PRE-MATRIX BALANCING OF CHALLENGING CHEMICAL EQUATIONS WITH A SIMPLE FORMULA REGISTER AND MIDDLE SCHOOL ARITHMETIC

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Abstract

This paper describes a procedure employing basic relational concepts (analogous to certain matrix algebra concepts), but structured by a simple formula register and using only middle school arithmetic to balance chemical equations ranging from easy to moderate to difficult redox reactions. This procedure allows average students and below average students to experience ready success in balancing, thus avoiding a traditional source of frustration and failure which might contribute to their losing interest in chemistry. One interesting serendipity of this procedure is how quickly it turns able pre-matrix students into extremely fast and accurate balancers.

Introduction

The balancing of chemical equations can be made much easier, especially for those who find it difficult, by moving the procedures toward the algorithmic and away from the heuristic. That is, a "step to step" procedure is simpler to master than is the haphazard hopping of inspection, even a highly refined inspection.

Computer-based balancing procedures use matrices. Excellent analogous procedures have been developed for humans, notably by Peterson, et al. [1] Unfortunately, a large number of students at the secondary and tertiary levels have no exposure to matrix algebra before they begin chemistry. Many of these, especially the lab focused rather than the theoretically focused, never do. Fortunately, there is a way to use matrix balancing concepts with a simple formula register and only middle school arithmetic to make balancing easy for almost everyone.

This allows average, and even low achieving students, a real chance at success. It can remove what is often a source of frustration and failure that turns students away from chemistry. Also, it allows the high achieving to become very fast and very accurate even with relatively difficult equations. A balancing technique based on quasi-matrix protocols is described below. Because of its unusual nature, it is best explained through demonstration; therefore, a balancing of three equations with the new procedure is presented. These equations will be shown in step-by-step instructional sequences that make all components of the new procedure explicit: new formats, new operations, and new terms. This technique provides students with a sharply delineated graphic organizer that improves their perceptions of quantity relationships within a chemical equation by expressing that equation in a quasi-matrix format called a register.

The Technique

First, the chemical equation is written with extra space, so that the formulas of its compounds are well separated.

$$Al_4C_3 \quad + \quad H_2O \quad \longrightarrow \quad CH_4 \quad + \quad Al(OH)_3$$

Then, the symbol for each element of the equation is written in a column beneath the yield sign. The order of the elements in this column, top to bottom, is the same as the order of the elements in the equation, left to right. If you use lined paper, skip a line between each element. Allow the equivalent space if you do not use lined paper.

Al_4C_3	+	H_2O	$\longrightarrow$	CH_4	+	$Al(OH)_3$
4			Al			1
3			C	1		
		2	H	4		3
		1	O			3

Balancing a register row means that coefficients are assigned (using the lowest common denominator) to its numbers so that the sum of the products of numbers on the left side of the element column, in that row, is equal to the corresponding sum on the right side.

$$Al_4C_3 \quad + \quad H_2O \longrightarrow CH_4 \quad + \quad Al(OH)_3$$

(1)4		Al		(4)1
3		C	1	
	2	H	4	3
	1	O		3

Once a row is balanced, then every number in any column having a number in the balanced row must be given the same factor as that column's number in that row. That is, all of the numbers in the column must be multiplied by equal factors. This is very easily done, but a little awkward to express. (For convenience, it might be called "equifacting.")

$$Al_4C_3 \quad + \quad H_2O \longrightarrow CH_4 \quad + \quad Al(OH)_3$$

(1)4		Al		(4)1
(1)3		C	1	
	2	H	4	(4)3
	1	O		(4)3

This pair of actions, balancing a row and equifacting its columns, constitutes a step in the balancing procedure. This two-part step might be called a "couplet." Once a couplet is completed, a new row to be balanced is chosen using the same criteria as before. When this new row is balanced,

Al_4C_3	+	H_2O	$\longrightarrow$	CH_4	+	$Al(OH)_3$
(1)4			Al			(4)1
(1)3			C	(3)1		
		2	H	4		(4)3
		1	O			(4)3

and its columns equifacted,

Al_4C_3	+	H_2O	$\longrightarrow$	CH_4	+	$Al(OH)_3$
(1)4			Al			(4)1
(1)3			C	(3)1		
		2	H	(3)4		(4)3
		1	O			(4)3

a new couplet is completed. Then a new row is sought (same criteria), then balanced, and so on until the entire register is balanced, as shown below. Each dotted line represents one step in a couplet, while a solid line represents the end of the couplet's procedure.

Al_4C_3	+	H_2O		CH_4	+	$Al(OH)_3$
(1)4			Al			(4)1
(1)3			C	(3)1		
		2	H	(3)4		(4)3
		(12)1	O			(4)3

(1)4		Al		(4)1
(1)3		C	(3)1	
	(12)2	H	(3)4	(4)3
	(12)1	O		(4)3

Al_4C_3	+	$12H_20$	$\longrightarrow$	$3CH_4$	+	$4Al(OH)_3$
(1)4			Al			(4)1
(1)3			C	(3)1		
		(12)2	H	(3)4		(4)3
		(12)1	O			(4)3

Once all couplets are completed, corresponding coefficients are placed in the original equation to complete the balancing.

It may be that equifacting the columns of a newly balanced row unbalances a previously balanced row. This is perfectly normal. Eventually, this unbalanced row will be selected for rebalancing. In theory, the same row might be balanced, unbalanced, and rebalanced more than once during this procedure. However, the row selection criteria suggest that balancing begins relatively high in the register and that priority be given to shorter compounds (number of elements) and to more complex compounds. With respect to the latter, complexity is proportional to the number of subscript coefficients distributed over some notationally defined set of elements and/or the number of elements in the set over which the subscript coefficient is distributed. Thus, $(HPO_3)_3$ is considered to be more complex than HNO_3.

Using the example and criteria above as a case in point, please consider that the first row (Al), the second row (C), and the fourth row (O) are tied for shortness. However, the first row is associated with the relatively complex compound, Al $(OH)_3$. Therefore, it was balanced first, then its columns equifacted. This completed the first couplet. A new row was selected (C) based on the original criteria, and the procedure continued until the register was fully balanced. Once completed, the factors of each column were transferred to their corresponding compounds (or elements) in the equation.

Most students respond very favorably to this technique. The method permits students to readily identify quantitative relationships. While easy balancing offers obvious cognitive advantages, it also offers subtle affective advantages (especially for uncertain or anxious students). That is, the method is simple and self-structuring, which helps students feel confident as they work thorough the problem. Haphazard guessing and desperate searching are virtually eliminated. A further example is provided below. This equation is usually perceived as more difficult than the first due to the relatively large numbers associated with its balancing.

Write the spread equation and its register.

$$HNO_3 \quad + \quad P_4O_{10} \longrightarrow (HPO_3)_3 \quad + \quad N_2O_5$$

1		H	3	
1		N		2
3	10	O	9	5
	4	P	3	

Then, proceed as explained above.

1		H	3	
1		N		2
3	10	O	9	5
	(3)4	P	(4)3	

1		H	(4)3	
1		N		2
3	(3)10	O	(4)9	5
	(3)4	P	(4)3	

(12)1		H	(4)3	
1		N		2
3	(3)10	O	(4)9	5
	(3)4	P	(4)3	

(12)1		H	(4)3	
		N	(4)9	2
(12)1		O	(4)3	5
(12)3	(3)4	P		

(12)1		H	(4)3	
(12)1		N		(6)2
(12)3	(3)10	O	(4)9	5
	(3)4	P	(4)3	

(12)1		H	(4)3	
(12)1		N		(6)2
(12)1	(3)10	O	(4)9	(6)5
	(3)4	P	(4)3	

$$12HNO_3 \quad + \quad 3P_4O_{10} \longrightarrow 4\,(HPO_3)_3 \quad + \quad 6N_2O_5$$

(12)1		H	(4)3	
(12)1		N		(6)2
(12)3	(3)10	O	(4)9	(6)5
	(3)4	P	(4)3	

Using the register to help teach equation balancing makes this part of chemistry far more algorithmic than heuristic and, consequently, easier for students to comprehend and master. Its steps are summarized below.

1) Write the equation, spread out.
2) Write the element column.
3) Write the element frequencies.
4) Choose the first row to balance.
5) "Run" the first couplet.
6) Continue choosing rows and "running" couplets until all rows are balanced and all columns equifacted.
7) Transfer column factors to their corresponding equation components.

The procedure is easily extended to the balancing of redox equations. It seems especially suited to combination with a modification of the "tried and true" half-reaction approach.

As before, write the equation with terms spread out.

$$C_2H_6O \quad + \quad K_2Cr_2O_7 \quad + \quad H_2SO_4 \longrightarrow Cr_2H_4O_3 \quad + \quad K_2(SO_4)_3 \quad + \quad H_2O$$

Then:

1) Separate the full-reaction equation into oxidation and reduction half-reaction equations.
2) Write relevant oxidation states above thee appropriate chemical symbols.

3) Calculate and show electron imbalances at the right of each half reaction.
4) Add water molecules to the right of the oxidation equation and hydrogen ions to its left.
5) Add hydrogen ions to the right of the reduction equation and water molecules to its left.

$$\overset{\boxed{-2+6-2}}{C_2H_6O} + H_2O \longrightarrow \overset{\boxed{0\ +1-2}}{C_2H_4O_2} + H^+ \qquad \boxed{-2e\text{- oxy}}$$

$$C_2H_6O + K_2Cr_2O_7 + H_2SO_4 \longrightarrow C_2H_4O_2 + Cr_2(SO_4)_3 + K_2SO_4 + H_2O$$

$$H + \overset{\boxed{-1\ +6\ -2}}{K_2Cr_2O_7} + H_2SO_4 \longrightarrow \overset{\boxed{+3\ -2}}{Cr_2(SO)_3} + K_2SO_4 + H_2O \qquad \boxed{+3e\text{- red}}$$

Figure 1. Oxidation.

Reduction

Write a reaction register for each half reaction. The reduction register is placed below its half reaction and the oxidation register is placed above its half reaction.

1		1	O	2			
6		2	H	4	1		
2			C	2			
-2 +6 -2				0 +1 -2			-2e- oxy
C_2H_6O +		H_2O	$\longrightarrow$	$C_2H_4O_2$ +	H^+		
C_2H_6O +	$K_2Cr_2O_7$ +	H_2SO_4	$\longrightarrow$	$C_2H_4O_2$ +	$Cr_2(SO)_3$ +	K_2SO_4 +	H_2O
	-1 +6 -2			+3 -2			+3e- red
H +	$K_2Cr_2O_7$ +	H_2SO_4	$\longrightarrow$	$Cr_2(SO_4)_3$ +	K_2SO_4 +		H_2O
1		2	H				2
	2		K	2			
	2		Cr	2			
	7	4	O	12	4		1
		1	S	3	1		

Now, chemically balance the registers for both half reactions using the new procedure.

1		1	O	2					
6		2	H	4		(4)1			
2			C	2					
-2 +6 -2				0 +1 -2					-2e- oxy
C_2H_6O	+	H_2O	$\longrightarrow$	$C_2H_4O_2$	+	H^+			
C_2H_6O +	$K_2Cr_2O_7$ +	H_2SO_4	$\longrightarrow$	$C_2H_4O_2$	+	$Cr_2\,(SO_4)_3$ + H_2SO_4 +			H_2O
	-1 +6 -2			+3 -2					+3e- red
H +	$K_2Cr_2O_7$ +	H_2SO_4	$\longrightarrow$	$Cr_2\,(SO_4)_3$	+	K_2SO_4	+	H_2O	
6)1		(4)2	H					7(2)	
	2		K	2					
	2		Cr	2					
	7	(4)4	O	12		4		1	
		(4)1	S	3		1			

Subtract water of the oxidation reaction (top left) from water of the reduction (bottom inset). Subtract hydrogen ions of the reduction reaction (bottom left) from hydrogen ions of the oxidation reaction (top right).

3(1)		3(1)	O	3(2)			
3(6)		3(2)	H	3(4)	12(1)		
3(2)			C	3(2)			
-2 +6 -2				0 +1 -2			-2e-oxy
$3C_2H_6O$	+	$\cancel{3H_2O}$	$\longrightarrow$	$3\ C_2H_4O_2$ +	$0\ 1\cancel{2\ H^+}$		
C_2H_6O +	$K_2Cr_2O_7$ +	H_2SO_4	$\longrightarrow$	$C_2H_4O_2$ +	$Cr_2(SO_4)_3$ +	K_2SO_4 +	H_2O
	-1 +6 -2			+3 -2			+3e-red
$0\ \cancel{12\ H}$ +	$2K_2Cr_2O_7$ +	$8H_2SO_4$	$\longrightarrow$	$2Cr_2(SO_4)_3$ +	$2K_2SO_4$	+	$11\ \cancel{14H}_2O$
12(1)		8(2)	H				14(2)
	2(2)		K		2(2)		
	2(2)		Cr	2(2)			
	2(7)	8(4)	O	2(12)	2(4)		14(1)
		8(1)	S	2(3)	2(1)		

Transfer all half-reaction coefficients to corresponding components or in the full-reaction equation. Note that hydrogen ions have been eliminated from both sides and water molecules from one side.

3(1)			O	3(2)			
3(6)			H	3(4)			
3(2)			C	3(2)			
-2 +6 -2				0+1-2			
$3C_2H_6O$			$\longrightarrow$	$3C_2H_4O_2$			
$\downarrow$				$\downarrow$			
$3C_2H_6O$ +	$2K_2Cr_2O_7$ +	$8H_2SO_4$	$\longrightarrow$	$3C_2H_4O_2$ +	$2Cr_2(SO_4)_3$ +	$2K_2SO_4$ +	$11H_2O$
	$\uparrow$	$\uparrow$			$\uparrow$	$\uparrow$	$\uparrow$
	-1 +6 -2				+3-2		
	$2K_2Cr_2O_7$ +	8H2SO4	$\longrightarrow$		$2Cr_2(SO_4)_3$ +	$2K_2SO_4$ +	$11H_2O$
		8(2)	H				14(2)
	2(2)		K			2(2)	
	2(2)		Cr		2(2)		
	2(7)	8(4)	O		2(12)	2(4)	14(1)
		8(1)	S		2(3)	2(1)	

Summary

This procedure seems to substantially facilitate the balancing of equations that, traditionally, have been considered difficult for many students. It is interesting that the more difficult the equation, the greater this facilitation appears to be. Moreover, the double registers appear to contribute to concept development in exactly the way an "exploded" assembly diagram does, by making every component relationship extremely clear.

The immediate importance of the procedure lies in the fact that it can remove the heuristic wall of haphazard inspection, replacing it with a near algorithmic procedure that virtually assures balancing success for average students and below average students. Also, it gives able students an unusual facility. A significant, but less immediate, advantage is the preparation the procedure could offer for future matrix techniques. ■

Reference

[1] L. Peterson, et al., "Chemical Balancing," *Project Intermath*, COMAP, Bedford, MA, 2002.