COUNTING ON COLLABORATION: A TRIANGULAR APPROACH IN THE EDUCATOR PREPARATION PROGRAM FOR TEACHERS OF MATHEMATICS

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ABSTRACT

This paper outlines the process of establishing a stronger and more reciprocal partnership for collaboration between an education preparation program and a local education agency. The essential partners identified included the College of Natural Sciences and Mathematics and the College of Education at Lee University and stakeholders in the local school district. First, this paper will discuss a theoretical framework that speaks to the importance of dialogue and a dialogic approach to teaching mathematics. Secondly, the processes and methods of the project involving collaboration through partnerships are described. These partnerships gave rise to the realization that coursework would be more effective if it mirrored the instructional practices of local education agencies. A detailed description of the process of changes to the coursework and initial outcomes of the project are outlined. Included are questions and recommendations for further collaboration.

KEYWORDS

collaboration, elementary mathematics, dialogue, problem solving

This paper addresses the impact of the Collaborative Research: A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) project at Lee University. SUMMIT-P is a multi-institutional project funded by the National Science Foundation that aims at revising lower division mathematics curricula through interdisciplinary collaborations, based on recommendations from the Mathematics Association of America Curriculum Foundations (CF) project (Ganter & Barker, 2004). As one recommendation, CF encourages the creation of Faculty Learning Communities consisting of mathematicians and faculty from other disciplines to help to implement the other CF recommendations in useful and practical revisions to mathematics courses. As a result of preparatory work and ideas gained from the CF Project (Ganter & Barker, 2004), we considered providing "[t]ools for teaching and learning, such as calculators, computers, and physical objects, including manipulatives commonly found in schools […] for problem solving in mathematics courses taken by prospective teachers" (p. 145). Another CF recommendation is that "[m]athematics courses for future teachers should provide opportunities for students to learn mathematics using a variety of instructional methods, including many we would like them to use in their teaching" (p. 145). This report provided a foundation on which to build professional development opportunities, course design, and pedagogical practices at Lee University.

The choice of collaborative partners in this project was based on those who are regularly involved in teaching mathematics educators. Selected participants were those who teach lower level mathematics courses in the Department of Natural Sciences and Mathematics (particularly those who teach mathematics educators), professors from the College of Education who teach courses in pedagogy, and the Coordinator of Professional Development in Mathematics from Bradley County Schools. Over the period of one year (2017 – 2018), discussions took place with all partners. Everyone agreed that, based on multiple data sources, there was a significant need to improve the pedagogical skills of teachers of mathematics. From general observation and research, it was established that improvement might rest on a connection between the theory and practice of dialogue, the importance of collaboration through partnerships, and the pursuit of effective practices such as the use of manipulatives. Out of the project there were initial outcomes and ideas for further collaboration.

The project partners decided that Concepts of Mathematics I and II, the primary courses for preparing pre-service teachers to teach mathematics in the P–8 setting, should be the focus of this project. Before the project began, each class was observed by the Principal Investigator (PI) for mathematics content and pedagogical practices and for the ways mathematics educators engaged in learning. Anecdotal data and the results of a mathematics manipulative tests taken by the students strongly indicated that these courses would benefit from review. Additionally, it was decided that other resources such as local experts involved in professional development in P–8 settings would have valuable input into raising the standard for teaching mathematics. Then during the next year (2018 – 2019), the professors who teach these two courses participated in intensive professional development with the local education agency. Details about our project are provided below.

The Importance of Dialogue

It has long been understood that language and communication are the basis for collaboration, partnerships, associations, and relationships. This is true in interpersonal cooperation but also in developing a cognitive understanding of specific concepts. The value of "talk" cannot be overestimated.

One essential aspect of this project was the realization that a way forward to improvement and enhancement of mathematics course work for elementary teachers was to emphasize "talk" (Meiers, 2010). Words such as talking, discussing, questioning, arguing, chatting, and conferring all conjure up the notion that discourse in the area of problem solving is a necessity. A recognition of the relationship between "talk" and solving problems relating to educator preparation was paramount in the minds of those involved throughout this project.

Problem Solving

The term "problem solving" is in and of itself a mathematical notion, but it does not imply that strict mathematical algorithms are the only methods to finding solutions. In fact, and more importantly, research increasingly suggests that solving "the problem" involves certain essential methods outside the perceived realm of mathematics, not least the idea of the necessity to talk.

The idea of problem solving through the use of discussion is not a new idea and has been used relatively often in the field of mathematics. The ancient Greek philosophers approached problems of mathematics and logic by posing and answering questions based on observation of the real word and on data. Over time it appears that this method was somewhat lost and replaced with rote learning and the memorization of processes. Essential elements of understanding were lost, particularly in P–8 classrooms. This project sought to revive and highlight the approach of collaboration through discussion as a way to solve problems. The courses under consideration were observed, data was gathered from each course, the performance of teachers in the field was investigated, and suggestions for improvement were recommended.

It was recognized, as already discussed, that there should be discourse between the departments involved and with the local education districts. This collaboration should be between all parties involved, with the sole objective of finding more effective ways to deliver instruction in the teaching of mathematics to the P–8 population.

Language and Problem Solving

In the relatively new field of human development, ideas were drawn from the work of theorists Freud, Piaget, Vygotsky, and Bakhtin for ideas related to affective and cognitive development. Considering the topic, "can talking solve problems?" a comment of note that was retrieved from *Psychology Today* (2017) frames a main premise that discourse is not only a way to problem solve but that this approach might also be curative. Conversely, this blog also suggests that the incorrect use of words, discussion, and discourse might also actually cause harm. The recognition that there are more helpful ways to talk through problems is valuable in all areas of life and education and cannot be overstated. Therefore, let us not underestimate the use of words in the realm of teaching mathematics and in the training of mathematics educators at all levels.

Piaget's stage theory of cognitive development has for many years had enormous impact on the teaching of mathematics. It assumes that certain concepts are acquired at certain stages that roughly correspond to age levels. In regards to language and its use in the mathematics classroom, it is generally believed that there is a close correlation to language development and the acquisition of certain mathematical skills. Although, in recent years, certain criticisms have been raised regarding Piaget and against a strict application of developmental stages, it might also be argued that within bands of development there is reason for adjusting and differentiating in the use of language as it relates to mathematical processes (Ojose, 2008).

An acceptance of Vygotsky's social learning theory and his work on language and thought brought to bear a consideration for effective questioning techniques, the increased use of language, the use of manipulatives, the role of the teacher, and collaborative learning within the courses and within the teaching of mathematics in P–8 classrooms (Vygotsky, 1986). The emphasis on the concrete, representational, and abstract steps in problem solving all rely on the essential connection between thought, language, and understanding within a social setting.

More recently, ideas from Bakhtin (1895 – 1975) on the radical importance of dialogue influenced thinking in a new way. In Wegerif's (2011) paper, "Towards a Dialogic Theory of how Children Learn to Think," he informs us that "learning to think" involves a dialogic space that has often been ignored in teaching in general and particularly in mathematics. Wegerif sought to discover why some groups of children were more successful at solving reasoning test problems than others. He observed the dialogue children used in relation to solving problems with seeing patterns, commutativity, and making a graph without instructions. He found that the more successful groups listened more to each other, asked each other for help, and were willing to change their minds as a result of seeing the problem through the eyes of another. Through his observations, Wegerif attributed the more successful activity of some groups to Bakhtin's notions of the ability to connect with a "dynamic continuous emergence of meaning" that depends on previous and succeeding knowledge that is mediated through the effective use of language in dialogue about representations and through posing questions.

It is the premise of this section that emerging educational theories of learning offer sufficient and necessary understanding of the importance of "talk" in understanding important elements in teaching mathematics.

Problem Solving Techniques

Today, there is common acceptance of the idea that all children learn differently and that all learning is a result of: shifts in thought that are properly mediated through language, the use of concrete representation (manipulatives), collaboration, and safe settings.

Advocated strategies such as the use of manipulatives, differentiation, "Accountable Talk," math journaling, math vocabulary, "Think Alouds," community of learners, and students connecting problems to self, others, and the world have all come to the forefront and offer promising results (Kazemi & Hintz, 2014). All these concepts, approaches, strategies, and shifts have emerged from observations of how children develop and learn. They relate directly to the theoretical framework for the Lee University SUMMIT-P project.

In discussion concerning the delivery of our courses, it was recognized that these shifts should receive a greater emphasis in the pedagogical approaches that are taught and modeled to those who will teach mathematics in P–8 classrooms. "Talk" is imperative in all classrooms and at all levels and is conceptually linked to understanding that is gained through active engagement that is brought about by "doing" (Smith & Stein, 2011).

Establishing and Strengthening Partnerships to Enhance Recommended Practices Underlying Rationale

If you make your way into any elementary or middle school, you will find that effective teachers of mathematics appear to have certain practices in common. One of the six Principles for School Mathematics (NCTM, 2013) states, "Research has solidly established the important role of conceptual understanding in the learning of mathematics. By aligning factual knowledge and procedural proficiency with conceptual knowledge, students can become effective learners" (p. 2). The National Council for Teachers of Mathematics also believes that "the foundation for children's mathematical development is established in the early years" (Seefeldt & Wasik, 2006, p. 249). If it is in fact true that conceptual understanding is vitally important in the learning of mathematics, then it seemed relevant to this study to first investigate current practices in teacher preparation that seek to address conceptual learning and second, to seek to improve upon these preparation practices to establish a foundation for the early years of development. We felt that three themes, problem solving, collaboration, and the use of manipulatives, held the keys for improving essential mathematics understanding.

The mathematical education community promotes hands-on learning and manipulatives. Companies such as ETA Hand2Mind, Learning Resources, and EAI Education distribute catalogues to educators advertising a variety of manipulatives. A mathematics educator can purchase products from an extensive list of manipulatives including patty paper, geoboards, counters, algebra tiles, and tangrams. However, if the educator has never learned mathematical concepts using these manipulatives or has never seen them used in mathematics instruction, they are left to wonder about the purpose, necessity, and benefit manipulatives bring to student comprehension. Implementing the manipulatives effectively is also a mystery to the educator that lacks experience and specialized training. Thus, it is imperative that teachers of pre-service teachers incorporate mathematical learning and teaching through manipulatives into course requirements. Recognizing this need, a relationship began between teachers and administrators with Bradley County Schools and the mathematics educators at Lee University to bridge the training gap as it relates to this method of mathematical instruction.

Manipulatives in the Mathematics Classroom

The important role that manipulatives play in the mathematics classroom cannot be overstated. Research shows that mathematics achievement levels increase with the use of manipulatives and learning is enhanced when students are actively engaged in the learning process. Stein and Bovalino (2001) concluded that manipulatives are important tools that can help students to think and reason in more meaningful ways. Sutton and Krueger (2002) found that manipulative use also increased mathematical interest among students. Manipulatives are a common instructional resource found in many mathematics classes. They can be used to model mathematical and often abstract concepts in order to support overall student understanding. Manipulatives can be a variety of objects such as coins, rods, paper clips, pieces of candy, or blocks. However, in recent years some classrooms have switched to using virtual manipulatives on tablets or computers (Uttal, 2003, p. 98). Kennedy (1986) defines manipulatives as "objects that appeal to several senses and that can be touched, moved about, rearranged and otherwise handled by children." He concludes that mathematical lessons should involve a variety of instructional methods. Integrating manipulatives along with other traditional teaching methods increases the likelihood that students will develop a solid understanding of the mathematical concept (p. 55).

According to the National Council of Supervisors of Mathematics in a 2013 statement, "[I]n order to develop every students' mathematical proficiency, leaders and teachers must systematically integrate the use of concrete and virtual manipulatives into classroom instruction at all grade levels" (p.1). NSCM's position statement is based on several research studies that support the practice of using manipulatives throughout classroom instruction. For example, in a study involving 8th-grade math teachers, Raphael and Wahlstrom (1989) concluded that the use of manipulatives along with "successful topic coverage by teachers" (p. 189) had a positive connection with the level of student comprehension. In 2013, a meta-analysis report was compiled involving research studies that had an emphasis on teaching mathematics with concrete manipulatives. Carbonneau (2013) specified that a primary requirement for inclusion in this study required assessment data from "an instructional technique that used manipulatives [and] a comparison group that taught math with only abstract math symbols" (p. 383). Out of 55 studies that were eligible for inclusion in this report, 35 came to the conclusion that students who were taught with manipulatives scored considerably higher on the unit assessment test when compared to those students who did not have access to manipulatives (Carbonneau, 2013).

While the use of manipulatives has been recognized to deliver positive results in many classrooms, it is necessary to highlight the probable explanations behind these results. In 2017, Willingham identified three likely theories for why manipulatives could be directly related to the increased assessment scores. First, manipulatives aid in learning because they require physical movement of the body, which some believe increases cognition. Another reason rests solely on the belief that children are concrete learners and that such learning leads them to understand the abstract. A final theory proposes that manipulatives are simply symbols for innovative mathematical ideas still to be learned in the classroom. However, if used incorrectly, manipulatives can cause difficulties for students to grasp the abstract concept they were intended to represent (Willingham, 2017, p. 26).

The way the teacher introduces and uses manipulatives plays a significant role in how well the mathematical concepts transfer to their students. Because manipulatives can represent abstract concepts, it is necessary that teachers understand how to appropriately use them during a lesson. Unfortunately, many difficulties with using manipulatives stem from a lack of familiarity on the part of the teacher. Kilgo and White (2015) recognized that "providing opportunities for pre-service teachers to use these [manipulatives…] will assist in building their confidence and encourage them to implement the aids in their own classrooms" (p. 217). Teachers have a responsibility to learn how manipulatives can bring about success while attempting to deter any complications that may set their students up to misunderstand a topic. Teachers should seek out opportunities to be trained in the use and functionality of different types of manipulatives. Waiting until days before a high-stakes assessment may result in confusion and frustration, both for the teacher and the students (Cope, 2015, p.17). Those teachers who have received clear directions and strategies for manipulatives are more likely to see positive results in their classrooms.

The Process

Bradley County Schools and Lee University have long been collaborative partners. With the increase of teacher accountability and high-stakes testing, a realization occurred that pre-service teachers enrolled at Lee University should be better equipped to demonstrate the most effective mathematical teaching practices. As a way of strengthening this partnership, an alignment of practices was believed to be essential.

The local school system has invested in curriculum adoptions that include classroom sets of manipulative kits. The district mathematics coordinator was able to utilize some of those sets to offer a hands-on professional development session for Lee University professional mathematics educators on how to effectively use manipulatives in the mathematics classroom. The professors provided the coordinator with course syllabi that included topics students would be learning throughout the courses. Two professional development sessions were designed by the district coordinator based on observational data of effective mathematics instruction with manipulatives in local classrooms, grade level standards analysis for Bradley County Schools from the Tennessee Department of Education, and topics from syllabi provided by the university professors. The district coordinator found natural links between the three pieces of data. Activities were designed to match manipulatives to conceptual understanding of mathematical concepts. Professors participated as learners and experienced learning mathematics with manipulatives, which assured them of the potential of these activities to leave students with long lasting understanding of mathematics at a concrete level. As a result, these professors left the session convinced of the need to incorporate learning mathematics with manipulatives into course requirements.

The development of understanding mathematics concretely is a process that can never be underestimated or overlooked. It is an important and necessary stage of development before a learner attempts to perform mathematics abstractly. In order to develop long-term comprehension, conceptual understanding, and procedural fluency, a mathematical learner must develop initial understanding at the concrete phase (the doing stage) before moving into the representational phase (the seeing stage) and the abstract phase (the symbolic stage). Unfortunately, the concrete understanding of mathematics is oftentimes underestimated and overlooked. Educators are not always equipped with the tools necessary to help students develop understanding at the concrete level, and many secondary mathematics educators do not see the need for it. The "I do, We do, You do" framework supports the belief that if a learner can see a mathematical process performed enough times then the learner will be successful performing the mathematical process alone. However, being fluent in mathematical concepts requires a concrete level of understanding, and learning with manipulatives can provide this type of understanding for students. New educators oftentimes walk into a classroom with cabinets full of manipulatives but with no understanding of how and when to use them. That is why courses for pre-service teachers must include learning and teaching mathematics with manipulatives.

Professors met with the coordinator twice. During the first session, participants explored how to use patty paper (i.e., small square pieces of wax paper) to model multiplication and division. The activities and problems were designed to enhance understanding of multiplication as an area model and division as partitioning. Participants also created hand-made fraction strips, which evoked a deep and specific conversation about the power in a learner creating fractional representations on equal-length strips of paper. Hand-made and store-bought fraction strips were used to create equivalent fractions, adding fractions, and multiplying fractions. Lastly, algebra tiles were introduced to participants as the key to developing number sense and a greater understanding of polynomials. Participants used the algebra tiles to build a foundation for the concrete understanding of the additive inverse property and the distributive property. Participants then modeled the technique of completing the square using the algebra tiles. The Bradley County

Schools Mathematics Coordinator had prepared many more examples and activities with manipulatives than time would allow, so a second professional development session was scheduled.

During the second professional development session, there was great discourse about the importance of concrete-representational-abstract learning. The coordinator shared documents created and produced by Mathematics Coordinators from the Tennessee Department of Education. These documents showed a variety of strategies students could employ in order to demonstrate understanding at each phase of concrete-representational-abstract learning. Participants then explored digital manipulatives such as Geometer's Sketchpad and Geogebra. Publications from Key Curriculum Press and the National Council of Teachers of Mathematics were shared with participants to provide a sample of resources and books with lessons and activities to support the use of Geometer's Sketchpad, an interactive geometry software. Geogebra, a free "dynamic mathematics software for schools that joins geometry, algebra, statistics and calculus through graphing and spreadsheets," was explored to enhance the concrete understanding of fractions, mean, and median (see www.geogebra.org). Participants also studied many pre-made lessons on desmos.com, such as linear functions, parabolas, slope, and graphing stories.

Professors had recently acquired class sets of Cuisenaire Rods and Base 10 Blocks, so a portion of the professional development session was spent using these two manipulatives to support concrete understanding of addition and subtraction, place value, multiplication and division, and the concept of regrouping. Participants finished the professional development session by playing a variety of games that develop and enhance number sense for students of all ages. Students must become comfortable strategizing with numbers.

Initial Outcomes of the Project

Observation Comments

The Principal Investigator (PI) for the SUMMIT-P project at Lee University had the opportunity to observe the two classes that are the subject of the collaboration between the mathematics division and the College of Education (COE): Concepts of Mathematics I and II. These are courses that future elementary and middle-school teachers are required to take for certification. She observed Concepts of Mathematics I in spring 2017 before any of the recommendations by the COE were implemented.

She then observed both courses in spring 2019 after the teaching faculty were provided with professional development opportunities about how to implement the recommendations, with a focus on how to use manipulatives in delivering the course material.

Observations after Professional Development

Concepts of Fractions and Representations

The Concepts of Mathematics I class the PI observed covered the topic Concepts of Fractions and Representations. The manipulatives used were Fraction Towers, consisting of interlocking blocks that indicate different fractions (see Figure 1). The instructor started with explaining the concept of unit fractions, fractions with 1 in the numerator. She then explained a fraction as a collection of equal-sized parts. For example, the 4 in the denominator of ¾ indicates how the whole is divided, and the 3 in the numerator indicates the number of equal parts of the whole we are considering. Students were given some exercises to solidify this concept.

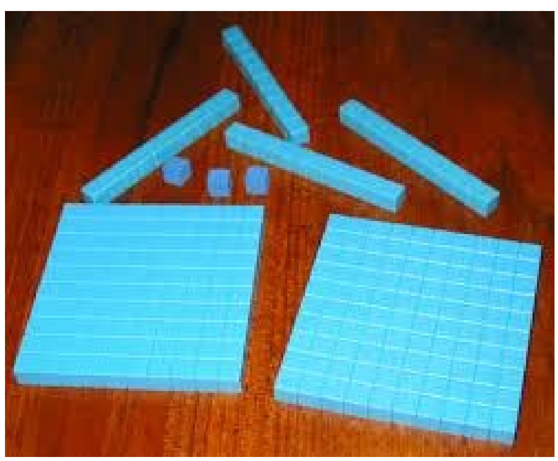
For the in-class activity students were asked to take a ½ tower and find all fractions that are equivalent to it. They then had to represent each equivalent fraction as part of a whole. Thus, they made the connection that 2/4, 4/8, 3/6 and 5/10 were all equivalent to the ½ tower (see right side of Figure 1).

Figure 1
Fraction Tower Manipulative Set Stacked to 1 and to 1/2

Some questions were posed to the students: why can we not use 1/3, or 1/5, creating self-discovery and critical thinking opportunities for students.

In another class in this course students used Base 10 Blocks to model division and multiplication. Base 10 blocks are made up of unit cubes. One unit cube is a 1; 10 unit cubes stacked up together is a rod of 10; 10 adjacent rods make a 100, a flat (see Figure 2).

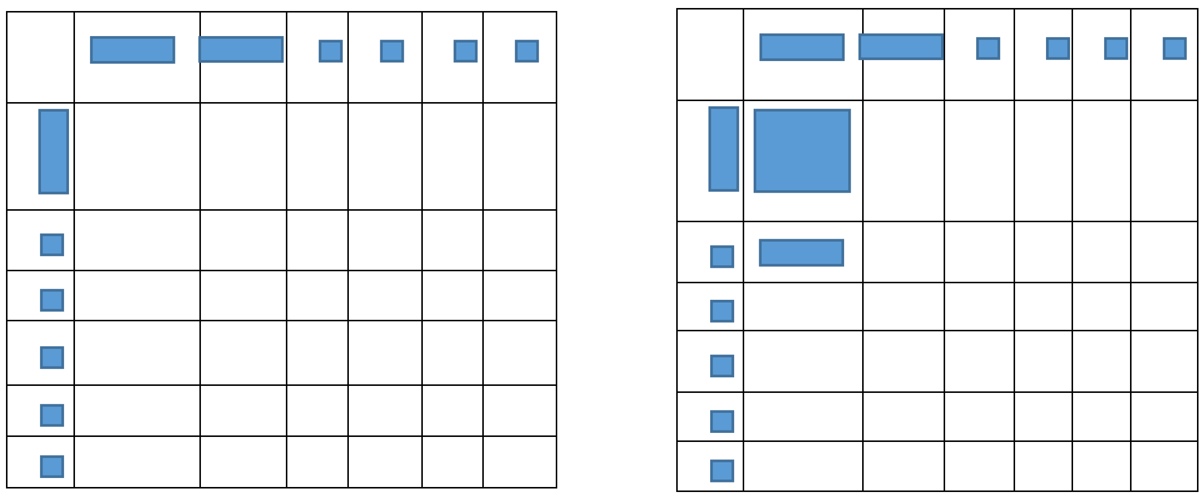
Figure 2
Base 10 Blocks



Activity 1

The students had to model 24 x 15 using Base 10 Blocks. They created a table structure where on the first row they placed two rods and four units, representing 24. On the first column they placed one rod and five units, representing 15 (see left side of Figure 3). They then proceeded to fill the table with appropriate Base 10 Blocks. The space below the first rod in the first row and adjacent to the rod in the first column was filled with a flat (a 100). The space below the first rod in the first row and adjacent to the first unit in the first column was filled with a rod (see right side of Figure 3), and so on.

Figure 3
Illustrations of the Unit Block Practice



After finishing the table, the students added up the Base 10 Blocks to get the answer for the multiplication.

Activity 2

The students had to model 736/3 using the flats, rods, and units. They started with 7 flats (= 700), 3 rods (= 30) and 6 units (= 6). They proceeded by dividing the 7 flats into groups of 3. This resulted in *two groups of 3 flats* each and one flat remaining. The remaining flat was broken to ten rods and added to the original 3 rods, resulting in 13 rods. The division process continued by separating the rods into groups of 3: *four groups of 3 rods* and a remaining rod. The remaining rod was broken then to 10 units and added to the original 6 units. The division process continued to give *5 groups of 3 units* and a *remaining unit*. Students were able to visualize that the result of the division was 245 and a remainder of 1.

Finding Area of Geometrical Figures

The PI observed one class of Concepts of Mathematics II. The day's topic was finding the area of geometrical figures. Each student was given a square and a rectangle cut out from card stock, a pair of scissors, and tape. The objective of the activities done in class was for the students to derive the formulas instead of memorizing them.
The instructor gave the definition of area as the measurement of the surface inside the boundaries of the geometric shape.

Activity 1

Students were asked to trace the square they were handed at the beginning of class on their notebook and find the area. Everyone knew the formula: area is base times height or length times width or $(\text{side})^2$ in this case. Some students had graphing paper, so they traced the square on their paper and were able to count how many squares were inside the boundaries.

Activity 2

The instructor asked the students to cut the square across the diagonal into two triangles and find the area of a triangle. Everyone realized that since the area of the whole square was base times height, and now the area is divided into two parts, then the area of each triangle is ½ base times height.

Activity 3

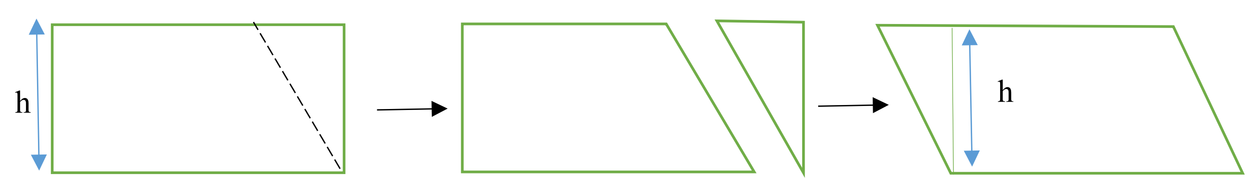
Similarly, students were asked to trace the card stock rectangle onto their paper and find the area. At this point they realized that the base times height, or length times width, formulas are applicable, but not the $(\text{side})^2$ formula.

Activity 4

To derive the area of a parallelogram, students were asked to cut the rectangle starting at any corner and cut off a corner, not necessarily through the diagonal, slide the cut-off triangle to make a parallelogram (see Figure 4) and tape it.

Figure 4

Making a Parallelogram from a Rectangle



In their exploration of finding the area of the parallelogram, they realized it was still base times height, as the area of the original rectangle. They became aware that now the height is not the length of the side of the parallelogram but what was the side of the rectangle.

Similar activities were completed to find the area of trapezoids and circles. These activities gave a deeper understanding of the mathematical concepts covered and revealed the logical reason behind mathematical rules and formulas. They also provided an excellent visual for the students. All students observed were paying attention in class and were rather amused when it came to the activities part. They used the manipulatives with ease, which indicated they have used them before and were comfortable manipulating them. Compared to the class observed before implementing the COE's recommendations, it was clear that the manipulatives kept the students actively engaged in the learning process and that they can create their own ideas for classroom materials they can use when they are in the workforce.

Conclusions and Recommendations

Initial outcomes appear to underline the significance and value of partnerships, close observations of current practices, and a subsequent willingness to dialogue with an eye towards revising approaches to established courses in Mathematics for teacher candidates. The opportunity provided by the SUMMIT-P Project at Lee University was the precipitating step to establishing a triangular approach to one important part of the Educator Preparation Program.

The relationship between Bradley County Schools Mathematics Coordinator and Lee University professors is new, unique, and unprecedented. The mathematical expertise that professors brought to the conversation was invaluable, and the experience from local classrooms that the district coordinator brought was meaningful. Both parties provided a lens through which mathematics education could be enhanced and improved. It is through this partnership that pre-service teachers at Lee University will be better equipped to teach mathematics with manipulatives to help students dialogue about and understand mathematical concepts in the concrete-representational-abstract phases.

Recommendations

The teaching of mathematics in the P–8 setting is often regarded as still "needing improvement." The experience and improvement gained through this project gives some guidance for endeavors for preparing future math educators.

First, the idea of mathematics professor educators learning from teachers in the field is novel. Attendance at trainings facilitated by local education agencies by university professionals appears to be an optimal way of learning the methods and processes that are used by teachers in the P–8 setting. It makes sense to continue this approach as new educators learn to teach in a way that promotes understanding and meets local needs.

Additionally, the consideration of theories of learning, particularly the importance of dialogue, between and within settings might be offered by education preparation programs as perspective for the importance but sometimes gaping nexus between theory and practice.

Consideration should also be given to expanding this process into other courses that address the principles of mathematics instruction and methods for teaching mathematics in clinical experiences.

Finally, observations of Lee University teacher graduates, their continued use of recommended practices in relation to teaching performance scores would strengthen the validity and reliability of this project.

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