INTEGRATIVE AND CONTEXTUAL LEARNING IN COLLEGE ALGEBRA: AN INTERDISCIPLINARY COLLABORATION WITH ECONOMICS

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ABSTRACT

Many students consider mathematics too abstract and useless for their academic and career goals. Meanwhile, instructors in quantitative disciplines such as economics find many students mathematically underprepared for their courses. The disconnect between students' perceptions of the utility of mathematics and their life and career may have contributed to some of the under-performance in learning mathematics. Addressing this problem requires collaboration across disciplines to develop an understanding of each other's needs, more specifically to develop an integrative platform that allows students to apply mathematical skills in interdisciplinary contexts (Ganter & Barker, 2004). We collaboratively designed and implemented an integrative platform that includes creation of assignments and resources that contextualize the course in College Algebra with applications of economics, facilitation of frequent interdisciplinary dialogues among faculty members, creation of a course pair, and expansion of the platform to include 15 sections of College Algebra. This paper describes the process of the design and implementation of the platform.

KEYWORDS

integrative, contextual learning, interdisciplinary, college algebra, economics

LaGuardia Community College (LaGuardia), with an enrollment of about 20,000 students, is part of the City University of New York (CUNY) system. The course discussed here, College Algebra and Trigonometry, hereafter referred to as College Algebra, is a required core course for most majors at LaGuardia. This includes many non-STEM majors such as Business, Tourism and Hospitality, Fine Arts and Design, and Commercial Photography. Approximately 40 sections of College Algebra are offered each semester. Many students consider this course to be too abstract and useless for their academic and career goals.

Meanwhile, instructors in quantitative disciplines such as economics find that many of their students are mathematically underprepared. Instructors find themselves devoting class time to teaching basic mathematics concepts and skills. Students are often late to discover that college algebra concepts and skills are in fact useful for advancing their academic and career goals.

This disconnect between students' perception of the utility of college algebra at different stages of their academic career is not just a problem at LaGuardia Community College. It is a problem observed nationwide, and combating this problem requires meaningful collaboration between partner disciplines (Ganter & Barker, 2004).

The main objective of our project was to make mathematics accessible, useful, and relevant to students while also developing their understanding of economics. Our project included creating mathematics assignments involving contexts in economics, piloting these assignments in a sizable number of College Algebra sections, and developing a course pair (defined below) between College Algebra and Microeconomics to help facilitate the use of these assignments. During the process, we held interdisciplinary conversations between mathematics and economics faculty, used surveys to gather information about the mathematical needs of students in introductory economics courses, developed a curriculum map of economics and mathematics courses at LaGuardia, utilized mathematics instructional resources in economics courses, and conducted a workshop to train mathematics faculty members to use the newly developed instructional platform before scaling up the use of the platform in all sections of College Algebra. These project activities are described in detail below.

Literature Review

Philosophers, educators, and social scientists have long been linking relevance of education to students' motivation to learn (Karabenick & Urdan, 2014; Lazowski & Hulleman, 2016; Albrecht & Karabenick, 2017; Albrecht & Karabenick, 2018). The National Research Council's Committee on Increasing High School Students' Engagement and Motivation to Learn (NRC, 2004) recommended the need for instructional programs to be relevant to and build on students' cultural backgrounds and personal experiences and to provide opportunities for students to engage with authentic tasks that have meaning in the world outside of school. "Relevance interventions" that inject an element of relevance into course curricula have resulted in promising, positive effects on academic achievement and motivation (Karabenick & Urdan, 2014; Lazowski & Hulleman, 2016). Although some recent work has produced null to negative results (Albrecht & Karabenick, 2017), academicians increasingly recognize the importance of contextual learning or situative learning, emphasizing how a person learns a particular set of knowledge and skills and the situation or a context in which a person learns (Lave and Wenger, 1991; Putnam and Borko, 2000). By providing meaningful contexts or situations that are relevant to students, contextual learning or situative learning allows students to construct the meaning of what they learn based on their own experiences.

This conceptual framework leads to integrated, interdisciplinary learning, also called connected learning, that incorporates contexts from different disciplines into a particular subject matter. In our case, the subject is mathematics. Many educators acknowledge the natural overlaps between mathematics and other disciplines in terms of real-world events. "Integrating mathematics and science shows students how applicable mathematics is in the natural world. They are using mathematics to make sense of the world around them," (Frykholm, & Glasson, 2005, p. 132). Our project is rooted in this framework by incorporating economics contexts in mathematics assignments.

Prior studies have found some positive effects of integrative approaches in STEM subjects on students' achievement and motivation (Riskowski et al., 2009; Sanders, 2009). While evidence of the effectiveness of an integrative approach on academic performance in mathematics is not as abundant, Farrior et al. (2007) found that this type of instruction could help students better understand real-world applications of mathematics. The increased intrinsic motivation may improve students' academic performance in the end (Farrior et al., 2007). Research on the effects of integrative and contextual learning on mathematics subjects has been limited, and mathematics is the STEM subject that benefits the least from these approaches (Becker & Park, 2011). The data that will be collected in the later stages of our project will add insight to this emerging field of research.

Creating Assignments

In the early 2000's, the Mathematical Association of America's (MAA) Curriculum Foundation Project (CF) led an effort to achieve the goal of increasing students' perception of the utility of college algebra (Ganter & Barker, 2004). Through a series of disciplinary workshops, participants learned about the mathematical challenges faced by partner disciplines (Ganter & Barker, 2004). The MAA recommended that direct and purposeful connections should be made between lower mathematics courses and introductory courses in other disciplines. With the use of the appropriate technology, mathematical modeling should be used to help students develop the necessary problem solving and communication skills for success in partner discipline fields (Ganter & Barker, 2004).

With the goal implementing the MAA's recommendations to improve lower level mathematics courses, the National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P), a National Science Foundation (NSF) funded project, was formed. As part of the SUMMIT-P project, mathematics faculty in each member institution conducted discussions with partner disciplines (Hofrenning et al., 2020). At LaGuardia, two mathematics faculty members partnered with two economics faculty members to understand how College Algebra could be redesigned to help students studying economics.

Our project started with interdisciplinary conversations to understand the mathematical needs of students enrolled in economics courses. We held multiple face-to-face discussion sessions between the mathematics and economics faculty. Similar to the structure of CF workshops, faculty in economics curated a "wish list" of mathematics topics. We also referred to two reports from the CF project (Ganter & Barker, 2004; Ganter & Haver, 2011) focusing on economics to develop the following list of topics that students need to be successful in introductory economics courses at LaGuardia:

- using basic arithmetic and algebraic skills such as equations and algebra, effects of changing parameters in linear equations,
- calculating areas of relatively simple geometric figures,
- plotting numbers and graphs, interpreting graphs,
- linear data and graphs, exponential data and graphs,
- graphing and interpreting linear equations, including understanding the slope of a line,
- solving simultaneous linear equations,
- total/average/marginal concepts,
- exponential functions.

In addition to the above list, we identified topics specific to the needs of LaGuardia students including absolute value inequalities, rational functions, and trigonometric functions. After the discussions, the economics faculty presented a full list of mathematical topics along with some detailed economics examples that could be used to illustrate the topics. The list can be found in Appendix A.

These discussion sessions helped us collect information on the mathematical needs of students in economics courses at LaGuardia and find connections between mathematics and economics. The conversations led us to a better understanding of the challenges inherent in knowledge transfer across disciplines. For example, presentation styles and notations differ significantly between disciplines. As a result, in order to develop a deeper understanding of the partner discipline's presentation style and how certain concepts appear in both disciplines, as well as other challenges students face when applying mathematics concepts and skills in economics contexts, the mathematics faculty visited the economics courses from time to time.

Input from Economics

In addition to discussions between the mathematics and economics faculty, we also gathered other data to help understand students' mathematical needs in economics courses. We conducted preliminary surveys on students' attitudes toward mathematics and the extent to which they connected mathematics with economics. We also conducted a preliminary assessment in an economics course to identify students' common weaknesses in mathematics.

From the previously mentioned interdisciplinary faculty conversations and class observations, we found that many students in economics courses struggled with quantitative skills. Many had trouble with the basics such as understanding tables, creating and interpreting graphs, and even working with fractions, decimals, and percentages. As a result, many students were not able to develop the ability to explain and apply quantitative concepts and interpret results, which are highly valued skills in economics. Such students needed a variety of basic mathematical knowledge to successfully complete their economics courses. We aimed to further study this issue in two steps. First, we assessed students' attitude towards mathematics and identified weaknesses in mathematical skills that are relevant to economics courses. Second, we intervened in an economics course by incorporating guided mathematics exercises and explicitly making the connection between these mathematical skills and relevant economics topics.

At the beginning of the semester, to identify students' mathematical weaknesses, all students in a microeconomics class took a diagnostic online mathematics pretest (see https://bit.ly/33n3h2m). The test was designed by faculty at Syracuse University as a general mathematics pre-assessment for economics students. No previous knowledge of economics is necessary to take this test. Mathematics skills that are assessed include fractions, decimals, percentages, ratios, proportions, graphs, linear and nonlinear relations, and basic algebra. We then mapped these basic mathematical skills to each microeconomics topic in the course syllabus. For each lesson, we listed both the economics and mathematical goals. We also experimented with an intervention strategy using guided mathematics exercises. A self-tutorial platform, Khan Academy (Khan Academy, 2018), was used for guided exercise to help students work on their weaknesses, as identified in the pre-assessment, and to apply basic mathematical skills, such as algebra, to the analysis and interpretation of quantitative information in the context of microeconomics. Students were required to watch short videos and complete weekly assignments on the Khan Academy website.

Qualitative and quantitative data on students' performance and attitudes towards mathematics were collected at the beginning and at the end of the semester. Students completed pre and post attitude surveys to determine if they were able to make the connections between their mathematical skills and what they were learning in the microeconomics course. A performance comparison was also conducted to ascertain the effectiveness of the intervention, using guided mathematics exercises. In terms of performance, the grade distributions of the class that had not been given guided exercises in spring 2017 (no intervention) and the class that had been given guided exercises in spring 2018 (with intervention) were collected and compared. From the data, there was no significant difference between the two grade distributions. However, the intervention did help some students recognize the relevance of mathematics to microeconomics as evidenced by the following comments by students in the 2018 group:

Comment 1: "The math that was required was not difficult but needed practice to remember. The math tutorials were crucial in equipping me with the necessary math tools for understanding microeconomics."

Comment 2: "I believe that in order to be successful in this course, you would need to have a good understanding of elementary math."

Comment 3: "The class helps me to see the usefulness of math in our life. We are using math to solve the problems in our life, and it is really helpful."

Comment 4: "Microeconomics is totally related to the math, so students should take MATH115 or 120 before this class."

Comment 5: "Being the type of student that strongly despises math, it is being so present in the microeconomics curriculum absolutely deterred me from the subject. My brain simply rejects the field of math. It is obviously up to me to rectify this behavior and put some efforts. But I do wish we were to study the field of economics with math."

Key Elements in the Design of Assignments

Through conversations with other mathematics instructors, we identified the following challenges when designing, implementing, and evaluating assignments that incorporated interdisciplinary contexts: (a) it is very time-consuming to find meaningful real-life problems to embed in mathematics teaching; (b) there is limited class time for students to develop a minimal level of familiarity with the interdisciplinary topics; (c) instructors need some training to design project assignments that address the components of one of LaGuardia's assessed core competencies, namely inquiry and problem-solving.

Noting these challenges faced by mathematics faculty members in general, we designed assignments with the intention that they eventually be used by all sections of College Algebra. In order to ensure a smooth scaling up of our platform of content to more sections in the later stage of the project, the content of the assignments should be accessible to all students and instructors. Very few College Algebra students and instructors have backgrounds in economics. With this in mind, we incorporated three pillars when designing our projects—relevance, scaffolding, and emphasis on written ability.

Relevance

Project assignments should be relevant to students with different academic backgrounds. We should motivate each mathematics and economics concept with real-world examples drawn from student life or subjects they are familiar with or can readily relate to.

Scaffolding

Faculty should be able to use our content with minimum preparation. In addition, we want our project assignments to be self-contained so that students can work independently if they need to. To this end, we scaffold our content. First, each project starts with small group discussions among students. Faculty members can then orient the class with the project topic through a class discussion. Students will have a chance to express their opinions about the key issues addressed in the assignment during the discussion. This sort of personalization will improve student motivation by giving them a sense of ownership over the topic. The group discussions also include technology demonstrations in the form of simple web-apps. Instructors can demonstrate these applications to help students develop intuition about the quantitative aspects of a project. Students can also play with these applications at home. These applications offer students a hands-on introduction to the topic and terminology without requiring them to fully understand the underlying mathematics. An example of one of our web-apps is an activity called Price and Profit (Henshaw, 2019). By varying parameters such as fixed and variable costs, students can see how prices are related to profits. In order to facilitate out-of-classroom learning, for each project assignment, we provide links to a set of videos on an online platform that demonstrate key concepts and techniques of mathematics and economics. Students can practice with the content presented in these videos by doing short sets of accompanying exercises. These sets of short-answer questions are graded automatically so students receive instant feedback.

Emphasis on Written Ability

The final component of each project assignment is an essay prompt which gives students an opportunity to communicate their ideas in writing. The essay is important because it helps students to self-identify areas of confusion or conceptual misunderstanding. It also allows students to meaningfully reflect on the opinions they expressed during the initial class discussion. Written communication makes an assignment memorable and engages students' quantitative reasoning skills.

Sample Assignment

Students apply quadratic functions and vertex formulas to maximize profit and revenue and minimize cost in one assignment we developed (see Appendix B). Topics such as profits, revenues and costs are relevant to many students enrolled in College Algebra as many of our students are business majors or are interested in starting their own business. LaGuardia has a relatively large percentage of non-traditional students, many of whom have experience working in or owning a business. We also attempt to emphasize the relevance of mathematical concepts by incorporating role-playing and using real-life examples. Students are asked to imagine themselves as an executive of a hypothetical smart phone company with a name that reminds students of a very familiar smart phone, the iPhone.

Because students and course instructors may not have rigorous backgrounds in economics, we scaffold the assignments. Students are introduced to economics concepts with question prompts as well as with video clips. They are guided though the activity with problems that require short answers. For example, students are introduced to the concepts of profit, revenue, and cost through the first few question prompts. Students watch video clips that explain these economics concepts and how quadratic functions and vertex formulas can be applied to the problem situation. They complete short online exercises requiring the use of quadratic functions and vertex formulas. By the end of the assignment, students are expected to solve more complex questions. For example, they are asked to solve a couple of maximization problems by applying quadratic functions and vertex formulas in the given scenario.

Finally, students are asked write an essay. Students are expected to include their answers to all of the questions in the assignment in the essay. By communicating their answers in writing rather than merely solving mathematical equations, students learned to integrate mathematics in an interdisciplinary context rather than through seemingly meaningless symbolic manipulations.

Implementation of the Assignments

At LaGuardia, College Algebra is one of the mathematics courses which assesses students' inquiry and problem-solving skills, one of the three core competencies in LaGuardia's general education program (see https://www.laguardia.edu/assessment/). The assessment is based on two to three required project assignments in which students are expected to apply mathematics knowledge to solve real world problems. The assignments we created help the mathematics faculty meet the requirements of this assessment. In part, as a result, the faculty are receptive to using our instructional platform.

As we did not expect all instructors to have an economics background, we offered a workshop to share the philosophy and help with the implementation of the projects. Twenty-six mathematics instructors attended the workshop. Throughout the workshop, the issues and strategies for implementing assignments were fully discussed. Participants cited time constraints as an issue. The additional amount of time needed to guide students through the completion of an interdisciplinary integrative assignment could result in instructors not being able to cover the entire course curriculum as required by the department. Furthermore, participants observed that many students were predisposed to perceive mathematics as a standalone subject and consider mathematics assignments that integrated contexts from another discipline as "extra work". Nevertheless, participants in the workshop agreed that integrative projects help students develop confidence and broaden their awareness. They suggested that these assignments should be assigned early in the semester whenever possible when both instructors and students were less busy and less stressed. They also suggested that instructors should provide students with guidance on the economics context and the available resources for completing the assignments. They also suggested that if these projects were given as group assignments, formal group roles should be assigned and each student should be held accountable.

During the workshop, we briefly described the SUMMIT-P project and LaGuardia's involvement in the project. We outlined the philosophy and goals of the SUMMIT-P project and demonstrated to participants how to import these projects into an online course platform. In order to gain insight into the student experience, workshop participants completed the project assignments in groups. Most participants agreed that these projects were appropriate for students in College Algebra and aligned with LaGuardia's inquiry and problem-solving core competency. In Spring 2019, these project assignments were piloted in 15 out of approximately 40 sections of College Algebra. The workshop was supported by the Center of Teaching and Learning at LaGuardia and CUNY Open Education Resources Initiatives (see https://www.cuny.edu/libraries/open-educational-resources/).

Learning Community: A Course Pair

In order to further expand the context of economics within the College Algebra curriculum, we developed a learning community in the form of a course pair between College Algebra and Microeconomics. The assignments we developed were incorporated into the course pair. Different higher education scholars and practitioners define learning communities differently. LaGuardia's learning communities are essentially curricular learning communities. The same group of students enroll in two or more courses from different disciplines thematically linked in such a way that students find increased coherence and integration across these courses (Lenning & Ebbers, 1999). At LaGuardia, this type of learning community is called a course pair if it consists of two courses and is called a course cluster if there are more than two courses involved. Students enrolled in the pair are to take both courses in the same semester.

A learning community that links courses and enrolls the same cohort of students has been found in previous research to help improve students' experiences in those courses, thus nurturing positive attitudes toward the subject matters (Lenning & Ebbers, 1999). The initial stage of developing our course pair involved reviewing the syllabi of both courses to map topics and timelines (see Table 1). The goal was to incorporate the context of microeconomics in College Algebra whenever possible. After the review, project assignments were designed for five chosen topics in College Algebra (see the Creating Assignments section above and Appendix B). The points in the semester at which the projects were assigned were carefully chosen so that the College Algebra topics aligned with the corresponding topics covered in Microeconomics. This helped to ensure that students were familiar with the related topics in Microeconomics while working on the projects in College Algebra. Students received course grades for the assignments in College Algebra.

The major challenge we faced when launching our course pair was enrollment. Due to low enrollment, we were not able to offer the course pair in Fall 2017 and Spring 2018 as we had initially planned. However, we successfully offered the course pair in Fall 2018 after adapting a strategy we learned from a SUMMIT-P project partner institution, Saint Louis University (SLU).

In April 2018, the LaGuardia SUMMIT-P team conducted a site visit to SLU, where we visited Professor Mike May and his project team. Similar to LaGuardia, SLU was also incorporating business and economics contexts in activities and assignments in College Algebra and Calculus. While visiting SLU, we learned that professional academic advisors promoted the newly-designed, SUMMIT-P-inspired college algebra course to students.

Learning from SLU's example, the LaGuardia team shared our project efforts with program directors and academic advisors. In particular, we promoted the course pair to Accelerated Study in Associate Programs (ASAP) at LaGuardia, a program that provides resources such as advising and financial assistance to help students complete their associate degree programs as soon as possible. This approach resulted in increased advisors' awareness of the pair and its mission and benefits. Conversations with students and advisors at ASAP led to the identification of several problems: (1) students were typically told that these two courses were “very difficult,” thus it was not advisable to take them together, (2) many students took College Algebra several semesters before they took economics courses, and (3) students were advised to take other mathematics or statistics courses instead of College Algebra for the purpose of transfer. After being made aware of these problems, we worked closely with advisors to help ameliorate some of these issues. Our efforts paid off and we were finally able to fill the pair with 29 students in each course section in fall 2018.

Table 1
Curriculum Mapping Between College Algebra and Microeconomics

Topics in College Algebra	Topics in Microeconomics
linear equations	equations of demand and supply curves
system of linear equations	market equilibrium
difference quotient	marginal value (marginal cost, marginal revenue, marginal product, marginal utility); elasticity
operations of polynomials	cost function, production function in polynomial functions
quadratic functions	cost and revenue functions
rational functions	cost function, production function in rational function
inverse function	converting demand/supply functions where the price depends on the quantity demanded/supplied
exponential function	compound interest; growth in macroeconomic variables such as GDP, price level and others. Exponential consumer utility function
exponential equations	consumer preferences
logarithmic functions	compound interest; growth in macroeconomic variables such as GDP, price level; logarithmic production function; logarithmic consumer utility function
logarithmic equations	consumer preferences; production decision

Course Pair Design

There were a few key elements essential to successfully implementing a course pair: (a) developing a sense of community across both courses, (b) maintaining regular communication between faculty members, (c) using similar language across disciplines and sharing resources, and (d) integrating the context in activities and assignments completed inside and outside the classroom. In order to support the sense of community in the pair, instructors in both sections frequently reminded students of the connection between topics and assignments in both courses. In addition, instructors of both courses communicated frequently to detect and discuss issues arising from the implementation so that we could fine-tune quickly. Each instructor used the partner instructor's nomenclature and integrated their course's resources whenever possible. This helped students to transfer contexts from one discipline to another more smoothly.

Also, co-curricular activities took place from time to time to further reinforce the economics context. For example, we organized a trip to the gold vault of the Federal Reserve of New York (FRBNY). FRBNY provided free guided tours for school groups, therefore it was logistically and economically accessible. Students were able to learn about the role of the Federal Reserve System and currency policies. More importantly, students learned about the importance and relevance of quantitative skills in economics policy-making. In addition, we also mentored a team of students in the course pair to compete in Math is Everywhere, a college-wide competition in which participating teams develop a research presentation that integrates mathematical skills with other disciplines. Our team developed the presentation "Quantifying Effects of a Minimum Wage," in which they applied college algebra skills to explain the quantitative implications of having a minimum wage. Last but not least, students in the course pair displayed their work at the annual LaGuardia Learning Community Showcase. These activities were very well-received. These opportunities further emphasized to students the importance and relevance of quantitative skills.

Discussions and Future Work

To date, we have created a series of college algebra assignments with economics contexts, experimented with these assignments in 15 out of approximately 40 sections of College Algebra at LaGuardia and also used the assignments in a course pair of College Algebra and Microeconomics. Throughout the process we learned that knowledge transfer across disciplines is not as straightforward as we had imagined. For example, both faculty members and students had difficulty switching between different notations and definitions used to refer to the same concept. We also observed that many students were not very motivated to complete the interdisciplinary integrative assignments because they tended to be more challenging and, perhaps more importantly, students had traditionally believed that mathematics was a standalone subject and perceived projects that integrated mathematics with another course discipline as unnecessarily burdening. We also speculated that other possible reasons were that students needed to take part in more in-class discussions, needed more guidance from instructors on how to complete the assignments, and needed more information on the economics concepts in each assignment as compared to regular assignments. In other words, they found it difficult to transfer knowledge from one discipline to another.

It was challenging for faculty to devote more time to these assignments as compared to their regular activities and assignments while also fulfilling department-related curriculum requirements. In addition, as most mathematics faculty members did not have much of a knowledge base for economics, it was challenging for faculty members to enrich the economics context with narratives and stimulate dynamic student engagement in class discussions.

We also found that learning communities such as a course pair, while useful in reinforcing the connection between mathematics and other disciplines, could also present challenges. Other than issues surrounding enrollment as described above, the performance of many students in the pair was worse than that of standalone courses. One of the reasons for this could be sample selection bias. As mentioned above, many students at LaGuardia tended to take College Algebra several semesters before they take Microeconomics. Those who waited until later semesters to take College Algebra might have been students who were weaker in mathematics and feared the subject the most.

We have continued to fine-tune our project to address the above challenges. First, we have converted notations and definitions in the assignments to better match what is used in economics. For example, in a demand curve, the Y variable in College Algebra typically denotes the price while the variable P is used in economics. Similarly, the X variable is used to denote the quantity in College Algebra while the variable Q is used in economics. We are encouraging College Algebra instructors to help students translate between the two sets of variables. Second, in order to mitigate issues related to the lack of economics background among students and mathematics faculty members, we are contemplating a crash course in sections of College Algebra taught by a guest economics faculty member twice during the semester. These crash courses may provide opportunities for students to solidify their understanding of the relationships between mathematics concepts and skills and the economics concepts in the assignment. Moreover, the guest lecturer could share narratives with the class that further enrich the context. Third, we are also contemplating making a collection of narratives on the economics concepts covered in the assignments readily available to mathematics faculty who teach College Algebra. Fourth, we plan to include extra-curricular activities such as those mentioned above to further raise awareness among students that mathematics is not just a standalone academic subject but is applicable to many fields and disciplines.

Despite all of the challenges outlined above, we found, through questionnaires and conversations with faculty members and students, that students in our SUMMIT-P college algebra project seemed to have an increased appreciation for the relevance and usefulness of mathematics. In the near future, piloting these assignments in more sections of College Algebra will allow us to quantitatively assess the effectiveness of these integrative projects. In particular, we believe the following research questions are worth investigating:

- What are students' perspectives on mathematics in relation to their academic and career goals?
- How does the experience of applying mathematics to solve real world problems change students' attitudes towards mathematics and mathematics learning?
- How has the pilot affected students' performance in current and future mathematics courses?

The data we collect will shed light on these questions. Moreover, we will collect feedback from course instructors and continually update these integrative projects to better accommodate the needs of both instructors and students.

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Appendix A

Economics faculty summarized their mathematical needs in economics courses as the following wish list.

1. Basic arithmetic and algebraic skills—equations and algebra, effects of changing parameters in linear equation.
2. Calculating area of relatively simple geometric figures.
3. Plot numbers, interpret graphs, linear/exponential data/graph.
4. Graphing and interpreting linear equations, slope of line, slope at a point, increasing at an increasing/decreasing rate.
5. Two linear simultaneous equations.
6. Total/average/marginal concepts.
7. Natural log for linearizing data.
8. Compound interest rate.
9. Using multiple variables/parameters, variables names other than x, y parameters can be stated numbers or alphabetical letters, practical limits on the values of variables.
10. Supply-And Demand Analysis:
    - Graphs (P is treated as the dependent variable and graphed on the Y-axis. Quantity Q is regarded as the independent variable and is placed on the X-axis).
    - Systems of Equations (implicit or explicit form) (Solving a system of supply and demand equations algebraically to find the equilibrium point (Price and Quantity).
11. Break-even Analysis:
    The breakeven point (Profit = 0) is found both algebraically and graphically by solving linear or/and quadratic functions (Revenue and Cost curves)
12. Income Determination Models:
    Y = C + I + G + (X –M) where Y= Income; C=Consumption; I = Investment; G = Government Expenditures; X = Export; M = Import. By substituting the information in a model, we solve for the equilibrium level of income. Aggregation of the variables on the right-hand side of the equation allows the equation to be graphed in two-dimensional space.
13. IS-LM Analysis:
    IS-LM analysis seeks to find the level of income and the rate of interest at which both the commodity market and the money market will be in equilibrium. This can be accomplished with the techniques used for solving a system of simultaneous equations. (Matrix Algebra).
14. Optimal Allocation of Scarce resources among Competing Products:
    Optimizing a profit or cost function subject to several inequality constraints by way of linear programing (using graphs).
15. Maximization and Minimization (Cost, Revenue, and Profit) :
    Math Concepts covered: Maxima, Minima, Increasing and Decreasing Functions, Concavity and Convexity, Quadratic and Cubic functions, inflection Points, and Optimization Functions. (Differential Calculus: Uses of the Derivative)
16. Production functions, Economic Growth, Homogeneous Function, Production curve:

Estimating growth rates from data points. Profit or Sales function growing consistently over time, annual growth rates can be measured and a natural exponential function estimated through a system of simultaneous equations.

17. Exponential and Logarithmic Functions.
18. Consumers' and producers' surplus:
    - Shaded areas under/above Demand/Supply curves below/above price line calculated mathematically by way of Integrals Calculus.
    - The shaded areas (triangles…) are used to analyze the market efficiency and the economic welfare of the society.
    - Integral calculus is also used to integrate a marginal cost function to find a total cost function or to integrate marginal revenue function to find a total revenue function.
19. Natural Growth of Income, Investment Theory, Capital Accumulation, and Marginal Efficiency of Capital and Consumer Demand Elasticity
    Math Topics covered: Natural log, Integration

Appendix B

Assignment: Analyzing Cost and Revenue

Introduction
Cost, revenue and profit functions may take parabolic forms. In many business and economics applications, our most important goal is to maximize revenue, profit or minimize cost. We may be able to find the price or the quantity of goods and services that maximizes profit, revenue and minimizes cost by using quadratic formula and vertex formula. The goal of this project is to enhance the understanding of quadratic functions and how to find the maximum/minimum.

Question 1:
If you have the chance to start a business, what business would you choose? Why?

Question 2:
Starting and running a business requires time, effort, hard work and in particular money. What kind of costs do you expect to have to pay in order to start and run your business? Please list them and explain why you need them.

Question 3:
Some costs are fixed, which are called **fixed costs**, such as equipment and buildings. Some cost are variable, which are called **variable costs,** such as labor and material. Please explain what costs in the Question 2 are fixed costs, and what are variable costs.
In general, the total cost consists of variable costs and fixed costs.

Question 4:
In order to keep your business running, you need to make **revenue**. Revenue is the money that comes into the business from customers. Suppose you know the number of products your business sold and the price you sold them at, how can you calculate the revenue? What strategies could you use in order to increase your business' revenue?

The revenue of a business may go up and down depending on many factors. For example, a business that sells ice cream will likely make more money during hot summer months. The profit of your business is the difference of the revenue and the cost. That is,

Profit = Revenue - Cost

If the profit is the positive, your business makes money. If the profit is negative, your business unfortunately makes a loss. If the profit is zero, that is the revenue is equal to the cost, it is called the break-even point.

Suppose that you were the CEO of a giant high technology corporation, Strawberry, Inc, manufacturer of the Strawberry Phone.

Question 5:

This month you have estimated the demand for the Strawberry Phone to be:

$$Q = 220 - 4P$$

where Q is the quantity demanded, and P is the price of a Strawberry Phone.

The cost of producing a phone is constant at $12, which is called marginal cost. The fixed cost that includes the cost spent on the factory, the equipment, among others is $1525. As a result, you have a linear cost function,

$$C = FC + (MC\ Q)$$

Where C is the total cost, FC is the fixed cost, and the MC is the marginal cost, and Q is the quantity as before.

Answer the following questions.

1. What is the price that maximizes the corporation's profit? (Hint: Profit = Revenue - Cost)
2. At what price does the corporation break even?

Question 6:

The other day, as you are burying yourself in the sea of data, you find out that at $500, your corporation sells on average 50,000 Strawberry Phones, the corporation's flagship product, monthly. Additionally, for every $50 increase in the price of a phone, the sales decrease by 1,000 phones. Based on this information, you are very interested in finding

1. the price point that will maximize the revenue, and;
2. what the maximum revenue is

Question 7:

From the two questions above, create a strategy to lower your cost and maximize your profit for the business you chose in question 1.

Essay: Write an essay that discusses the answers to the questions above include a detailed description of your business ideas and how it is possible to use maximization of quadratic functions to find the maximum profit.