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Mathematical Modeling of Tracheal Luminal Size Change under Angioedema-Caused Stiffness Alteration

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Summary: Tracheal angioedema is a pathology of the airway caused by soft tissue swelling due to fluid leakage from the blood vessels [1]. This pathology can suddenly change the normal tracheal luminal size and cause breathing difficulty for an medical emergency. The extra fluid accumulation inside the tissue can also alter the stiffness of the tissue, and make the luminal size change more complicated. We set up a model using continuum mechanics to understand how the angioedema swelling extent can quantitatively change the trachea luminal size particularly under the tissue stiffness modification. Interestingly, the swelling may not always shrink the tracheal lumen, and may expand it for proper parameter values. This model can assist conducting more appropriate medical treatment for tracheal angioedema.

The Trachea is modeled as a two-layered cylindrical tube following [2, 3]. The inner layer consists of soft tissue where angioedema occurs, and the outer layer is mainly composed of harder cartilaginous tissue allowing no angioedema syndrome [4]. One family of longitudinally oriented fibers is also incorporated in the inner layer. The outer layer is modeled by the neo-Hookean model as

$$W_o = \frac{\mu_o}{2}(I_1 - 3), \tag{0.1}$$

where μ_o is the shear modulus of the outer layer, and I_1 is the first invariant of the right Cauchy-Green tensor $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$ for the deformation tensor $\mathbf{F}$. The inner layer is modeled by a generalized neo-Hookean material model with fibrous energy as

$$W_i = \underbrace{\frac{\mu_i \nu^{q-2/3}}{2}(I_1 - 3\nu^{2/3})}_{\text{neo-Hookean energy}} + \underbrace{\frac{\gamma}{2}(I_4 - 1)^2}_{\text{fiber energy}}, \tag{0.2}$$

where μ_i is the shear modulus of the inner layer, γ is the fiber elastic modulus, and ν is the swelling parameter. Here q is a parameter to indicate that the original shear modulus μ_i of the material can be updated to $\Lambda_i = \mu_i \nu^{q-2/3}$ according to how angioedema alters the stiffness. Three cases are of special interest: (1) $q \to -\infty$ making the shear modulus annihilated; (2) $q = 2/3$ making the shear modulus identical to the original one; (3) $q \to \infty$ making the shear modulus turns to be infinitely large.

For the outer layer, the material is incompressible satisfying $\det\mathbf{F} = 1$. The inner layer is modeled as being volume-specified satisfying $\det\mathbf{F} = \nu$. The inner and outer boundaries of the trachea are modeled as being traction free. The longitudinal length is fixed. The undeformed and deformed radii are denoted by R and r, respectively. The radii of the undeformed inner boundary, the interface, and the outer boundary are denoted by R_i, R_m and R_o, respectively. After the angioedema-caused deformation, R_i becomes r_i. The deformation is taken to be axisymmetric. The tracheal body is under equilibrium, satisfying $\mathrm{div}\mathbf{T} = \mathbf{0}$, where $\mathbf{T}$ is the Cauchy stress tensor. The parameter values used in this article are $R_i = 8.85$ mm, $R_m = 9.15$ mm, $R_o = 11.45$ mm, $\mu_i = 0.0429$ MPa, $\mu_o = 0.58$ MPa, and $\gamma = 0.0429$ MPa. We study how r_i changes with respect to the swelling parameter ν when the inner shear modulus Λ_i is altered by the swelling.

Fig. 1a shows r_i vs. ν curves for several q values as $q \to -\infty$. The limit case means the shear modulus Λ_i is zero, that is, as angioedema occurs, the expansion of the inner layer has no effect on the outer layer. Therefore, the interface radius is not changed, and the swelling makes the inner boundary move inward until the lumen is completely filled. For other small q values, the curves decrease initially and become

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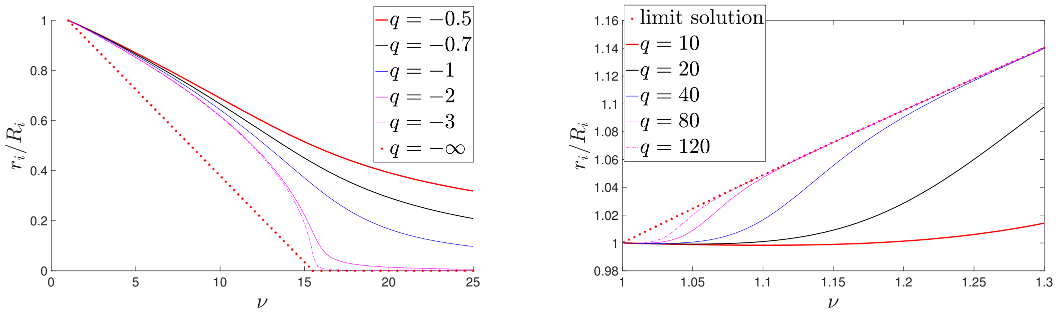

(a) Results for $q \to -\infty$. As $q = -\infty$, the graph is given by the lowest curve. As q is decreasing, the curve $r_i(\nu)$ approaches the limit curve.

(b) Results for $q \to \infty$. The most upper one is the limit curve for $q = \infty$. As q is increasing, the curve $r_i(\nu)$ approaches the upper limit curve.

Figure 1: Graphs of r_i (normalized by R_i) vs. ν when $q \to -\infty$ and $q \to \infty$.

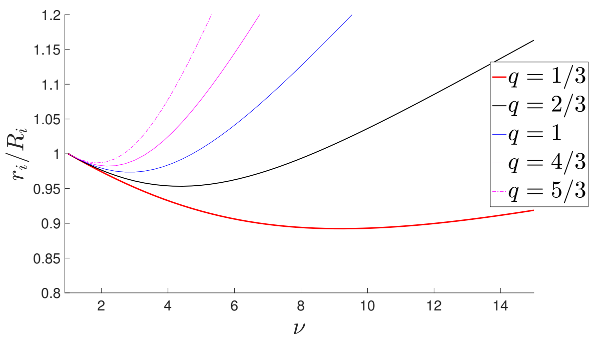

Figure 2: Graphs of r_i (normalized by R_i) as a function of ν for various regular (not too small or too large) q values. Here $q = 2/3$ is the defaulted value representing that the material stiffness is not affected by swelling. The graph shows that for these q values, the curves decrease initially and then increase. As q is increasing, the minimum point of each curve is shifted to the left and also elevated, shortening the ν interval for decreasing.

very flat gradually. Fig. 1b shows the curves as $q \to \infty$, under which the shear modulus turns to be infinitely large giving strong resistance to any force imposed on the boundary of the inner layer by the outer layer. All curves show upward concavity. As ν increases, r_i initially decreases and then increases. Larger q makes the turning point occur at smaller ν. The limit curve is close to a straight line.

The modeling results interestingly show that the lumen may not always shrink when the trachea encounters angioedema. Instead, the lumen may expand for proper ν and q values. The lumen size change is a complicated effect determined by the swelling extent, how swelling alters the shear modulus of the inner layer, and other geometrical and mechanical parameters.

References

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