrium constant, K_{eq} . For water $K_{\text{eq}} = 1.0 \times 10^{-14}$ as the relation

$$[\text{H}^+][\text{OH}^-] = 1.0 \times 10^{-14}. \quad (2)$$

Thus, $[\text{H}^+]$ and $[\text{OH}^-]$ are expressed by the special symbol “[]” as moles/liter in solution (one mole is 6.0225×10^{23}) and when $[\text{H}^+] = [\text{OH}^-]$ they each equal the square root of K_{eq} as 1.0×10^{-7} and a pH of 7. When $[\text{H}^+]$ increases, $[\text{OH}^-]$ decreases and equation (2) is maintained, leading to pH lower than 7 for acid solutions. Conversely, the pH is above 7 in a basic solution. Mathematically, it is possible for highly concentrated acids to have a pH below 0 (12 molar HCl) or above 14 (moistened pellets of NaOH), but in a typical aqueous environment the range of pH is from 4 to 10. This material is fundamental to any discussion of chemistry on a planet like Earth on which about 71% of the surface of the planet is covered by water; especially when about 65% of our human bodies are also water. The topic of pH is simply essential to much of chemistry and yet the commonly measured quantity involves logarithms. It is important to also define $\text{pOH} = -\log_{10}[\text{OH}^-]$ and it can be seen from taking the log of equation (2) that $\text{pH} + \text{pOH} = 14$ and that in solutions where $[\text{OH}^-]$ is larger than $[\text{H}^+]$ (a “basic” solution) $\text{pH} = 14 - \text{pOH}$. These relationships are trivial in courses for chemistry majors and yet the use of logarithms causes difficulty with the non-science majors.

One way to introduce logarithms is to show that simple concentrations such as $[H^+] = 0.001$ from a strong acid such as HCl in water can be written as $0.001 = 1.0 \times 10^{-3}$ which leads to $pH=3$ and that the logarithm (base 10) is simply the power of 10. In the case of negative exponents, the definition of pH simply discards the minus sign due to the definition of pH in equation (1). When HCl dissociates in water, the Cl^- merely becomes a “spectator ion”; it is the H^+ ion that causes the acidity. It is important to *slowly* show several examples, such as for 0.1, 0.01, 0.001, 0.0001 etc., but this is sufficient to teach the concept. In a second lecture, the case of numbers in between exact powers of 10, such as $[H^+] = 0.05$, can be illustrated using calculators with \log_{10} functions to obtain $pH=1.3$.

Table 1
pH and pOH of Simple Acids and Bases in Water

Solute	Concentration (moles/L)	pH	pOH	Common Source
HCl	0.1	1	13	Stomach Acid
HCl	0.01	2	12	
HCl	0.001	3	11	
HCl	0.0001	4	10	
H_3O^+	0.000063	4.2	9.8	Acid Rain (SO_2)
H_3O^+	0.00001	5	9	Normal Clean Rain
H_2CO_3	0.0000025	5.6	8.4	Carbonated Water
H_2O	55.51(*)	7	7	Pure Water (*)
NaOH	0.0001	10	4	
NaOH	0.001	11	3	
KOH+NaOH	0.01	12	2	Wood Ashes
NaOH	0.1	13	1	

(*) Bulk pure water = $(1000g/L / 18.016g/mole) = 55.51$ moles/L, but only dissociates slightly so that $[H^+] = [OH^-] = 1.0 \times 10^{-7}$ moles/L

The issue here is to slowly present simple examples with motivation as to the importance of understanding aqueous pH. By using the correct mathematics on a few chosen topics of importance, a more thorough presentation of acid-base chemistry can be presented in spite of student reticence regarding logarithms.

Another topic in which the concept of pH is critical is the definition of acid rain as rainwater with a pH below that of natural clean rain [3] at $pH=5.0$. One might expect the pH of rain to be

about 5.6 due to dissolved carbon dioxide forming the weak acid carbonic acid, H_2CO_3 , but worldwide the pH of clean rain is slightly lower. Acid rain can have a pH as low as 4 and typically has a pH of 4.2 to 4.4; almost ten times the $[\text{H}^+]$ as normal rain (note each unit on a logarithmic scale is a factor of 10). The use of logarithms for discussion of pH is one application of basic mathematics which has sufficient significance as to justify some extra class time spent in going over base 10 logarithms.

Temperature

Temperature scales are another area of chemistry where simple mathematics is applicable. Converting Fahrenheit (F) degrees to Celsius degrees (C) and then to the Kelvin scale of absolute temperature (K) is a trivial example of simple algebra as in equation (3):

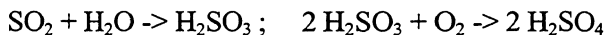
$$F = (9/5)C + 32 \text{ and } K = C + 273.15 . \quad (3)$$

It is possible to purchase a single thermometer with both scales written side by side to convince students that the (9/5) ratio is due to 180 degrees F but 100 degrees C for the same physical conditions of ice water and boiling water (at one atmosphere external pressure); $(180/100)=(9/5)$. The physical interval is the same in terms of energy, but arbitrary human scales have resulted in different incremental degrees. It is important to go very slowly in a lecture when showing that the two scales yield the same value at -40 degrees. This is easily done by setting $F = C$, but the algebra which follows can cause consternation to an English major because ninth grade algebra was the last time that student did such a calculation. We should add that such a student may be brilliant in verbal reasoning and fully prepared to take a course in the tax code in law school, but he/she may have not done algebra for a long time and may have delayed taking a required science course until the senior year of university education! One motivating example is to convert normal human body temperature of 98.6 degrees F to the Centigrade equivalent of 37 degrees C and then show that is about 310.2 Kelvin. Our conclusion from teaching *CHEM 112* for over five years is that university education should not avoid algebra in one variable, but that is about the limit of algebraic complexity to demand from students with primary aptitude in verbal reasoning. There are some students in this class intending to earn a law degree; those students need simple but accurate mathematical treatments of basic chemistry if they plan to specialize in environmental law.

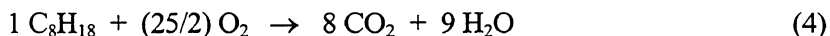
Stoichiometry

After using *SIM-EARTH* simulations and discussing several articles on atmospheric effects of SO_2 leading to acid rain [4] and global warming [5] due to increasing amounts of CO_2 , a question

arises. How much of these gases are due to natural causes such as volcanos compared to human activities such as burning fossil fuels, some of which contain sulfur which lead to SO_2 as well as CO_2 ?



Quantitative use of balanced chemical reactions to calculate mass of reactants consumed and products produced is called stoichiometry. Here we use it to bring factual information to a controversial political question. We can gain insight to the issue of human responsibility for global warming by calculating the pounds of CO_2 formed from one gallon of octane (C_8H_{18}) hydrocarbon, the principle component of gasoline. It is also important to note that in physical science, many numerical values also carry units.



CHEMSITE molecular modeling can be used in the lecture to show the structure of several isomeric forms of C_8H_{18} , such as the linear form of n-octane or the branched form used as the standard of octane rating for internal combustion engines, 2,2,4-trimethylpentane and there are other isomers. The linear octane has a density [6] of 0.7028 grams/ml at 20 degrees C and the branched isomer has a density [6] of 0.69194 grams/ml at 20 degrees C. Since there is a mixture of isomers in gasoline, we will use a value of 0.70 grams/cc. We can also use the atomic weights of C=12.011, H=1.00794, and O=15.9994 grams/mole with the molecular formulas to get the gram molecular weights of the compounds per mole. For instance, the gram molecular weight of CO_2 is the sum of one C (12.011) and two O (15.9994) = 44.0098 grams/mole. Similar calculation yields a gram molecular weight of 114.23092 for C_8H_{18} (the answer will be rounded to two significant figures at the end of the calculation due to the use of 0.70 for the density). Thus using units:

$$\frac{(1 \text{ gal.})(4 \text{ qt} / \text{ gal.})(946 \text{ ml} / \text{ qt})(0.70 \text{ g} / \text{ ml})}{(114.23092 \text{ g} / \text{ mole} \text{ C}_8\text{H}_{18})} = 23.1881 \text{ moles} \text{ C}_8\text{H}_{18}$$

This means that one gallon of octane equals 23.1881 moles of octane. From the balanced chemical reaction in (4), there are eight moles of CO_2 produced for every one mole of C_8H_{18} consumed.

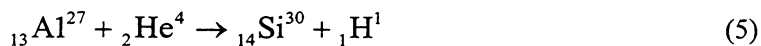
$$\frac{(23.1881 \text{ moles} \text{ C}_8\text{H}_{18})(8 \text{ moles} \text{ CO}_2 / \text{ mole} \text{ C}_8\text{H}_{18})(44.0098 \text{ g/mole} \text{ CO}_2)}{453.6 \text{ g/pound}} = 17.9983 \text{ pounds} \text{ CO}_2$$

Rounding to two significant figures yields an easily remembered fact that one U.S. gallon of octane will produce eighteen pounds of CO₂ when burned (in an engine or by any other means) and ten gallons of gasoline will produce 180 pounds of carbon dioxide when burned. Using real units adds to the credibility of the result. Note that every quart oil container has 946 ml or 946 cc written on it and that one pound equals 453.6 grams. Such simple factors as 1 quart = 0.946 liters and 1 pound = 0.4536 kilograms should be helpful in dealing with metric units throughout our society.

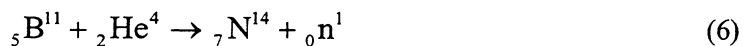
Nuclear Reactions

There is one surprise from our experience in *CHEM 112*. When *CHEM 112* was organized, a text [7] was developed based on modernizing a soft cover study guide, *General Chemistry*, by George and Richard Sasin and written at Drexel University in 1958. The text was brought up to date by adding several new chapters and some fifteen magazine articles were inserted near related chemistry sections. The Sasin text includes a short chapter on nuclear reactions. After discussion of simple elementary particles, such as protons, electrons, neutrons and positrons, the class was assigned to read about nuclear power as an energy source [8]. The Sasin Chapter 52 is only three pages, but it gives examples for nuclear reactions which can occur in accelerator beams as well as nuclear decay. For instance:

Alpha-particle (${}_2\text{He}^4$) Bombardment with Proton emission



Alpha-particle (${}_2\text{He}^4$) Bombardment with Neutron emission

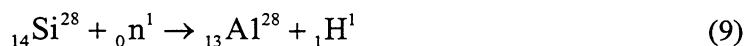


Proton Bombardment (${}_1\text{H}^1$)

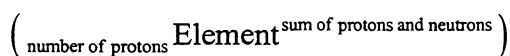


Deuteron Bombardment (${}_1\text{D}_2 = {}_1\text{H}_2$)

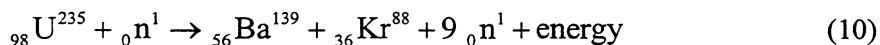


Neutron Bombardment (${}_0n^1$)

Here we use the convention



for each isotope. The main point here is that the non-science students seemed to be more interested in this topic than others and they quickly became adept at balancing the number of neutrons and protons on both sides of the nuclear reaction. The class was then able to easily appreciate one typical chain reaction of a nuclear fission reactor as in (10) where just one neutron splits ${}_{92}\text{U}^{235}$ and releases nine neutrons which can cause further reactions. Many other examples were discussed regarding nuclear energy processes.



Equation (10) is only one of a number of possible nuclear fission reactions in which neutrons are produced to accelerate the reaction and class discussion was held on the use of Cd rods, C graphite and heavy water (D_2O) to slow the neutrons for control of the reaction.

In reviewing the student interest in nuclear reactions, it is noted that students generally are concerned about nuclear weapons and nuclear waste disposal from reactors. They are also concerned about global politics based on petroleum energy sources. That is, the topic itself is of concern and interest to non-science students. We report here that students are able to consider such a politically important topic in science and not be encumbered by any mathematics more than adding and subtracting integers and simply balancing the total number of protons and neutrons on each side of a nuclear reaction. While this may seem trivial, the topic itself is so important that it is worth pointing out that nuclear reactions can be treated in a non-science majors class more effectively than might be supposed given the aura of difficulty commonly associated with nuclear physics.

Conclusion

Teaching rudiments of chemistry to non-science majors is very important to improving science literacy in a society that is increasingly dependent on technology. Chemistry is a “central science” and chemical literacy in the general population is an important component of political

judgment. Our position is that the mathematics must not be neglected, and that carefully chosen portions of mathematics can be taught with motivation to solve chemical problems. Minimal mathematics includes base 10 logarithms for use with pH, simple algebra in one variable for temperature conversions, use of common conversions between metric and British units in quantitative calculations and balancing reactions for both chemical and nuclear processes. Observations based on teaching non-science majors for five years and developing a text [7] for their use indicate that they have much greater skills and abilities in writing and reading comprehension than in mathematics. However, it is our position that some limited mathematics must be used to make chemistry meaningful at a level useful to the general population in order to make judgments regarding political questions with a component of technology.

Summarizing positive outcomes from *CHEM 112*, we note especially the already established verbal ability of non-science students to *stimulate critical thinking* regarding science and technology in our society. Use of mathematical concepts (emphasized here) was no more than 15% of *CHEM 112*. Readings explored the lives of Albert Einstein and Madame Curie as well as some recent controversies in the chemistry of Gallium. The students learned from reading selected articles that scientists are humans, but that they try to use quantitative relationships to understand nature. The class discussions bring out student experiences. One profound discussion produced accounts from a student who had worked in Africa in AIDS education following a class-assigned reading on the AIDS virus. Discussion of nuclear power brought renewed interest in two nearby nuclear power plants, and a very interesting discussion followed reading the history of Love Canal in New York and kepone pesticide contamination of the James River in the 1970s which led to a long fishing ban on the James. Discussing articles regarding the tragic history of Easter Island and mudslides in Haiti, both probably caused by unwise depletion of trees, and the controversy of causes and effects of global warming favored environmental concerns. However, there was contrasting analysis of the economic impact of the Kyoto treaty on the U.S. as out of proportion to the effect on China and India.

It should be clear that students in *CHEM 112* learned about some of the global implications of natural and manmade chemistry. Some chemistry majors also took *CHEM 112* as an elective and were enthusiastic about “seeing the big picture” after focusing on specific molecules in other courses. Our recommendation is that carefully motivated use of mathematics brings the full quantitative nature of chemistry to bear on political questions regarding technology and should not be avoided. ■

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