INVESTIGATIONS IN GEOMETRY—A HANDS-ON COURSE FOR GRADES 6-8 PRE-SERVICE TEACHERS

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Abstract

This paper describes a geometry course at Virginia Commonwealth University for pre-service middle school teachers that features hands-on activities, group collaboration, and technology. Details of the topics and activities covered in this course are provided. Feedback from the first group of students to complete the course was gathered throughout the semester and student responses to the class format and topics are also discussed.

Introduction

Geometry is a fundamental topic which pre-service middle school teachers must develop a solid understanding of before entering the classroom. As a result, a course focusing on geometry is a key component of middle school teacher preparation programs in Virginia. The National Council of Teachers of Mathematics (NCTM) advocates the use of hands-on activities, collaborative exercises, and technology in the instruction of mathematics at the pre-college level [1]. Teachers are better able to present material and cover topics in an activity- and technology-based environment if they have participated in similar experiences during their educational career. Therefore, the geometry course for pre-service middle school teachers at Virginia Commonwealth University (VCU) has been recently redesigned to feature hands-on activities, group work, and the use of a software package specifically designed for the study of geometry. Course topics and activities, as well as student feedback on the course, are discussed below.

Course Description

The course, *MATH 303 - Investigations in Geometry*, is a three-credit course that meets two days a week. One seventy-five minute class period is spent with students engaging in hands-on and group collaboration activities. During the other seventy-five minute class period, students conduct geometric explorations with *The Geometer's Sketchpad* software program [2]. Student collaboration is encouraged during all activities, including those that require *The Geometer's Sketchpad*. The topics covered by this course include measurement/area, constructions, pattern exploration, transformations, and symmetry. Details about these topics are provided below.

The educational philosophy behind the design of this course is based on the theory that students learn mathematics best by doing mathematics. Therefore, very little time is spent in a traditional lecture format. The instructor acts as a facilitator for activities that lead students from one geometric discovery to the next. The textbooks were carefully chosen for the topics covered and the probing nature of the dialogue that prompts students to engage in activities to develop an understanding of the concept [3,4]. Whenever possible, more than one instructional medium is used to address a topic: such as constructions with patty paper, as well as ruler and straightedge; and, pattern exploration with mathematics manipulatives and *Sketchpad*. This approach allows students to solidify their understanding of the topic by looking at it from more than one point of view. It also gives students the opportunity to analyze the similarities and differences between instructional methods and to explore the limitations of some methods. For example, it is not possible to trisect an angle with ruler and straightedge, but the construction is possible with patty paper and quite simple with *Sketchpad*. A summary of the activities for each topic appears below.

Measurement/Area — This topic is introduced by helping students develop a foundation for measurement systems by using the lengths of body parts (hand span, arm length, etc.) to measure different objects. The concept quickly progresses from one-dimensional to two-dimensional measurement with the cutting of different shapes (triangles, rectangles, parallelograms, circles) to derive the formulas for area. Formulas for different shapes are also compared through this process (e.g., cutting the end from a parallelogram and moving the piece to the other side reveals that the area of a parallelogram is equal to the area of a rectangle). There are a variety of *Sketchpad* activities that can be used to further study the concept of area [4]. For example, students explore the ratio of different areas by carefully constructing and animating a square nested inside another square and a triangle within a triangle. They discover the rectangle with maximum area through another animated *Sketchpad* project. Lastly, volume formulas are determined with the use of relational geosolids and Polydron™ shapes.

Constructions — Three instructional mediums are used in the study of geometric constructions: compass and straightedge, patty paper, and *Sketchpad*. Most basic constructions can be created with *Sketchpad* in two or three clicks of the mouse. Therefore, most of the time devoted to this topic is spent constructing with compass and straightedge, and patty paper. Patty paper is a thin square of translucent paper that is easily creased so that the line formed by the crease remains

when the paper is unfolded. Students discover several constructions that can be done with patty paper, but are not possible with compass and straightedge. In writing, they reflect on the advantages and disadvantages of different construction methods. This topic concludes with a *Sketchpad* discovery of the golden ratio.

Pattern Exploration — This topic requires extensive drawing of objects on grid paper and dot paper. *Symmetry, Shape, and Space* devotes several chapters to the discovery of patterns in ways that are not covered in more traditional geometry textbooks [3]. In particular, the path of a ball on differently shaped billiard tables, the construction of Celtic knots, and the generation of star polygons on circles with a given number of evenly spaced dots are central to the study of patterns. Through the development of many examples, students discover generalizations for the patterns they study. For example, drawing numerous examples of the path of a billiard ball shot at a 45° angle from the lower left-hand corner of a billiard table allows students to determine which size tables will result in: 1) the ball landing in a particular corner; 2) the ball's path resulting in a symmetric pattern; or, 3) the ball passing through every square of the grid paper representing a billiard table. The remainder of this topic is the study of regular, semi-regular, and dual tilings using regular polygons. The discovery of different tiling patterns is conducted with regular pattern blocks and pattern tessellation blocks. This topic concludes with students developing their own Escher-style patterns using paper and pencil and *Sketchpad.*

Transformations — Patty paper and *Sketchpad* are featured in the analysis of this topic. Patty paper allows students to physically move an object to see the result of a reflection, rotation, or translation. Sketchpad allows students to generate more than one example at a time by clicking and dragging different objects (the translation vector, the angle of rotation, the object of the transformation, etc.), and immediately viewing the results of the movement. Lastly, students use Sketchpad to design an amusement park ride in which transformations are necessary to put the seats on the ride and to put the ride into motion through animation.

Symmetry — Students make discoveries about two-dimensional and three-dimensional symmetry with pattern blocks and Polydron™ shapes. The physical movement of a regular polygon or a Polydron™ object results in students determining the lines of reflection and the angles of rotation. They also explore the composition of two movements (i.e., reflection along line X followed by a rotation of Y degrees). If the line of reflection is a line of symmetry of the regular

polygon and the angle of rotation is the same as the measure of the polygon's angles, then the composition of two movements does not result in a new line of reflection or a new angle of rotation.

Assessment

The bulk of the course grade is not based on traditional forms of assessment. Sixty percent of a student's grade is based on in-class and homework assignments, with half of the assignments requiring *Sketchpad* and the other half activities requiring paper and pencil or a math manipulative (pattern blocks, ruler and straightedge, patty paper, etc.). Homework assignments are gathered for grading on a random basis with roughly half of the assignments being graded throughout the semester. The due date of a *Sketchpad* assignment is announced and the student submits the completed assignment in the form of a *Sketchpad* file as an attachment to an e-mail sent to the instructor. The assignments are graded with the *Sketchpad* program, with teacher comments and point distribution highlighted in red, and the graded file is returned then to the student through e-mail. In-class assignments are similar to the types of activities students engage in for homework, but they are much shorter. They are graded near the end of the same class period in which they were assigned.

Class participation comprises 10% of the semester grade with group interaction, time on task, and class attendance being contributing factors. Students work in groups of three or four on non-computer activities while groups of two are encouraged to work on computer activities. The remaining 30% of the semester grade results from a midterm exam and a final exam. The exams cover all aspects of the class including the use of manipulatives and *Sketchpad*. The *Sketchpad* questions require students to exhibit that they can perform basic constructions with the software program and are familiar with all of the available tools. Each completed exam file is transferred by the teacher from the computer to a floppy disk for grading.

Student Response

The course was first taught with this format in Spring 2003 to a large class of forty students. Throughout the semester, the instructor elicited feedback from the class about the instructional format, the topics being studied, and the role of technology in the classroom. While hesitant of learning through discovery at the beginning of the semester, students gradually warmed to the idea and most of them were positive about the whole experience by the end of the

semester. They liked the topics covered in the class but were surprised by some of the things they studied, especially items relating to pattern exploration. Several mentioned that they would like to do similar activities in their future classrooms, but felt that due to the significance of the Standards of Learning (SOL) testing in Virginia, time was not available to engage in this type of pattern exploration. The comments about patty paper were mixed. About half of the class liked doing constructions with the paper, but the other half favored ruler and straightedge constructions. However, nearly all of the class felt students should be exposed to basic patty paper constructions.

Students consistently expressed positive comments throughout the semester for *Sketchpad* and group collaboration. Many stated that the types of activities and the way the textbooks present material are perfect for collaboration. On most days, students moved into groups before the instructor had a chance to announce that it was time to work together on the day's lesson. Using *Sketchpad* was the best part of the class for all but a few class members. Most of them liked using the software from the very beginning, and by the end of the semester felt that it was the most interesting and useful part of the class.

Conclusion

Based on the opinions of the first class, the changes to the course were deemed an initial success. The course appears to be doing what it was designed to accomplish. Students get a semester-long experience in a course that completely avoids the traditional lecture format. They are exposed to a wide variety of geometry topics through hands-on activities, group collaboration, and the use of technology. Students are assessed in the same manner in which they learn. This course is based on the principles outlined by the NCTM [1] and gives pre-service teachers experience with teaching and assessment methods they will be expected to employ once they become middle school mathematics teachers. ■

References

[1] *Principles and Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 2000.

[2] *The Geometer's Sketchpad, ver. 4*, Key Curriculum Press, Emeryville, CA, 2003.

[3] L.C. Kinsey and T.E. Moore, *Symmetry, Shape, and Space: An Introduction to Mathematics through Geometry*, Key College Publishing, Emeryville, CA, 2002.

[4] D. Bennett, *Exploring Geometry with* The Geometer's Sketchpad, Key Curriculum Press, Emeryville, CA, 2002.