May 30th, 10:30 AM - 11:00 AM

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Mathematical Modeling of Tracheal Luminal Size Change under Angioedema-Caused Stiffness Alteration

Kun Gou*

Summary: Tracheal angioedema is a pathology of the airway caused by soft tissue swelling due to fluid leakage from the blood vessels [1]. This pathology can suddenly change the normal tracheal luminal size and cause breathing difficulty for a medical emergency. The extra fluid accumulation inside the tissue can also alter the stiffness of the tissue, and make the luminal size change more complicated. We set up a model using continuum mechanics to understand how the angioedema swelling extent can quantitatively change the trachea luminal size particularly under the tissue stiffness modification. Interestingly, the swelling may not always shrink the tracheal lumen, and may expand it for proper parameter values. This model can assist conducting more appropriate medical treatment for tracheal angioedema.

The Trachea is modeled as a two-layered cylindrical tube following [2, 3]. The inner layer consists of soft tissue where angioedema occurs, and the outer layer is mainly composed of harder cartilaginous tissue allowing no angioedema syndrome [4]. One family of longitudinally oriented fibers is also incorporated in the inner layer. The outer layer is modeled by the neo-Hookean model as

\[
W_o = \mu_o \frac{2}{I_1 - 3},
\]

(0.1)

where \(\mu_o\) is the shear modulus of the outer layer, and \(I_1\) is the first invariant of the right Cauchy-Green tensor \(\mathbf{C} = \mathbf{F}^T \mathbf{F}\) for the deformation tensor \(\mathbf{F}\). The inner layer is modeled by a generalized neo-Hookean material model with fibrous energy as

\[
W_i = \mu_i \nu^{\frac{-2/3}{2}} (I_1 - 3\nu^{2/3}) + \frac{\gamma}{2} (I_4 - 1)^2,
\]

(0.2)

where \(\mu_i\) is the shear modulus of the inner layer, \(\gamma\) is the fiber elastic modulus, and \(\nu\) is the swelling parameter. Here \(q\) is a parameter to indicate that the original shear modulus \(\mu_i\) of the material can be updated to \(\Lambda_i = \mu_i \nu^{q-2/3}\) according to how angioedema alters the stiffness. Three cases are of special interest: (1) \(q \to -\infty\) making the shear modulus annihilated; (2) \(q = 2/3\) making the shear modulus identical to the original one; (3) \(q \to \infty\) making the shear modulus turns to be infinitely large.

For the outer layer, the material is incompressible satisfying \(\det \mathbf{F} = 1\). The inner layer is modeled as being volume-specified satisfying \(\det \mathbf{F} = \nu\). The inner and outer boundaries of the trachea are modeled as being traction free. The longitudinal length is fixed. The undeformed and deformed radii are denoted by \(R\) and \(r\), respectively. The radii of the undeformed inner boundary, the interface, and the outer boundary are denoted by \(R_i\), \(R_m\) and \(R_o\), respectively. After the angioedema-caused deformation, \(R_i\) becomes \(r_i\). The deformation is taken to be axisymmetric. The tracheal body is under equilibrium, satisfying \(\text{div} \mathbf{T} = 0\), where \(\mathbf{T}\) is the Cauchy stress tensor. The parameter values used in this article are \(R_i = 8.85 \text{ mm}, R_m = 9.15 \text{ mm}, R_o = 11.45 \text{ mm}, \mu_i = 0.0429 \text{ MPa}, \mu_o = 0.58 \text{ MPa}, \text{ and } \gamma = 0.0429 \text{ MPa}\).

We study how \(r_i\) changes with respect to the swelling parameter \(\nu\) when the inner shear modulus \(\Lambda_i\) is altered by the swelling.

Fig. 1a shows \(r_i\) vs. \(\nu\) curves for several \(q\) values as \(q \to -\infty\). The limit case means the shear modulus \(\Lambda_i\) is zero, that is, as angioedema occurs, the expansion of the inner layer has no effect on the outer layer. Therefore, the interface radius is not changed, and the swelling makes the inner boundary move inward until the lumen is completely filled. For other small \(q\) values, the curves decrease initially and become

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(a) Results for $q \to -\infty$. As $q = -\infty$, the graph is given by the lowest curve. As $q$ is decreasing, the curve $r_i(\nu)$ approaches the limit curve.  

(b) Results for $q \to \infty$. The most upper one is the limit curve for $q = \infty$. As $q$ is increasing, the curve $r_i(\nu)$ approaches the upper limit curve.

Figure 1: Graphs of $r_i$ (normalized by $R_i$) vs. $\nu$ when $q \to -\infty$ and $q \to \infty$.  

Figure 2: Graphs of $r_i$ (normalized by $R_i$) as a function of $\nu$ for various regular (not too small or too large) $q$ values. Here $q = 2/3$ is the defaulted value representing that the material stiffness is not affected by swelling. The graph shows that for these $q$ values, the curves decrease initially and then increase. As $q$ is increasing, the minimum point of each curve is shifted to the left and also elevated, shortening the $\nu$ interval for decreasing.

The modeling results interestingly show that the lumen may not always shrink when the trachea encounters angioedema. Instead, the lumen may expand for proper $\nu$ and $q$ values. The lumen size change is a complicated effect determined by the swelling extent, how swelling alters the shear modulus of the inner layer, and other geometrical and mechanical parameters.

References


