GEOMETRY EXAMPLES ENCOUNTERED IN VARIOUS EVERYDAY EXPERIENCES

R.W. FARLEY
Professor Emeritus of Mathematics
Dept. of Mathematics and Applied Mathematics, Virginia Commonwealth University
Richmond, VA 23284-2014

Introduction

A student who is learning or relearning elementary geometry can benefit from applications that occur in everyday life. By explaining these applications, the study of geometry is not only enhanced, but the student is left with an appreciation for their relevance. Some of these applications can be used in a pedagogical content course. They range from the household to the construction industry to landscaping to hydraulics. All involve elementary geometry mainly involving areas and volumes.

Finding the Rectangle

Early in their study, Mathematics Specialist trainees learn that a necessary and sufficient condition for a parallelogram to be a rectangle is that the diagonals are equal. So, how is this interesting fact useful in daily work? One practical example is a method used by builders and involves ensuring that the foundation for a rectangular building, including many houses or rooms or parts thereof, is indeed rectangular. First, stakes with boards are installed on each corner. Then strings are tightly strung around the site in a pattern around nails on the corner boards (see Figure 1).

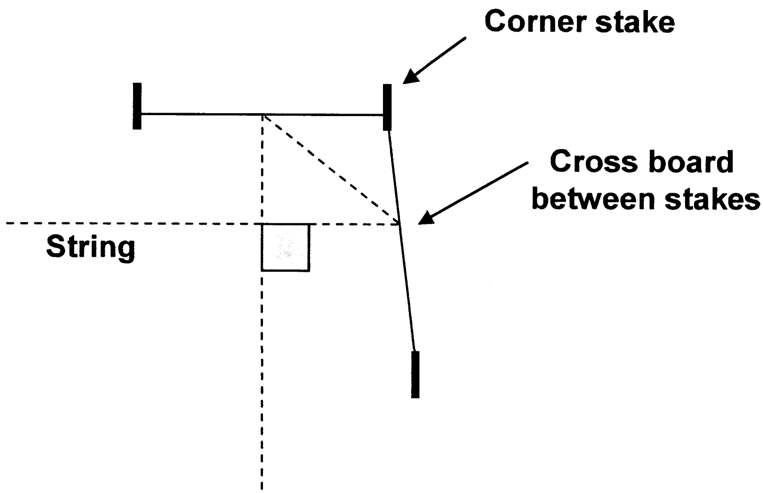

Figure 1. Aerial view of a foundation.

These string lines are used by the foundation bricklayers to plumb the necessary rectangular foundation. The problem is that using a square to measure the corner angles marked by the string is difficult to produce the necessary accuracy. So, the builder gets the string configuration reasonably close to producing the corner right angles, then measures diagonals for equality, and refines the string configuration until the diagonals are equal. This geometry has been used for decades by many builders. Students in several Mathematics Specialist geometry classes have enjoyed using an electronic range finder to measure outdoor "rectangular" regions.

"Would You Like Shag or Berber?"

A household application can be as simple as ordering the correct number of square yards of carpet to cover a 9' x 12' room. While students may experiment correctly using the measurement in feet to produce $9'\times 12' = 108$ square feet, they must then convert to square yards by recalling that $3'\times 3' = 9$ square feet which is one square yard. Then, they divide 108 by 9 and correctly obtain 12 square yards. Another method of calculation would be to convert the original measurements to 3 yards by 4 yards and easily multiplying to obtain 12 square yards.

"How *Much* Paint?!"

Another simple household application involves how many gallons of paint to buy in order to paint the four walls and ceiling of a rectangular room measuring $10'\times 14'\times 8'$(high). Instructions on the gallon paint can state that normal coverage is 275 square feet. Of course, two walls measure 14' x 8' each and the other two measure 10' x 8' each. Adding the 384 square feet of wall space to the $14'\times 10' = 140$ square feet of ceiling surface yields 524 square feet. Since the room has a 4' x 7' door in one wall which will not be painted, we can be comfortable with buying two gallons of paint.

Pouring the Footing

An additional building example involves the measurement in cubic yards of the amount of concrete to order to pour the "footing" on which the brick foundation will be laid for constructing a 60' x 120' ranch style house. The footing consists of pouring cement into a 16" wide trench so that the depth of the cement is uniformly 8" deep. Converting these measurements into feet, the resulting calculation will be $\frac{16"}{12"} = \frac{4'}{3}, \frac{8"}{12"} = \frac{2'}{3}$ so that the volume of concrete in the trench is $\frac{4'}{3} \times \frac{2'}{3} \times 360 = \frac{8}{9}$ square feet times 360 feet long = 320 cubic feet. Recalling that a cubic yard is $3'\times 3'\times 3' = 27$ cubic feet, the resulting division, $320 \div 27$, yields slightly less than 12 cubic yards.

An interesting question results from our calculation. Since the depth and width of the poured concrete is constant for most normal building applications, can we simplify repeated calculations by finding a constant *K* (which we will call "the footing constant") which can always be multiplied by a varying length to quickly produce the number of cubic yards of concrete needed? Students can decide whether to work with units of either feet or yards. One solution is to notice that our calculation above involves $\frac{8}{9}\times 360'\,(\text{length}) \div 27$. So, if we use $\frac{8}{9} \div 27 \times \text{length} = \frac{8}{243} \times \text{length}$, the result will be the cubic yards required. Thus, the constant *K* is $\frac{8}{243} \approx .033$. So, for example, if we need to pour 260' of uniform footing, we need $.033 \times 260 = 8.58$ cubic yards of concrete. Effectively, we will order 9 cubic yards of concrete.

The calculation requires the calculation $\frac{8}{9} \times 360' \div 27 = \frac{8}{9} \div 27 \times 360'$. Use number properties to verify this equality.

Circular Logic

Landscaping offers further opportunities to explore geometry. The problem: how many Belgian path stones 8” long will we need to form a circular mulch border of a 10' radius from the center of a tree which has a 12' circumference at its base?

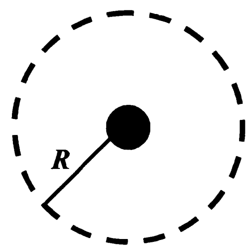

Figure 2. The tree’s radius.

We can calculate the circumference of the circle as $2\pi \times 10' = 20' \times 3.14 = 62.8'$ or approximately $62\,{}^2\!/_3$ feet. Dividing by 12, we determine that each stone is 2/3 foot long so that $62\,{}^2\!/_3 \div \frac{2}{3} = \frac{188}{3} \bullet \frac{3}{2} = 94$ stones. This answers the original question, but to construct the circle accurately, we need the radius *R* measured from the outside of the tree. Here, $2\pi R = 12'$ so that *R* = 1.90'. Thus, the radius of the tree can be reasonably approximated as 2' so that the working radius *R* from the outside base of the tree is 8'.

If a Tree Falls...

We encountered a similar problem when cutting down a large oak tree by using an available chain saw which has a 20” blade. The radius of the tree could not exceed 20” so that the tree could be cleanly cut through the center and not left standing precariously. Using a tape measure, we measured the circumference near the base to be 12'. However, this yields $2\pi R = 12'$. Without a calculator available, we decided to use $\pi \approx 3$, so that $R = \frac{6'}{3} = 2'$. Unfortunately, this cannot be cut with a $20''$ blade. Since we know $\pi = 3.14$ is a slightly larger divisor which will produce a slightly smaller actual radius, we are erring on the side of caution. The solution here was to move the tape measure up the tree until we reached a point where the circumference was 10'. Then, $2\pi R = 10'$ produced a $\frac{5'}{3}$ radius. Converting the fraction to inches produced a $20''$ radius which provided a safe cutting margin for a perfect cut.

From the Dock, Part I

Oyster (or crab) floats are used to nurture and protect seedling oyster "spats" until they mature, at which time they become marvelous cleansing agents for river and bay water. A quick plumbing problem, related to these floats, involves 4' lengths of capped PVC pipe. How many 2" diameter pipes would it take to replace one 4" diameter pipe for water flow or buoyancy if sealed for air space? Let's consider uniform lengths of 1'. Hence, the 4" pipe has a radius of 2" and a cross section of $2^2\pi = 4\pi$ square inches which yields a volume of 48π cubic inch per foot. For the 2" pipe, the radius is 1". Thus, the cross section area is $1^2\pi = \pi$ square inches, so that the four 2" pipes will produce the same volume per foot. It is interesting to try to draw the four 2" pipes fitting inside of the 4". The four circles inside the 4" diameter circle will overlap in a manner that makes the physical areas difficult to compare. Physically, the four 2" pipes will not fit inside the 4" pipe, thus making physical comparisons non-obvious for a mathematically easy problem.

A similar example involving a comparison of 1" and 2" pipes requires work with fractions, since the radius of the 1" pipe is ½". The resulting 4 to 1 ratio of the number of pipes is the same as in the previous comparison.

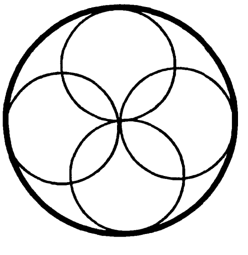

Figure 3. Theoretical PVC pipe placement.

From the Dock, Part II

The aforementioned oyster float part poses an interesting geometrical construction problem. Such floats can be constructed from relatively "stiff," 1"× 1" mesh wire by making careful bends, using a rubber mallet and table edge, to form edges, and then cutting and discarding some corners to allow for closed corners. For starters, think about making an open box by cutting square corners from a rectangular sheet of paper and folding up the edges. The difference here is that the float which will result when the capped PVC tubes are later attached to the sides for flotation must be a closed rectangular solid. We have the capability of clamping edges together with "pig rings" which also can be used to hinge a cut piece of wire on one end to form a "door flap." One suggested design is similar to the one shown in Figure 4.

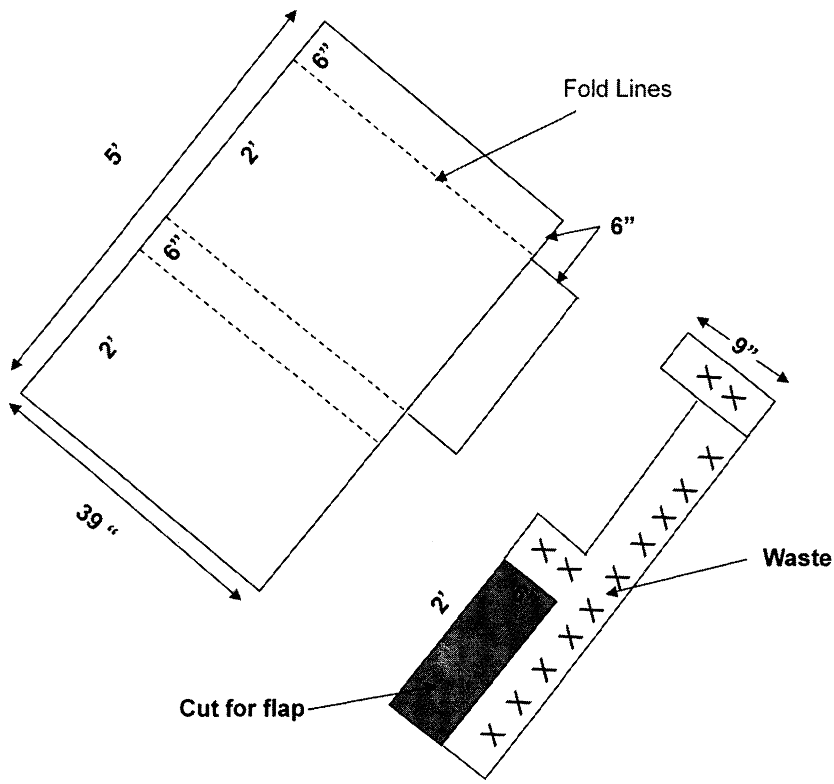

Figure 4. Aerial view of oyster float plan.

When cut, bent, and assembled, what is the inside volume of the float? Calculation in inches yields $24" \times 39" \times 6" = 5615$ cubic inches.

Can you construct a model of similar design using a $5' \times 4'$ piece of 1" coated wire which still has a $6"$ cage height, but yields a larger enclosed volume? Hint: one solution can be achieved by folding from the $5'$ side and using a cut from one end to make the flap on the other end. This will produce a volume of 5,832 cubic inches. Although the volume is slightly increased, there might be other dimension considerations which could be pertinent. Can you devise this design or better diagram by folding and cutting a $5" \times 4"$ sheet of paper?

Poolside Problem

A simple volume problem which involves filling a swimming pool with water leads to an interesting gallon-per-cubic-foot equivalence. Consider a swimming pool which has rectangular dimensions of $18' \times 36'$ with an average depth of 6 feet. Multiplying these dimensions in feet yields 3,888 cubic feet, but the problem is that water is delivered in gallons. If you try to guess this relationship, you will probably not be close to the actual answer. An employee of the MathScience Innovation Center in Richmond, Virginia built a reinforced cubic foot model for our geometry class, since no model of a cubic foot could be found on site where metric dominated. The students used a quart container to pour water into the model. They researched the answer and were convinced that 7.48 gallons filled the cubic foot container. Thus, the swimming pool required $3888 \times 7.48 = 29082.24$ gallons of water.

Shingling the Roof

An area problem occurs repeatedly in roofing construction. This problem is usually effective after discussions of the Pythagorean Theorem. How many "squares" of shingles are needed to roof a building with an A-frame roof if the view of the building is like that in Figure 5?

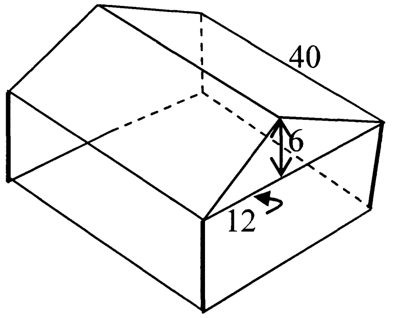

Figure 5. Aerial view of an A-frame roof.

The slope or pitch of this roof is called a 6/12. Using the Pythagorean Theorem rather than making somewhat dangerous measurements from atop the roof, we find the rafter length as $\sqrt{12^2+6^2} = \sqrt{144+36} = \sqrt{180} = 13.446'$. Since the rafter overhangs the side by $6''$, we use $14'$ for the rafter length. Now $14' \times 40' = 560$ square feet which yields 1,120 square feet for both sides of the roof. Since a square of shingles is 100 square feet, we need 11.2 squares. Effectively, we will order $12\ {}^1\!/_3$ squares to allow for a 10% cutting loss. This will require thirty-seven bundles of dimensional shingles which require three bundles per square.

House of the Two Gables

Siding the gable ends of a garage with a A-frame roof as in Figure 5 poses an interesting observation which can save half of the possible waste of $4' \times 8'$ sheets of T-111 siding. The sheets of siding have an outside (finished side) which has grooves cut every twelve inches and an inside which is smooth. Thus, the sides are not interchangeable. Obviously, six sheets of siding will cover the 24□ span on the building front, cutting the tops off on the proper angles. Note that the cuts from the right front will not work on the left front. Can you show, using a piece of paper with lines on the finish outside, how the cut off pieces can be used on the gable on the building back so that we can eliminate waste and need not order six more sheets for the back gable?

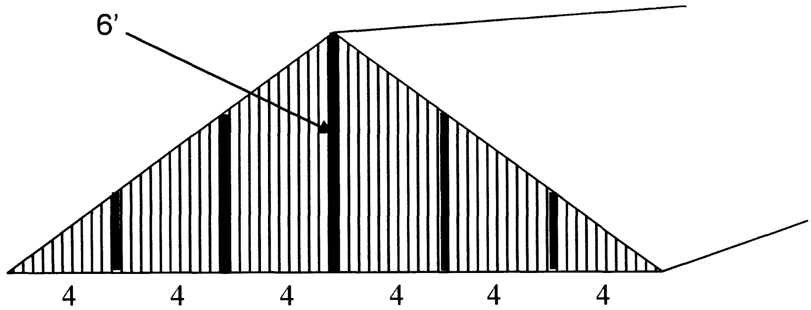

Figure 6. Gable end.

Let It Rain

As a final example, we investigate an actual rain gauge that will measure more than 5" of rain so that the scale in inches is marked approximately $4\,{}^{3}\!/_{8}$ inches per inch. Since the gauge is nearly 2□ tall, with this large scale per inch, it can be read from a distance of 10-20 yards. The problem is for students to determine the accuracy of the scale by performing actual measurements on the gauge which has a circular collection top much larger than the tube it feeds water into (see Figure 7).

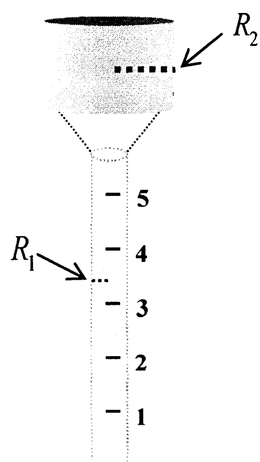

Figure 7. Oversized rain gauge.

Some discussion about how to measure rainfall is useful. Assuming rainfall is uniform in a certain locale, we could use a pot of any dimension, so long as the edges are vertical, to measure that an inch of rain in the pot represents a rainfall of one inch. Similarly, if the **level** of water in a swimming pool rises one inch, we received an inch of rain, even if the pool is kidney shaped or otherwise. The problem here is to compare the area of the collector top to the area of the collector tube to determine the scale on the tube representing an inch of rain. It is difficult to measure the diameter of the closed tube even when a centimeter tape measure is used. Students usually resort to a somewhat more accurate measurement of the circumference in both places and converting the measurement of the corresponding radii. This provides a good millimeter problem to find each radius since the ratio of the square of the corresponding radii will yield the ratio of the corresponding areas $\left(\frac{\pi R_2^2}{\pi R_1^2}\right)$ and hence, the ratio producing the inch scale on the tube. A follow-up discovery question is to find a ratio $\frac{\pi R_2^2}{\pi R_1^2}$ which will give an exact inch scale of 4:1. If the student is familiar with square roots, this will not be difficult. Perhaps a patent could be acquired on a new rain gauge.

Conclusion

While not all of these applications might be interesting to every student, they do illustrate a wide range of mathematical applications. By using these geometrical applications in a practical, hands-on manner, teachers gain confidence in their problem solving abilities which they will, in turn, pass on to their own students. Enthusiasm is infectious, and these students may need little encouragement to find examples of geometry in their own lives. If students are persuaded to present their "findings" during classroom presentations with the promise of extra credit, they may surprise themselves with their own ability to teach.