LESSONS LEARNED IN A MATH EXCEL WORKSHOP: THE IMPORTANCE OF MAINTAINING HIGH COGNITIVE DEMANDS

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Abstract

Uri Treisman's Emerging Scholars Workshop model has been implemented on many college campuses over the last twenty years. The Treisman model is based on groups of students meeting regularly in a social atmosphere to work collaboratively in solving challenging mathematics problems related to their introductory coursework. Emerging Scholars Programs (or Math Excel as it is called in many settings, including ours) have been particularly successful in increasing the academic success and participation of underrepresented groups in mathematics. The primary responsibilities of a workshop leader include the design of a session's worksheet, as well as the facilitation of students' problem solving efforts during the workshop session itself. In this paper, we discuss a mathematical tasks framework proposed by researchers in the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project that may be especially helpful to workshop leaders in making a successful implementation of Math Excel. This framework emphasizes the notion of the *cognitive demand* of a mathematical task. The level of cognitive demand is not a static attribute and may well change as students undertake a task in a classroom setting. QUASAR researchers noted how the initially high demands of a task may not be maintained in the classroom, and how teachers' actions may lower the demands and consequently limit learning opportunities for students. Although the QUASAR project involved middle school mathematics instruction, we believe that this mathematical tasks framework can provide valuable lessons for Math Excel workshop leaders, and it suggests how critically important both the choice of problem tasks and the workshop leaders' facilitation of student work can be. In this paper, we review the mathematical tasks framework and illustrate its application to scenarios actually encountered in our Math Excel workshops.

What Exactly Is Math Excel?

In solving a murder mystery, detectives look for *motive* and *opportunity*. Those are also two crucial ingredients in a successful Math Excel program. Students must provide the motive, whether it is directed toward an extrinsic goal of improved grades or a more intrinsic goal, such as a deeper understanding of the course material. Math Excel workshops provide the opportunity in terms of a structured schedule (one to three meetings per week in addition to the regular class meetings) where students can work in small collaborative groups solving challenging problems related to the work they are doing in their regular classes. Neither ingredient should be taken for

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granted. Students' motivation must be sufficient for them to make a commitment to the extra time demanded by the workshop schedule. In turn, the workshops must provide problems that are clearly relevant to the current coursework and demanding enough to stimulate student discourse in a setting where students receive the encouragement and support to persevere.

The particular administrative logistics of implementing a Math Excel program can differ widely from setting to setting. The workshops themselves may be led by course instructors, graduate students, or by undergraduate peer leaders (often alumni of previous Excel classes). The workshops may be formally "attached" to a special section of a course (for example, Math Excel workshops may take the place of a recitation meeting for students electing that section) or may be offered as an "add-on" separate course carrying additional credit. Common to most implementations is a strict requirement of faithful attendance and participation by the students. The primary activity in a workshop session is that of students working together in small collaborative groups on a worksheet, i.e., a collection of problems.

The key elements of a successful Math Excel program are the people (students and leaders), the process (collaborative learning in a supportive social atmosphere), and the problems (worksheets providing rich and substantive problem-solving opportunities). In this paper, we take a closer look at the interactions between students and leaders in a Math Excel workshop session. In particular, we want to emphasize the critical role that the workshop leader plays in facilitating fruitful student discourse, and how easy it can be for a leader to inadvertently limit opportunities students have for learning.

Background: Treisman's Emerging Scholars Workshop Program

In 1975-76, Uri Treisman conducted a study at the University of California, Berkeley, in which he documented the study habits of a group of twenty African-American and a group of twenty Chinese American students enrolled in *Introductory Calculus* [1]. Treisman found that the most striking difference between these two groups were in how they viewed what "studying math" meant. The African-American students tended to work in isolation, rarely consulting with other students or teaching assistants. In effect, these students had compartmentalized their daily life into academic and social components. In contrast, the Chinese American students often met in peer study groups and had integrated this activity into their social lives.

Out of this experience, Treisman developed the Mathematics Workshop Program to provide supplementary peer collaborative problem solving experiences in a social atmosphere for students enrolled in *Introductory Calculus*. Now called the Emerging Scholars Program, it has enjoyed success in increasing the representation of African-American and Latino mathematics majors (Treisman replicated the program at University of Texas at Austin starting in 1988). The model has been adapted at many other campuses with a similar goal—to increase both the success and the participation of underrepresented students in mathematics. What constitutes an underrepresented target group varies—it could be female students, students of color, students with disabilities, students from rural backgrounds, etc. Bonsangue found that the minority students in the Emerging Scholars workshops at University of California, Pomona, when compared to minority students not enrolled in the workshop, achieved significantly higher grades in the calculus course [2,3]. At the University of Kentucky, Michael Freeman founded the Math Excel program based on Treisman's model, with the target population consisting of students from predominantly rural communities. Freeman found that the students enrolled in these Treisman style collaborative workshops consistently achieved higher grades than students not in the workshops [4].

The Math Excel program at Oregon State University began in 1998 with initial funding from Oregon Collaborative for Excellence in the Preparation of Teachers (OCEPT) and was patterned closely on the University of Kentucky implementation. Math Excel workshop sessions at Oregon State University are currently offered twice a week for *College Algebra*, *Precalculus*, *Differential Calculus*, and *Integral Calculus*. Duncan and Dick documented the success of the program over nineteen different sections of Math Excel across all four courses during the first two years of the program [5]. According to their study, student achievement averaged approximately half a grade point higher than predicted (by mathematics SAT scores).

Leading a Math Excel Workshop: Using QUASAR's Mathematical Tasks Framework

Despite the initial success of Math Excel at Oregon State University, the care and nurturing of the program requires continuing attention and ongoing efforts. It is clear to us that the role of the workshop leader is critical to the success or failure of the model. However, adequately communicating the distinguishing characteristics of an effective Math Excel leader can be difficult. To assist workshop leaders, one must move beyond vague general directives, such as "show that you care about your students." The nuts and bolts of a good workshop lie in the details of worksheet preparation and workshop facilitation; prospective leaders need specific advice on both problem selection, as well as techniques for encouraging fruitful student discourse.

A framework that we believe is helpful for elaborating on this discussion is the Mathematical Tasks Framework described by researchers involved in the QUASAR project during the early 1990s [6]. Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) was a national project aimed at improving mathematics instruction to middle school students in economically disadvantaged communities. The project was funded by the Ford Foundation and directed by Ed Silver at the Learning Research and Development Center at University of Pittsburgh. Although QUASAR concerned middle school mathematics instruction, its emphases on critical thinking, reasoning, problem solving, and the communication of mathematical ideas are entirely consistent with the goals of a Math Excel workshop session. The researchers in the QUASAR project developed the Mathematical Tasks Framework to guide their analysis of observed classroom lessons. They found the Framework useful not only as a research tool, but also as a tool for teachers who "began to use it as a lens for reflecting on their own instruction and as a shared language for discussing instruction with their colleagues." [6] We would propose that the Mathematical Tasks Framework is also well suited for Math Excel leaders to reflect on their worksheet preparation and workshop facilitation. The Framework provides a useful vocabulary for leaders to discuss with each other the dynamics of a workshop session—what went "right" and what went "wrong"—in terms of accomplishing their goals.

Description of the Mathematical Tasks Framework

A central idea of the Mathematical Tasks Framework is that of the *cognitive demand* of a task. Different mathematical problems require different kinds of thinking from students in order to solve them. Moreover, the cognitive demand of a particular task should not be considered a static attribute of the task—the level of cognitive demand of a task can shift as students work on it, and teachers (leaders) can have a great influence on this shift of level. Stein and Smith [7] identify three phases that tasks pass through:

<u>Phase one</u> — as they appear in curricular/instructional materials

<u>Phase two</u> — as they are set up by the teacher (leader)

<u>Phase three</u> — as they are implemented by students

The level of cognitive demand can shift from its originally intended level (Phase one) at either Phase two or Phase three. The teacher can influence this shift not only at Phase two, but also through the type of assistance or direction provided to students during Phase three. These shifts, in turn, have consequences ultimately in student learning outcomes.

What are the different levels of cognitive demand that a mathematical task can have? The Mathematical Tasks Framework identifies two lower levels, *memorization* and *procedures without connections,* and two higher levels, *procedures with connections* and *doing mathematics.* These categories were used to analyze hundreds of middle school mathematics lessons during the QUASAR lesson and are illustrated in detailed case studies [6]. However, we find that the framework works very well for other levels of mathematics. Below, we identify some of the key features of each level using example tasks from *Introductory College Calculus.*

Memorization — Memorization tasks involve simply reproducing previously learned facts, rules, formulae, or definitions (or committing these to memory). These tasks can be performed without making any connections to underlying concepts or meanings.

Example: What is the derivative $\frac{dy}{dx}$ of each of the following functions?

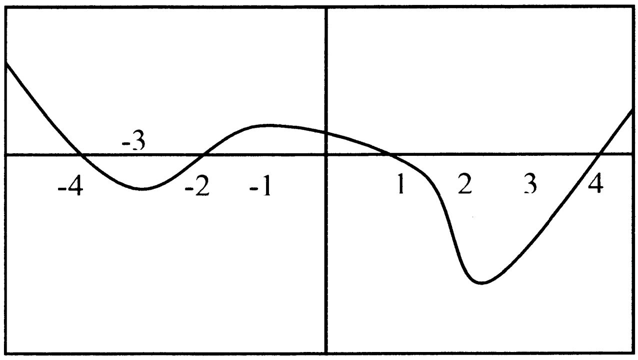
a) $y = \sin x$ b) $y = \cos x$ c) $y = \tan x$ d) $y = \sec x$

Procedures Without Connections — These are algorithmic tasks that are focused on producing correct answers. There is no ambiguity in what steps need to be performed and they can be successfully completed without making any connections to underlying concepts or meanings.

Example: Find an equation for the tangent line to the curve $y = x^3 - 4x^2 + 10x - 7$ at the point (2,5).

Procedures With Connections — These tasks involve procedures, but students need to engage with the underlying concepts and meanings in order to successfully complete the task. They often involve multiple representations and require making connections. These tasks are intended to develop deeper understanding of the underlying concepts and meanings.

Example: Below is a graph of the function $y = f'(x)$. If $g(x) = f(x^2)$, for what values x does g have a relative minimum?



Doing Mathematics — These tasks require complex, nonalgorithmic thinking and there is not a predictable, well-rehearsed path suggested by the task, instructions, or by previously worked example. Such tasks require students to explore and understand the nature of mathematical concepts, processes, or relationships and to analyze and actively examine task constraints. They may involve some level of anxiety or frustration for the student due to the unpredictability of the solution process.

Example: Graph $y = \cos(x^{2/3})$ on a graphing calculator. Is $y = \cos(x^{2/3})$ differentiable at $x = 0$? How can you reconcile the results of the chain rule with your graph?

Applying the Mathematical Tasks Framework to Math Excel

Researchers in the QUASAR project noted in their middle classroom studies that the cognitive level of a task originally appearing or set up at higher cognitive levels could be lowered by the teacher. We have found that this phenomenon aptly describes what can go awry in a Math Excel workshop session. For example, consider the example calculus task given as an illustration of *procedures with connections*. Assuming that the students have the requisite knowledge of the chain rule and the first derivative test for ***extrema***, there is a procedure they can follow to solve the task. However, carrying out this procedure will require students to connect the graphical representation conceptually to both the chain rule and the first derivative test. If one or more groups of students is struggling with the task, a Math Excel leader might be tempted to illustrate the procedure with a different example. However, such a move may well lower the cognitive demand of a task to that of a *procedure without connections*—students may be "successful" (in the sense of getting the correct answer) by mimicking the leader's example, but perhaps miss out on the opportunity to grapple with the representational connections.

A preferable alternative is to employ what the QUASAR researchers call *scaffolding*—questioning that supports student reasoning without simplifying the task at hand. For example, a leader could suggest that the students think about how they would approach the task if they had an explicit formula for g or f, and encourage them to look for ways that the given graph of f' could be exploited to yield similar information.

As students make promising steps toward a solution, it is also important for the leader not to lapse into the role of a "certifying authority." It is important to hold students accountable for their reasoning, and continually ask for justifications and explanations. The leader who answers questions with questions initially may be a source of frustration to students, but is more likely to be successful in maintaining high cognitive demands.

Discussion and Concluding Remarks

We believe that the Mathematical Tasks Framework provides not only a helpful vocabulary for highlighting key characteristics of a successful Math Excel workshop, but also a means by which workshop leaders can reflect on and analyze their practice. Indeed, the Framework directly touches on two of the most important responsibilities of a workshop leader: the preparation of an appropriate worksheet of problems (Phase two) and the facilitation of student work on those problems during the workshop session (Phase three).

To be sure, the Framework does not address all aspects of implementing a successful Math Excel workshop. Another important responsibility of the workshop leader is in setting and maintaining expectations of the students for collaborative learning. Students bring varying degrees of experience with collaborative learning to a Math Excel workshop. Thus, it is important to spell out early the expectations the students should have *of each other:* showing respect for other members of the group, coming prepared to work and participate, being an active contributor and listener, and providing encouragement for one another's efforts are the most essential. Specific rules for group work should also be laid out explicitly. For example, it may be permissible for a group to choose to work on the day's worksheet problems in some other order than presented, but this should be a group decision and all members of the group should be working on the same problem at the same time. While a social atmosphere is welcomed in Math Excel workshops, students may stray into too much off-task conversation. The leader's presence can help students stay on task. A leader also plays the role of cheerleader. This becomes especially important as fatigue sets in toward the later part of the term or semester and students may need extra encouragement to persevere.

The fuel that makes a Math Excel workshop run is the problem worksheet. Putting together a good worksheet is one of the most important duties of a Math Excel workshop leader. The Mathematical Tasks Framework highlights the importance of setting up and maintaining high cognitive demands. Most worksheet problems should be challenging enough to stimulate students to work together and discuss them. Some fairly routine problems aimed at building basic skills are fine, especially as early "warmups." However, student discussion of such problems tends to be limited to comparing individual answers. At the other extreme, including one or two very difficult problems is appropriate, especially to challenge the better students, but too many of these on a worksheet can be discouraging.

Two other important attributes of good worksheet problems are: **relevance**—students should be able to tell at a glance that most of the problems on the worksheet pertain to material they are studying *currently* in the corresponding class; and, **variety**—a mix of problems helps keep students engaged (problems that illustrate applications, require interpretations of graphical or tabular displays of data or quantitative relationships, or questions that expose commonly held misconceptions are great types of problems for Excel worksheets). Finally, an especially difficult problem or two at the end can ensure that even the best students do not finish early. There is no quota of problems to be finished on an Excel worksheet. The aim is to keep all students actively and productively engaged in problem solving throughout the workshop.

Adapting to the role of a facilitator is perhaps hardest for leaders who are experienced lecturers. It can be difficult to fight the urge to demonstrate solutions, especially to a group of students who are frustrated and struggling with a problem. Asking the right question at the right time (the art of scaffolding) is the most valuable help that a Math Excel leader can provide. When a group of students is "spinning their wheels," the challenge is to find just the right helpful push rather than to serve as a tow truck.

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Bio

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