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COLLABORATIVE EXPLORATIONS

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PART I: SPECIAL ISSUE

A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P)

PART II: REGULAR JOURNAL FEATURES

Virginia Mathematics and Science Coalition

JOURNAL OF MATHEMATICS AND SCIENCE: COLLABORATIVE EXPLORATIONS

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A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P)

PART II: REGULAR JOURNAL FEATURES

THE NEED FOR INTERDISCIPLINARY COLLABORATIONS

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ABSTRACT

The challenge faced by developers of collegiate mathematics curricula is to determine—and then provide—the mathematical experiences that are true to the spirit of mathematics yet also relevant to students' futures in other fields. The Curriculum Foundations Project (CF) of MAA/CRAFTY was designed to gather input from partner disciplines through a series of 22 two- to three-day workshops. Each workshop resulted in a report directed to the mathematics community, summarizing the workshop's recommendations and conclusions. One message from the partner disciplines appeared again and again: introductory collegiate mathematics courses should focus on giving students an understanding of fundamental mathematical topics while grounding the discussions in context. The National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) is a group of 16 institutions working to implement the ideals from the CF recommendations. Full participation from partner discipline faculty in this process is a key ingredient in successfully redeveloping introductory mathematics courses in a way that incorporates the contextual needs of other disciplines. The papers in this special issue speak to the work of the SUMMIT-P consortium, focusing on the processes used for successful interdisciplinary collaboration.

KEYWORDS

interdisciplinary collaboration, MAA/CRAFTY, Curriculum Foundations, SUMMIT-P

Mathematics plays a critical role in undergraduate education. In fact, employers have emphasized for years that they seek individuals who can think mathematically, reason through problems, and work effectively on interdisciplinary teams (Singapore Ministry of Education, 2018; Steen, 2001). As such, graduates who have meaningful mathematical experiences are better able to face the challenges of careers in both mathematics and other disciplines—including those in non-scientific areas. Additionally, students who are equipped to use technology appropriately, model complex situations, and apply specific mathematics to the work within their chosen fields will be well on their way to a successful career—no matter what their chosen field may be.

This being the case, the real question becomes: How are "meaningful mathematical experiences" defined, and how can (and should) they be measured? Most educators would agree that many mathematics courses are not designed and executed in ways that create such experiences (Lederman, et al., 2013; Walker & Sampson, 2013; Blair, 2006). Students leave these courses with a set of skills they are unable to apply outside of the classroom and for which they do not appreciate the relevance to their future careers. Such experiences do not endear students to the importance of mathematical thinking, creating an urge to finish with required mathematics courses as quickly and painlessly as possible. That attitude often translates to faculty members outside mathematics, with the perception that the mathematics community is not interested in the needs of non-mathematics majors, especially those in introductory courses.

This is a long-standing issue for the mathematics community—and for the discipline of mathematics broadly defined. Educators continue to agree that mathematical thinking is an important skill, as demonstrated by the continuation of mathematics requirements at all academic levels. However, what isn't universally clear is exactly *why* and *in what ways* these mathematical skills are important to the 95% of students in first-year mathematics courses who go on to major in other disciplines. The challenge, therefore, is to determine—and then provide—the mathematical experiences that are true to the spirit of mathematics yet also relevant to students' futures in other fields. The question then is not whether they need mathematics, but what mathematics they need and in what context.

Twenty Years in the Making

In the late 1990s, the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America (MAA) began discussing the preparation of its next Curriculum Guide (CUPM, 2004, 2015), a document published once each decade since CUPM's formation in 1953. The purpose of the Curriculum Guide is to assist college mathematics departments in the on-going development and improvement of their undergraduate programs. Historically, this document had focused on the traditional mathematics major, with little attention to alternative courses and programs and virtually no mention of mathematics courses for non-mathematics majors. However, it became clear that this important document could no longer ignore the wealth of new programs, courses, and materials resulting from the reform movement in the undergraduate mathematics community. In particular, the dramatic changes being implemented in introductory college mathematics courses, including precalculus, calculus, and differential equations, needed to be studied and directly addressed in the recommendations of the Curriculum Guide.

As a result, in 1999 CUPM initiated a major analysis of the undergraduate mathematics curriculum. As the subcommittee of CUPM concerned with the first two years of college mathematics programs, the Committee on Curriculum Renewal Across the First Two Years (CRAFTY), has a major role in analyzing and formulating recommendations concerning the foundational years in mathematics instruction. Moreover, given the impact of mathematics instruction on the sciences and quantitative social sciences—especially instruction during the first two years—there was a need for significant input from these partner disciplines. Therefore, CRAFTY was charged with gathering this necessary information for the "mathematics intensive" disciplines (e.g., physics, chemistry, and engineering). Thus, the Curriculum Foundations Project was born.

The Curriculum Foundations Project

The Curriculum Foundations Project was designed to gather input from partner disciplines through a series of workshops. The original workshops with colleagues from 17 mathematics-intensive disciplines were held across the country between 1999 and 2001, culminating in a final summary conference in November 2001. A second set of workshops was organized with five additional disciplines (agriculture, arts, economics, meteorology, and social sciences) between 2005 and 2007, creating a combined set of recommendations from representatives of 22 distinct disciplines. These recommendations and the 22 disciplinary reports were published by MAA in two documents: *Curriculum Foundations Project: Voices of the Partner Disciplines* (Ganter & Barker, 2004) and *Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra* (Ganter & Haver, 2011). The results also contributed significantly to the content of the two most recent Curriculum Guides (CUPM, 2004; 2015).

Each Curriculum Foundations (CF) workshop lasted two to three days and consisted of 20−35 participants, the majority chosen from the partner discipline(s) under consideration, the remainder chosen from mathematics. The workshops were not intended to be discussions between mathematicians and colleagues in the partner disciplines, although this certainly happened informally. Instead, each workshop was organized in a "fishbowl" style that encouraged dialogue among the representatives from the partner disciplines, with mathematicians present only to listen and serve as a resource when questions about the mathematics curriculum arose. A set of guiding questions was used to motivate the conversation and produce responses that could be combined across disciplines. Each workshop resulted in a report directed to the mathematics community, summarizing the workshop's recommendations and conclusions. The reports were written by representatives of the partner disciplines, ensuring accurate reporting of the workshop discussions while also adding credibility to the recommendations.

The host institutions funded most of the workshops. Such financial support indicated the high level of support from university administrations for such interdisciplinary discussions about the mathematics curriculum. Workshop participants from the partner disciplines were extremely grateful—and surprised—to be invited by mathematicians to state their views about the mathematics curriculum. That the opinions of the partner disciplines were considered important and would contribute to national mathematics policy only added to their enthusiasm for the project as well as their interest in continuing conversations with the mathematics community.

In addition to the workshop reports, the CF project resulted in a number of publications that describe the workshops, their outcomes, and related work. These publications include articles in journals of the disciplinary societies as well as the general press. Conversations also continued via panels and invited colloquia at professional meetings, both in mathematics and the partner disciplines.

Recommendations for the Undergraduate Mathematics Curriculum

Whether the workshop focused on physics, engineering, economics, or the arts, the message from the partner disciplines was repeated again and again: introductory collegiate mathematics courses should focus on giving students an appreciation and understanding of fundamental mathematical topics while grounding the discussions in context. The specific topics are not as important as 1) technical confidence; 2) the application of mathematics to a variety of contexts; and 3) the ability to choose appropriate tools for modeling, evaluating, and communicating mathematical results (Ganter & Barker, 2004; Ganter & Haver, 2011). Specifically, the collective CF reports recommend emphasis on the following:

Conceptual Understanding

- Focus on understanding broad concepts and ideas in all mathematics courses during the first two years.
- Emphasize development of precise, logical thinking. Require students to reason deductively from a set of assumptions to a valid conclusion.
- Present formal proofs only when they enhance understanding. Use informal arguments and well-chosen examples to illustrate mathematical structure.

Problem Solving Skills

- Develop the fundamental computational skills the partner disciplines require, but emphasize integrative skills: the ability to apply a variety of approaches to single problems, to apply familiar techniques in novel settings, and to devise multi-stage approaches in complex situations.

Mathematical Modeling

- Expect students to create, solve, and interpret mathematical models.
- Provide opportunities for students to describe their results in several ways: analytically, graphically, numerically, and verbally.
- Use models from the partner disciplines: students need to see mathematics in context.

Communication Skills

- Incorporate development of reading, writing, speaking, and listening skills into courses.
- Require students to explain mathematical concepts and logical arguments in words.
- Require students to explain the meaning—the hows and whys—of their results.

Interdisciplinary Priorities throughout Content and Courses

- Strive for depth over breadth.

- Offer non-calculus-based descriptive statistics and data analysis.
- Develop curricular materials within calculus and linear algebra that are appropriate for the needs of partner disciplines.
- Replace traditional college algebra courses with courses stressing problem solving, mathematical modeling, descriptive statistics, and applications in the appropriate technical areas.
- Pay attention to units, scaling, and two- and three-dimensions (Ganter & Barker, 2004).

Because the CF recommendations were compiled from the collective input of 22 disciplinary working groups, they are broadly defined and encompass a wide variety of perspectives. Therefore, effective implementation of the recommendations requires continuous conversations with the partner discipline faculty, allowing them to collaborate with mathematics faculty in the development of curricula that include disciplinary context. For example, the CF recommendation that students create, solve, and interpret mathematical models as applied to chemistry would involve utilizing solutions that are highly visual; i.e., students must be able to visualize structures and atomic and molecular orbitals in three dimensions (Ganter & Barker, 2004, p. 29). The same recommendation applied to civil engineering might take a more analytical or numerical path, utilizing technology-based mathematical techniques to arrive at a solution and determine its limitations (Ganter & Barker, 2004, pp. 58 – 59).

SUMMIT-P: Promoting Collaborations across Disciplines

While the effective implementation of the CF recommendations requires collaboration across a large and diverse set of disciplines that are making ever greater use of mathematics, no mathematics department can offer a different set of mathematics courses for each partner discipline. Therefore, it is critical to rethink and revise the most common introductory mathematics courses. Since the broad categories of conceptual understanding, problem solving, mathematical modeling, and communication cut across the recommendations from all partner disciplines, introductory mathematics courses should be redeveloped in ways that incorporate these universal needs.

The Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships consortium (SUMMIT-P) is a nationally-distributed group of 16 institutions[1] working to implement the ideals from the CF recommendations through cooperation with a variety of partner disciplines. Full participation from partner discipline faculty in this process is a key ingredient in successfully redeveloping introductory mathematics courses in a way that incorporates the contextual needs of other disciplines. As such, the consortium's first task was to find ways to best engage colleagues in the partner disciplines. Initial conversations at SUMMIT-P meetings led to activities that experimented with a variety of mechanisms for that purpose.

[1] At the time they were written, the papers in this special issue describe the work of multidisciplinary curriculum development teams at nine institutions, supported by a Project Management Team involving four additional institutions. The projects at the nine implementation sites are briefly described in *Curricular Change in Institutional Context: A Profile of the SUMMIT-P Institutions*. Subsequent to the writing of the papers in this issue, three other institutions—Embry-Riddle Aeronautical University, University of Tennessee at Knoxville, and Humboldt State University—have joined the SUMMIT-P project, bringing the number of multidisciplinary curriculum development teams to twelve.

Specifically, because this collaborative approach for curriculum development is being implemented at a variety of institutions, each institution has 1) used locally appropriate strategies, 2) engaged faculty from locally-selected partner disciplines, and 3) focused on mathematics courses selected by that institution. However, each SUMMIT-P institution has undertaken the following processes:

- Faculty have studied the CF recommendations and the relevant disciplinary reports.
- Opportunities have been provided for partner discipline faculty to describe to mathematics faculty which of these CF recommendations are most important to their students, often through "fish bowl" activities.
- Partner discipline faculty have developed wish lists of mathematical topics and experiences their students need, and mathematics courses are being changed in response to these lists.
- Faculty have participated in local professional development experiences (through seminars and learning communities) and SUMMIT-P professional development (through webinars, poster presentations, panel discussions, and the development of this special issue).
- Courses are being developed, piloted, and refined in a collaborative fashion involving faculty and students from mathematics and partner disciplines.
- Mathematics and partner discipline faculty have visited courses offered outside of their departments.
- Faculty have participated in SUMMIT-P "course clusters" that frequently bring together institutions that are working on similar mathematics courses to discuss implementation strategies and outcomes.
- Teams from other SUMMIT-P institutions as well as project leadership and evaluation personnel have visited each SUMMIT-P institution to 1) attend classes, 2) interact with faculty and administrators, and 3) talk with students; these site visits are organized using a protocol designed to help the host institution plan for and conduct the visit in a way that engages the broad college community.
- In turn, mathematics and partner discipline faculty at each institution have visited other SUMMIT-P institutions.

JMSCE Special Issue Articles

The papers in this special issue speak to the work of the SUMMIT-P consortium as the project reaches the four-year mark, focusing on the processes used for successful interdisciplinary collaboration (as opposed to the curricular changes that have and will be implemented). Indeed, the SUMMIT-P participants believe that these processes constitute the most important contribution of the SUMMIT-P work to the national effort called for by CF and the CUPM curriculum guides.

Some of the processes deployed by members of the consortium are described in this issue as follows:

Fishbowl Discussions: Promoting Collaboration between Mathematics and Partner Disciplines describes how "fishbowl" discussions were used to enable mathematics faculty to understand the perspective of faculty in partner disciplines. It also describes how "wish lists" were developed to enunciate the needs of students studying topics in the partner disciplines.

Using Site Visits to Strengthen Collaboration describes the power of site visits for strengthening the collaboration among faculty from different disciplines. Site visits allow curriculum developers and implementers to narrow the focus of their efforts, contribute to community building, support cross-pollination of ideas, and provide dedicated time to reflect on the ongoing work.

Structured Engagement for a Multi-Institutional Collaborative to Tackle Challenges and Share Best Practices highlights two protocols that provide a structured format to give feedback to partners who are seeking advice on a challenge they are experiencing and to enable partners to share their success stories. While collaboration among different institutions and different disciplines is extremely powerful, it also can be challenging; formal protocols can be a useful tool for supporting this collaboration.

Paradigms for Creating Activities that Integrate Mathematics and Science Topics describes how mathematics and partner discipline faculty from three universities developed integrated activities that illustrate real-world applications of the mathematics topics being studied in Precalculus and Calculus. These activities address the statement in the CF recommendations that undergraduates need to see connections and applications of mathematics across and among their quantitative reasoning courses.

Using a Faculty Learning Community to Promote Interdisciplinary Course Reform describes the development and implementation of a faculty learning community that enabled faculty from mathematics, nursing, social work, and business at one institution to redesign mathematics content and vertically integrate mathematics into the partner discipline programs. Intentional efforts are required to support collaboration among disciplines; different SUMMIT-P institutions used different mechanisms and this paper reports on one successful approach.

The Roles and Benefits of Using Undergraduate Student Leaders to Support the Work of SUMMIT-P provides examples of how undergraduate peer leaders at different universities provided support in creating and implementing interdisciplinary lessons to enhance lower division mathematics courses. The engagement of the peer leaders is beneficial to faculty, to students in the target courses, and to the undergraduate peer leaders as well.

The Process and a Pitfall in Developing Biology and Chemistry Problems for Mathematics Courses explains the process used for developing applied problems from biology and chemistry for use in a Differential Calculus course. It includes a discussion of the role that peer students played but also deals with how these problems will be used by instructors who did not participate in their development.

Counting on Collaboration: A Triangular Approach in the Educator Preparation Program for Teachers of Mathematics describes collaboration among college mathematics faculty, education faculty, and teachers and administrators in a local school district. One goal was to ensure that teachers of pre-service teachers incorporate mathematical learning and teaching through manipulatives in their pedagogy courses.

Integrative and Contextual Learning in College Algebra: An Interdisciplinary Collaboration with Economics reports on how faculty at one community college used the CF reports to initiate discussion between economics and mathematics faculty members. Based upon these discussions and a review of the literature, student activities with real world applications are being developed to enhance the college algebra course and plans were made for professional development for all instructors.

Promoting Partnership, Cultivating Colleagueship: A SUMMIT-P Project at Norfolk State University describes a strong partnership developing at one university between mathematics and engineering faculty. It also reports on how a SUMMIT-P site visit played a crucial role in providing focus to their collaborative work.

Designing a Student Exchange Program: Facilitating Interdisciplinary, Mathematics-focused Collaboration among College Students shares the details of a student exchange program providing interdisciplinary experiences for students majoring in mathematics, statistics, or social sciences. The paper describes how the program is proving to be valuable not only for the participants but also students in statistics classes and both statistics and social science faculty.

From Creative Idea to Implementation: Borrowing Practices and Problems from Social Science Disciplines discusses the challenges to developing a mathematics course in collaboration with partner discipline faculty, with particular attention to portability. As faculty beyond the original collaborators and developers teach the course, attention needs to be paid to instructor familiarity with applications, varieties of assessment styles, and grading consistency.

Good Teachers Borrow, Great Teachers Steal: A Case Study in Borrowing for a Teaching Project describes how a set of courses at one university is being developed based on continual borrowing and stealing of ideas. Borrowing takes place from textbooks and CF reports as well as by joining the SUMMIT-P consortium, collaborating with faculty from partner disciplines, and visiting other sites and hosting site visits.

Evaluating a Large-Scale Multi-Institution Project; Challenges Faced and Lessons Learned reports on the on-going evaluation of SUMMIT-P and describes how the evaluation provides a birds-eye view of the work that those entrenched in the project are not able to see. It also discusses lessons being learned as the evaluation continues that could be valuable to others involved in multi-site evaluations.

Curricular Change in Institutional Context: A Profile of the SUMMIT-P Institutions provides a brief description of the context and work being conducted at each of the SUMMIT-P institutions. The authors invite faculty at other institutions interested in conducting similar work to peruse the descriptions and find familiar institutions, geography, curriculum goals, and mathematical topics and then to reach out to these institutions for support and further collaboration. This paper concludes with a "getting started" check-list.

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FISHBOWL DISCUSSIONS: PROMOTING COLLABORATION BETWEEN MATHEMATICS AND PARTNER DISCIPLINES

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ABSTRACT
A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships project (SUMMIT-P) is a collaboration of institutions focused on revising first- and second-year mathematics courses with the help of partner disciplines with prerequisite mathematics courses. This paper describes the fishbowl discussion technique used by the consortium members to encourage interdisciplinary conversation. Vignettes describing the results of conversations that occurred at several consortium member institutions are provided by the co-authors.

KEYWORDS
Curriculum Foundations report, SUMMIT-P, interdisciplinary collaboration, fishbowl discussions

Many disciplines use mathematics, but rarely do faculty from mathematics and other disciplines engage in meaningful conversation about how the subject is taught and used in the undergraduate curriculum. Faculty from other, nonmathematical disciplines can be important partners in developing a more robust mathematics curriculum. A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P), a project funded by the National Science Foundation (NSF), is a nationally distributed group of institutions focused on revising first- and second-year mathematics courses in collaboration with partner disciplines with prerequisite mathematics courses. Such revisions and interdisciplinary partnerships allow faculty to encourage broader and more successful participation in science, technology, engineering, and mathematics (STEM) learning, especially as it relates to learning in undergraduate mathematics courses. The goal of these partnerships is to build stronger support for partner disciplines and to encourage critical thinking skills in all fields while empowering sustained growth in the STEM workforce.

The nine original institutions of SUMMIT-P are Augsburg University, Ferris State University, LaGuardia Community College, Lee University, Norfolk State University, Oregon State University, Saint Louis University, San Diego State University, and Virginia Commonwealth University. As of January 2020, three additional institutions, Embry-Riddle University, Humboldt State University, and University of Tennessee-Knoxville have joined SUMMIT-P. Conducted across disciplines at each institution, SUMMIT-P meetings build on the strength of collaboration between a variety of partner disciplines, including nursing, economics, business, biology, chemistry, engineering, physics, and social work. Participants from partner disciplines work with mathematics faculty to discuss appropriate measures for updating mathematics curricula to make them more relevant to students majoring in the partner disciplines.

This paper will describe the partner-discipline conversations, known as fishbowls, which have been conducted by several institutions as part of the SUMMIT-P project. These conversations were modeled on the basic methodology established at the inaugural SUMMIT-P meeting in 2016. After describing the background and methodology used in developing these conversations, we will present illustrative vignettes to summarize the experience of using the fishbowl discussion technique at six of the partner institutions. These discussions have generated a viable roadmap for developing expanded mathematics curricula based on the wish lists created through the partner-discipline conversations.

Background

The National Science and Technology Council (NSTC) report stresses the importance of STEM in helping the United States develop a competitive economy (National Science Council, 2018). STEM knowledge is a critical component for an innovative workforce. In fact, employers place a premium on employees conversant in mathematical skills. An analysis of résumés and salary reports by the jobs and recruiting firm Glassdoor (Berry, 2018) finds that the best-paying jobs are those that require mathematical knowledge and skills. However, according to the President's Council of Advisors on Science and Technology (PCAST), fewer than 40% of students who plan to major in a STEM field actually complete programs and graduate with a STEM degree (PCAST, 2012). PCAST reports that it is necessary to increase the retention of STEM majors to reach the goal of producing one million more college STEM majors by the next decade in order for the United States to maintain its excellence in science and technology (PCAST, 2012).

Attracting and retaining students in STEM programs has proven challenging. In fact, there are many reasons students abandon STEM majors. Students express that they find STEM courses uninspiring and uninteresting; some indicate that the mathematics required in these courses can be difficult, leading them to give up on their programs and pursue a different major. To reverse this situation, the PCAST report recommends improving STEM teaching methods through the use of evidence-based approaches to engage students in “active learning,” which could lead to an increase in the number of students in STEM majors; Braun et al. (2017) provide recent examples of active learning techniques. However, many teachers are unfamiliar with these approaches or lack experience in teaching using such methods (PCAST, 2012).

Work on understanding and addressing these issues was conducted in the early 2000s. The Mathematical Association of America (MAA), through the Curriculum Foundations (CF) project, conducted a series of national workshops to facilitate discussions with non-mathematics faculty from November 1999 through February 2001. In these workshops, participants from 17 disciplines provided their insights on the mathematics curriculum in order to help create meaningful and relevant content for students majoring in their disciplines. Later workshops including five additional disciplines were held between 2005 and 2007. The combined results of these workshops were summarized in two reports that offered recommendations for departments interested in updating college mathematics courses (Ganter & Barker, 2004; Ganter & Haver, 2011). The CF reports’ findings revealed that faculty in disciplines that include mathematics seek to emphasize conceptual understanding and problem-solving skills in their introductory mathematics courses. At the same time, the connection between mathematics and the students’ chosen field is often not made clear from the outset, leading to confusion and frustration. For this reason, the reports concluded that mathematicians and partner-discipline faculty should work together on curriculum development to demonstrate to students the essential connection between mathematics and their discipline. The CF recommendations emphasize creating successful experiences for students by making mathematics content relevant to students’ lives and future studies.

The SUMMIT-P consortium has worked to amend the mathematics curriculum of the participating institutions based on the CF recommendations. These changes have been implemented in ways that support improved STEM learning. In addition, the interdisciplinary collaboration within each member institution and across the SUMMIT-P member institutions creates a network that supports transformative institutional change. This network of institutions can share challenges, successes, and ideas to further promote interdisciplinary partnerships within other institutions. This can lead to improved teaching and learning in undergraduate mathematics courses and ultimately to improved STEM learning for all students.

Development of the SUMMIT-P Fishbowls

All nine institutions of SUMMIT-P participated in structured conversations, called fishbowl discussions or simply fishbowls. The fishbowls employ a discussion technique (Priles, 1993) that allows for rich interaction between groups, where one group (the partner-discipline faculty) responds to questions while the other, silent group (mathematics faculty) observes the discussion. In this way, the dynamics of discussions are focused on a partner discipline’s needs and allow for honest conversation by and between the non-mathematics faculty.

While the original CF fishbowl discussions of 1999 – 2001 were held by the Mathematical Association of America (MAA) across the country at various academic institutions, the SUMMIT-P discussions were conducted at each of the participating SUMMIT-P institutions. The ground rules established for the discussions incorporated recommendations in the CF project reports (Ganter & Barker, 2004; Ganter & Haver, 2011). The context of discussions varied however, as each SUMMIT-P institution was unique in terms of student population, size of institution, culture, partner disciplines participants, and curriculum (Beisiegel & Dorée, 2020). In each case, the SUMMIT-P project institution revisited CF report recommendations and considered whether, and to what degree, they remained applicable within their own institution, and modified them accordingly. A set of discipline-specific "wish lists" was generated collaboratively from these fishbowl conversations, where the participants sought to capture what they felt were important mathematical concepts that should be included in their revised curricula. The creation of these lists constitutes an important element in the success of SUMMIT-P's work, as the wish lists could be implemented by each institution and each discipline to map both the mathematics and partner discipline course learning objectives. We describe these institution-specific fishbowl discussions in the vignettes below.

Fishbowl Structure

The first project-wide SUMMIT-P meeting in 2016 gathered all project participants from the nine institutions, including principal investigators (PIs), co-PIs, evaluators, and a project management team (administrators of the NSF grant) representing a variety of disciplines.

To prepare for the inaugural fishbowl, all SUMMIT-P members read the CF report that was relevant to their discipline and reviewed the original recommendations in that report. In addition, questions to be used in the fishbowl were provided to participants ahead of time (see Table 1). The fishbowl participants were first asked if the recommendations from the original CF report still rang true and if there were any points missed by the recommendations or learning skills omitted from the original summary CF reports. The other questions were arranged in four categories: understanding and content, technology, instructional interconnections, and instructional techniques. Table 2 provides a list of the participants and the roles necessary for a fishbowl discussion. Since there were over thirty SUMMIT-P project participants, two fishbowl discussions occurred simultaneously, one with faculty from the physical and natural sciences and engineering and one with faculty from the social sciences, business, nursing, accounting.

The fishbowl facilitators had an important role in guiding the conversation. The discussions in the initial SUMMIT-P fishbowls were led by two non-mathematics SUMMIT-P participants (engineering and economics) who posed questions, directed discussion, and monitored the time. Significantly, facilitators were chosen from a discipline other than mathematics to ensure that faculty from the partner disciplines would feel comfortable answering questions about the mathematics skills needed for their own disciplines. At the same time, it was deemed important for the partner disciplines to be aware of and acknowledge mathematics faculty members' opinions. For this reason, mathematics faculty were expected to observe the conversations and serve as a resource if there were any questions. After about 20 minutes, the discussion was opened up to the mathematics faculty observing the fishbowl.

The participants of the two initial SUMMIT-P fishbowls then participated in fishbowl discussions at their respective institutions as part of their institutional SUMMIT-P projects. In this way, the activity at the SUMMIT-P meeting proved to be a beneficial modeling exercise for learning and experiencing the process firsthand.

Table 1
Fishbowl Activity Questions

General 1. As you read the CF report, do the recommendations still ring true? 2. Do you believe there are topics unique to your discipline that are not reflected in the summary report?
Understanding and Content 1. What conceptual mathematical principles must students master in the first two years? 2. What mathematical problem-solving skills must students master in the first two years? 3. What broad mathematical topics must students master in the first two years? 4. What priorities exist between these topics? 5. What is the desired balance between theoretical understanding and computational skill? How is this balance achieved? 6. What are the mathematical needs of different student populations and how can they be fulfilled?
Technology 1. How does technology affect what mathematics should be learned in the first two years? 2. What mathematical technology skills should students master in the first two years? 3. What different mathematical technology skills are required of different student populations?
Instructional Interconnections 1. What instructional methodologies relative to teaching mathematical concepts in your discipline would you like to be made aware of?
Instructional Techniques 1. What kinds of mathematical technologies do you use in your discipline? 2. What mathematical technologies should students develop?

Vignettes

The vignettes presented here provide summaries of fishbowl discussions conducted from 2016 – 2017 at six SUMMIT-P institutions: Augsburg University, LaGuardia Community College, Lee University, Norfolk State University, Oregon State University, and Virginia Commonwealth University. The authors describe the experience of using the fishbowl discussion technique, including the preparations for, participation in, and outcomes derived from the conversations that took place on their respective campuses. As these vignettes show, even though the basic methodology was modeled at the 2016 SUMMIT-P meeting, each institution customized the technique to suit their institutional profile and specific programmatic goals and needs.

According to the rules laid out in the initial SUMMIT-P meeting, depending on the size of the group and the number of disciplines involved in the discussion, an institution could choose to run a fishbowl with any number of partner disciplines. For example, Oregon State University ran a fishbowl with faculty from biology while Lee University facilitated a fishbowl discussion with chemistry and biology. Augsburg University held seven separate fishbowls (a separate fishbowl for each discipline).

In the vignettes that follow, Rhonda Fitzgerald from Norfolk State University and Joan Kunz from Augsburg University explain the process of conducting fishbowls at their universities. Tao Chen from LaGuardia Community College and John Hearn from Lee University describe the table mapping exercises by which they connect the wish lists to course learning objectives. Lori Kayes from Oregon State University provides a vignette describing the dialogue between biology and mathematics. Rebecca Segal and Afroditi Vennie Filippas describe how information and wish lists were obtained at Virginia Commonwealth University.

Table 2
Fishbowl Participants and Their Roles

The discussion technique used to create conversation and share ideas is called a fishbowl. One group (usually 5 – 8 people) sit in a circle or at a table, conversing in full view of another group of listeners (also 5 – 8 people).

Participants	**Roles**
Mathematics Faculty	Observers who serve as a resource if questions arise about mathematics curriculum
Partner-Discipline Faculty	Discussants who answer questions posed by the facilitator
Facilitator	Participant from a non-mathematics discipline who poses questions, directs discussion, provides a summary or recap after each question, and keeps time
Note Taker	Participant assigned to take notes and record comments of discussants and any new questions posed

Augsburg University

Augsburg University is a small private university located in Minneapolis with a diverse student population. The Augsburg SUMMIT-P team includes three faculty members from the mathematics department, one faculty member from chemistry, and one from economics. The goal defined by the Augsburg SUMMIT-P team was to revise three calculus courses (Calculus I, II, and III) to align with the CF recommendations.

The team organized multiple fishbowl conversations about calculus between the SUMMIT-P team and various partner-discipline departments: biology, chemistry, economics, mathematics education, environmental studies, and physics. They followed the traditional fishbowl format with minor adjustments. Members of the SUMMIT-P team made arrangements for the meetings and asked the partner discipline departments to consider several questions before arriving for the fishbowl event. There was one fishbowl conversation for each discipline for a total of seven fishbowls. The questions posed included the following:

- How does the faculty use calculus in their major?
- How does the current calculus delivery and sequence serve their major?
- How does the current calculus delivery and sequence not serve their major?

The fishbowl meetings were held in person with the partner discipline departments and a non-mathematics facilitator from economics (also a participant in the SUMMIT-P project) leading the discussion. The rest of the Augsburg SUMMIT-P team sat outside the inner table, took notes, and ensured they did not interrupt the flow of the conversation.

Each conversation usually started by having the partner discipline faculty describe which courses in their major use calculus and how it is used in each course. The conversation then segued into what is currently working and what is not working within the present calculus sequence. After 20-30 minutes of discussion among the partner discipline participants, the mathematics faculty were brought into the conversation. This provided a format for the Augsburg SUMMIT-P team to evaluate the strengths and weaknesses of the current system from multiple perspectives and use this information to decide what changes should be made to the calculus courses.

The main challenge in these discussions was to avoid pitfalls such as disparaging some students' apparent inability to perform basic functions of mathematics or algebra; the facilitator helped keep the discussion on track to prevent this from happening. Surprisingly, the most common source of errors was determined to be deficient skills in algebra as opposed to calculus; it was therefore determined that partner disciplines should ensure student competence in this area by employing sufficient "drill" homework that builds mastery in this algebra.

Another challenge revealed through the conversations was the divergent use of vocabulary; according to the participants, partner disciplines often use language that differs from that used in mathematics to describe a skill or concept. As one participant stated, "We still need to figure out how to help our students 'translate' between the two languages."

After the fishbowl was completed, the Augsburg SUMMIT-P team created wish lists for each partner discipline based on the conversations. These lists were then compared for commonalities. From there, the team worked on revising the calculus topics sequence to address the concerns raised in the discussions and wish lists. One key outcome of the fishbowl discussions provided significant impact to chemistry students who need multivariable calculus and partial derivatives but not some of the other topics covered by the calculus curriculum. By rearranging topics in Calculus I, II, and III, the Augsburg SUMMIT-P team provided a direct pathway from Calculus I to Calculus III. These students are thus able to succeed in Calculus III with a prerequisite of Calculus I. The Augsburg team plans to observe student performance by reviewing course evaluations and monitoring grades in both Calculus III and in the partner-discipline courses (especially physical chemistry) that use multivariable calculus.

LaGuardia Community College

LaGuardia Community College (LAGCC) is part of the City University of New York university system, which serves a diverse population (43% Hispanic, 21% Black) of 50,000 students. Many students are first-generation college students and come from low-income families. The goal of the LAGCC SUMMIT-P project was to improve students' quantitative and digital reasoning by revising College Algebra to include applications from business and the social sciences. Each semester, more than 40 sections of College Algebra are offered. College Algebra serves to assess students' inquiry and problem-solving competencies. Student responses to surveys about College Algebra found that they have negative attitudes towards the course since they do not recognize its usefulness. In addition, instructors of economics courses teach mathematics skills in their courses because of their students' diverse mathematics proficiency. In order to address this disparity, the LaGuardia team consisting of two mathematics faculty and two economics faculty are contextualizing College Algebra with economics by soliciting mathematical needs from economics faculty, exchanging and implementing mathematics resources in economics courses, piloting a course pair of College Algebra and Microeconomics, developing mathematics projects contextualizing economics, and implementing these projects in multiple sections of College Algebra.

In order to obtain authentic insights into the mathematical needs of economics courses, the LAGCC SUMMIT-P team held a face-to-face fishbowl discussion session between mathematics and economics faculty. The two CF reports were distributed among economics faculty with particular attention on the economics chapter from the report *Partner Discipline Recommendations of Introductory College Mathematics and the Implication for College Algebra* (Ganter & Barker, 2004).

The fishbowl exercise, led by an economics faculty member, began with a discussion about the reports, and all agreed that most of the mathematics needs summarized in the report also applied to their courses, specifically the following concepts from the CF reports:

- Basic arithmetic and algebra skills—equations and algebra, effects of changing parameters in linear equations;
- Calculating the area of relatively simple geometric figures;
- Generating and interpreting graphs for linear and exponential data: calculating and interpreting the slope of a line and the slope at a point on a non-linear graph, calculating rates of change;
- Two linear simultaneous equations;
- Total/average/marginal concepts;
- Compound interest.

The economics faculty indicated that additional mathematical concepts are needed for LaGuardia students in economics courses. These additional concepts include: absolute value inequalities, rational functions, and trigonometric functions.

The fishbowl discussion helped the LaGuardia mathematics faculty understand the mathematics needs of the economics courses offered at the school. One challenge with the discussion was in communicating across the two disciplines. Economics faculty presented materials in a different manner than mathematics faculty. As a result, the discussion sometimes turned into an impromptu lecture, allowing faculty from the two disciplines to understand the differing notation and concepts. Similar to the conclusions from the original CF report, the discussion revealed that mathematics is widely applied across the economics curriculum at different levels.

After the discussion, economics faculty were invited to summarize the needed mathematical topics and to provide some related economics examples in detail. In order to truly understand the application of mathematics in economics courses, a mathematics faculty member visited economics courses to observe how mathematics is applied in these courses and to witness the challenge students face when they apply mathematics in these courses. Moreover, faculty from economics and mathematics matched the syllabi of two courses, College Algebra and Microeconomics, to develop a detailed mapping of concepts and to create a timeline so that students are mathematically ready for all economics topics that will be taught. Table 3 provides a mapping of concepts between College Algebra and Introductory Microeconomics.

Table 3
LAGCC: Mapping Curriculum between College Algebra and Introductory Microeconomics

Topics in College Algebra	Topics in Introductory Microeconomics
linear equations	equations of demand and supply curves
system of linear equations	market equilibrium
difference quotient	marginal value (marginal cost, marginal revenue, marginal product, marginal utility); elasticity
operations of polynomials	cost function, production function in polynomial functions
quadratic functions	cost and revenue functions
rational functions	cost function, production function in rational function
inverse functions	converting demand/supply functions where the price depends on the quantity demanded/supplied
exponential functions	compound interest; growth in macroeconomic variables such as GDP, price level, and others; exponential consumer utility function
exponential equations	consumer preferences
logarithmic functions	compound interest; growth in macroeconomic variables such as GDP and price level; logarithmic production function; logarithmic consumer utility function
logarithmic equations	consumer preferences; production decision

The LAGCC SUMMIT-P fishbowl showed that some quantitative skills are essential, for example, in economics courses, understanding tables of data and creating and interpreting graphs. From the experience of economics instructors, students who lacked these skills tended to perform poorly in economics courses. Therefore, the exchange of ideas across these two disciplines was felt to be vital to understanding the mathematical needs supporting the subjects in these courses.

Lee University

Partner-discipline faculty from chemistry (John Hearn), psychology (Brian Poole), and education (Jason Robinson) worked with mathematics faculty (led by Caroline Maher-Boulis) to revise Algebra for Calculus, College Algebra, Concepts of Mathematics, and Statistics. Each of the partner disciplines conducted separate fishbowls, and we present here the results from discussions between science (chemistry and biology) and mathematics faculty regarding Algebra for Calculus. This course was selected because it is a foundational course for much of the chemistry and biology curriculum. The fishbowl was jointly facilitated by John Hearn and Caroline Maher-Boulis.

Prior to the fishbowl exercise, the CF recommendations were presented to chemistry and biology faculty by a SUMMIT-P team member and they were asked which recommendations they would give highest priority. The Lee University team then deviated slightly from the traditional format during the actual fishbowl exercise. Partner-discipline faculty were given the current Algebra for Calculus syllabus as well as relevant information from the CF reports. These documents were used to keep the conversation focused and relevant.

As part of the fishbowl exercise, partner-discipline faculty discussed the CF recommendations and generated a wish list based on those recommendations. In addition to the algorithmic skills of algebra (e.g., solving for *x*), partner-discipline faculty wanted students to have a solid understanding of the characteristics of functions and relations and their similarities and differences. The CF recommendations were particularly relevant in regards to two statements: (1) students majoring in biology need to understand the meaning and use of variables, parameters, functions, and relations, and (2) students majoring in chemistry need to be able to follow and apply algebraic arguments in order to understand the relationships between mathematical expressions, to adapt these expressions to particular applications, and to see that most specific mathematical expressions can be recovered from a few fundamental relationships in a few steps (Barker & Ganter, 2004).

Partner-discipline faculty drafted a wish list consisting of nine statements. For students to have a "good understanding" about functions and relations, they should:

1. Be able to generate mathematical expressions from fundamental relationships or through logical deduction,
2. Be able to work with equations without the letters "*x*" and "*y*,"
3. Know the difference between dependent and independent variables,
4. Be able to regroup variables and constants to simplify expressions,
5. Be able to manipulate and quantitatively explain rational relations,
6. Understand numeric, algebraic, and graphical relations and their interrelationships,
7. Be able to move beyond language to conceptual thought,
8. Be able to interpret graphs, and
9. Be able to manipulate equations with confidence.

Since the recommendations would be implemented at Lee University, the team was able to take the fishbowl exercise a step further and conduct a syllabus review. After the faculty discussed the CF recommendations and drafted a wish list, they reviewed the current Algebra for Calculus syllabus. The mathematics faculty member answered questions about the course with the goal of mapping the wish list items to the general course objectives and topics (see Table 4). The wish list items were not mapped to the specific behavioral objectives (e.g., solve quadratic equations of one variable) that are used as the primary measurable outcomes of the course. Instead, each wish list item noted above was mapped to one or more learning objectives or course topics (see Table 4). Such mapping provided the necessary feedback for determining whether the curriculum needed to be modified in any way. The mathematics faculty concluded that all of the wish list items were addressed somewhere in the Algebra for Calculus course.

Table 4
Wish List Items Mapped to Algebra for Calculus Learning Objectives and Topics

	Wish List Item								
General Learning Objective	1	2	3	4	5	6	7	8	9
Acquaint the student with the processes for determining the correct algebraic model from a given set of data.	X								
Acquaint the student with the processes for determining a locus or graph for a given algebraic equation or function.								X	
Acquaint the student with the processes of using algebraic models to solve everyday types of problems.	X	X					X		X
Course Topic									
Algebraic equations and inequalities									
Functions and graphs			X			X		X	
Polynomial functions: zeros and graphs						X		X	
Rational functions					X				X
Exponential and logarithmic functions									X
Systems of equations and inequalities				X					X

One of the course topics not listed in Table 4 is "equations in one variable." This topic caused some confusion among partner-discipline faculty because the word *variable* in science has the precise meaning of something that can change. When an algebra textbook says, "an equation in one *variable*," a scientist may say, "an equation with one *unknown*." As the conversation progressed, the partner-discipline faculty asked whether such a topic was needed, since equations with two variables become equations with one unknown when the value of one of the variables is specified. This change would allow more time to be devoted to more advanced algebraic concepts, such as exponential and logarithmic functions. At the conclusion of the fishbowl activity, the algebra faculty member decided to remove that topic (equations with one variable) from the course curriculum.

Following the fishbowl exercise, partner-discipline faculty drafted several problem sets involving applications of algebra in biology, health science, and chemistry. These problem sets were intended to serve as a resource for algebra instructors. The instructor could show students how they may be asked to apply the algebra content in later discipline-specific courses. In addition to generating direct input into the algebra course, the fishbowl discussions revealed areas where partner-discipline faculty could improve their courses by helping to bridge the gap between mathematics and science. These outcomes, however, were not systematically documented.

The Lee University fishbowl led to the following two results. First, while Algebra for Calculus already addressed all the topics outlined in the wish list items drafted by partner discipline faculty, the time devoted to these topics was revised so that logarithmic and exponential functions could be covered near the end of the course. Second, partner-discipline faculty garnered a better understanding of the language differences between mathematics and science courses. This awareness can improve mathematics education while also advancing science education.

Oregon State University

At Oregon State University (OSU), the SUMMIT-P discipline partners are biology and chemistry. Only the biology partnership will be discussed here. The biology faculty involved directly with the project are two senior instructors who teach either a very large enrollment (~1000 students per term) introductory biology course for life science majors or a large enrollment (~200 students per term) upper division human anatomy and physiology course. The mathematics faculty member is a tenured associate professor in mathematics education. The OSU team is working on discipline integration into Differential Calculus courses. Differential Calculus was chosen for a number of reasons: 1) Calculus (Differential and Integral) is required for biology majors, and 2) there has been a lot of focus on improving college algebra courses using adaptive online and active in-class learning that is concurrent with the SUMMIT-P project, and the OSU SUMMIT-P team wanted to avoid conflicting projects. The modified fishbowl activity was facilitated by the SUMMIT-P biology co-PIs.

The biology co-PIs engaged the entire biology department in a modified fishbowl activity to determine if their faculty's mathematics topics for biology wish list was similar to the CF recommendations. Prior to the fishbowl activity, the OSU SUMMIT-P biology team compiled a list via email of critical mathematics skills that faculty wanted students to be prepared to use in their biology classes. Using the wish list, the OSU SUMMIT-P team led a faculty meeting where faculty 1) determined if the wish list was complete, 2) indicated the mathematics skill levels they expected students have before and during the specific biology courses that individual faculty were responsible for teaching, and 3) indicated whether biology faculty used the mathematics skills in their biology courses. In this way, the fishbowl was modified to meet the needs and the availability constraints of biology and mathematics faculty.

A number of biology faculty also contributed ideas via email; the OSU SUMMIT-P team utilized this feedback to spur conversation during the faculty meeting. At the faculty meeting, the OSU team asked biology faculty to work in small groups to review the wish list and answer questions by filling out an evaluation. Biology faculty were very interested in talking about the uses of and need for mathematics in their biology classes. A noted benefit was that faculty engaged in conversations about curricula, something they rarely do. Focusing on the necessary mathematics concepts made the conversation feel less threatening to biology faculty than a conversation about biology topics. Some of the challenges in implementing the fishbowl technique included finding ways to include biology faculty who do not teach biology majors in the conversations and reaching faculty in different units who teach core biology courses. Additionally, the biology faculty largely found the OSU students lacking in generalized mathematical skills (i.e., proportions, fractions and probabilities) that are not taught at the collegiate level.

In general, the wish list created by the OSU biology faculty contained topics at lower mathematics levels than the concepts and skills outlined in the CF report. The OSU biology faculty also largely desired that students have strong sense of quantitative literacy (Steen, 2004). They also desired that all students and instructors use clear methods to incorporate symbols in activities and assignments that represent and model biological phenomena. Additionally, the OSU team found that very few biology faculty members actually use mathematics (except for statistics) in their biology courses. The lack of mathematical utilization appears to be somewhat sub-discipline specific; for example, the ecology and genetics focused courses were much more likely to utilize mathematics in their biology courses than physiology or organismal focused courses.

The OSU SUMMIT-P team used the adapted fishbowl protocol described above to have additional conversations about the mathematics concepts and skills they should be focusing on in the project to help identify courses in the biology curriculum that require the use of mathematics and places in the biology curriculum where they might engage biology faculty in implementing more mathematics. The OSU team recently used the adapted fishbowl protocol to encourage curricular conversations around teaching mathematics and, more broadly, the importance of mathematics in the OSU biology curriculum. Additionally, they have used the adapted fishbowl protocol to gather feedback on the quantitative aspects of the biology curriculum. The team's next steps include talking to OSU biology students about their experiences with mathematics in their biology courses. They would also like to get students to share any biology-related experiences that take place in their mathematics courses to see if they can identify any patterns and to help faculty discuss mathematics skills and concepts in a similar manner in both disciplines.

This experience taught the OSU team that the fishbowl can be useful for disciplines even without engaging the mathematics faculty in the process and that biology faculty really enjoyed discussing mathematics topics in their curricula. The OSU SUMMIT-P team were able to leverage the faculty meeting to get broad participation from faculty within a variety of sub-disciplines in biology and also engage the majority of the biology faculty. By linking the fishbowl back to the biology major curriculum and the required quantitative literacy, the OSU team was able to get buy-in from the department chair to use faculty meeting time.

The team gathered information in two ways: first through an online survey to develop an initial list of topics and then during the faculty meeting. During the meeting, worksheets were distributed with the categories and topics that had been gleaned and summarized from the online survey. Faculty completed the worksheets in small groups. The worksheets were collected after the fishbowl activity and compiled and synthesized into a final wish list.

One of the lessons learned from implementing the adapted fishbowl protocol is the need to prepare a list of mathematics concepts ahead of time. Additionally, having biology sub-disciplines (e.g., physiology, ecology, and genetics) work together so that each sub-discipline had a voice in the process and could share their specific mathematics desires was important. The OSU SUMMIT-P team was pleasantly surprised by how much the faculty thought about and cared about the mathematics skills required in their major programs. These curricular conversations around mathematics and biology continued in subsequent meetings and resulted in modifications in the biology major to expand the types of quantitative courses offered in the major.

Norfolk State University

Norfolk State University (NSU) is a public, historically black university (HBCU) in Virginia serving 5,100 undergraduates. The goal of the NSU SUMMIT-P team is to broaden the participation of African-Americans in the STEM workforce. Mathematics and engineering faculty are partnering to redesign Calculus I and II because these two courses are identified as roadblocks for students interested in majoring in science.

In May 2017, the mathematics and engineering faculty at NSU joined together for an end-of-year, half-day faculty retreat to discuss how to best serve the engineering majors at the university. To initiate the dialogue, the faculty participated in a fishbowl activity, a first of its kind at NSU. Faculty from both departments have had similar discussions; however, never had the majority of faculty from both departments been in the same room at the same time for a discussion like this. Prior to the retreat, the faculty were emailed the CF report on electrical engineering from *Voices of the Partner Disciplines* (Ganter & Barker, 2004). Printed copies were also available at the retreat.

The fishbowl activity was new to the faculty in that the engineering faculty were able to have a discussion about what they felt was important to them without having input from the mathematics faculty. The discussion was led by engineering professor Dr. D. Geddis, former SUMMIT-P co-PI, who had a list of questions through which to navigate. The second half of the morning involved working groups in discussion and producing recommendations for engineering application problems to be included in a newly designed section of Calculus I to be taken only by engineering majors.

As the conversation in the fishbowl kicked off between the engineering faculty, many of the statements were not surprising, and they aligned with the CF report. As the mathematics faculty looked on, several things became clear. First, there were a few items that engineering faculty mentioned as important that mathematics faculty may overlook or not emphasize. Second, mathematics faculty realized that there were misconceptions among engineering faculty about when certain topics were covered in the mathematics curriculum. Engineering faculty mentioned several topics that they deemed important which were not covered in the actual required mathematics courses; this opened the eyes of many mathematics faculty. At the end of the discussion, the mathematics faculty were able to suggest elective courses that would help support the engineering students.

In the end, all participants of the fishbowl discussion agreed that the CF recommendations still hold true but that there was much work to be done. Mathematics and engineering faculty need to work together to make sure that courses are aligned to better support students. The engineering faculty developed a wish list that contained the items they felt to be important. For their part, the mathematics faculty reviewed the list and discussed ways to implement the wish list items, agreeing that such conversations should be held more often. It is everyone's responsibility to ensure that courses are aligned and to give students the tools they need to be successful.

Virginia Commonwealth University

At Virginia Commonwealth University (VCU), the main focus of the grant effort has been to strengthen student ability to transfer mathematics knowledge about differential equations to the engineering courses that build on differential equations. VCU is a large, urban research university serving 22,000 undergraduates. Approximately 7,000 students per year take mathematics and almost 80% of these students are STEM majors. Thus, better aligning mathematics courses to STEM disciplines can improve STEM learning. The VCU College of Engineering offers eight different undergraduate degree programs, and all but one requires differential equations. Because the faculty is so diverse and large, it would be challenging (read: impossible) to get input from all of the programs at once in a single fishbowl activity. In the CF projects (Ganter & Barker, 2004; Ganter & Haver, 2011), each engineering discipline offered their own report; thus, to prevent VCU mathematics faculty from having to facilitate seven different fishbowl activities, VCU took a multilevel approach to gathering information.

Similar to OSU, VCU used a modified fishbowl approach, beginning with a survey to gather information from as many faculty members as possible, using an online form. This form contained a list of all the major topics typically contained in an introductory differential equations course. The survey was sent to all the engineering faculty, who were asked to rate the importance of each topic to the courses they teach. The ratings were compiled and circulated to the mathematics faculty in preparation for their actual fishbowl meeting.

At the official fishbowl, VCU had representatives from four engineering programs as well as mathematics faculty who were current or recent instructors of differential equations. Armed with the list of the highest ranked topics, as well as the discussion questions from the CF report, VCU held a fishbowl style meeting with the engineering faculty. Some back and forth conversation did take place, but there was also significant listening on the part of the mathematics faculty.

Following the fishbowl meeting, a survey was circulated through the engineering faculty to request engineering application problems that incorporate differential equations concepts and skills. The mathematics faculty also met separately to determine the final list of topics to be covered in the course. The current syllabus is a pared-down version of that list and is a direct result of the fishbowl. Having heard that the laundry list of methods that the course used to cover is not useful for students, this survey offered a clear direction for the mathematics faculty to pursue as a way to better serve the engineering faculty.

VCU mathematics faculty took a multilevel approach to gathering information from colleagues in engineering. They conducted an online survey of all faculty to gather as much input as possible and to prime the faculty for thinking about mathematics topics. This approach worked well for this large institution, which meant that the VCU SUMMIT-P team collected broader input than would be possible with face-to-face meetings. Interested engineering faculty were then invited to participate in the fishbowl. The mathematics faculty primarily listened to the input from the engineering faculty. The VCU mathematics faculty were surprised to learn that streamlining the course content would serve engineering students better, and this information led to syllabus revisions.

Fishbowl Technique in Practice

As can be seen from the vignettes shared above, the fishbowl discussion technique used in the initial meeting and the subsequent SUMMIT-P fishbowls conducted at each member institution offered informative results and, not surprisingly, participants agreed with the CF project's original recommendations. The participants generally agreed that mathematics and non-mathematics disciplines regularly use different vocabulary to describe the same mathematical concepts. There was also agreement that the fishbowl discussions helped faculty to see and think beyond their own disciplinary silos and encouraged better communication across disciplines. Since only non-mathematics faculty participated in the fishbowl discussions, reflecting and responding to questions, partner-discipline participants felt more at ease and engaged in sharing their experiences and perspectives. In addition, mathematics faculty found it helpful to look at partner-discipline curriculum broadly to identify which mathematical concepts were used and where in the curriculum those concepts were introduced (for example, concepts introduced in upper-level courses versus introductory courses).

Conclusion

The fishbowl discussion technique provides a framework for discussion across disciplines. As seen in the vignettes, the original fishbowl technique was modified specifically for each institution. In order to move from a national model to an institutional model, these vignettes create the foundation for an institutional fishbowl framework or protocol that contains five steps:

1. Craft survey questions using the CF report with reference to the specific topics covered in the mathematics course or courses under study or slated for revision; distribute this survey to partner disciplines
2. Analyze survey responses to create a new, common starting point for the fishbowl discussion, which is shared with the partner-discipline fishbowl participants prior to the fishbowl.
3. Hold the fishbowl meeting following a structure similar to the original fishbowl, making sure the right participants are included in the fishbowl discussion.
4. Create partner-discipline wish lists and a syllabus map in which the topics identified in the survey and fishbowl are actually mapped onto the syllabus of the course slated for revision.
5. Compile and create exercises and examples with partner discipline input that utilize the mathematical concepts identified in the fishbowl exercises. Shared materials can be used in the classroom. In this way, collaborations among partner disciplines and mathematics can lead to substantive changes in the classroom curriculum to benefit student learning.

For institutions interested in conducting their own fishbowl discussions and implementing CF recommendations, it is important to obtain buy-in from partner disciplines who will fully participate. Some colleges and universities have a history of collaboration across their campuses while others do not. Emphasizing the need for reform in the content and pedagogy of mathematics courses and highlighting the potential increase in retention of students in STEM majors and improved student learning is therefore a great motivator for participation. Communication is critical: sharing and disseminating information to partners and keeping them in the loop further encourages partner disciplines to buy-into the project. Finally, it is important to have a person or group of people who advocate and champion for collaboration among partner disciplines and mathematics in order to sustain the conversations. The customized fishbowl technique fosters productive collaboration on course development and has proven to be instrumental in the success of the SUMMIT-P project.

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USING SITE VISITS TO STRENGTHEN COLLABORATION

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ABSTRACT
The SUMMIT-P project is a multi-institutional endeavor to leverage interdisciplinary collaboration in order to improve the teaching of undergraduate mathematics courses in the first two years of college. One goal of this work is to establish collaborative communities among the institutions involved. As part of the project, institutions visit one another on site visits that are structured according to a common protocol. The site visits have been valuable to the project. Participating institutions report the exchange of actionable ideas and feedback; members of the grant leadership team have used the site visits to direct the overall project, and evaluators have refined questions and identified trends that will help their assessment of the project. At a deeper level, the site visits have created a strong sense of community among those involved in every aspect of the SUMMIT-P project.

KEYWORDS
campus visits, collaboration, interdisciplinary, professional development

An overarching goal of the NSF-IUSE funded project, A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P), is the development of robust collaborations between higher education mathematics departments and the partner discipline departments who use mathematics courses as pre- or co-requisites. The project involves teams of educators at nine institutions that are working to improve mathematics curricula and instruction in lower-level courses. Each team (hereinafter referred to as "institutional teams") consists of at least one mathematician and at least one faculty member from a partner discipline. The institutions range from small, private colleges to large, research-intensive universities.

One of the key features of the SUMMIT-P project is the exchange of knowledge and resources across institutions as they implement the recommendations of the Curriculum Foundations (CF) reports (Ganter & Barker, 2004; Ganter & Haver, 2011). The CF recommendations resulted from extensive conversations among mathematicians and various partner disciplines from the humanities to the natural sciences. Each conversation with a partner discipline resulted in a separate report. The introduction to the first report (Ganter & Barker, 2004) indicated that there was a great deal of consistency across the recommendations presented in the different partner discipline reports. The recommendations included incorporating mathematical modeling in activities and assignments, emphasizing conceptual understanding over computational fluency, and increasing the use of active learning.

The project has many systems in place to encourage collaboration across the institutions, from annual meetings at a national conference to regular virtual meetings to the submission of institution reports. The most extensive system in the project is the site visit pairings. Each participating institution agreed to host two site visits over the five-year project. Additionally, each institution agreed to visit two different partner institutions. The site visits are not symmetric, that is, each visit is a stand-alone experience and is not necessarily reciprocated. In this paper, we will share how the site visits provide a valuable mechanism for showcasing and reinforcing the project activities to the visiting institution, the project management team, the evaluators, and the various stakeholders at the institution being visited. We believe that the site visits have provided a means for substantial work to happen in a shorter time span than would have occurred without the visits. We believe that other large-scale projects spanning different departments and institutions can benefit from incorporating a site visit component.

In what follows, we detail the goals of the site visit along with the structured planning that goes into making a successful visit. We illustrate the value of the site visits by analyzing the responses to post-visit questionnaires, sharing perspectives from different types of participants, and providing a series vignettes from a variety of institutions that highlight the benefits gained by hosting or attending a site visit. The paper is intended to be helpful for planning a site visit and also for motivating the use of site visits to strengthen collaborative projects.

Why Site Visits?

A goal of the SUMMIT-P project is to go beyond creating interdisciplinary course materials by establishing professional development communities that will outlast the grant funding period. The project involves a mix of online professional development with face-to-face site visits.

Online professional development experiences take place through various "clusters" of institutions. The clusters consist of two or three institutions that have a common project attribute, such as the target mathematics course level or the type of partner disciplines involved in the institutional project. In a narrative study of seven college or university educators, Teras (2016) found that online professional development can result in significant change in participants' perception of education. However, Teras found that the different expectations and preferences of participants were more challenging to navigate in an online environment than in a face-to-face environment. This was partly due to a difference in the sense of community. The study indicated that successful online professional development requires de-emphasizing the accommodation of different participant preferences and instead paying more attention to the participants' self-regulation as well as an intentional emphasis on the quality of facilitation in an online environment. In the SUMMIT-P context, online clusters helped to initiate collaboration across institutions, and the discussions helped to define questions that were further examined during site visits and beyond.

We always expected that building trust and forming sustainable communities would require more than online meetings. In addition, gathering information, whether for purposes of informing one's own work or for evaluating projects, required deeper and more intensive interaction. In order to achieve this quality of professional community development, site visits were built into the design of the SUMMIT-P project. The project included dedicated funding to support site visits in order for participants to share ideas, observe the implementation of institutional projects, and refine strategies for interdisciplinary collaborations. In addition, site visits allow members of the project management team and the evaluation team to observe project activities at each of the institutions. During site visits, the evaluation team conducts focus groups with both faculty and students. The faculty involved in the focus groups are those who are not part of the institutional team of PIs and co-PIs but instead are those using the products that were developed through the institutional team's collaboration. Students who participate in focus groups are taking or have taken the courses that have been modified as a result of the collaborative efforts. Face-to-face focus groups facilitate flexible discussions that permit the evaluators to explore teaching and learning interventions in depth with both faculty and students involved (see Greenbaum, 2000 for more information on focus groups).

An Overview of SUMMIT-P Site Visits

The first round for site visits in the SUMMIT-P project is illustrated in Figure 1. Prior to the visit, the host and the visitors exchange goals. The host plans an agenda for the visit which includes classroom observations, focus group discussions with students, conversations between and among mathematics and partner discipline faculty, sharing teaching materials, discussions with host institution administrators, and, of course, meals. The primary hosts, and the group largely tasked with organizing the visit, are the mathematics faculty. They are most familiar with which courses should be observed, the meeting space to be used, the other individuals on campus who will be interested in the project, and they have access to the students taking the mathematics courses. Although there is a significant time investment in preparing for a site visit, the mathematics faculty have the most to gain from the event. Their courses can be modified and further improved based on observations and reflections from the visitors: they are able to hear what students think of the project, they get to showcase their work to the broader university community, and they may gain buy-in from tangentially involved faculty at their institution. Following the visit, participants write a structured reflective review of the experience.

Figure 1
Map of the First Round of Site Visits.

For example, Virginia Commonwealth University (VCU) volunteered to host the first visit in early November 2017 at the start of the second year of the project. They were visited by Norfolk State University (NSU). This required the participants to start planning for the visit in late August. A general schedule of the site visit activities was outlined in September. Since this was the first visit, several meetings were held to discuss the structure. The planning stage of the process has now been formalized in a protocol, which has streamlined the planning process for all subsequent visits. For VCU, the main scheduling constraints were key administrators' availability and the specific schedules for the courses to be observed. All other components could be arranged around these times.

The Site Visit Protocol

In order to provide consistency among the site visits, the Project Management Team (PMT) developed and revised a common protocol based on the initial VCU visit. The protocol includes required features for each site visit as well as standardized questions to be used for follow-up reporting. After the visit, the host, visitor, a PMT representative, and a project evaluator each submit a post-visit report. Following the first round of site visits, the protocol was modified. The second iteration of the protocol included a small amount of preparatory work for each site visit. The site visit protocol is stated below.

The preparatory work is as follows:

1. The host team prepares a summary of their SUMMIT-P work to date and sends it to the visiting project members.
2. Based on the summary, each team prepares goals for the site visit and distributes the goals to the other site visit participants.

3. Based on the goals, the host prepares, circulates, and modifies (as necessary) a schedule and itinerary for the site visit.

The site visit lasts two days and includes the following components:

- Project highlights presentation: Typically, this is a presentation to the university community about the SUMMIT-P work and consists of an overview of SUMMIT-P from the PMT representative along with presentations by both the host and the visiting institution teams. The host invites administrators, including deans and the provost, along with faculty from the mathematics and partner discipline departments to attend. One of the purposes of the highlights presentation is to impress upon administrators the innovative nature of the work as well as the national scope and significance of the work going into this project.
- Classroom visits: Typically, there are at least two opportunities for participants to observe the mathematics classes that are being modified through the SUMMIT-P project. One of these classes should be taught by a faculty member who is not a PI or co-PI from the host institution. This gives the team an opportunity to see how well the course materials work when taught by faculty members who were not directly involved in their development.
- Conversations with students: Two conversations take place with students taking the modified mathematics courses. One conversation involves the visitors and PMT representatives. The evaluator may or may not be present. This conversation provides the visitors and the PMT representative with an opportunity to learn about the host institution's work from the students' perspective. The second conversation is a focus group with the SUMMIT-P project evaluator. The evaluators use common protocols for this conversation which provides data for the project evaluation.
- Various conversations among faculty: Faculty are matched in different configurations to talk with one another. This includes conversations with faculty who are not part of the host institution's leadership team and conversations among partner discipline faculty.
- Inspecting course materials: Some institutions provide time for visitors to inspect and work through sample course materials. This provides the visitors with a more comprehensive look at the results of the host institution's collaboration in order to identify ideas that they can borrow or steal. Indeed, several of the examples of borrowing in May et al. (2020) resulted from site visits.
- Social events: Meals are critical opportunities for participants to talk about the institutional projects in a casual setting in which everyone "has their guards down." This provides an opportunity to discuss issues and discover concerns that have not yet been voiced and builds a sense of community among participants. Some institutions have also found time to include a tour of campus for the visitors. Augsburg University and NSU both provided tours of new buildings housing the mathematics and partner discipline faculty. Ferris State University (Ferris State) provided tours of the Museum of Sexist Objects and the Jim Crow Museum. Tours help to provide "breathing space" on days that are often densely packed, help the host show off unique features of their institutions, and provide further opportunities for community building.

Each of the visit components has associated challenges. The most significant challenge is getting those from outside of the project to participate, including administrators (who have been mostly interested but are very busy), faculty, and especially students. Some institutions have accomplished this easily; others have had to be creative in order to fit in as many components as possible. Scheduling meetings over meals and offering refreshments can help improve faculty and student attendance. Indeed, serving food and refreshments is a strategy for success in convincing participants of the difference between professional development and a committee (Cox, 2004). Ferris State made sure to get the highlights presentation on administrators' calendars early. Some teams were creative about scheduling class observations. For example, at LaGuardia Community College, the modified class normally takes place in a two-hour time block, so the host team planned one of the observations for one hour of lesson time and utilized the second hour for discussion among visitors and students.

Figure 2
Post-Site Visit Questionnaire

PI of host institution

1. Please discuss challenges/successes about your SUMMIT-P work, relative to what you learned during the site visit.
2. What did you learn about planning and coordinating a site visit? Include suggestions for others who will host site visits. Include suggestions for the PMT for future site visits.
3. Describe the new ideas that you learned during the site visit that you plan to apply to your SUMMIT-P work.

PI of visiting institution

1. Describe the new ideas that you learned during the site visit that you plan to apply to your SUMMIT-P work.
2. Provide advice to the host institution regarding what you learned about the work they are doing. Include ways you think the project work could be enhanced.

Representative of the Project Management Team

1. How well are mathematics department faculty and partner discipline faculty at the host institution communicating with each other?
2. How well did the site visit protocol work? How should the protocol be changed for future site visits?
3. Provide advice to the host institution regarding what you learned about the work they are doing. Include ways you think the project work could be enhanced.

Representative of the Project Evaluation Team

1. Describe the evaluation team's focus group with students. What were some highlights?
2. How well are mathematics department faculty and partner discipline faculty at the host institution communicating with each other?
3. What did you find particularly noteworthy about the work being conducted at the host institution?

Post-visit reflection and discussion are built into the protocol. Each party completes a report after the visit. The report includes an assessment of the value of each component of the visit with space for comments along with specific questions for each party (see Figure 2). The site visit reports are combined by the PMT and shared with the PIs across all SUMMIT-P institutions. These reports strengthen the collaboration among institutions by allowing visitors to reflect on what they learned through the visit and ideas they can incorporate into their own projects. The host institution is provided with valuable feedback to help improve their work. The reports are also used by the evaluators as part of their project evaluation and by the PMT as they monitor the direction of the project as a whole.

The Value of the Site Visits: Analysis of Questionnaires

While it is too early to tell whether the collaboration among institutions will continue after the project funding period, we will share how the site visits have been uniquely valuable for participants during the course of the SUMMIT-P project. This article will illustrate the value of the site visits in three ways. First, we will analyze what participants shared in their post-visit questionnaires. Then we will share perspectives on the site visits from participants in each role. Finally, we will share some of the individual stories that have arisen out of the site visits in a series of vignettes.

Analyzing responses to the questionnaires suggests that the extended time together on campus resulted in the kind of sharing of actionable advice and deep reflection that comes from meaningful collaboration. By *actionable* we mean advice or ideas that fall within a SMART framework–advice that is specific, measurable, attainable, relevant, and time-bound–a framework regularly used in educational settings (see, for example, O'Neill, 2000).

Both hosts and visitors indicated that there were valuable take-aways from the site visits. Some examples of useful feedback are as follows:

- A visiting team noted that the integration of partner discipline applications in the hosts' mathematics classes were only used in a few, seemingly isolated in-class labs. In order to improve the impact on students, the visitors recommended finding ways to integrate partner discipline applications throughout the mathematics courses.
- A PMT representative recommended that non-tenure track faculty and graduate teaching assistants, as well as more partner discipline faculty, play a larger role in the development of course materials.
- The hosts of a site visit learned that the feedback provided by students was consistent and matched the goals of the course they are working on. They also learned that there will be non-trivial challenges to supporting new instructors teaching the course, especially non-tenure track faculty. This led to a conversation about having faculty who have taught the course serve as mentors for those who are teaching it for the first time. Conversations like this would not be possible without the on-site, collaborative nature of the institution visits.

After the first round of site visits, seven out of eight host PIs described an actionable new idea from the visiting institution that they planned to implement in their own project. These ideas matched with the seven out of the eight visiting PIs who contributed actionable advice in their post-visit reports. For the visitors, six of the eight reports described an actionable idea that they will implement back at their home institution. In addition, six of the eight reports described the ability to see the host's project in action through classroom visits, conversations with students, and meetings with faculty as critical to providing a basis for exchanging actionable ideas. In terms of moving the project forward, six of eight reports provided critical reflections on the project's next steps. To see how some of the ideas were implemented, see May et al. (2020).

The PMT representative's and the evaluator's roles were to observe what was taking place at the host institution. In seven of the eight post-visit reports, both the PMT representative and the evaluator indicated that quality communication took place. The PMT representatives tended to characterize the communication as *strong* or *excellent* whereas the evaluators tended to describe the communication as *good*. This is an indication of the difference between the leadership role of the PMT representative and the more scholarly and summative role of the evaluator. Further to this distinction, the PMT representative was asked to provide advice to the host team, and seven of the eight reports included actionable advice. In half of the reports, the evaluators noted that students in focus groups indicated that they can better see how mathematics is used in real life as a result of the host institution's actions to improve mathematics courses. In five of the eight reports, the evaluator described a unique, valuable, and scalable achievement of the host institution, such as adding a lab hour to an existing mathematics course like College Algebra.

The reports describe benefits of the site visits that could not occur simply through online meetings and discussions. The extended time focused on understanding the host institution's project and seeing it in action through class visits and discussions with students were critical to the exchange of actionable ideas that could be used to move both projects forward. In one case, an institution that was struggling to meet project goals was able to use the site visit as an opportunity to bring the team together. By engaging with the visiting institution, the PMT representative, and the evaluator, they were able to reinvigorate their project. The observations and advice from the PMT representative and the observations from the evaluator would also be very difficult to develop outside of the site visit framework. The SUMMIT-P project is, at its heart, about collaboration. The nature of the collaboration and communication among members of a team requires extended observations over time and in context. This is an opportunity that uniquely occurs through site visits.

The reports also include some of the challenges involved in site visits. In one case, a host PI had difficulty getting students and faculty from outside of the grant to participate in the site visit. We learned from this experience about the importance of quality food and the use of calendar invitations. Another site visit was interrupted by a blizzard, and the participants learned how to improvise as best they could.

The analysis of the post-visit reports and the perspectives offered above illustrate the value of different roles among the site visit participants. Both visitors and hosts are looking for something that they can use in their projects. Evaluators and leadership representatives are interested in the project as a whole– the quality of the collaboration and the impact of the nature of each institution on the project. Each perspective is enriched by participating in the site visit together.

The Value of the Site Visits: Perspectives by Role

The site visits involved many individuals coming together to make the interactions a success. At a minimum, the participants for each site visit include the host mathematics PI, a host partner discipline co-PI, visiting faculty from an outside institution who include mathematics and partner discipline faculty, a representative from the PMT, and a project evaluator. The various roles a participant plays in a site visit each provide a different perspective about what is learned during the experience. Below, we provide comments contributed by participants about the different tasks and goals of a site visit. These comments are from a representative sample of the different roles faculty assumed as they participated in a visit. The perspectives represent a PI from mathematics and co-PI from a partner discipline for a host institution, a PI from mathematics and a co-PI from a partner discipline for a visiting institution, a representative of the PMT, and a representative of the evaluation team. Although the site visits discussed below took place at different institutions, the reader should see the diverse benefits that result from including the variety of roles in the design of the site visit, including the differing perspectives between mathematics faculty and partner discipline faculty.

Host Institution—Virginia Commonwealth University (VCU)

Virginia Commonwealth University is a large, public institution. The mathematics department is housed in the College of Humanities and Sciences and teaches courses for students in that college, as well as students in the School of Engineering, the School of Business, the School of Education, and students in pre-health majors on the Medical Campus.

Mathematics Faculty—Rebecca Segal, VCU PI.

VCU's efforts have focused on collaborating with the engineering faculty on revising the Introduction to Differential Equations mathematics course. This course is taken by students majoring in engineering, mathematics, physics, and chemistry.

As mentioned above, VCU served as the host institution for the first SUMMIT-P site visit. Although the project was still in the early stages, the site visit allowed the PI to (1) pull the team together, (2) strengthen the project activities, (3) rally the faculty and administration behind the grant-funded work, (4) encourage meetings with and buy-in from the partner disciplines, particularly from engineering, and (5) exchange ideas and get feedback from site visit participants. Although the planning and delivery of the visit was time-intensive, the visit positively influenced the work. It created a more immediate need for time to be spent on the project, and the time invested in the visit paid off by generating a boost in activity both before and after the visit.

Partner Discipline Faculty—Afroditi Filippas, VCU Co-PI (Electrical Engineering).

There were multiple components of the site visit that the engineering faculty played a large role in. The engineering faculty had the opportunity to describe perceived challenges for students to the visitors as well as to faculty from the VCU mathematics department. While the conversation was centered on Differential Equations, some side conversations identified student difficulties in Calculus and other mathematics courses that are required courses in the engineering curriculum. The discussions confirmed that the visitors from NSU shared the same challenges. Further discussions revealed similar trends in thinking among faculty.

Classroom visits were instrumental in allowing engineering faculty to witness both the quality of mathematics teaching (high to very high) and the students' abilities to comprehend the material and manipulate differential equations (again, high to very high). This is important for engineering faculty to witness, as it shifts at least some of the onus away from the mathematics faculty to teach the material and onto the engineering faculty to create better and more intentional links to review the material that students have already mastered.

Preparing for the highlights presentation was also very useful, as it provided the engineering faculty with an opportunity to re-focus their attention on their initial project objectives and review how the activities up to that point were informed by those objectives. It also provided an infrastructure through which the engineering faculty could make an initial assessment on whether the designed activities led to the desired outcome. Through the presentation and a recap at the end of the visit, the engineering faculty confirmed their commitment to this project, reconnected with colleagues, and evaluated and improved the plan for further work on the project.

Visiting Institution—CUNY LaGuardia Community College (LAGCC)

LaGuardia Community College (LAGCC) is a Hispanic-serving institution with a diverse 50,000-student population (45% Hispanic, 20% Black), most from families in poverty and many in the first-generation of their family to go to college.

Mathematics Faculty—Tao Chen, LAGCC PI.

At LAGCC, a group of mathematics and economics faculty are integrating economics into College Algebra. They are also creating a “paired course learning community” with College Algebra and Introduction to Microeconomics. This means that a cohort of students take the two courses together, and the faculty design common assignments to be used in both courses.

The faculty from LAGCC visited St. Louis University (SLU) during Spring 2018. SLU mathematics faculty are also collaborating with economics and business faculty in order to contextualize mathematics in College Algebra as well as in Calculus.

LAGCC’s goals for their visit to SLU were:

- Learn how Excel was embedded in mathematics instruction and course activities.
- Observe the interaction between the instructors and students from different majors.
- Learn from student feedback about the courses being redesigned.
- Learn about discipline-specific applications being incorporated into College Algebra and Calculus.

The visitors noted that the economics and business contexts are well integrated with the mathematics content in SLU courses. In particular, Microsoft Excel use is seamlessly integrated into the courses, and the mathematical needs of students taking business courses are comprehensively addressed. Moreover, Tao learned the following during the visit:

- Contextualized mathematics boosts students' interest in the course work.
- The introduction of partner discipline applications helps to facilitate students learning.
- Working with academic advisors is necessary to increase enrollment in revised courses.

Thanks to what they learned during the visit, the LAGCC team worked closely with academic advisers to promote the course pair successfully. Moreover, assignments in the course pair are designed to be relevant to students from different academic backgrounds and also be self-contained activities so that students can work independently if they need to.

Partner Discipline Faculty—Soloman Kone (Economics).

Tao was accompanied on the visit to SLU by Soloman Kone, an economics professor from LAGCC. As part of the LAGCC project, Soloman developed a “wish list” of mathematical skills that need to be applied when learning economics, developed theoretical and applied activities and assignments for College Algebra linking mathematics to economics, and collected qualitative and quantitative assessment data from his students taking Introduction to Microeconomics.

Solomon found the visit to SLU to be very productive and beneficial. His goals were to learn about some best practices through the project highlights presentation of the host institution, to see examples of business content integrated into College Algebra through class visits, to hear the students' perspectives through the focus group meetings, and to learn about approaches to collaboration by meeting with the SLU project mathematics-business working group. He learned that (1) some of the best practices at SLU were portable and could be replicated at LAGCC, and (2) the mathematics and partner discipline faculty were communicating very well at SLU. He was also impressed by the enthusiasm of SLU students and the hospitality of the host institution.

Project Management Team (PMT)—Rosalyn Hargraves, VCU

As a PMT member, Rosalyn participated in the visit by LAGCC to SLU. Her goal for the visit to SLU was to observe how the project was progressing and to provide any advice or new perspectives on the work of the team. The most beneficial components of the site visit were meeting with the partner discipline faculty and attending the student focus groups. While the classroom observation was informative, the highlight was talking to students afterwards to inquire about their reasons for taking the course and the benefits of the revised content. In conversations with students, she found that a number of students in College Algebra that were not majoring in business were struggling with linking the mathematics concepts learned in their classes to what they may need in the other courses required for their majors. However, they did believe that the content that they were learning (such as the time value of money) may be valuable at some time in their future lives when they need to know about retirement funds, taxes, and mortgages. Therefore, they did not necessarily see the importance of having examples linked specifically to their major. It was surprising that many of the College Algebra students in the focus group were not business majors but still appreciated having "business" type examples linked to real life instead of examples related to their social science or life science majors. Furthermore, because they could not predict what direction their career path may take, they could not be certain the content would not be relevant in the future. In both College Algebra and Calculus, students saw the value of learning Excel as a computational tool and potentially for jobs in the future. Learning Excel was one of the best features of the course even though it placed an additional workload burden on the students compared to the non-modified sections of Calculus and College Algebra.

One of the most noteworthy aspects of this site visit was seeing the institutional commitment to the work of the project. For example, the dean of the School of Business matches the salary supplement for participating mathematics-business working group faculty. This type of monetary commitment was not seen in any other site visit. This incentive may be one of the reasons the monthly meetings with mathematics and business faculty are attended and valued by faculty from the two disciplines. These monthly meetings allow for all of the faculty to come together to discuss issues pertinent to the project. This type of close collaboration across the campus facilitates what appears to be a strong connection between the participating mathematics and partner discipline faculty, which is one of the desired outcomes from the SUMMIT-P project.

Evaluation Team—Jack Bookman, Duke University

Jack participated in Oregon State University's (OSU) visit to San Diego State University (SDSU), VCU's visit to Augsburg, and Ferris State's visit to LAGCC. In general, as an evaluator, he observed the differences in the engagement and awareness of faculty in other disciplines in addition to the mathematics faculty not directly involved with the project. The site visits give a glimpse of the variety of ways institutional projects have been implemented along with the unique challenges faced by each institution.

During his various visits, he has observed that the size and mission of an institution can have an impact on the implementation of the project. He also noted that the level of experience of those teaching the modified courses can be an important element of early successes. Finally, Jack observed that even at a very large institution, a well-developed culture of student learning communities can contribute positively to the implementation of the project.

Collectively, Jack has found it invaluable to observe classes and see how students interact with the content and pedagogy developed through the SUMMIT-P project. One also gets a snapshot of what the culture of a campus is and how that can affect the success of a project like this. Talking with students in the focus groups is important, but getting students to participate in these focus groups is difficult, and those who do are unlikely to be representative of the entirety of the student experience.

The Value of the Site Visits: Highlights, Vignettes, and Challenges

The benefits and challenges we identified from the post-visit reports are well illustrated by the following vignettes. Although each site visit followed a consistent protocol, the nature of the visit was influenced by the type of institution being visited, the interests and personalities of the participating faculty, and the nature of the curriculum work being undertaken. We try to capture the range of experiences in the site visits and highlight some of the interesting interactions. Each of the contributors below is either a PI or co-PI and has involvement both on curriculum work in the classroom and with the interdisciplinary collaborations.

The institutions featured in these vignettes range from large, public universities, to small, private colleges. The diversity of the institutions represented in these vignettes illustrates that the benefits of the site visits are not limited to one type of college or university. These stories provide the reader with a few episodes in which they can peek at what happens during the site visits. Some of what we see is expected–such as learning a new approach to teaching a topic. Others are unexpected and lead to the kind of social bonds that become stronger than the grant funding itself. In all cases, these vignettes give examples of the human stories of the site visits, stories that illustrate the high level of interaction and engagement among colleagues and friends and illustrate the power the site visits have for building community among university teams and among colleagues at different institutions.

Sharing Teaching Approaches Across Disciplines—John Hearn (co-PI), Chemistry, Lee University

Lee University is a private, four-year, liberal arts college in East Tennessee serving 4,000 undergraduates. During a visit by Ferris State to Lee University, John met with Victor Piercey from Ferris State and discussed what Victor was doing in his quantitative reasoning class. Students have always had trouble with algebraic manipulation in John's chemistry classes, and Victor described some of the "moves" that he defines in his class (see Piercey, 2017). Victor uses the term "moves" as a student-friendly way to describe the types of steps that are needed during algebraic manipulations. The move that John has used in his teaching since is the "swap move," in which the unknown in the denominator is swapped with the other side of the equation (if $a =$

b/c, and we want to solve for c, we can "swap" a and c to create $c = b/a$). Thus, a chemistry faculty member was able to take a concrete teaching tool from the visiting mathematics faculty and implement it in his course.

Feedback from Classroom Visits—Rebecca Segal (PI), Mathematics, VCU

Because the VCU team was hosting a site visit in the early stages of the SUMMIT-P project, they decided to have their own engineering faculty visit some sections of Differential Equations just prior to the site visit in order to solidify connections between mathematics and engineering faculty. This turned out to be a great boost to their collaboration. It allowed the engineering faculty to experience the course in ways that would not have occurred during a typical meeting conversation. The mathematics faculty gained valuable feedback from their engineering colleagues. Some specific suggestions related to a comparison between how problems are presented in a mathematics textbook and how they are presented in an engineering textbook. The authors of the mathematics book vary the units in examples and problems, often using slugs and feet. The engineering faculty revolted when they saw this, explaining that they never use anything except metric units. As a result of the classroom visits, the VCU team was able to make some immediate adjustments to the way material was being presented in Differential Equations. Planning for the site visit prompted a beneficial level of interaction among faculty at the institution, interaction that was not in the initial plan. This helped to energize the project activities during and after the site visit.

Scheduling Challenges—Victor Piercey (PI), Mathematics, Ferris State

Ferris State is a public university in Michigan serving 14,000 undergraduates and is a popular transfer destination from community colleges. When Ferris State hosted SLU during a site visit in 2018, the hosts planned several events and hoped to include a broad spectrum of individuals involved in the project. They especially wanted to include the mathematics, social work, business, and nursing faculty involved in a faculty learning community (FLC) (see Bishop, Stone, & Piercey, 2020). Having the SLU visitors observe a session of the FLC would have been ideal, but the time of the visit could not be scheduled during any of the times that the FLC met. Consequently, Ferris State planned multiple opportunities to meet and, more importantly, eat together. Unfortunately, there was not much participation from faculty involved in the FLC – with the exception of a dinner at one of Big Rapid's finest eateries. Where was everybody? As the reader likely knows, scheduling is nontrivial. But in this case, there were added challenges. Most of the FLC participants (along with many faculty at Ferris State) commute from Grand Rapids–which is about a one hour drive away from campus. They also have significant family and other responsibilities at home. It is not easy for them to be spontaneous. On top of this, the Ferris Faculty Association (the union for tenure and tenure-track faculty) was mired in contract negotiations that had already produced several work actions, including a strike at the beginning of the academic year. As a result, morale on campus was quite low. From this experience, the Ferris State team decided to use calendar invitations for the next time they host a site visit.

Team Bonding—Debbie Pike (co-PI), Accounting, Saint Louis University

SLU is a private university in Missouri serving 8,000 undergraduates. Debbie could recite many benefits that SLU's SUMMIT-P project received from the site visits, such as (1) identifying content to be shared, (2) developing a diverse network of contacts with shared interests across universities, and (3) increasing visibility of the project to SLU administration. None of these benefits are likely to be surprising in the context of this project. However, during a visit to Ferris State, Debbie found an unexpected benefit to the site visit. During a three-hour car drive from the airport to the Ferris State campus, she got to know her colleagues better and more personally. The deepening friendship strengthens her resolve to do her part for the success of the SUMMIT-P project because it is *the team's* shared success. She doesn't ever want to let her friends down.

The time away from day-to-day to-do lists gave the team a much-needed opportunity to really think about where they were with the project so far and, more importantly, some ideas of where they could take it. Finally, the most unexpected benefit of all, in the course of conversation, the team happened to discuss a current business school discussion item: strengthening curriculum around business data analytics. The chance to have a meandering conversation led directly to Debbie being much more informed for the ensuing discussions within her partner discipline.

Inter-institutional Bonding—Kathy Williams (co-PI), Biology, San Diego State University

SDSU is a public, Hispanic-serving, research university with 29,000 undergraduates, over 34% from underrepresented groups. A team from SDSU visited Unity College to observe their work with the biology department. During the course of the visit, weather intervened to provide an opportunity for a community-forming experience. Due to Nature's intervention, this carefully planned visit turned out to be a richer experience for all. Janet Bowers (PI, mathematics) and Kathy Williams (co-PI, biology) from SDSU were to meet with Augsburg colleagues Joan Kunz (co-PI, chemistry), Jody Sorensen (co-PI, mathematics), and Su Doree (PI, mathematics) at Unity College, where Carrie Diaz Eaton (PI, mathematics) and Emma Perry (co-PI, biology) teach.

The teams had a great visit to campus during the first day, observing classes, talking about teaching and learning with faculty and students, touring the campus, and even visiting the animal care facility while building on ideas and discussing what elements of the programs might transfer. Unfortunately, a blizzard hit on the second day of the visit and Unity cancelled classes. Snowbound at their hotel, the visiting teams held scheduled meetings with Unity students and faculty online via Zoom that were wonderfully rich, with all of us caught in the same unexpected situation. Even though we were locally "apart," we were together in a way that would not have been the same if we were at our home campuses. The storm broke the next morning; after digging cars out of the snow, the Augsburg team (skilled in snow travel) led the SDSU team (from southern California) along the snowy turnpike to the Portland airport, driving behind snow plows the whole way. What team work!

This visit encapsulates what the SUMMIT-P project has been to many participants: a shared adventure—not knowing what was coming and adapting as necessary, and always looking for opportunities to learn from each other as we share our discoveries about improving our students' successes.

Conclusions

The site visits are expensive in terms of both time and money and may only indirectly advance the project goals in terms of curriculum development and collaboration between mathematics and partner disciplines within an institution. But they accomplish a great deal at a much deeper level. The site visits are very focused events and *contribute to community building, the cross-pollination of ideas, and provide time to reflect on the SUMMIT-P projects*. Some institutions have found hosting site visits to be critical moments to regroup as a project flounders. They are eye-opening and powerful, and there is simply no substitute for face-to-face visits.

For future visits, the SUMMIT-P project management group is considering some minor changes to the protocol which include:

- preparing students to participate in the visit by explaining to them what is different about the courses they are taking compared to prior course offerings;
- including visits to courses taught by partner discipline faculty, if applicable to the goals of the visit; and
- focusing the site visits on plans for sustaining the project beyond the period of grant funding.

The series of site visits serves to strengthen the SUMMIT-P project community. One host PI wrote in their post-visit report that:

> [T]he most important part [of the site visit] was the relationship building. While content is important, I think that the site visits help the institutional PIs create relationships that can be built on for future curriculum development.

Hosting a site visit is a vulnerable experience, but in each instance, the host institution has risen to the occasion to showcase their project and their institution. It has allowed for a significant exchange of ideas as well as first-hand experience of the differences in institutional cultures. It has allowed for deepening of professional relationships during down times between sessions. Colleagues who have travelled together to site visits have also had quality time together outside of the normal, hurried day-to-day interactions. By facilitating sharing and collaboration via intensive visits in ways that are not possible with virtual meetings, the site visits are the heart of the SUMMIT-P project.

For a large, multi-department and multi-institution project, site visits provide a means of community building and substantial exchange of ideas beyond traditional meetings. Engagement and longer contact time with faculty and students surrounding the project enables better exchange of information. Although individual experiences were different with different site visit institutions, and sometimes the visit did not go as planned, all participants left the visits energized for their own project, having learned valuable information and built stronger ties with their colleagues.

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Structured Engagement for a Multi-Institutional Collaborative to Tackle Challenges and Share Best Practices

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Abstract

A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P), funded by the National Science Foundation, is a multi-institutional consortium with members from twelve institutions. The consortium adapted two protocols developed by the School Reform Initiative to: 1. provide advice on challenges or dilemmas a consortium member is facing and 2. share project successes with consortium members. The two protocols—a Modified Descriptive Consultancy protocol and a Modified Success Analysis with Reflective Questions protocol—provide a structured format for these discussions. This paper provides an in-depth description of the two protocols and how they have been used for this project. Examples demonstrating the impact of the protocols are provided by the co-authors.

Keywords

collaboration, continuous improvement, interdisciplinary, protocols,

The success of interdisciplinary, multi-institutional collaborations hinges upon how well the partner institutions engage with and support each other. The National Science Foundation often funds projects which feature multiple institutions. One such grant is the National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships or the SUMMIT-P project (National Science Foundation, 2019, SUMMIT-P, 2019). SUMMIT-P was established to refresh lower division undergraduate mathematics curriculum (e.g. college algebra, pre-calculus, calculus, differential equations, etc.) based upon the expressed needs of other disciplines which require mathematics as part of their curriculum, i.e., the partner disciplines (e.g. biology, business, chemistry, economics, engineering, nursing, physics, social work, etc.). SUMMIT-P was built upon the work of the Curriculum Foundations (CF) project (Ganter & Barker, 2004; Ganter & Haver, 2011) which gathered input from partner disciplines on the mathematics topics students needed to master to be successful in their respective field of study. The CF project resulted in 22 reports, which the SUMMIT-P project used as a springboard for implementing the CF recommendations across 12 institutions and a variety of disciplines.

A project that originally included ten institutions, the SUMMIT-P Consortium now includes Augsburg University, Embry-Riddle Aeronautical University, Ferris State University, Humboldt State University, LaGuardia Community College, Lee University, Norfolk State University, Oregon State University, Saint Louis University, San Diego State University, University of Tennessee Knoxville, and Virginia Commonwealth University. The consortium has more than 50 team members across approximately 20 disciplines and from 15 institutions. These include the aforementioned institutional members in addition to the evaluation team member institutions: Appalachian State University, Duke University, and Virginia Polytechnic Institute and State University. With such a large group of participants geographically located across the United States, it is necessary to establish an effective means of sharing successful practices and facilitating peer learning. Drawing on the experience of some consortium members in K-12 education, the project management team identified two protocols which provide a format for effective and fruitful discussion during the principal investigator (PI) meetings, which occur over the internet once a semester, and the annual project meetings, which occur in person at the Joint Mathematics Meetings. The two protocols, the Descriptive Consultancy protocol and the Success Analysis with Reflective Questions protocol, were chosen because of documented effectiveness in educational settings (Yau & Lawrence, 2020; Mindlich & Lieberman, 2012; Bryk, 2010) and the experience of selected team members in the use of the protocols. The protocols were modified slightly to tailor them to the needs of the SUMMIT-P project.

The two revised protocols—a Modified Descriptive Consultancy protocol and a Modified Success Analysis with Reflective Questions protocol—provide the SUMMIT-P project a structured format for (a) feedback to partners who are seeking advice on a challenge they are experiencing and (b) partners to share their success stories.

Each of the activities adhere to a schedule to prevent discussions from spiraling off topic and to respect the participants' time. This paper provides an in-depth description of the two protocols and how they have been used for this project. Examples from co-authors Janet Bowers from San Diego State University, Mary Beisiegel from Oregon State University, Victor Piercey from Ferris State University, Stella Hofrenning from Augsburg University, and Erica Slate Young from Appalachian State University are also shared.

Overview of the Protocols

Developed by the School Reform Initiative (SRI) (McDonald, Mohr, Dichter, & McDonald, 2013), the two protocols, Descriptive Consultancy protocol and Success Analysis with Reflective Questions protocol, have historically been applied in the K-12 education community (Yau & Lawrence, 2020). The Descriptive Consultancy protocol (Mohr, Parrish, & Taylor, 2019), originally developed by Nancy Mohr and revised by Connie Parrish and Susan Taylor in August 2013, was modified by McDonnough and Henschel (2015) and has been adapted for this project to help presenters think more expansively about a particular, concrete dilemma and get advice from other teams members on how to resolve it. This Modified Descriptive Consultancy (MDC) protocol has two main purposes—to develop participants' capacity to see and describe the dilemmas that arise in their work and to help each other understand and deal with them and thus lead to positive outcomes. The Success Analysis with Reflective Questions protocol (Johnson, 2019), developed by Vivian Johnson, was created to help groups or teams learn from successes and share best practices through structured discussion. The protocol was modified for this project by changing the reflective questions and adjusting the time allotted for each protocol activity to give the presenter and the consultants an opportunity to think about the context and environment in which the successful practice was executed and also how the practice can be adapted and implemented in the partner institutions' classrooms, departments, or projects.

The MDC protocol allows the problem to be framed and then reframed to enable and empower the presenter to move towards a focused solution. As stated by Mohr, Parrish, and Taylor (2019), the protocol "recognizes that the best advice is the least advice, and that helping to define and set the problem is what is truly helpful in reaching resolution.…It asks us to practice being more descriptive and less judgmental" (para. 1). The Modified Success Analysis with Reflective Questions (MSA) protocol gives the presenter an opportunity to share a success that proved to be highly effective in achieving an outcome important to them. As stated by Grove (2019), protocols like MSA work "in the spirit of appreciative inquiry," to allow presenters to "share professional successes with colleagues in order to gain insight into the conditions that lead to those successes, so participants can do more of what works" (para. 1). These two protocols differ in their format and function; however, together they create a comprehensive framework for structured engagement. The MDC protocol helps address issues a team member is facing, and the MSA protocol allows for a best practice to be shared and potentially adapted and implemented at other partner institutions. Both the MDC and MSA protocols give all participants an opportunity to reflect on the information shared and consider how that knowledge can be adapted to or used directly in their own institutional context.

Modified Descriptive Consultancy Protocol

During the Modified Descriptive Consultancy protocol, the "presenter" and the "consultants" follow a timed, eight-step process (see Table 1) during which the problem is shared by the presenter first. Then clarifying questions are posed by the consultants in order to uncover different dimensions of the problem and potentially reframe the problem. The presenter then answers the questions, and the consultants follow with a description of what they understand the problem to be and pose any additional questions they may have. Next, the consultants engage in a brainstorming session of possible solutions. The presenter reflects on the ideas. The session ends with all members engaging in a debriefing, where the facilitator asks participants about the MDC process and the roles they assumed during the protocol. In addition, the participants share their thoughts about the exploration of the problem, the proposed solution, and how the information they learned may apply to their own context. The activity provides direct advice to the partner presenting the challenge and also gives other participants an opportunity to reflect on issues at their own institutions and how they might address similar situations.

Table 1
MDC Protocol

Step	Presenter	Consultants	Time
1	Describes the problem and lays out all different dimensions. For example, did you attempt to address it already? If so how and what were the results?	Listens quietly and takes notes as needed.	3 min
2	Quietly listens and takes notes, if needed.	Asks questions of the presenter. Considers what information is missing in order to address the problem.	2 min
3	Responds to questions.	Listens quietly and takes notes as needed.	3 min
4	Listens quietly and takes notes as needed.	Each of the consultants describes what he or she heard in the presentation of the problem with statements like: • What I heard you say was… • It is still unclear because… • I would like to know more about… The consultant may pass if his/her reflection has already been shared.	5 min
5	Responds to the consultants' expressed understandings and provides further clarification of the problem if needed.	Listens quietly and take notes as needed.	3 min

Step	Presenter	Consultants	Time
6	Listens quietly and takes notes as needed.	Brainstorms possible solutions or next steps. Presents ideas in the form of questions like: • What if…? • Have you thought about…? • Would ____ be a possible solution? • I heard/read about…	10 min
7	Reflects on the advice of the consultants. Shares ideas by responding to questions like: • How might you be thinking now as a result of what has been said? • Did you gain any new insights? However, the presenter does not need to answer any questions.		5 min
8	Presenter and/or consultants share what they learned that would be useful to their institution or project		5 min

Modified Success Analysis with Reflective Questions Protocol

The MSA protocol (see Table 2) follows a similar structure during which a "presenter" shares a successful practice they have implemented at their institution and the "consultants" provide contextual feedback. The presenter gives an overview of their successful activity, which is followed by group analysis and observation about what made it a success. All observations are compiled and reflected upon by the group (presenter and consultants) to envision how the activity or practice could be duplicated within their own institutional setting. Furthermore, the MSA protocol is used to more fully understand why a specific practice works, and it thus allows individuals to apply this understanding to their future practice.

As stated previously, these protocols have been used at each of the annual meetings of the SUMMIT-P consortium and during several of the PI meetings. While the MSA protocol has not been used as frequently as the MDC protocol in the SUMMIT-P projects, consortium members appreciate both protocols and the resulting positive influence the discussions have had on the work of SUMMIT-P. Six vignettes from consortium members are shared below, describing their dilemma or success and what they gained from the experience by being a "presenter" in the protocol.

Table 2
MSA Protocol

<table>
<tr><th>Step</th><th>Presenter</th><th>Consultants</th><th>Time</th></tr>
<tr><td>1</td><td>Shares a case of a successful practice in the implementation of the grant (e.g., activity, lab, project, collaboration, or assessment).</td><td>Takes notes.</td><td>5 min</td></tr>
<tr><td>2</td><td>Engages in discussion on insights about what made this a successful practice.
Answers any questions about the environment, incentives, context, etc.</td><td>Offers insights into what made this case of practice successful for the presenter and their institution. Share information like:
• What you think the presenter may have done to contribute to the success?
• What other factors may be involved?</td><td>5 min</td></tr>
<tr><td>3</td><td colspan="2">Compile a list of characteristics which contributed to the success of the case including:
• <u>Tactics</u> - activities, actions (e. g. classroom visits, weekly meetings, etc.)
• <u>Behaviors</u> - ways in which you do your work (e. g. build trust, follow-through, responsive, being respectful, shared priorities)
• <u>Environment</u> - (e. g. freedom to experiment in your classroom, institutional support)</td><td>10 min</td></tr>
<tr><td>4</td><td colspan="2">Reflect and discuss how might we apply what we learned in this protocol to other parts of our work.</td><td>5 min</td></tr>
</table>

Modified Descriptive Consultancy Protocol Vignettes

The three vignettes presented in this section provide examples of problems shared during the MDC protocol (see Table 1) and the resulting impact the activity had on the presenters' projects. Two vignettes are from consortium institution PIs (one presenter for each case), and the third is from the SUMMIT-P evaluation team (three presenters). The consultants, usually 10 – 15 people for each vignette, were project management team members, institution PIs and co-PIs, and other consortium team members.

San Diego State University Example—You Are Not Alone

Within the SUMMIT-P project, the work at San Diego State University (SDSU) is focused on two mathematics courses: Precalculus (for students completing an engineering track) and Calculus (for students completing majors in the life sciences). The SDSU team's goals are (a) to identify mathematical topics from partner disciplines that instructors indicate students are weak in or to identify discipline-specific areas where mathematics is used to model phenomenon or systems, and (b) to improve students' view of mathematics as a modeling language for science. Both courses are taught in large lecture formats (approximately 150 students per lecture) with small breakout sections that meet once per week. The question for which the presenter and SDSU PI, Dr. Janet Bowers, sought guidance was how to design and implement engaging labs for the breakout sections of Precalculus that met the goals outlined above.

During the first step of the protocol, Dr. Bowers described the challenges she faced while implementing the project at SDSU. She presented four aspects: (a) the large number of students who take the course, (b) varied instructor participation, (c) a required time limitation of 50-minutes per lab, and (d) the low level of student engagement. Precalculus is taken by about 850 students at SDSU in the fall semester and the course is taught primarily by part-time lecturers. The low level of student engagement is attributed to several factors including common freshmen issues with adjusting to the university experience and students' beliefs that they have learned all of the mathematics concepts covered in the course in high school. The strategy of the SDSU SUMMIT-P team was to work with client discipline partners to develop and offer modeling-based labs for the course and to provide students with opportunities to hear from the "faces of the future"—that is, the professors that will teach the courses in their majors. These professors present short lectures on subjects (e.g., pH-levels) that the students will encounter in their future courses and subsequent careers. Bowers shared that the SDSU team had collected data on students' reactions (mediocre engagement, mild interest, not connecting as anticipated with the content, etc.) to the labs and shared the data during the session with the group.

After hearing the case, the consultants asked clarifying questions during steps 2 – 4 of the protocol (see Table 1). From Bowers's perspective, this phase of the protocol was amazingly generative. The consultants listed more constraints than she had considered. For example, one consultant pointed out that the lab instructors might need some professional development. Dr. Bowers agreed. The labs are taught by undergraduate learning assistants (ULAs) who are math majors. Another consultant pointed out that the labs might need a hook to catch students' interest by presenting a "need to know" for the content. The facilitator (who also participated in the session as a consultant) pointed out that the diversity of majors in the course—for example, some students are studying engineering while others are biology majors—might cause some students to be less interested if the application is not directly applicable to their major. Another consultant suggested that the SDSU team might want to create more realistic examples that practitioners (scientist, engineers, etc.) may or have encountered in their work. Some of the comments shared during this step were both humorous and enlightening.

The iterative process of asking clarifying questions and responding to those questions, and then going through the process again before offering solutions, provided more opportunities for reflection and allowed for a deep exploration of the full scope of the issue. In the end the clarifying questions seemed to coalesce around two different issues: (a) the selection of lab topics that are fabricated and discipline-specific versus those that feature real life situations and (b) the focus on the study of applications versus understanding abstract mathematics concepts. Once the problem was narrowed down to these two issues, the brainstorming session (see Table 1, step 6) generated several ways for the SDSU team to address the problem. Two consultants pointed out that sometimes a puzzle—even if it is abstract—can be cognitively appealing for students. There is an intrinsic motivation to solve or work on something that seems reachable. The key is to create problems that focus on precalculus content and also have "low floor, high ceiling" properties so that all students have an "on-ramp" to solving the problems, and yet there are also many ways to develop solutions. Another consultant noted that there are topics that may resonate more with 18 – 24-year-olds. In other words, she suggested that the presenter consider whether students would rather investigate questions such as “Why do you vomit blood?” or “How long does one drink take to metabolize in the body?” instead of pH levels in a pond or some other abstract concept. At the end of the session, Dr. Bowers shared during the reflection stage that both of the “real life” topics one consultant mentioned (i.e., vomiting blood and metabolizing drinks) are currently addressed in the Calculus course. While Dr. Bowers’s focus is on discipline specific, modeling-based labs, she was encouraged by some suggestions in the discussion that, if they are designed well, some labs designed on topics that are not directly connected to a content discipline can still be useful. After the session concluded, Dr. Bowers took an additional step by classifying all of the comments she had received. She organized them in three groups: (a) topic selection, (b) implementation issues, and (c) deeper engagement with the client disciplines in the course labs.

As a result of the MDC protocol, the SDSU team was able to make a number of changes to the Precalculus course. In particular, the team has developed PowerPoint presentations for the Precalculus instructional leaders to use during the laboratory session, and they also have arranged to have the laboratories co-led by experienced and novice ULAs together. Finally, given that the focus for the SDSU team during year three (with an eye toward sustainability) will revolve around the creation of Learning Glass (Learning Glass, 2020) videos that feature either professors or near-peers describing an application of mathematics in their fields, the insights garnered during the MDC protocol will inform the development of content for the Learning Glass videos.

In reflection, Dr. Bowers’s take-away message from the overall MDC experience was that, while the suggestions were very helpful, the most valuable part was the personal aspect of the experience. The whole group that engaged in the protocol session was listening and empathized with the problems she faced. Dr. Bowers reflected that, even when working as part of a team, a faculty member can feel very isolated and that egos or professional expectations can prevent colleagues from admitting that they are struggling. The opportunity to feel comfortable while 12 knowledgeable, busy colleagues actively engaged in the session was a profound and professionally enriching experience.

Oregon State University Example—The Importance of Talking to Colleagues

Oregon State University’s (OSU) original focus in SUMMIT-P was on a college algebra course. More than 40% of students who enroll in College Algebra at OSU are pursuing STEM majors. Historically, College Algebra at OSU and at other institutions across the country has not included meaningful connections between mathematics and other STEM disciplines. Since Dr. Mary Beisiegel (the PI for the SUMMIT-P OSU project) was involved in the redesign of the course from 2012 to 2015, it seemed to be a good focus for the OSU SUMMIT-P work. In 2016, the structure of College Algebra had become a patchwork of mathematical concepts lacking continuity and not efficiently aligned with the content a student needed to master to be successful in subsequent courses. OSU’s project goals were to improve the connections between the content in College Algebra and the content covered in biology and chemistry courses. In their first steps, the team sought to understand which mathematical skills the partner disciplines wanted students to develop, what mathematical topics could be easily exemplified using content examples from biology and chemistry, and what mathematics concepts are important for work in biology and chemistry. However, the OSU PI was struggling to engage the OSU partner disciplines in the project, thus making only minimal progress on the goals identified by the OSU SUMMIT-P team. In addition, a number of other partner disciplines required College Algebra, thus other stakeholders had interest in the content of the course. By January 2018 at the SUMMIT-P annual meeting, it was clear that the project could use some help from others on the SUMMIT-P team. Thus when Dr. Hargraves and Dr. Hofrening, SUMMIT-P co-PIs who are also faculty in partner disciplines, asked at the annual meeting if anyone from the consortium wanted to present a challenge they were facing using the MDC protocol, Dr. Beisiegel volunteered to present the challenges she faced while implementing her project.

During the protocol Dr. Beisiegel shared the major outside influences on College Algebra that made it seem no longer feasible to work on the course, the difficulty she had engaging with partner discipline colleagues at OSU, and her struggle to understand the role of the site PI and what she could do to make progress on this project. During the first four steps of the protocol (see Table 1), Dr. Beisiegel described the issues and the consultants asked clarifying questions. As Dr. Beisiegel reflected on the experience, the consultant questions that had the most significant impact on her included: (1) Can you find a passion for something you have more control over than College Algebra? (2) Is the work on the course important and meaningful to the collaborators? What time and resources do they have to devote to the project? (3) Are there other avenues you could use to get the work done?

During the brainstorming stage of the protocol, some of the important and encouraging advice she received was to rely on students to assist with the work, keep up with meetings and meeting reminders (including using a system to automatically remind people about meetings), and develop a management skill set. Based on the feedback during the protocol, Dr. Beisiegel reconsidered the College Algebra focus and instead considered focusing on a different course which would allow the project to incorporate the existing structures at OSU and involve other OSU colleagues who would be enthusiastic partners on the project.

The OSU team is now working on a differential calculus course. An in-depth explanation of the switch in focus and the outcomes of the OSU SUMMIT-P project can be found in Beisiegel (2020). Differential Calculus is required for engineering majors, the largest group of majors in the College of Science, and for other majors in public health fields. Differential Calculus is taught in large lecture sections (approximately 110 to 150 students per section) that meet for three, 50-minute periods each week. The lectures are supplemented by an 80-minute recitation section that meets once each week. The OSU team hired three undergraduate students (majoring in bio-health sciences and engineering) to develop in-depth problems for the course, primarily to be used during the recitations. Faculty from the partner disciplines provided feedback to the problem development team. They described how calculus concepts and skills could be used to solve problems in their disciplines. Then the faculty reviewed and revised the problems. The OSU team then implemented the problems in the course. From Dr. Beisiegel’s view, the discussion and feedback that she received through the MDC protocol was invaluable. The consultants were able to see paths that she was unable to see. Through the experience, she felt supported, encouraged, and empowered to move forward. After changing the focus of the OSU SUMMIT-P work from College Algebra to Differential Calculus, Dr. Beisiegel was pleased with the progress the team made in the subsequent year.

Evaluation Team—When Just Asking Isn't Enough

The focus of the SUMMIT-P evaluation is on documenting how faculty members change in their instructional practice, teaching philosophy, and engagement with faculty in other disciplines as a result of participating in the project. To understand this change, the project evaluation team is examining data from site visits, interviews, focus groups, and surveys but most importantly from a set of "prompts" sent a few times a year to project participants—the PIs and co-PIs—that require them to write one or two paragraphs in response to the questions. Some examples of the prompts include "What was your biggest challenge to overcome while working on the project so far?" and "Think back to your early teaching experiences compared to the present. Describe a way in which your teaching has changed. What were the reasons for that change?" By examining the responses to these prompts over time, they intend to explore the conditions that contribute to or inhibit different aspects of change in the faculty.

At a virtual meeting of the project PIs in summer of 2018, the evaluation team presented a challenge through an MDC protocol session. During the first stage of the protocol, the evaluation team described the challenge. Simply stated: they were getting poor response rates to the required prompts from the participants. Typical response rates were about 25%. This was measured by determining who responded by the due date without needing to be reminded by the evaluation team. While this rate may be acceptable for surveys for which there is no incentive to participate or for participants without a vested interest in the outcome of the project, it was not acceptable for this project. When individuals agreed to participate in the SUMMIT-P project, the evaluation team assumed it was clear that all investigators would contribute by responding to the prompts. Multiple reminders were sent to individuals and institutional PIs. The evaluation team explained the need for full participation during several online meetings, but none of those efforts resulted in significantly higher participation rates.

From the evaluators' perspective, participating in the MDC protocol produced a very successful outcome. During both the clarifying questions stage and the brainstorming stage, the process was uncomfortable at times for the evaluation team. Some of the suggestions were critical of the evaluation team and were difficult to hear, but what emerged from the activity was a very practical approach to the problem. The participant response rates have significantly improved. The consultants identified a concern with the original response system. The PIs did not know which of their institution participants had responded to a prompt. They also noted that prompts being sent via email were more likely to get lost in the shuffle and could be difficult to find when an individual sat down to respond to a prompt.

Through the MDC protocol, the idea emerged to create a real-time response submission system using the Google Suite. A Google Form was developed to simplify the submission process. The result was the use of a single web link where participants could go at any time to write a response and also, when necessary, submit responses to past-due prompts. The evaluation team also created a digital check-in system for PIs so they would know who on their team had submitted responses to all available prompts in real-time. The new system provided transparency for the project participants and shifted the accountability for faculty participation to the PIs of the partner institutions. This allowed the evaluation team to focus on analyzing the content of the responses instead of collecting late submissions. Many of the participants still require reminders before completing their submissions; however, the reminders now come from the PIs at the institutions instead of from the evaluation team. The evaluation team believes that these reminders are more useful because they are coming from someone the participant knows and works with on a daily basis.

Modified Descriptive Consultancy Summary

As can be seen from the vignettes presented in this section, team members are facing challenges in the implementation of their projects. With a large, multi-institutional consortium, these challenges can be disparate, and team members can feel isolated. However, by using the MDC protocol, which can be applied in various contexts, the SUMMIT-P team members are able to draw on the collective and varied expertise of the consortium members to make substantive progress in identifying potential paths toward to achieving the consortium goals.

Modified Success Analysis with Reflective Questions Protocol Vignettes

The three vignettes presented in this section provide examples of problems shared during the MSA protocol (see Table 2) and the resulting impact the activity had on the presenters' projects. The vignettes are from consortium institution PIs and co-PIs. Either one person (the PI) or a team of people from mathematics or the partner disciplines presented the successful practice. The consultants, usually 10 – 15 people, for each vignette were project management team members, institution PIs and co-PIs, and other consortium team members.

Ferris State University—A Spin-off of Backward Design

Through the SUMMIT-P project, Ferris State University (FSU) is revising its two-semester quantitative reasoning course sequence called Quantitative Reasoning for Professionals (QRP). QRP was originally designed for business students, and the course sequence was developed through a partnership between mathematics and business faculty. The course is inquiry-based and uses scaffolded explorations for each daily lesson. Each exploration is couched in one or more realistic applications. Through the SUMMIT-P project, FSU is broadening the audience for QRP to include students completing degrees in the Health Professions and Social Work. Rhonda Bishop is the co-PI representing Nursing and Mischelle Stone is the co-PI representing Social Work. The project goals include (a) adding social justice and health contexts into QRP and (b) adding the mathematics concepts and contexts from projects developed for QRP into the partner discipline courses.

During an enactment of the MSA protocol, the FSU team shared one of their greatest SUMMIT-P project successes: developing rich contexts for their QRP course. As the FSU team started their work, they decided they were going to add case study explorations and role-playing simulations to the course sequence. During their first three-hour project meeting, the team members brainstormed possible contexts and applications related to business, health, and social justice. In that first meeting, the team ended up filling two whiteboards with ideas. They found that the richest applications integrated all of the disciplines. They selected those applications to use in case studies and simulations. They then considered the characteristics of good simulations and classified the simulations and their respective case studies based on those criteria.

By sharing this success, the presenters and consultants discovered a practical, collaborative approach to finding realistic contexts for teaching mathematics concepts. The observation stage of the MSA protocol (see Table 2) uncovered that the FSU approach started with brainstorming possible applications in the partner disciplines instead of initially focusing on mathematics concepts for which to find applications. The FSU team explained that one reason for this approach was pragmatic: that summer Dr. Bishop and Dr. Stone were in the process of learning about the QRP course sequence, which is quite unique. They further shared that a more fundamental reason was based on the “applications first” ethos inherent in quantitative reasoning. The team has extended their success with this approach at FSU by incorporating it into the work of a faculty learning community (Bishop, Piercey, & Stone, 2020). During the MSA reflection step, in which participants discuss how this success can be adapted to their context, the FSU team and consultants discussed ways in which others can take a similar “application first” approach. For flexible courses such as quantitative reasoning, they expected that this will be quite effective. For other courses, such as calculus, that have a more rigid syllabus, they hypothesized the approach may not work as well.

Augsburg University—Making It Relevant

At Augsburg University mathematics faculty are partnering with faculty in chemistry and business to revise the three-semester calculus sequence based on the CF recommendations (Ganter & Barker, 2004). Two of the themes identified from the recommendations are contextualizing problem solving and active learning, which align with Augsburg’s curriculum and commitment to student learning. The goal of the collaboration with partner disciplines at Augsburg is to increase the relevance and frequency of applications involving contexts from the partner disciplines in the calculus courses. The Augsburg team consists of a core group of mathematics faculty and two partner discipline faculty (one from chemistry and one from economics) who meet regularly to develop and adapt materials to be used in the quantitative labs. The purpose is to have the labs be a significant feature of the calculus sequence.

During the MSA protocol (see Table 2) session, mathematics faculty member Dr. Jody Sorensen and chemistry faculty member Dr. Joan Kunz shared their work to develop materials for a new lab for Calculus 1. An important factor which aided in the development of the new materials is the culture of cooperation which exists at Augsburg across disciplines. Augsburg University is a small liberal arts university where most faculty know each other by name and collaboration across disciplines is promoted and valued by the institution. For example, the tenure policies and procedures at Augsburg allow promotion and tenure committees to acknowledge, recognize, and give weight to these collaborations. Another factor which led to the success of the work by the Augsburg team is that the work was completed in the summer. This allowed for participants to focus on the work, with meetings taking place every day between mathematics and partner discipline faculty. The focused conversations led the Augsburg team to examine the ordering of topics in the calculus course sequence and make changes to correspond with when students would need to use the calculus knowledge and skills in partner discipline courses. Finally, the success achieved by Dr. Sorensen and Dr. Kunz in this work was based on finding an interesting topic in chemistry which then led to discussions about the mathematics concepts needed to understand the topic. Thus, it was the partner discipline topic which drove the course material development. The mathematics concepts and skills were incorporated in the lesson or activity as tools for students to understand the topic.

Through the probing questions section of the protocol (see step 2 in Table 2), the consultants identified several features which contributed to the success of the work at Augsburg University. First, a campus culture that is steeped in collaboration creates an environment where cross disciplinary collaborations are rewarded and can thrive. While the consultants could not replicate this unique climate at their institutions, they could create opportunities for collaboration to occur, for example during the summer, when schedules are more flexible and there are fewer competing commitments for faculty. They could also think about ways to report the project work on their annual evaluations so that the department chair or a supervisor would recognize the value of the SUMMIT-P work in terms of promotion and tenure metrics. Finally, the consultants recognized how working on common ground by exploring compelling and interesting topics in the partner disciplines resulted in a nexus point at which the partner discipline and mathematics faculty could develop relevant examples for teaching concepts important to both disciplines.

Saint Louis University—Giving Them What They Want

The Saint Louis University (SLU) mathematics department has a long-standing partnership with the SLU Chaifetz School of Business. For over a decade, Dr. Michael May, a faculty member in the mathematics department and PI on the SLU SUMMIT-P project, has partnered with his colleagues in business to develop a Business Calculus with Excel course for students majoring in business fields (accounting, business, finance, etc.). This collaboration provided the foundation upon which the current SUMMIT-P work at SLU is built. Dr. May is partnering with business faculty to develop other required mathematics courses for students majoring in business fields. Given the success of prior collaboration, Dr. May volunteered to use the MSA protocol (see Table 2) to share the activities and processes by which the new College Algebra for Business was developed.

During the first step of the MSA protocol, Dr. May described the origin of the collaboration with the School of Business. He then described the process by which the focus on College Algebra for the SUMMIT-P project was determined. The mathematics faculty and business faculty created a workgroup during which they would discuss topics that the business students needed proficiency in to be successful in their business classes. During the meetings, faculty also identified gaps in the students' skills that needed to be addressed. Initially Dr. May believed that the SUMMIT-P project would focus on Precalculus, but through the faculty workgroup, he learned that most of the topics identified were actually covered in College Algebra. Furthermore, since the business faculty were primarily leading the workgroup, the dean of the School of Business was supporting the work by providing a salary supplement for participating faculty.

Through the probing questions section of the protocol (see step 2 in Table 2), the consultants identified several features which contributed to the success of the work at SLU. Consultants believed the administration's support of the partnership was vital. The monthly meetings and the engagement of multiple faculty in both disciplines facilitated the success of the project.

Modified Success Analysis Summary

The opportunity to share, adapt, and implement best practices is important to the continuous improvement of the SUMMIT-P project. The consortium achieves this in a variety of ways, including site visits, publications, presentations, poster sessions, webinars, etc. The MSA protocol is yet another way to share, adapt, and implement best practices. As seen in the vignettes, the MSA protocol provides opportunity for the group to collectively explore successful practices and consider ways to adapt them to their context.

Summary

As can be seen from the vignettes presented above, the MDC protocol provides a structured format for PIs to address what may seem to be intractable challenges in their projects. By providing space, time, and structure for the presenter to both flesh out and reflect on the challenge through a process involving probing questions, the presenter and the consultants are able to drill down into the challenges. At times during an unstructured back and forth exchange, neither party fully hears what the other person is saying and thus is unable to get to the root of the challenge. This protocol addresses that issue. In this protocol all of the questions are posed at one time. The presenter must write them down before responding. This also allows the consultants to think through several questions and the presenter to reflect on possible answers before responding. During the brainstorming stage of the protocol, time is devoted to open and uninhibited ideation, and throughout the process, ideas build upon each other. Furthermore, by closing the session with an opportunity for reflection, the consultants are able to think about how the ideas posed can be used within their own institutional contexts.

The MSA protocol provides a structured format for sharing successful practices. Most academics are used to this information being conveyed in a poster, presentation, TED Talk, paper, etc. This protocol allows for the traditional sharing of information but also for the consultants and presenter to think about why this worked so well within their institutional contexts. By engaging in that process, including the consultants describing the specific behaviors and underlying principles that contribute to the success of the practice, all of the participants are able to reflect on and think about how to implement or possibly adapt all or aspects of the example at their institutions.

These two protocols provide a structured format for the SUMMIT-P multi-institutional consortium to provide feedback to partners who are seeking advice on a challenge they are experiencing and for partners to share their success stories.

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PARADIGMS FOR CREATING ACTIVITIES THAT INTEGRATE MATHEMATICS AND SCIENCE TOPICS

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ABSTRACT
Research has shown that undergraduate students benefit from seeing examples of mathematics applied to real-world situations. This article describes three different paradigms for how math and discipline partner faculty worked together to create mathematical activities that illustrate applications of the topics being studied in precalculus and calculus. All three examples are discussed within the framework of PDSA cycles to describe the process by which the teams collaborated to plan, enact, study, and refine their lessons. Findings discuss both the difficulties of creating integrated activities (differences in terms and definitions between mathematics and science faculty, different foregrounding of mathematics versus science among faculty), and the value of the resultant lessons, such as increased level of student engagement, higher cognitive demand, and the role that relevant applications can play in piquing student interest in STEM.

KEYWORDS
chemistry, integrated activities, mathematical biology, pH

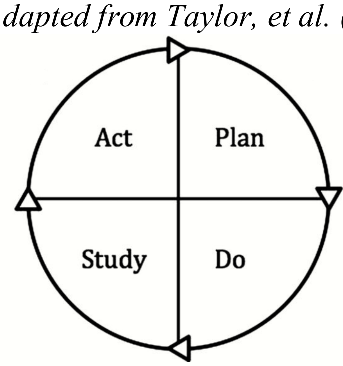
During the early 2000's, mathematics faculty held discussions with faculty from a variety of client disciplines to identify mathematics concepts usually taught in lower division mathematics classes that deserved either increased or decreased emphasis based on their use in successive courses (Ganter & Barker, 2004). As a follow up to this work, a consortium of ten universities across the U.S., collectively referred to as A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P), is now working to implement the recommendations at various large and small institutions around the country. One of the methods for accomplishing this implementation involves having mathematics faculty at each of the ten institutions form Faculty Learning Communities (FLCs) with faculty from various partner disciplines to create integrated lessons. This article describes how that collaborative process played out at three SUMMIT-P institutions, San Diego State University (SDSU), Augsburg University, and Oregon State University (OSU).

We first describe a theoretical framework outlining how curricular design processes can be characterized as passing through four stages. We then use this framework to describe how the three universities each engaged in these stages in different ways. The first example describes how the FLC at SDSU developed integrated lessons to engage students in the study of exponential and logarithmic functions in a precalculus class. The second example describes how the FLC at Augsburg University developed integrated activities from biology and chemistry to demonstrate critical ideas in calculus. The third example describes how the FLC at OSU sought to find local biological models to apply to calculus. We conclude by comparing and contrasting how the processes played out in each of the universities studied and offer some "lessons learned" to inform other interdisciplinary teams.

The Process of Design: PDSA Cycles

When discussing the development of integrated lessons, all of the authors of this article agreed that the process can be messy, time-consuming, and, at times, frustrating. However, we also agreed that when a lesson goes well, it is one of the most rewarding experiences for educators. Even though there was no single path that any of us took to design lessons, we did agree that our efforts could be retroactively captured using the Plan-Do-Study-Act (PDSA) model shown in Figure 1. This model has been used for Quality Improvement in the healthcare field (Taylor, et al., 2014) and adapted by researchers in the Improvement Sciences (Lewis, 2015; Struchens, Iiams, Sears, & Ellis, 2016).

Figure 1
The Four Phases of the PDSA Model Adapted from Taylor, et al. (2014).



Within the SUMMIT-P adaption, the “Plan” phase begins when the FLC identifies a problem that might be addressed through curricular change. This might involve conducting a needs-assessment in order to identify goals and constraints or asking questions such as “What are we trying to accomplish?” (Taylor, et al., 2014). Successive steps in this phase involve proposing possible lessons. In the “Do” phase, the team works to create an initial prototype and implement it. This could be done as a small pilot or enacted in the target classes at large. The “Study” phase occurs as the lesson is being implemented and can continue with follow up interviews or surveys. At some sites, members of the team study lesson implementation by noting students’ ways of engaging with the material, their approaches to problem solving, their level of engagement, and any intended and unintended comments and solutions. In addition, the development team notes their impressions of the instructors and any observers. In some cases, dependent measures may also include items on a test, final exam, or student survey that refer back to the particular application.

The “Act” phase of the PDSA cycle involves making changes to the lesson based on the feedback from the enacted lesson collected during the “Study” phase. This could be revising the materials or re-thinking the discipline application context more broadly within the larger context of the mathematics curriculum it was designed to augment. Collaborations during this phase have also been aided by conducting site visits to other SUMMIT-P schools to share ideas. The “Act” phase completes just one cycle of the process. The full PDSA model envisions a series of micro-cycles until the final outcome is reached (TKMG, 2012).

We first present three examples from three SUMMIT-P institutions. They highlight a few of the unforeseen issues that emerged as the groups progressed through the PDSA cycles. We then discuss some commonalities that emerged across sites, such as differences in emphasis and terminology between partner science faculty and mathematics faculty. We conclude with some suggestions for other integrated teams.

PDSA Cycle Vignettes

Vignette 1: Precalculus Students Need to Learn About Comparing Numbers with Logarithms

Step 1: Plan. Phase I of SDSU’s collaborations was initiated by a mathematics faculty member teaching precalculus. The mathematics faculty (Bowers, first author of this article) worked with faculty from three partner disciplines (Williams from biology, Smith from chemistry, and Luque from mathematical biology) to brainstorm where they see deficits in their students’ mathematical knowledge. Williams had done some prior research in this area and pointed out that many of the most significant deficits science faculty see involve skills such as interpreting graphs and solving algebraic equations, both of which are taught in lower-level courses such as algebra (see Williams et al., 2018). However, all discipline faculty agreed that students needed a stronger understanding of exponents and logarithms because these concepts are frequently used in science to compare very small or very large numbers.

The lesson planning phase began by discussing the parameters of the situation. SDSU teaches Precalculus in large lectures three times per week with required small breakout sections that meet for 50 minutes once per week. The team decided that the lesson would be taught during the 50-minute break out sessions that are run by undergraduate TAs (See Bowers, et al., 2020). Next, the team focused on the nature of the activities the students would engage in with a specific emphasis on active learning. Two activities emerged from these discussions: a virus spread simulation lab to model exponential growth and a pH lab to model logarithmic conversions. While the group had hoped to use authentic tools such as litmus paper (chemically treated paper that changes color based on the pH of a substance it touches) for the pH lab, the restrictions of time and materials forced us to choose the simplicity of worksheets.

Step 2: Do. The pH lesson was implemented in 23 recitation sections (hereafter referred to as "labs" to indicate the intention of more active learning). The worksheets included three parts. Part I involved having students use Desmos or Excel to graph very small numbers (concentrations of hydronium in various substances that ranged from 0.0000000000316228 to 1.0). Once they realized that base-10 scales were problematic for showing differences of very small numbers, the instructors of the breakout sessions introduced the idea of log scales for the y-axis. Part II involved applying the laws of logs the students had learned in lecture to convert concentrations of hydronium to pH values and vice versa. For example, given that canned artichokes have a concentration of hydronium (denoted $[H+]$) = 0.000034, students use the formula $pH = -\log[H+]$ and their knowledge of the laws of logs to compute a pH value of 4.463. Part III involved comparing magnitudes of other logarithmic measures such as decibels and the Richter scale.

Step 3: Study. SDSU's FLC team sought three types of feedback: reactions from the instructors, impressions from the students, and performance data on related items from the final exam. The lab instructors stated that the pH lesson was challenging for them to teach because none of them had known exactly what pH measures are or felt prepared to field students' questions regarding the surrounding science. In this way, several admitted that teaching the lesson helped *them* strengthen their understanding of logarithms beyond the rules they had dutifully memorized because they appreciated the role that the rules played in the conversion algorithm. To the pleasant surprise of the lab instructors and the coordinator, the students were engaged in ways that they had not observed during other worksheet-based labs. They noted that students commented on how "cool" it was to be studying pH, a concept many of them had been discussing in their chemistry class.

Results from an end-of-course survey revealed that the students rated the virus and pH labs very highly. In addition, the students performed well on a related task in which they were asked to compare two pH values. Unfortunately, results from an exam item requiring the procedural implementation of the laws of logarithms to expand expressions into addition and subtraction revealed that many made algebraic errors.

Step 4: Act. Based on the results of the study phase, the lesson was revised to strike a better balance between sufficient science (e.g., enough to explain what pH stands for and what the small numbers represent) with the need to keep the mathematics relatively close to the mathematics teachers cover in their lecture. In subsequent semesters, the lesson has been front-loaded with an activity focusing on the more calculational process of applying the laws of logarithms, noting that the students will need these skills for calculus because they need to learn logarithmic differentiation which will then be used in chemistry.

Vignette 2: Discipline Faculty at Augsburg Present Ways that Calculus Is Used in Biology and Chemistry

In redesigning Calculus I at Augsburg, the FLC included partner disciplines that provided examples of how they use calculus content in their classes that require calculus. Specifically, the mathematicians asked their SUMMIT-P discipline partners to present examples from chemistry, environmental science, and economics calculus.

Step 1. Plan. The calculus instructors were looking for applied settings that could be used in a calculus class with a broad audience. For them, this meant interesting problems that could still be understood by students of varied backgrounds. One example that resulted from these discussions was the titration lab. The team was particularly interested in this process because identifying the inflection point in the data set is a crucial part of identifying unknown solution concentrations. The team used this to create an activity where students worked with spreadsheets containing a real data set to approximate first and second derivatives to identify an unknown acid. The data contained an independent variable, the volume of the titrant in mL, and a dependent variable, the pH of the solution being titrated. After the discipline partner (Kunz, the chemist) presented the problem to the mathematics faculty (Sorensen and others) and provided the data set, Sorensen then wrote a draft of the activity. Working through the lab, Sorensen made several changes, such as asking Kunz for additional data near the inflection point for a more precise analysis. In the final phase, Kunz read the draft lab to suggest correct wording and possible extensions.

Step 2: Do. The titration lab was one new lab in a completely redesigned course full of active learning and applied examples of calculus. This activity has been taught by two different faculty members teaching Calculus I at Augsburg University. The students were asked to talk about the shape of the plot that emerged from the graphs they made with Excel and then to use their new vocabulary to describe it. They used mathematical terms such as increasing/decreasing, concavity, and limits. The partner discipline faculty have occasionally attended class during relevant labs/activities, but the development team decided it was more important that they evaluate the language and questions in the activity before the class.

Step 3: Study. When talking with instructors, Sorensen learned that they enjoy teaching the new course, and student engagement seemed to be high. For example, the instructors reported that it was fun to see students who had done titration in a chemistry class react to the lab and to be able to explain some of the ideas to their peers. But the instructors also noted that even students who had not studied chemistry succeeded and remained engaged.

Step 4: Act. The titration lab, along with myriad other new applied activities and labs, will continue to be used in our calculus courses each semester. The mathematics faculty have a process for re-evaluating and editing the materials during the semester. They also seek feedback from instructors, partner disciplines, and students.

Vignette 3: Incorporating Activities from Biology and Chemistry into Differential Calculus

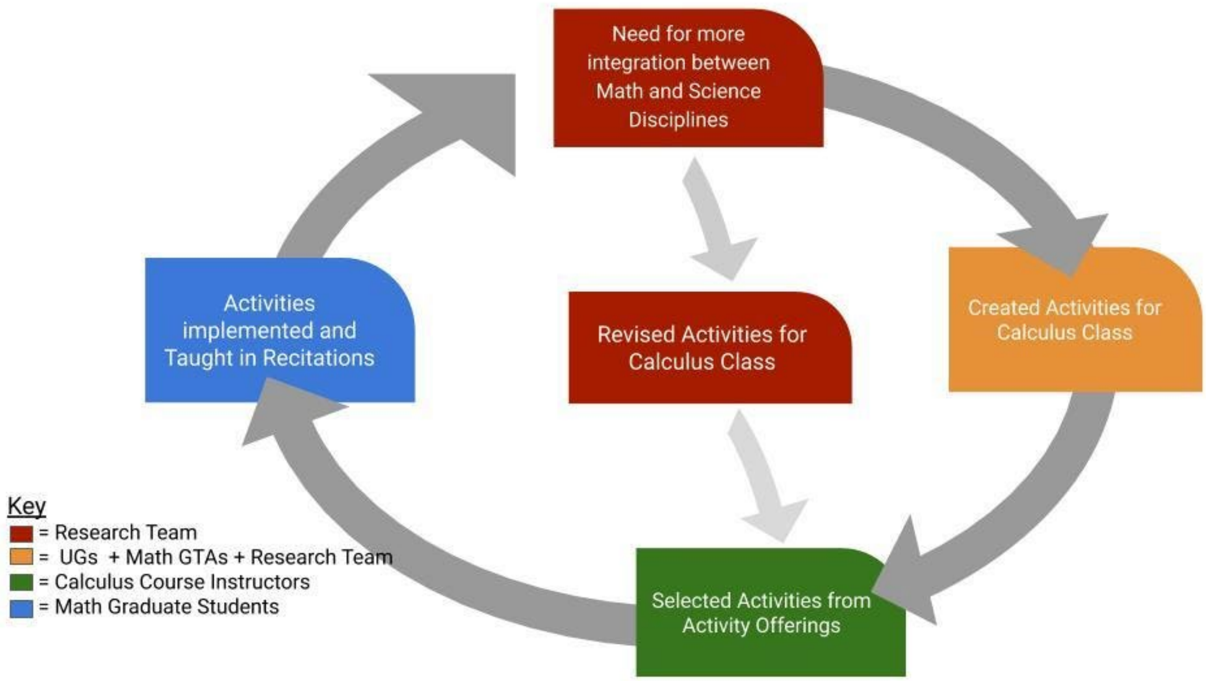
At OSU, a cross-curricular FLC acknowledged the need for more integration between mathematics and partner disciplines for science students. After spending time examining existing curricula in disciplinary courses (biology and chemistry), the FLC decided to craft relevant activities (applied problem sets) for incorporation into the ten-week Differential Calculus course.

Step 1: Plan. To create the activities, two undergraduates from biochemistry worked with the OSU FLC to create, revise, and align applications for the Differential Calculus course. Broadly, the FLC considered all the topics from the Differential Calculus course (limits, continuity, derivatives, related rates, optimization) and sought to use phenomena from biology or chemistry to suggest the need for calculus concepts. Approximately 30 problems were developed by the two undergraduates. Calculus course coordinators and instructors then selected activities from the list of problems for use in recitations taught by graduate students.

Activity creation occurred iteratively (see Figure 2). First, the two undergraduate students used topics they remembered well from biology and chemistry to craft initial calculus activities.

Then, working with the OSU FLC, the students explored and aligned the mathematical components of each problem to the existing calculus courses by topics. For example, a predator-prey model from biology was examined for the mathematics that would be used to answer questions about rates of change, as well as the relationship between minima and maxima for the predator and prey populations. Next, the FLC consulted with two undergraduate students and calculus instructors to ensure accuracy of disciplinary concepts, provide background information that would be relevant for answering questions, and revise problems for shared language between the partner disciplines and mathematics.

Figure 2
Iterative Cycle of Lesson Development at OSU



The activities that were developed and revised by the FLC with the help of the two undergraduates were not only focused on certain disciplinary topics (e.g., predator-prey interactions); they also considered locally relevant species (e.g., Pacific Northwest organisms) or socially important topics (e.g., effects of radiation from Chernobyl). The final drafts of activities contained questions that covered calculus concepts from the entirety of the 10-week term. By developing activities in this way, they could be used in recitations multiple times during a term.

Step 2: Do. Once the activities were planned, the lead calculus instructor (the calculus course is team taught and coordinated by a lead instructor) selected activities that would be used during the term. The activities were then discussed with other calculus instructors before being deployed to recitations taught by graduate teaching assistants.

Step 3: Study. When the activities were deployed in recitation sections, the recitation GTAs reported an increase in student engagement perhaps attributable to increased critical thinking (GTA interviews). Whereas in the past, recitation activities consisted of strictly computation mathematics problems, the new activities required students to interpret and understand the application of the mathematics concepts before solving. One GTA theorized that, with less critical thinking required, the students were more likely to diverge into social conversations with each other, causing a decrease in student engagement. However, in the new activities, they had to comprehend the meaning of the activity, relate it to a real-world application, and determine which mathematical equation to use. This allowed students to not only understand the real-world application of the mathematics but to also have "a sense of discovery" (GTA interview) while solving the activities, which was lacking in the prior worksheet style.

Other feedback from GTAs stated that, on average, the students thought the activities were more challenging and took longer to solve than the less integrated recitation worksheets. However, one GTA considered this a strength because the activities required students to remain working in the lab throughout the recitation and helped to further develop the students' understanding and appreciation of the real-world application of mathematics in the biology and chemistry disciplines.

Step 4: Act. The activities are planned to be deployed in future calculus courses at OSU. Due to the feedback of the course instructors and GTAs, instructional guidelines will be provided to the instructors before the implementation of the activities in future calculus courses. These instructional guidelines will contain activity background information, rationale for the science topic, science and mathematics terminology, common science misconceptions, and the representation of math in the activity's topic. Other revisions to the activities include removing excess background information in the activity for English language learners and implementing the activities earlier in the calculus course.

Discussion: Common Themes Across the Approaches

Although each of the sites had slightly different approaches to engaging in the PDSA cycles, there were several themes that emerged though each collaboration process. In particular, faculty at all three sites noted differences in terms of language use (e.g., what is a "line") and professional approach (e.g., "what is a conceptual definition?"). We believe that commenting on these differences might inform readers who are attempting to design integrated activities. Despite the differences, we found agreement among all faculty that these lessons are helpful and hold promise for greater transfer of skills from the mathematics to science classes. We also found agreement from students who said that they appreciated the efforts to illustrate real-world uses of the topics they were learning.

Differences Between Mathematics and Science

Using Differing Terms and Definitions

One of the common issues that arises when departments cross-collaborate is the interpretation of terms. For example, in the titration example at Augsburg University, what the mathematicians would call an *inflection point* is referred to as the *equivalence point* in chemistry. Working through this and highlighting the difference to students was an intentional part of the activity that was created. More broadly, science faculty at SDSU have often stated that they focus on systems that seek balance and *equilibration* rather than static *equivalence* of two sides of an equation.

Another example of a term that carries different meanings across disciplines is the term *line*. Mathematicians distinguish between *linear functions* (often called “lines”) and nonlinear functions (often called “curves”). In contrast, many biologists call any graphed relationship between two variables a “curve” even if it is linear.

Different Foci from Mathematics to Client Lessons

A second issue we found is a mismatch in terms of emphasis or process. For example, client disciplines often rely on mathematics to model data and hence begin with numbers rather than equations, whereas mathematics instructors usually focus on known functions. The two institutions that developed activities that focused on pH both encountered this. When chemistry instructors introduce the idea of titration, they are plotting pH as a function of acid added to an unknown substance. The focus of the process is to identify the inflection point *of the data* because it indicates the amount of acid that causes the pH of the unknown acid to suddenly rise. In order to model authentic practices, mathematics faculty at Augsburg University used data and then found a best fit function to illustrate how the mathematical operations could be used to identify the inflection point. Similarly, at SDSU, the lessons shifted from focusing on functions and processes to operations on data (e.g., molar concentrations of given substances or data collected during the virus spread simulations).

At OSU, discussions with calculus course faculty (lead and teaching faculty) revealed that while the project’s goals (integration between mathematics and partner disciplines) were recognized, subtleties of the activities were not. Partner faculty noted that mathematics faculty changed the problems as they perceived they needed but often edited or removed components deemed important to disciplinary partners. For example, in an activity in which students consider the effect of temperature on sex determination in turtles, the original activity expected the students to work more like biologists and consider the implications of the relationship between temperature, turtle sex, and climate change conceptually before starting to use calculus to explain the relationship. However, the lead mathematics faculty revised the activity so that students were expected to engage in computations on functions (such as calculating instantaneous rates) without first predicting or exploring the relationship between the variables.

Working with Undergraduates

Several of the sites in the SUMMIT-P project have found that undergraduate learning assistants can be great resources for developing cross-disciplinary activities. For example, as described, the undergraduate biochemistry majors at OSU identified the activity topics by recalling activities in which they had used mathematics in their biology and chemistry classes. Once topics were identified, these undergraduates worked with a team of mathematics and partner discipline faculty to modify the activities. The undergraduates were then tasked with ensuring the accuracy of disciplinary concepts and providing background information, and perhaps most importantly, they were able to recognize and resolve differences in terminology use between the discipline faculty and mathematics faculty who implemented the lessons. Both of these roles for undergraduates were shared with the SDSU FLC during a site visit. Faculty from OSU provided some insights regarding the ways in which OSU employs undergraduates as learning assistants in their curriculum development process and as helpers who sit in on lecture classes to encourage students to work problems as they are posed during lectures.

At SDSU, undergraduates were also critical in the lesson development and implementation processes. They provided insights from their perspective as students who took the courses and also provided guidance and monitored what "would fly" in terms of activities that would appeal to undergraduates. For example, while planning the logarithm lesson, two undergraduates working on the project said that they had used logarithmic transformations while interning in a virology lab on campus to illustrate differences in very large numbers. They made a video to illustrate this use, and it was added to the pH lesson materials distributed to each lab instructor (see Bowers et al., 2020).

Student Engagement

Reports from all three sites indicated healthy and often enthusiastic student engagement. We believe that the reasons can be organized in two general categories: increased cognitive demand (that differs from what general worksheets require) and the use of practical, relevant problems.

Increased Cognitive Demand

One of our goals is to craft lessons that engage students in non-traditional practices of learning mathematics including hypothesizing, modeling, and making and critiquing arguments. Although not all sites reported hitting all of these goals, each team mentioned some aspects of increased cognitive engagement. The team at OSU reported that the integrated activities required students to interpret and understand the application of the mathematics concepts before solving, which differs from activities consisting of strictly computation problems. The team at SDSU reported that some students were able to understand logarithms at a deeper level by seeing their utility for measuring differences of magnitude.

Practical Applications

Reports from instructors at all three institutions indicate the value of relating content to real-world applications. As one OSU GTA noted, this allowed students to not only understand the real-world applications of the mathematics but to also have "a sense of discovery" (GTA interview) while solving the activities, which was lacking in the prior style of worksheet. At SDSU, the pH lab was voted as the top lab three semesters in a row. When asked on an end-of-semester survey why they liked it the best, student comments fell into three categories: (1) Pertains to a current/future class (e.g., "It helped me define the relationship between pH and logs, and seeing as I'm currently taking chemistry it was really interesting to make that connection and figure out easier ways to solve pH problems without having to use a calculator"); (2) Real life applicability (e.g., "I liked this lab because logs can be difficult to understand, but since this lab did a great job of tying it together with real life examples it made it a lot easier to understand."), or (3) Helped clarify a mathematics concept (e.g., "I struggled with Log functions so it was very helpful for me.").

Faculty at Augsburg University who taught the titration lab also felt that it was well received by the students. They noted that students recognized the idea from previous courses, and even if they did not recognize the topic or were not interested in it, they were still able to understand the mathematics that the application was designed to highlight. These challenges have the potential to pique students' interest in other STEM applications of mathematics in the biology and chemistry disciplines.

Conclusion

Developing and integrating activities from partner disciplines is a challenging but rewarding endeavor. The process benefits faculty and students. Mathematics faculty are challenged to move beyond their silos to learn different terms, applications, and ways of using mathematics that they may not have considered before. In addition, the collaborations also support curricular decisions, such as what topics to emphasize based on STEM utility rather than just the mathematicians' views of conceptual development. The applications can be beneficial to students for several reasons: (1) they can serve as productive disruptors in mathematics classes because they challenge students' expectations for what it means to learn mathematics; (2) they provide a new, more concrete way of looking at concepts that may have appeared only calculational in nature; and (3) they provide insights into how the mathematics they are learning will be used in subsequent classes or in "real life."

It is also critical to be aware that such endeavors may not be widely appreciated or successful. In particular, we found that if students' perceptions of how college mathematics classes should be conducted are limited to introductory lectures in which instructors talk about manipulating symbols, then using applications might be stretching the students beyond their comfort zones. Or, they may resent learning material that "won't be on the test." In addition, if the applications require a good deal of science understanding, some students not majoring in a science discipline may discount the relevance of the particular application. From the perspective of the discipline faculty, although they appreciate being asked to identify areas of weakness, they often indicate that their particular needs rest in general procedural skills such as proportional reasoning and graph interpretation.

Finally, we believe that the PDSA cycle can be a useful way to lay out a plan for moving forward. In particular, the "Plan" phase of the cycle should include both discipline partners and mathematics faculty. While the original intent of our work was for discipline partners to completely lay out the skills they desire students to have, it is also the case that mathematics faculty teaching lower-division courses may be bound to cover certain topics for successive courses. Thus, the planning phase may involve developing a compromise to meet the needs of all involved. All three sites found the "Do" and "Study" phases of the work to be somewhat intertwined. Reflecting on how the mathematics faculty do (or do not) maintain the scientific content of the applications can inform future iterations of the "Act" phase of the cycle. Moreover, while these integrated activities were generally well received, they will become more effective if the partner discipline faculty can "Act" in the future by referring *back* to these experiences when the topics—titration, for example—are discussed in subsequent courses. We look forward to following these students and researching how these ideas proceed and develop over time.

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USING A FACULTY LEARNING COMMUNITY TO PROMOTE INTERDISCIPLINARY COURSE REFORM

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ABSTRACT

As part of a multi-institution, National Science Foundation (NSF) grant-funded project, Ferris State University (FSU) joins a national effort to reform mathematics curricula. Researchers from FSU developed and facilitated a faculty learning community (FLC) as one strategy to redesign the traditional approach to the quantitative reasoning skill development of students in the departments of mathematics, nursing, social work, and the College of Business. Over the course of one academic year, the FLC provided an interdisciplinary faculty connection to develop pedagogical approaches that integrated cross-curricular concepts and context from each discipline. The FLC not only produced uniquely designed, learning-centered approaches to teaching quantitative reasoning but created a sense of community and camaraderie that promoted faculty development and the scholarship of teaching.

KEYWORDS

Faculty Learning Community, mathematics

In 1990, Boyer characterized higher education as a series of department, discipline, and curricular silos. This fragmented approach to teaching and learning often results in a student experience that lacks coherence and relevance. Boyer challenged the academic community to think outside of these silos by focusing on discovering the most effective way to teach that also produced genuine student learning. Since Boyer's challenge, faculty have learned much about effective teaching and learning. They have learned that teaching is more than telling students what they need to know and that authentic learning occurs by engaging in real-world issues and solving relevant problems. A host of scholarly work reveals that faculty have experimented with a variety of strategies to enhance student learning. Unfortunately, there has been little incentive to cross academic boundaries and engage other departments and programs to reform curricula. The need for curricular reform so that higher education relates to the realities of society, the business world, and many professions has never been greater. College graduates need to know how to solve real problems, communicate effectively, work collaboratively, use technology, lead, and demonstrate professionalism (The National Association of Colleges and Employers [NACE], 2020). Thus, to produce career-ready college graduates, higher education must redesign curricula so that students engage real-world problems across their educational experience.

College mathematics courses are often considered a prerequisite to higher-level course work. This curricular structure expects that students carry over foundational quantitative reasoning skills into future course work and ultimately their future careers. However, some students are not able to carry over the needed skills into future courses, nor do they "see the connections between mathematics and their chosen disciplines" (Ganter & Barker, 2004). In addition to this disconnect, the mathematics content in prerequisite courses may not be relevant to the students' chosen field of study (Ganter & Barker, 2004). Although the mathematical skills students need in the non-mathematics majors vary, all students need a conceptual understanding of basic mathematics tools (Ganter & Barker, 2004).

As part of the Synergistic Undergraduate Mathematics via Multi-Institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) Project, three faculty researchers from Ferris State University (FSU), a public university in central Michigan, have undertaken an interdisciplinary endeavor to reform mathematics education for the students completing majors in partner disciplines. SUMMIT-P is a multi-institution, National Science Foundation funded project to improve undergraduate mathematic courses. The work of SUMMIT-P is based on recommendations outlined in the *Curriculum Foundations Project* (CF) (Ganter & Barker, 2004) and focuses on reforming mathematics courses by emphasizing the conceptual understanding of mathematics as related to the partner discipline needs. Instructional methods feature active learning that is grounded in career-focused problem-solving skills, mathematical modeling, and communication. One element of this work was the development and implementation of a faculty learning community (FLC). Mathematics faculty along with faculty in the partner disciplines of nursing, social work, and business worked together to redesign how mathematics content is taught and to vertically integrate mathematical concepts into the partner discipline programs. FLCs are commonly used to facilitate faculty development of the scholarship of teaching and learning (SoTL). While that was a focus of this FLC, known as the *Mathematics and Partner Disciplines FLC*, it was also the vision of the facilitators that long-term partnerships would produce a sense of community and camaraderie among the participants and continue the effort to break down the department and subject-matter silos that exist at FSU.

This article describes the process undertaken at FSU to develop and implement a multi-disciplinary FLC to reform the approach to mathematics instruction. Consistent with the literature on FLCs, the *Math and Partner Disciplines FLC* was characterized by (1) the role of the facilitators, (2) the development of goals and outcomes, (3) the approach to choosing participants and team division, and (4) the process for designing sessions and deciding on the deliverables. Each of these will be described in detail below. Finally, based on a review of participant feedback, we will reflect on the lessons learned and describe the next steps in our project.

Faculty Learning Communities

According to Cox (2004), FLCs are:

> a cross-disciplinary faculty and staff group of six to fifteen members (eight to twelve members is the recommended size) who engage in an active, collaborative, yearlong program with a curriculum about enhancing teaching and learning and with frequent seminars and activities that provide learning, development, the scholarship of teaching, and community building (p. 8).

There are many benefits to the FLC model over other forms of professional development. FLCs allow for faculty to provide direction and consequently deal with issues relevant to the cohort in real time (Daly, 2011). When someone facilitates an FLC and has no authority over the participants' advancement within the organization, it provides a safe atmosphere for discussion, vulnerability, and growth (Cox, 2003b; Daly, 2011; Bickerstaff, Lontz, Cormier, & Xu, 2014). FLCs help encourage experimentation in teaching and learning (Bickerstaff, Edgecombe, & the Scaling Innovations Team, 2012) in a context in which participants tend to find their internal motivation and take ownership of their growth as instructors (Daly, 2011). FLCs offer a productive environment for the development of teaching projects that address real problems (Cox, 2007). FLCs can also guide faculty in the work that develops competence in and produces SoTL (Cox, 2003a, Cox, 2007).

The Mathematics and Partner Disciplines Faculty Learning Community

The project team, consisting of one faculty member each from the collaborating disciplines (mathematics, nursing, and social work), worked together to revise the existing mathematics curriculum. We started by exploring the role of quantitative reasoning in the nursing and social work professions. This exploration lead to identifying common quantitative reasoning skills that were embedded in discipline-specific courses. The purpose was to intentionally introduce the concepts and skills in the mathematics courses taken as prerequisites to discipline-specific courses. As the project team continued analyzing the current mathematics curriculum and brainstormed possible revisions, it became apparent that the team needed buy-in and assistance from other faculty in the partner disciplines to complete the task. Subsequently, the project team envisioned an FLC in which partner discipline faculty would assist with the revision of the mathematics curriculum and with the intentional incorporation of quantitative reasoning into their respective courses. The FLC was conceptualized in spring 2018. During the summer of 2018, the project team prepared for the FLC that would be implemented during the 2018 – 2019 academic year.

Reflecting on the process of planning, implementing, and evaluating the FLC has been useful in determining its impact on the mathematics curriculum and the curricula of the partner disciplines as well as on the participants and facilitators. We believe the participants and facilitators engaged in professional development that culminated in SoTL. Richlin (2001) refers to SoTL as a cycle that begins with scholarly teaching and evolves into scholarship. Prior to the development of the FLC, the authors identified the opportunity to change mathematics content delivery for partner discipline students. Development of the FLC assisted the authors, who also served as FLC facilitators, to engage in the process of scholarly teaching, and writing about the process has helped us transition into the scholarship phase of the cycle.

Faculty Center for Teaching and Learning

The Faculty Center for Teaching and Learning (FCTL) at FSU offers a host of resources to faculty and the university community. One such resource is the opportunity to participate in or facilitate an FLC. Consistent with Cox's (2004a) description of an FLC, FCTL supports both discipline-based and interdisciplinary faculty groups that have a defined focus or purpose. In spring 2018, the project team developed and submitted a proposal to the FCTL for the *Math and Partner Disciplines FLC*. The proposal was evaluated against predefined criteria, including how the FLC aligned with FCTL values, the measurability of the overall project outcomes, descriptions of the outcomes, activities, and assessments for each session, the expected deliverables or end products, and the assessment plan for the deliverables (FSU, n.d.). Once approved, the facilitators participated in FCTL training and planning sessions during summer 2018 to prepare for the year-long endeavor.

In addition to the mandatory training, the FCTL also provided resource support throughout the FLC. The FCTL helped with reserving rooms and equipment, ordering meals, and making copies for the FLC sessions. Professional development funds were available for participants who completed the FLC as demonstrated by consistent attendance and submitting deliverables. This was a significant factor to the success of our program. The co-facilitators did not receive professional development funds, but instead received a stipend that compensated them for the time invested in overseeing and running the FLC. The FCTL required attendance reports and periodic updates on the progress of the project which helped keep the project team accountable for all aspects of the FLC.

Purpose and Goals

The purpose of the FLC was to engage faculty in the work of transforming mathematics education at the university under the leadership of the SUMMIT-P project team. Specifically, the vision was to reduce barriers that prevented students from using the concepts and skills learned in mathematics courses in their respective majors. Based on the principles of the Mathematical Association of America Curriculum Foundations Project (Ganter & Barker, 2004), the overarching goals for the project were to use the FLC sessions and activities to determine which mathematics concepts and skills to cover in courses in each of the partner disciplines and to develop an understanding of relevant and practical discipline-specific contexts in which to embed the mathematics concepts and skills.

Outcomes

The facilitators approached the FLC from a teaching and learning perspective. By applying established best practices in course design, the identified session and terminal outcomes were made observable and measurable, thus identifying behaviors that evidenced learning (McDonald, 2014). Each of the outcomes, therefore, established the level of success of the FLC activities. First-semester session outcomes began with a focus on active learning and teaching styles. In the second semester, the session outcomes evolved to produce active learning materials or exercises and a re-evaluation of teaching styles, and it culminated with a capstone presentation of a discipline-specific mathematics activity. The FLC design allowed faculty ample opportunity to reflect on and evaluate their own teaching practices in order to improve their practices. In fact, a goal of the project was to determine how participating in the FLC would influence classroom practices. In addition to specific assignments completed between sessions, participants needed to complete a set of deliverables.

Role of the Facilitators

The project team, acting as co-facilitators, each had different experiences with FLCs; one had participated in and facilitated previous FLCs, another had attended several FLCs, and the third had no sustained experiences with an FLC. Because of the diverse experiences with FLCs and the importance of the facilitator role to the FLC process, it was very apparent that facilitator preparation would be a critical element of the FLC process. As a stipulation of FCTL approval, the co-facilitators attended a campus based, two-day FLC facilitator workshop. The activity-based workshop explored Cox's (2004) definition of an FLC and Ortquist-Ahrens and Torosyan's (2009) work on the role of the FLC facilitator. The co-facilitators also completed Sandell, Wigley, and Kovalchick's (2004) goals inventory, and the results lead to identifying key outcomes for the FLC. Other activities in the workshop included the intentional development of the outcomes, relevant evidence, and the facilitator and participant activities for each FLC session. Ultimately, the workshop aided in understanding the purpose of an FLC and the role of a facilitator, determining and dividing the facilitator responsibilities, and intentionally creating space for planning the details of the FLC. After the workshop, the co-facilitators had an in-hand plan and framework for implementing the FLC.

Defining the roles of the FLC facilitators included the division of both task and process responsibilities and ultimately aligned with the roles of a champion, organizer, and energizer (Petrone & Ortquist-Ahrens, 2004). However, we found it necessary to include a fourth role in the process—the role of an analyst. Each of these roles will be described below.

Understanding that a sense of shared responsibility would evolve from using a team approach, it was essential to engage each other with open and frequent dialogue, mindfulness, and flexibility, and to capitalize on individual strengths. The open and frequent dialogue encouraged collegiality, a non-threatening and engaging atmosphere, and genuine reflection on the FLC process. Approaching meetings with mindfulness produced clarity of communication and increased productivity. As tenured faculty, each of the facilitators held various leadership responsibilities and demands. Thus, it was important to be considerate of each other's time and maintain a flexible attitude, which demonstrated a commitment to the FLC process.

The Role of Champion

The FLC facilitator acts as a champion by making connections from actions to outcomes and being a catalyst for change (Petrone & Ortquist-Ahrens, 2004). The champion role evolved from the mutual vision for changing the approach to quantitative reasoning in mathematics, nursing, social work, and business. Ready with content resources and department-specific insight, the co-facilitators shared the responsibility for championing the effort. Collaboratively, we arranged the time, space, and resources to develop simulations, case studies, and assignments that involved mathematics concepts and would be embedded in both mathematics and the partner discipline curricula. A light meal and informal conversations created a nonthreatening climate, interpersonal connections, and a sense of community. Stories of family, children, pets, and the challenges of Michigan winters created commonality among all participants. The champion also works to create a challenging climate (Petrone & Ortquist-Ahrens, 2004). The facilitators accomplished this by preparing prompts and resources that stimulated the participants to think in terms of another discipline or to review the ongoing work from yet another perspective. Integral to the success of the envisioned curricular changes, each of the facilitators initiated ongoing communication with departments, deans, and advisors. It was essential to the success of the project to not only advocate for the FLC program but also to communicate to the university community about the cross-disciplinary work that was underway. This was accomplished in several ways, but one of the most significant was a visit by a partner SUMMIT-P institution during one of the early meetings of the FLC. During this visit, FSU administrators and other stakeholders attended a briefing session about the work the FLC was accomplishing.

The Role of Organizer

The organizer "focuses on the operational and logistical aspects" of the FLC (Petrone & Ortquist-Ahrens, 2004, p.65). Through a collaborative effort, the organizer's responsibilities evolved into three categories, each assumed by one of the co-facilitators. The mathematics facilitator prepared the content for each session. The social work facilitator communicated to the FCTL staff for reserving rooms, ordering food, and making document copies. The nursing facilitator communicated reminders to participants and monitored the completion of session assignments and deliverables. Associated with the responsibilities of this third category was the development of an FLC course in the university learning management system (LMS). Framing the FLC as an academic course allowed information to be available through the FSU LMS. Participants could also submit deliverables as assignments in the LMS. This helped the facilitators to easily track participant completion of tasks and gather qualitative feedback about the FLC. The discussion board feature of the LMS was useful for exchanges between participants or between participants and facilitators. The LMS gradebook and messaging system also helped facilitate direct communication with participants.

The Role of Energizer

Petrone and Ortquist-Ahrens (2004) defined the role of energizer as one who monitors and directs the interaction of participants. This role, although shared by all three facilitators, tended to find focus in the high energy and humorous personality of the mathematics partner on the team. In the FLC sessions, the facilitators each joined a workgroup and participated in the ongoing process. As embedded team members, the facilitators would listen attentively to workgroup dialogue, ask qualifying questions, and model effective communication skills. With the goal of nurturing a climate of collegiality, the process required carefully listening to the voices of participants as the workgroup explored how mathematics concepts are embedded in the other disciplines and discussed the discipline-specific language used to describe mathematical ideas.

The Role of Analyst

The role of analyst evolved as we collected feedback from participants about the FLC sessions, activities, and evaluated assignments. With expertise in qualitative data analysis, the social work partner assumed this role. The analyst collected and analyzed participant feedback to help the facilitators make mid-year adjustments and organize content for subsequent FLC sessions. The analyst also provided periodic reports to the FCTL on the progress being made in the FLC. In the end, analysis of the final feedback facilitated the assessment of outcomes.

Choosing Participants

The facilitators initially planned to select participants for the FLC through an application process. However, because of the nature and purpose of this particular FLC, the facilitators decided to intentionally recruit key faculty from each of the partner disciplines. Ideally, there should have been equal representation from each of the participating disciplines, but recruiting efforts resulted in three faculty from mathematics, two from social work, two from nursing, and two from business. Although not an element of the original project proposal, business faculty were recruited because of previous collaborative work between the team leader and the business department to create a quantitative reasoning course for business students. A total of 12 faculty (nine participants and three facilitators) participated in each FLC session.

Design of Sessions

For each session, the facilitators identified outcomes for the session, the evidence to be produced by participants that demonstrated meeting the outcomes, and the facilitator and participant activities that would produce the expected evidence. The sessions were two hours in length and started with lunch and conversation. Pre-defined session activities gave participants time to explore thoughts and processes in a team environment. During the initial session, the schedule for the FLC sessions was developed to best align with participant schedules. In-person attendance during sessions was a critical element for the FLC, which required a significant amount of collaborative work. However, due to circumstances related to weather, illness, child care, or professional responsibilities, some participants did attend sessions virtually using video technology.

Teamwork

Teamwork was central to the goals and outcomes of the FLC. A goal of the curricular reform project was to embed experiential learning activities into the mathematics courses that included concepts from more than one partner discipline. The goal was to first introduce partner discipline concepts through active learning exercises in the mathematics courses and then revisit the concepts in the discipline-specific course work. Thus, the facilitators chose to divide the FLC participants into three teams each with representatives from three different disciplines. The interprofessional teams combined their skills, knowledge, and resources during the FLC sessions to complete activities that produced high-quality deliverables and modeled the university core value and general education competency of collaboration. Collaborative work was concentrated in the pre-defined working sessions and deliverables, and minimal teamwork occurred outside an FLC session. However, individual deliverable expectations did require out-of-session work.

Deliverables

Activities that produce growth and development are a fundamental component of FLCs. These kinds of activities also provide evidence that learning is taking place during the FLC. In the case of the *Math and Partner Disciplines FLC*, the purpose of the activities was for participants to demonstrate their plans to make changes to course content and instruction. By the end of the FLC, each participant produced five deliverables.

Types of Deliverables

Syllabus. A syllabus, by definition, includes course outcomes, learning activities, and a schedule of those activities (Gunert-O'Brien, Mills, & Cohen, 2008). Each participant submitted a syllabus for one course that demonstrated how active learning was incorporated into the course and assessed.

In-class Activities. During the fall semester, the initial FLC sessions focused on team building exercises and arriving at an understanding of how mathematics concepts and skills are used in the partner disciplines. After establishing a sense of community among participants, session activities primarily involved participants collaborating in teams to produce a learning activity that involved using mathematics concepts in a partner discipline context. The activities that were developed would be integrated into mathematics and partner discipline courses. Each FLC session included activities in which participants worked together in large and small groups to refine the developing mathematics scenarios. The fall sessions laid the foundation for the development of the final deliverables by facilitating a review of pedagogy and supporting literature such as the CF reports (Ganter & Barker, 2004; Ganter & Haver, 2011; Pratt's, 1988; Teaching Perspective Inventory (TPI), and the Taxonomy of Significant Learning, Fink, 2013). Other in-session activities included scheduling peer observations in mathematics and partner discipline courses, developing discipline-specific class activities that incorporate the mathematics scenarios, analyzing course outlines to identify where to best include the activities being developed, and reflecting on the learning taking place during both in-class and out-of-class activities. Groups were paired to critique each other's scenario and provide feedback on revisions and refinements. The FLC culminated with participants simulating the in-class activities in the capstone session. A detailed example of a scenario is provided below.

Journals. Throughout the FLC, participants completed reflective journaling assignments as a way for them to share their thoughts and feelings about class materials, identify how their participation in the FLC was influencing their practice, and identify which concepts they understood. The journaling was also used as a guide for facilitators to gather feedback and focus participant learning in future sessions.

Peer Observation Reflection. During the spring semester, each participant conducted two classroom teaching observations of their FLC peers. These peer observations provided participants with opportunities to learn from each other about learning and instruction in the partner disciplines. The observer provided feedback to the participant being observed. Each participant observed with the intent of learning about the partner discipline, the pedagogy of the host instructor, and the class content for the observation period. Each visitor provided a written reflection of their observation and thoughts to the host instructor.

Teaching Perspective Inventory. The participants completed a pre- and post-survey Teaching Perspective Inventory (TPI) designed to help understand their perspectives on adult learning. The five non-mutually exclusive categories of teaching perspectives are: Transmission, Apprenticeship, Developmental, Nurturing, and Social Reform (Pratt, 1998). After completing the survey and analyzing the results, participants discussed their beliefs, intentions, and actions based on their particular perspective. For example, a social work faculty who identified with a "developmental" teaching perspective focused on teaching that centered on those aspects of the assignments that allowed students to demonstrate their thinking, reasoning, and judgment. In this perspective, students are evaluated in large part on how they subjectively create individual and sometimes overlapping groups of knowledge or meaning, and the role of the instructor is to help guide students toward a goal of making deep meaning. On the other hand, nursing is a profession that places emphasis on having students master a body of knowledge that is taught in a "step-by-step" manner by a "content expert" and has a strong emphasis on student performance (i.e., meeting pre-established criteria or standards). In this perspective, the teacher has mastery over content and is expected to deliver that content in a way that transfers the mastery of an objective body of knowledge and set of skills to the student. It should be noted, however, that the two nurses who participated in the FLC do not fit this framework; instead, both identified with the Nurturing perspective in both their pre- and post-TPI. It is also of note that these two faculty members came to the teaching profession after working in the field as nurses for several years, where nurturing and empathy are as central to the job as being able to perform a technical skill such as detecting an irregular heartbeat. It might be that because of their applied experience in the field, their pedagogical focus on technical skill, while strong, is accompanied by an even stronger emphasis on learner efficacy and self-esteem because they view it as central to a student's ability to acquire the requisite skill set. As one nurse put it, "I did not change my dominant areas. I think this lends itself to the profession of nursing and how it is taught. Several nursing theories mention nurturing and caring, which is evident in my teaching style." For the other participants from nursing, her pre- and post- TPI scores remained exactly the same, again with the Nurturing perspective being the highest.

Traditional mathematics education is more similar to nursing than it is, for example, to social work. As Pratt (1998) notes, it, too, has a fixed set of rules and facts that apply to "…a fixed body of knowledge and core skills…" (p. 179); interestingly, only one mathematics faculty was identified in both his pre- and post-TPI as having the Transmission perspective. The other math faculty identified with the Apprenticeship perspective in his pre-TPI and the Transmission and Developmental perspectives in his post-TPI. He noted, "I believe my experiences with other faculty [in the partner disciplines] have changed my thoughts…this (post-TPI) was a dramatic shift from my previous report."

Though this current work does not focus specifically on shifts in faculty teaching perspectives, preliminary results suggest that significant shifts did occur for some faculty, and that even when shifts in teaching perspectives were not made, faculty reported that they embraced aspects of other teaching perspectives as a result of their collaborations with faculty from other disciplines.

In addition to revealing shifts in teaching perspectives, results of the TPI also served as a discussion point for pedagogical issues that arose during FLC sessions and helped to establish community among the participants. During one of the first meetings of the FLC, the participants discussed their TPI results and the connections between their teaching perspective and their discipline. This was an important step in forming our community and the FLC interdisciplinary working groups. It created an understanding and empathy for each participant's perspectives and the needs of students in their discipline.

Example of a Deliverable

An in-class activity produced by one FLC team (comprised of mathematics, nursing, and social work faculty) simulated running an emergency shelter for hurricane victims. This real-life scenario was designed to evolve over several weeks through different activities in a course in each of the respective disciplines. Students consider issues that plague an area that had been devastated by Hurricane Katrina in 2005. The focus of the scenario is an emergency shelter that can serve up to 100 victims. Poor, predominantly African American communities had more difficulty recovering as compared to more affluent white communities, which had better infrastructure and more resources to help with recovery. To provide a foundation for understanding the issues in the learning activity, students are encouraged to watch the film *Trouble the Water* (Lessin & Deal, 2008). Each of the partner disciplines on the team adapted the in-class activity to explore discipline-specific issues in their respective courses, although each version was slightly different in focus and presentation.

Social work. The in-class Hurricane Katrina activity is used in a beginning level course that explores the values and ethics of social work. The activity begins with exploring the primary mission of the profession: to enhance the well-being and meet the basic needs of all people, with particular attention to those who are vulnerable, oppressed, and living in poverty. After studying the National Association of Social Workers (NASW) values and ethical responsibilities for the profession, the students analyze the scenario from macro, mezzo, and micro levels to identify the embedded ethical principles of service, social justice, dignity and worth of a person, as well as the importance of human relationships, integrity, and competence. Next, the students compare racial demographics and poverty statistics between 2015 and 2017. The activity culminates with students exploring both the evident and probable ethical issues that occurred before and after Hurricane Katrina devastated New Orleans and surrounding areas.

Nursing. The Hurricane Katrina scenario is incorporated in a first-semester nursing course in which students explore the roles and responsibilities of the professional nurse. Through the scenario, nursing students are introduced to leadership concepts, collaboration, nursing theory, evidence-based practice, principles of patient-centered care, professional standards and values, and the use of the nursing process to guide their critical thinking. In small groups, students first consider the ethical, logistical, and legal issues that might be initially and subsequently encountered.

As the scenario evolves, the students simulate the role of a charge nurse who is working with untrained workers to receive displaced residents after Hurricane Katrina. The nurse leads a team to determine needed supplies and the quantities required to offer aid and comfort to 100 victims. Students use the mathematics concepts of linear functions, units, and proportional reasoning to determine the dosage and quantity of antimalarial medication tablets needed for shelter residents for ten days. To ensure medication safety for all concerned, the team writes a summary to be used as a guide for untrained aid workers and to educate the shelter residents. Another element of the scenario allows students to explore infection control principles when the class teams analyze an outbreak of gastrointestinal symptoms after shelter residents consume a chili dinner prepared by volunteers. The scenario concludes with a postmortem team debrief to discuss the lessons learned and what could have been done differently, including ethical and legal concerns.

Mathematics. The hurricane scenario was also adapted for a quantitative reasoning course for business, social work, and nursing students. The scenario begins with students considering the ethical and legal issues encountered by social work students and the medication calculation scenario for nursing students that is described above. The scenario is revisited over several weeks and introduces various functions. After completing the ethical and legal issues and medication calculation scenario, the mathematics students complete an activity based on managing the finances of the shelter and trying to recoup costs after the disaster. They raise $15,000 and want to invest it in a bank account. With the introduction of exponential functions, teams must calculate various types of interest. In a follow-up assignment, logarithms are used by the students as they develop a plan for the shelter's future. They must consider purchasing and financing an additional building and calculate a monthly mortgage and a payoff time frame. The final situation of the scenario introduces linear analysis. In this situation, students compare the cost of operating two different kitchens and find the minimum cost of producing 460 beef meals and 340 chicken meals. The students must define the variables, write an objective function, identify constraints, and solve the presented problem. Students conclude the scenario by writing a summary for colleagues that explains how to minimize kitchen costs.

Assessment of Outcomes

Assessing FLC outcomes and specific deliverables can be complex (Goto, Marshall, and Gaule, 2010) and perhaps especially difficult in multi-disciplinary FLCs. In an attempt to minimize the impact of the assessment challenges presented by the interdisciplinary nature of the FLC, our approach involved measuring the extent to which participants met pre-defined learning objectives. However, it should be noted that the facilitators understood, and even expected, that participants would use a constructivist path to accomplish the objectives. For example, the course assignments that included the scenarios developed by participants were not defined by the facilitators but were created by individual faculty. In doing so, participants were able to develop discipline-specific content for their courses. As noted by Goto, et al. (2010), the very nature of FLCs is such that they are rarely organized in a hierarchical fashion around a single authoritative leader; instead, authority is dispersed among the participants. This structure for the FLC was appealing for several reasons; chief among them was that each participant was considered a discipline-specific content expert. While facilitators required specific deliverables from the participants and provided a guiding framework for developing the deliverables, participants were free to work among themselves to develop both the structure and content of their products. Since no two deliverables might look the same, the creative freedom of this expectation presents a challenge for assessment. While it is true that deliverables could vary significantly even within one discipline, several activities were assigned each week to assess the extent to which participants met the pre-defined objectives.

Participant Feedback

Participants provided rich feedback during and at the conclusion of the year-long FLC. The concluding reflection prompted participants to consider what they learned about themselves and their teaching. A qualitative review of the journal entries and narrative reflections by the co-facilitators revealed that these assignments not only met the original goals but that participants valued the transformation they experienced. Our qualitative analysis suggested six themes present in the reflections: relevance, applicability, learning to learn, similarities and differences in course challenges, teaching styles, and language. These themes were affirming to the process and overall objectives and capture the meaning of the FLC experience for the participants.

Relevance and applicability were closely related themes that emerged from the reflections. Participants spoke of how the FLC content and expectations helped them change their thinking about what is important in a course. One participant also shared that he had learned a lot about choosing which topics were most relevant in his courses because it challenged the way "we often teach as though everything is equally relevant." For one participant, the focus of student learning evolved from the ability to solve a mathematics problem to "what students really need to know to be successful in their other courses." Another participant indicated a heightened awareness for "the life lessons we are trying to impart." The participants wanted students to learn how to transfer their knowledge from one situation to another. Thus, it is clear that the FLC participants learned that the infusion of mathematics concepts into a course was a vehicle for helping students succeed not just in other areas of academic study but also in life.

Nearly all participants addressed the commonality of the challenges they experienced in teaching courses. Attendance issues, classroom engagement, completion of assignments, and how to help students apply previous knowledge and learn new skills are examples of topics that often filtered into the work of developing cross-disciplinary assignments. One participant noted, "We all seemed to talk about how students learn, and there were a lot of similarities regardless of discipline." Another participant said

> I especially enjoyed, overall, that we were able to share our challenges in the classroom, which turned out not to be really, very discipline-specific. Engaging students in their learning process was a topic we discussed quite a bit, and I found this helpful and affirming.

Several exercises and assignments required the participant to align their TPI results with their teaching style(s). It was fascinating to see how each participant favored a teaching style that aligned with the subject matter they taught and how they became aware of other styles they unknowingly used. After reviewing the feedback from an in-class observation, a participant stated,

> I knew early on that a lot of my teaching focused on transmission, but I didn't realize until I was observed that I also do a lot of modeling when it comes to ….critical thinking and problem-solving when I share 'tricks of the trade.' I don't think I put this together before and I think it can be useful in helping to take the fear out of math.

Another theme that emerged was how each of the participants used discipline-specific language to communicate mathematics concepts. One of the mathematics faculty addressed this variation and the importance of grasping the impact language has on the understanding of concepts, saying "I learned the importance of language and feel a little embarrassed that I didn't realize my partner colleagues have difficulty understanding what is meant by a concept, then my students must really struggle at times." Another participant was "amazed at how discipline-specific our language is. In order to accomplish cross-over, it is important to be aware of this."

Finally, many of the participants spoke of learning several things, some of which were about themselves. Through the collaborative process, participants said they learned "a lot about how students learn by participating," and did so by "watching other colleagues teach." One commented that they learned "about integration from Fink's taxonomy; that is where I think this FLC was most helpful." The emergence of this theme of mutual learning clearly highlights the importance of breaking down discipline, department, and college barriers to learning about the needs of students from other disciplines. This theme supports the underlying tenet of the SUMMIT-P project and aligns with the nationwide call to rethink the nature of mathematics courses and consider how students could benefit from a curriculum that is both rigorous and relevant (Ganter & Haver, 2011).

Lessons Learned

The facilitators learned a great deal from the FLC experience, some of which was related to FLCs themselves. For example, having a well-organized and funded center for teaching and learning contributes significantly to the success of an FLC, both in terms of training for the facilitators and providing professional development funds for participants, meals, space, and other infrastructure needs for the FLC to conduct its work. The FCTL is one of the institution's strongest assets. As such, it was natural for the facilitators to turn to the FCTL as a means to extend the SUMMIT-P collaboration beyond themselves. Readers who are considering interdisciplinary projects may want to identify the assets on their own campus that will enhance their work and strengthen interdisciplinary collaborations.

In addition to having funding support from and access to space at the FCTL, the co-facilitators were also provided two-day training by FCTL staff that focused explicitly on facilitating an FLC. While receiving guidance from the staff, participating in the training also allowed the co-facilitators to clarify goals and objectives and outline the activities to meet them.

The sheer joy experienced by the co-facilitators that resulted from inter-disciplinary collaboration cannot be overstated. Academics often teach and research in silos. Moving beyond those bounds and interacting with so many different people from different disciplines was both instructive and joyful. While developing a shared understanding of course content across disciplines was no easy feat, it proved to be one of the most beneficial outcomes of the FLC. This shared understanding allowed the FLC participants and co-facilitators to bond in ways that were unanticipated at the start of the process, and the richness of the course content that emerged from the scenarios was an indication that there had been a significant increase in understanding across disciplines. This understanding not only facilitated academic development, but also personal development among both the participants and the co-facilitators.

Despite all of the benefits of participation, there were some unanticipated "hiccups" that arose throughout the course of the year. One of the most significant of these was the discipline-specific languages that were spoken by the participants (and at times, the co-facilitators). For example, the term "variable" is defined and used differently in accounting than it is in mathematics or social work. While a mathematician refers to the slope or rate of change of a function, it may be referred to differently in another discipline. For example, within business disciplines, reference to marginal costs describes slope. These language challenges were significant enough near the end of the first semester that the co-facilitators adjusted the session schedules to accommodate a more robust discussion of the topic. After addressing language barriers, the facilitators and the participants were surprised that, in addition to finding connections between their disciplines and mathematics, they also found connections among the partner disciplines.

Some unforeseen challenges were more impactful than those presented by language differences. There was one participant who was "arm-twisted" by the department chair into participating, and this was evidenced by a lack of enthusiasm for the project and a lack of engagement (e.g., a number of sessions were missed). That participant did not end up completing the FLC.

Despite the challenges identified above, the facilitators learned a great deal about the faculty within their university community. It was clear that all of the participants were dedicated to student learning, even participants who did not finish the FLC. Conversations within the FLC were free of "disciplinary microaggressions," as they were focused on enriching the student learning experience. The core that connected all of the disciplines—business, mathematics, nursing, and social work— was student learning.

Conclusion

FLC's benefit facilitators and participants when executed effectively. While SoTL is often a product of FLC's, the *Math and Partner Disciplines FLC* used a team approach to reform mathematics education at FSU to establish a foundation for ongoing partnerships across the university. The growth and development in the SoTL, cross-disciplinary connections, and sense of community that resulted from participation in an FLC cannot be understated. As supported by the goals of FLC's outlined by Cox (2004) and in the cases presented in this article, a greater understanding of a discipline-specific subject resulted in in-depth learning across both mathematics and partner disciplines.

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The Roles and Benefits of Using Undergraduate Student Leaders to Support the Work of SUMMIT-P

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Abstract

The article by Poole, Turner, and Maher-Boulis (2020) describes one way in which undergraduates have been used to support the SUMMIT-P goal of investigating examples of how mathematics and statistics are applied in partner discipline courses. Two other universities in the SUMMIT-P consortium, San Diego State University and Oregon State University, also use undergraduates in different ways to support the work of integrating science applications into math classes. In this article, we compare and contrast these three uses to further highlight this somewhat untapped resource.

Keywords

integrated activities, peer leaders, undergraduate learning assistants

The Synergistic Undergraduate Mathematics via Multi-Institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) consortium is a group of mathematics faculty from ten institutions who work with faculty from partner disciplines to implement curricular updates in lower division mathematics courses. The goal of this article is to argue that undergraduate students can be excellent resources for helping with this task because they bring initiative, knowledge about content being taught in partner disciplines, and a much-needed student perspective into the curriculum development process. In what follows, we support this argument by presenting three cases that describe the different ways in which undergraduates were involved in the creation of integrated activities at three different institutions involved in SUMMIT-P. In all cases, the results created a win-win-win situation for faculty, students in the target math classes, and the undergraduate students serving as leaders.

The first case describes how undergraduate STEM majors at San Diego State University (SDSU) serve as teaching assistants for a precalculus course. Their main responsibility is to teach the integrated lessons during course breakout sessions, but they also have a great deal of input into how the lessons are created and what examples are included. The second case describes a Student Exchange Program (SEP) at Lee University that is designed to pair mathematics majors with non-mathematics majors who are enrolled in an introductory statistics class. The third case describes how undergraduates at Oregon State University (OSU) worked to find biological studies examples to illustrate the central ideas in Calculus I. We conclude by noting that, in all three cases, the undergraduate teaching assistants were involved in the design of integrated activities and brought new perspectives that enhanced the project in ways that faculty alone could not have achieved.

Examples

Example 1: Undergraduate Teaching Assistants at SDSU

At SDSU, Precalculus is taught in a large lecture format that is augmented by weekly recitation sections. The lecture classes, which meet for three 50-minute sessions per week, are held in an auditorium that accommodates roughly 125 students. The material is generally delivered in a straight-forward manner, although some active learning is encouraged through the use of “clicker” questions. On the other hand, the recitation sections, or “labs,” are capped at 25 students and are designed to maximize opportunities for active, collaborative learning. It is important to note that we chose to refer to the recitation sections as “labs” in order to emphasize that the overall goal is not simply to provide rote practice but instead to engage students in active learning lessons that integrate science applications with the particular mathematics topics being studied in the course.

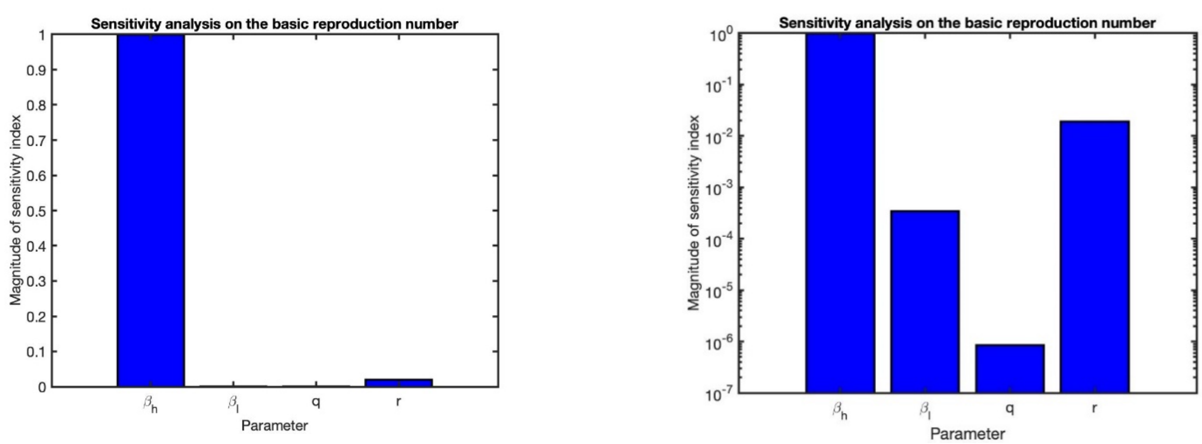
The most unique aspect to the approach taken at SDSU is that these labs are taught by undergraduate students rather than Graduate Teaching Assistants. These undergraduates, often referred to as Undergraduate Learning Assistants (ULAs), are carefully selected to be very strong mathematicians who also have enough self-confidence to talk in front of peers. The group (roughly 7 – 9 ULAs depending on the semester) meet weekly with the coordinator to debrief the prior lesson and discuss the upcoming lesson. During these meetings, the group is introduced to the draft PowerPoint presentation that the director has created for the lesson. The team then debates design, content, and pedagogical ideas to enhance how they will implement the lesson during the following week. Once the lesson has been finalized, the director makes final touches to the shared PowerPoint slides and any handouts. The director also visits the lab sessions on a regular basis to provide content and pedagogical guidance as needed.

Each ULA generally teaches two sections of 25 students during 50-minute lab periods. They are also encouraged to observe other lab sections so that they can gauge time allocation decisions. Although the ULAs are not instructors of record, they are responsible for taking attendance and leading the class in the activity that is described on the PowerPoint slides used to organize the lesson. This span of work enables the ULAs to contribute to just about every aspect of the SUMMIT-P work. Not only do they teach the lab sections, they play a strategic role in transforming the application idea from the client discipline into an active learning lesson with pedagogy that gets students involved in social interaction as well as learning the mathematics and science.

One example that highlights the role that ULAs have played in developing integrated activities for the SUMMIT-P project occurred when the team was discussing the pH lab, which was developed by Bowers (first author of this paper) and a biologist and chemist who are also SUMMIT-P partners. Bowers told the group of ULAs that both the chemist and the biologist emphasized the importance of helping students understand how logarithms are used to transform data containing a lot of very small or very large numbers to one scale so that values can be compared and analyzed. Two of the ULAs (also co-authors on this paper) stated that they used this process while working as interns in a virology lab on campus. The group hypothesized that if the ULAs could create a video explaining their work in their own words, the students in Precalculus could get a better understanding of how logarithms are used *by their peers* in university research. As they describe in the video, the application they explored required comparing the relative impact of various parameters on the progression of AIDs. The work required the use of logarithmic transformations because the data set consisted of extremely small numbers. Thus, nuanced differences between the quantities were difficult to see. The “before” and “after” graphs showing how logarithmic transformation was used to compare the relative contribution of various parameters labeled $\beta_h, \beta_1, q,$ and r in a biological model of AIDS propagation in a virology lab featured in the video are shown in Figure 1.

Figure 1

Two Graphs Created to Demonstrate the Use of Logarithmic Transformation



A second example of how the ULAs have been helpful for implementing the integrated lessons involves their ability to predict how the lessons will be received by the students in Precalculus and also to reflect on what went wrong and could be improved for future semesters. For example, another integrated lab developed through the SUMMIT-P project involves simulating the exponential spread of a virus. This lab has been tweaked many times based on the feedback ULAs provided in discussions after their classes. These tweaks included ideas to accommodate more interaction between students and ways to expedite the process for collecting and modeling the data.

These contributions illustrate how leveraging the ULAs' research experiences, relationships as fellow undergraduates to the students in the class, and perspectives on learning enabled us to create more student-friendly (and hence effective) lessons.

Example 2: Undergraduate SEP Participants at Lee University

Figure 2

A Page of the Resource Website with Information About Terminology Differences

Math-Stats Differences

Statistical Terms	Psychological Definition	Mathematical Definition
Independent variable	A measure that is manipulated or recorded in a study to see how it impacts or influences something else	A value that may be chosen regardless of the value of any other variable.
Dependent variable	A measure that is impacted or influenced by the independent variable, usually interval or ratio data	A variable that depends on the value of one or more other variables.
p-value	This value shows how likely the null hypothesis is in being true. If the *p*-value is less than the alpha value, then you can reject the null	The probability of seeing a result at least as extreme as the actual result for when the null hypothesis is true.
Nominal data	This is categorical data without true numerical value. This is usually seen in groups like male or female; or Republican, Democrat, or Independent	Categorical Data without true numerical value just allocating groups to distinct categories regardless of any measuring value.
Ordinal data	This data has an order to it, but the spacing between each number might not be even (such as first, second, or third in a race)	Has an order to it but a specific type of ordering in that things are ranked.
Interval data	The spacing between each number is even, but has no absolute zero (a good example is Fahrenheit; there is no "real" zero)	Data that is measured in groups where the individuals are equally spaced out inside the group of data, and there is no absolute zero.
Ratio data	The spacing is even between each number and an absolute zero exists (such as weight, age, or height)	There is an absolute data and everything can be measured in relation to other data points because of this.

As Poole, Turner, and Maher-Boulis (2020) describe, the Student Exchange Program (SEP) at Lee University has been designed to provide collaborative learning opportunities about the uses of statistics for pairs of students, one majoring in mathematics and another majoring in a social science discipline (e.g., psychology, political science). Through this collaboration, students extend their knowledge of statistics and the application of statistics in the social science discipline by assuming responsibilities similar to those of the ULAs at SDSU. At Lee this includes (a) attending statistics course lectures and, upon invitation of the instructor, facilitating demonstrations for students enrolled in the class; (b) tutoring students and holding weekly recitation sections; (c) working through problem sets to facilitate discussions about the overlap of statistics and social sciences; (d) conducting interviews with the partnering student on topics related to statistics; and (e) designing instructional supports for student use.

Like the undergraduate participants in SDSU's program, the SEP participants at Lee University were asked to create resources to enhance student learning. For example, SEP participants were tasked with designing and building an instructional website, including finding and posting content to the site (see Figure 2). Importantly, they collaborated in pairs to determine which information would be useful for students accessing the website, including those taking the statistics course and those who sought a review of topics covered in the course. The program directors routinely reviewed the materials before the SEP participants posted them to the website. We provide additional details on these activities in another article in this volume (see Poole, Turner, & Maher-Boulis, 2020).

Example 3: Undergraduate Leaders at OSU

As described by Beisiegel, Kayes, Quick, Nafshun, Lopez, Dobrioglo and Dawkins (2020), the SUMMIT-P team at OSU are working to develop content-rich problems for Calculus I. Undergraduates on this team play a critical role because they are often enrolled in partner discipline courses and, therefore, are more familiar with the content being covered in the partner discipline courses and the relevant and important ways that mathematical topics are used in the different courses. The undergraduates that worked with the OSU faculty were particularly industrious and self-motivated. They worked to find over 20 examples of how local biological scenarios such as local predator-prey concerns and environmental changes impact the region.

One unique innovation that this team implemented is re-introducing a context several times over the course of the semester. For example, in one scenario describing glucose absorption, the exponential function is first introduced early in the term when covering the idea of limits. Later, it is discussed again to explore an application of the first derivative. Finally, the context is brought up a third time in relation to the second derivative. It is critical to note that the applications are authentic. For example, when examining the first derivative, this example required students to explore how the rate of change can be computed as a function of a particular parameter. This small nuance reflects the critical significance of working with client disciplines: mathematicians might not have known that such a shift in focus was needed. These nuances, combined with the intentional repetition, not only solidify the importance of the topic from a biological perspective, they also provide a new way to look at the particular mathematical concepts that transcend the calculation approach used to solve simple practice problems.

Benefits of Involving Undergraduate Students in SUMMIT-P

The goal for describing these three cases has been to call attention to the value of involving undergraduates in projects like SUMMIT-P. The three examples we have presented describe different ways in which students have contributed invaluable insights to the development and teaching of integrated lessons. It is interesting to note that all three participating SUMMIT-P universities, OSU, SDSU, and Lee University, describe the contributions of undergraduate students in similar ways. At each institution, lesson development began by working with partner discipline faculty to find an application that could be used in the mathematics course under study. Then, the SUMMIT-P faculty worked with undergraduates to hone the lessons to work with the instructional constraints. For example, at OSU, the undergraduate students found applications in biology that illustrated the need for exponential and logistic function models in the Calculus I classes. Similarly, undergraduates at SDSU were asked to hone and teach a lesson about pH—a science topic most were not intimately familiar with—in order to create a deeper understanding of how logarithms can be used to compare very small numbers during the lab sessions. Undergraduates at Lee University were asked to find applications of statistics concepts in the field of psychology to be shared among majors and non-majors via a website they created. In each case, the undergraduate teams created a win-win-win situation for faculty, students, and student leaders themselves.

Win for the Mathematics Faculty

These examples have illustrated ways in which the faculty gained critical help from the undergraduate students. At SDSU, the ULAs are actually teaching the labs, and hence the peer leaders are providing instruction which helps faculty communicate important mathematics ideas and concepts. Similarly, the undergraduates in the SEP program at Lee University are extending the teaching efforts of the faculty by mentoring their peers in ways that students may respond to on a more personal level. At OSU, the faculty were rewarded with excellent applications that enlivened their teaching.

Win for the Students Enrolled in the Course

Even though the Common Core Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) have been in place for almost a decade, we have found that many students still believe that mathematics should be taught in isolation from other subjects. To combat this, all three of these SUMMIT-P programs encourage the students to think about how the mathematics they are learning applies to disciplines outside of pure mathematics and, most notably, to their own future careers. Having undergraduates design examples, teach recitations, and serve as mentors allows the students in courses to interact with peer role models early in their academic careers. By meeting with undergraduate teachers, talking with them, and often struggling with ideas they present, students are able to envision themselves in similar peer leadership roles in the near future. Moreover, they are able to get a sense of what college life can be like when students are excited about what they are learning through their undergraduate majors and are passionate about a field of study.

Win for the Undergraduate Leaders

All three of these cases illustrate what Kuh (2008) calls "High Impact Practices." The teaching assistant experiences described here are most closely aligned with what Kuh labeled as "Collaborative Assignments and Projects," which involve "learning to work and solve problems in the company of others, and sharpening one's own understanding by listening seriously to the insights of others, especially those with different backgrounds and life experiences" (Kuh, 2008). As Kuh notes, such experiences have been correlated with higher graduation rates, more reported satisfaction with college experience, and more successful job placements.

Our findings regarding the benefits to the undergraduate leaders also align with the literature focusing on undergraduate peer leadership roles, which reveals that participants report a number of benefits, including (a) a great deal of applied content knowledge (Fingerson and Culley , 2001), (b) "soft skills" that can translate to the workplace (Lockie & Van Lanen, 2008), and (c) a sense of belonging by participating in this kind of work (Marx, Wolf, and Howard, 2016). These results, in turn, can support the students' efforts to stay connected to the STEM fields (Stout & McDaniel, 2016; Blackwell, Katzen, Patel, Sun, & Emenike, 2017). Based on feedback from our undergraduate peer leaders, it is clear that they have benefitted from these experiences in terms of content and personal as well as professional growth.

Conclusion

This article has presented three cases describing how undergraduate leaders were used to support faculty as they developed integrated activities for lower division mathematics courses. Both SDSU and OSU are large, state universities where change can be difficult to initiate and sustain because of the large-scale tasks involved in developing materials for so many sections of a class. Bringing undergraduate peer leaders onto the team has been a critical component of the integration effort. Undergraduate peer leaders contributed to the development and teaching of the activities without much cost or disruption to the university scheduling system. In contrast, Lee University is a very small institution where change is somewhat easier to implement because faculty and students from all disciplines are interconnected. During a SUMMIT-P site visit, the research team was able to talk with some of the SEP participants. These students described their experiences passionately and remarked that the program has produced deep, interdisciplinary learning and transferrable soft skills for the students learning social sciences and their math major partners as well. Although the roles of the undergraduates differed in the three institutions, the results indicate that these high-impact practices were mutually beneficial to the faculty, students enrolled in the classes, and undergraduate learning assistants.

Acknowledgment

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THE PROCESS AND A PITFALL IN DEVELOPING BIOLOGY AND CHEMISTRY PROBLEMS FOR MATHEMATICS COURSES

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ABSTRACT
In this paper, we describe our process for developing applied problems from biology and chemistry for use in a differential calculus course. We describe our conversations and curricular analyses that led us to change from our initial focus on college algebra to calculus. We provide results that allowed us to see the overlaps between biology and mathematics and chemistry and mathematics and led to a specific focus on problems related to rates of change. Finally, we investigate the problems that were developed by the partner disciplines for use on recitation activities in calculus and how those problems were modified by the calculus coordinator. We compare what partner disciplines emphasize in scientific applications with what mathematics instructors emphasize in calculus and consider what that means for students' understanding of science in mathematics. We also describe the role of the students, partner discipline colleagues, and calculus instructors in the development, refinement, and use of the problems.

KEYWORDS
biology, calculus, chemistry, partner disciplines

The National Consortium for Synergistic Undergraduate Mathematics via Multi-Institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) project brings together institutions and disciplines in order to improve undergraduate experiences in mathematics courses. The project is based on work by the *Committee on Curriculum Renewal Across the First Two Years* of the Mathematical Association of America and uses recommendations from the *Curriculum Foundations Project* (CF) (Ganter & Barker, 2004) to inform collaborations among disciplinary partners. Based on this prior work, the SUMMIT-P project is designed to strengthen the connections between mathematics and partner disciplines in institutionally—and programmatically—relevant ways. For example, at Ferris State University, mathematics faculty have partnered with nursing and social work faculty to meet specific programmatic goals through a quantitative reasoning course. As another example, mathematics faculty at LaGuardia Community College have partnered with business and economics faculty to revise a college algebra course to meet broader institutional goals. (Information about these two projects and many others that have taken place through SUMMIT-P are outlined in other papers in this issue.)

Oregon State University (OSU) is one of the eleven participating SUMMIT-P institutions. OSU is a land grant, research intensive university with approximately 27,000 undergraduates, and more than 3,200 of those students intend to earn a degree in one of the sciences. The Department of Mathematics at OSU offers lower-division courses including College Algebra, Trigonometry, and a calculus sequence that consists of Differential Calculus, Integral Calculus, Infinite Sequences and Series, Vector Calculus, and Differential Equations. In a given academic year, about 800 students enroll in College Algebra and approximately 2,800 enroll in Differential Calculus. Many students enrolled in College Algebra and Differential Calculus also co-enroll in a biology course or chemistry course. Because of the focus on science, the OSU SUMMIT-P team chose biology and chemistry as disciplinary partners. The goal of OSU's SUMMIT-P team is to improve connections between mathematics and the disciplines of biology and chemistry (Ganter & Haver, 2020). In this paper, we describe the four-step process we followed in doing this work (i.e., creating a content map, developing in-depth problems for use in Differential Calculus, implementing the problems, and revising the problems).

Our SUMMIT-P team consists of a mathematics educator in the Department of Mathematics (Beisiegel), two instructors from the Department of Integrative Biology (Kayes and Quick), and an instructor from the Department of Chemistry (Nafshun). Over the course of the project, our team also included a graduate student in mathematics (Michael Lopez; during summer 2017) and two undergraduates (Steve Dobrioglo and Michael Dickens; beginning in summer 2018). Because of the importance of college algebra (Ganter & Haver, 2011), our initial focus was on the College Algebra course, which was in the midst of a significant curricular and pedagogical redesign at OSU and seemed to be a good fit for the SUMMIT-P project. However, our focus eventually turned to the Differential Calculus course and developing rich, in-depth biology and chemistry contexts that required differential calculus concepts to answer questions about those contexts.

In this paper, we first share some background on the importance of providing students with connections between mathematics and partner disciplines, along with some information about the Committee on the Undergraduate Program in Mathematics (CUPM). Then we report the steps we took in the process of conducting our work. We first describe the step of creating a content map that resulted from our conversations about which mathematics concepts and skills are used in biology and chemistry and how the mapping helped us to better understand the connections between mathematics, biology, and chemistry. We then provide details about how we developed the in-depth differential calculus problems with biology and chemistry contexts along with some examples. Once the problems were developed, our next step was to incorporate the problems into the differential calculus course. We describe how that process occurred. Based on the implementation of the problems in the third step, we explain our fourth step which was revising the problems. Finally, we reflect on our experiences, including what were positive features of the experience and what we learned by going through the process.

Background

Introductory mathematics courses, such as College Algebra or Pre-Calculus, and subsequent courses, like Calculus, have historically been problematic for students (Fairweather, 2008; Seymour & Hewitt, 1997). In particular, college mathematics courses are "frequently uninspiring, relying on memorization and rote learning" (President's Council of Advisors on Science and Technology, 2012, p. 28). To add to this issue, faculty in departments of mathematics do not often collaborate with faculty outside of mathematics on curriculum development. As a result, students are unable to "see the connections between mathematics and their chosen disciplines; instead they leave mathematics courses with a set of skills that they are unable to apply in non-routine settings" (Ferguson, 2012, p. 187).

For several decades, the Mathematical Association of America (MAA) has aimed to address these issues through CUPM. As part of its charge, CUPM provides recommendations to help mathematics departments design meaningful materials for undergraduates taking mathematics courses. The *2004 CUPM Curriculum Guide* (Barker, Bressoud, Epp, Ganter, Haver, & Pollatsek, 2004) offered six recommendations, three of which were the focus of the OSU SUMMIT-P project. Our goal was to "continually strengthen courses and programs to better align with student needs, and assess the effectiveness of such efforts" (p. 1), to "promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects" (p. 2), and to "encourage and support faculty collaboration with colleagues from other departments to modify and develop mathematics courses" (p. 2).

The Process of Our Project

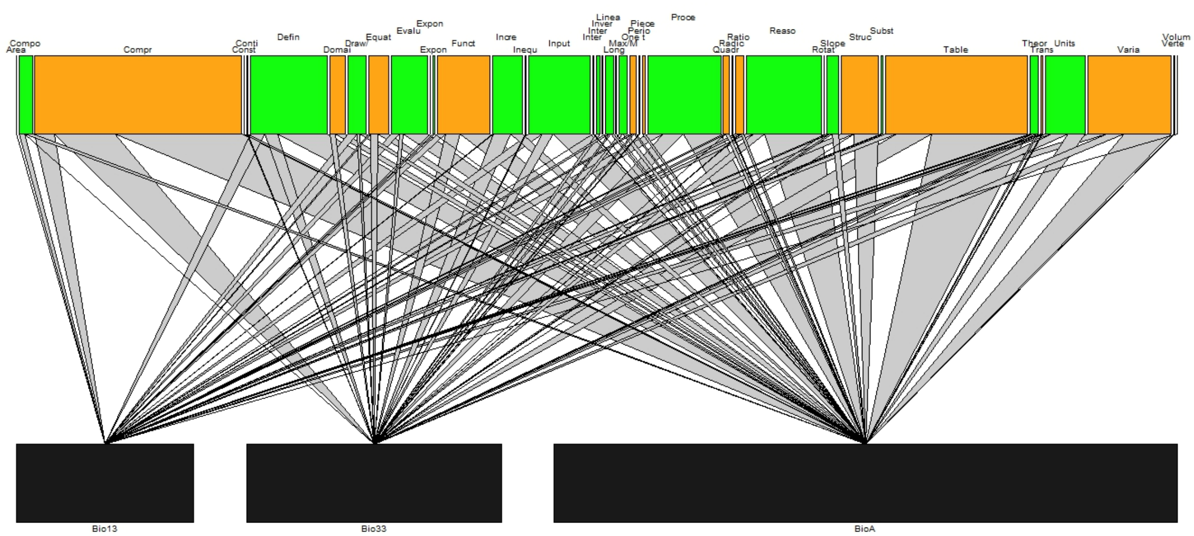
Step 1: Content Mapping

From the outset of the project, the OSU team focused on the partner disciplines of biology and chemistry. In the first year of the project (2016 – 2017) Beisiegel, Kayes, Nafshun, and Quick met bi-monthly and looked specifically at whether or not different mathematics topics appeared in biology and chemistry courses. If concepts were included in a course, we analyzed how those concepts were described and used in the partner discipline curriculum. We learned about what was important to the partner disciplines on a national level by reading publications of the CF project (Ganter & Barker, 2004) and locally via fishbowl discussions with OSU partner discipline faculty.

In spring 2017, Kayes from biology and Nafshun from chemistry met with faculty members from their departments using the fishbowl protocol (Hofrenning et al., 2020) and developed lists describing the mathematical "needs" for biology and chemistry courses. We noticed that the list outlined in CF reports (Ganter & Barker, 2004) and the one developed through the fishbowl activity aligned with the content covered in mathematics courses. Many mathematical concepts that the partner disciplines noted as important were concepts that were emphasized in mathematics courses. For example, the chemistry fishbowl included a discussion about proportional reasoning being a key skill that chemists hoped students would learn in mathematics courses. However, after several discussions about specific places in the mathematics curriculum to include problems with science contexts, we realized that language was a barrier to doing this. We noticed the different ways that mathematics describes and uses concepts compared to other disciplines like biology and chemistry.

After the biology and chemistry fishbowl conversations, one of our main questions became: What specific mathematics concepts are important to the study of biology and chemistry? After some discussion, an equally important question arose: Do these two disciplines talk about and use mathematical concepts in similar ways? To answer these questions, a mathematics graduate student (Lopez) conducted a rich, in-depth analysis of the mathematical concepts, skills, and vocabulary that are included in the biology and chemistry curriculum materials. As a first step, the OSU team listed the most common mathematical terms and concepts that are used in courses such as College Algebra, Trigonometry, and Differential Calculus. Examples of these terms include function, variable, input, composition, graph, and intercept, among many others. Mr. Lopez then examined OSU biology and chemistry materials as well as Advanced Placement materials, which are used as a guide for biology courses, to look for instances of these terms.

Figure 1
Map Between Biology Content and Mathematical Terms



Figures 1, 2, and 3 illustrate a mapping between mathematical terms and concepts found in biology and chemistry curriculum materials. The width of a rectangle at the top of the figure is a representation of how often a term appeared in the science content. For example, in Figure 3 (which is a zoomed in corner of Figure 1) the terms 'constant' and 'continuous' are represented by very narrow rectangles indicating that these terms appear very infrequently in the biology content. The width of the rectangles at the bottom of each figure illustrates the extent to which the set of curricular materials contained mathematical concepts and language. In Figure 1, Bio A represents the activities, and Bio 13 represents the lecture notes that are used in Principles of Biology at OSU; Bio 33 represents the materials used in Advanced Anatomy and Physiology at OSU. Note that the width of the Bio A rectangle compared to the width of the Bio 13 rectangle in Figure 1 reflects that mathematics terms appear much more frequently in activities than in the lectures in Principles of Biology.

Figure 2
Map Between Chemistry Content and Mathematical Terms

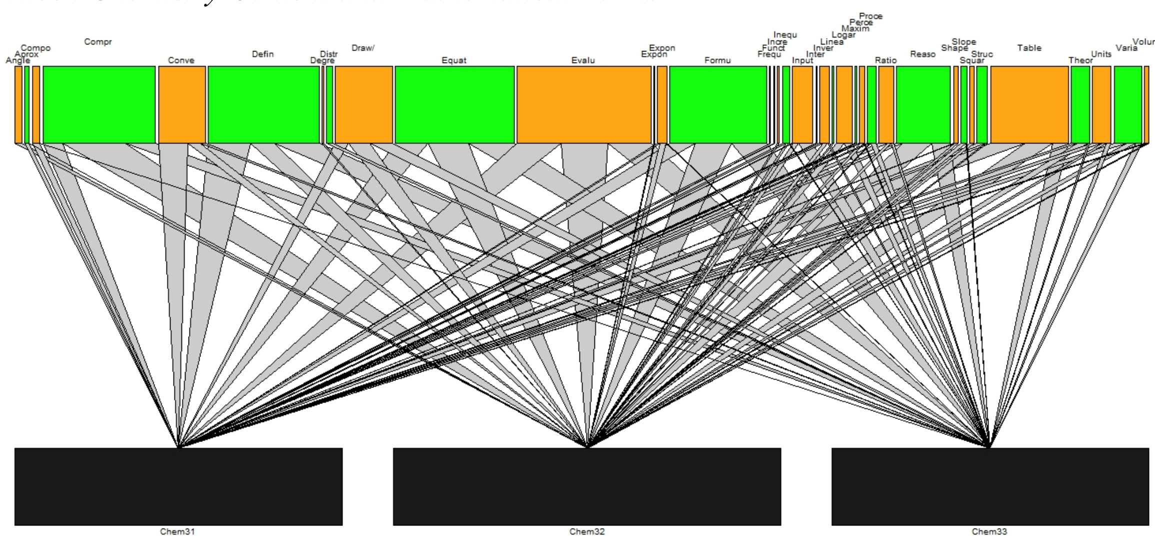
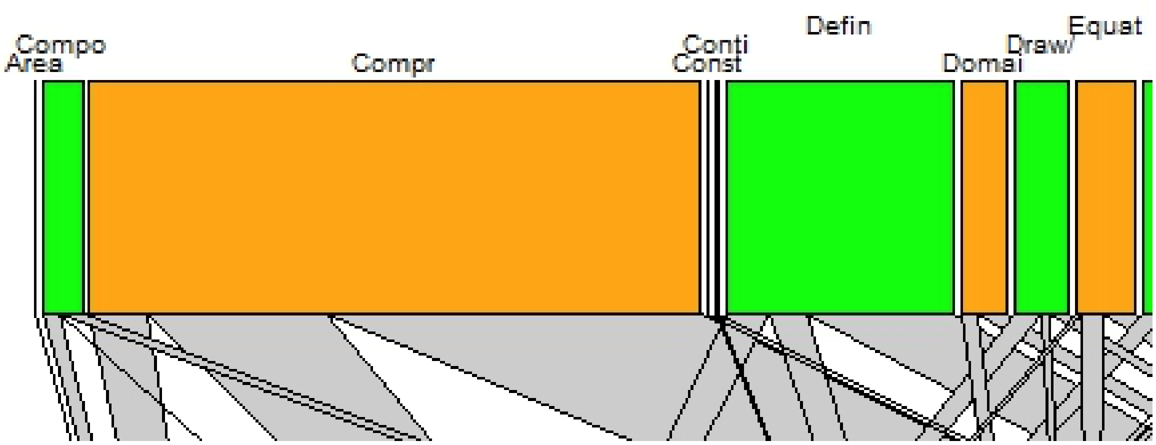


Figure 3
Zoomed in View of the Map Between Biology Content and Mathematical Terms



These figures provided insight into where we could explore the correlations between College Algebra and partner discipline content instead of wading through the content without any specific direction. It also allowed us to explore ideas that appear to be very important in mathematics but on the surface appear to be not as important to the disciplinary partners. For example, "function" is a salient concept in mathematics courses, and yet it did not appear as important in biology or chemistry. Notice the somewhat narrow rectangle representing the function concept in Figure 1 and the extremely narrow rectangle representing the function concept in Figure 2. In our discussions about this discrepancy, we found that biologists and chemists might use the terms ‘formula’ or ‘equation’ instead of ‘function.’ More importantly, these discussions allowed us to understand some essential differences in how mathematical concepts are addressed in the partner disciplines. This was critical to our understanding of how biology and chemistry problems could be incorporated into a mathematics course in a meaningful way.

Through the analysis of terms and concepts and the ensuing conversations, we found the concept in the partner disciplines that would be most appropriate to highlight in mathematics courses was *change*; in particular, we saw that biology and chemistry problems that address change (e.g., rates of change) provided a strong connection between mathematics and the sciences. While linear slope is a rate of change that is studied in College Algebra, it is only highlighted briefly in the course. Given that we wanted to create in-depth, robust connections between mathematics and biology and chemistry, we decided to move to project’s focus to the Differential Calculus course, in which change is a predominant theme.

Step 2: Developing In-depth Biology and Chemistry Contexts for Use in Differential Calculus

In the second year of our project (2017–2018), our goal was to develop specific problems and activities for Differential Calculus that were strongly rooted in the biology and chemistry content. During the academic year, we met monthly, and as we began our work in earnest, we realized that involving undergraduate students who had taken courses in all three disciplines would be advantageous for the project. Students with experiences in these courses would have unique insights that the faculty might not have. At the beginning of summer 2018, we met with the students (Dobrioglo and Dickens) to share the purpose, goals, and intended outcomes of the SUMMIT-P project.

We described the vision of developing calculus problems that could be addressed from both the mathematical and partner discipline (biology or chemistry) perspectives. We posed the following questions to the students: How do mathematics and biology or chemistry faculty represent problems in class? What questions do they ask during class? How are the representations in mathematics different or similar from the representations in biology or chemistry? What different terminology do they use? For example, in one of the initial meetings with the Dobrioglo and Dickens, we talked about the purpose of the logistic function in calculus compared to how the function is presented and used in biology courses. More broadly, we talked about wanting to develop a specific set of applied problems that could be explored at different points in Differential Calculus. In this way, students would understand how different calculus concepts can be used to understand different parts of contextually rich problems. We felt that this would provide a much deeper experience for students than the more typical “one-off” problems.

We met with the students weekly and gave them “homework assignments.” Their first assignment was to explore the logistic function and to find contexts in biology, chemistry, general science, and other human-interest situations of interest to undergraduates that could be used to introduce the function in Differential Calculus. In addition to emphasizing contexts from partner disciplines, the problems that were developed also highlighted the importance of understanding the behavior of this function by determining its derivatives. We created a Google document that the entire team could access as we developed the set of problems. In subsequent meetings, we discussed the problems and contexts the students found, the terminology used in different applications, the significance of those terms in the partner disciplines, and the appropriateness of those applications for use in Differential Calculus.

By the end of summer 2018, our team had developed over 25 problems with contexts in biology or chemistry for use in Differential Calculus. We provide two examples here:

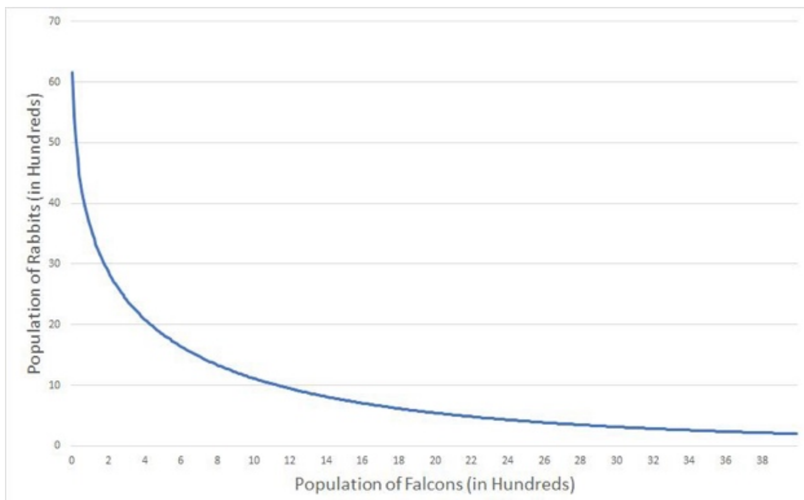
Example 1: Falcon-Rabbit, Predator-Prey in Biology

There is a large population of Mountain Cottontail rabbits in the woods of Oregon. A family of falcons (of the peregrine variety) moved into the area and preyed on the population of rabbits, devastating the rabbit population. The function below (see Figure 4) represents the number of rabbits, with *f* representing the number of falcons. The graph illustrates the change in the rabbit population. Use the function and data to determine how devastated the rabbit population was by the introduction of the falcons into their habitat.

Figure 4

Population of Falcons and Rabbits

$$r(f) = -\frac{200\, ln\left(\frac{f}{40}\right)}{f+20}$$



1. Is the function continuous for all real numbers?
2. Find and interpret the value of $\frac{dr}{df}$ for $f = 10$.
3. Find and interpret the value of $\frac{d^2r}{df^2}$ for $f = 10$.
4. For what population of falcons will $\frac{dr}{df}$ be greatest?

This problem met our established goals for incorporating it into the course: (1) the problem could be used at different points across the ten-week term, and (2) it required concepts and skills covered in three chapters in the course textbook. The question about continuity could be addressed when instructors are developing the concepts of limits and continuity, the questions about the first and second derivatives could be addressed when the derivative is defined and the rules for differentiation are introduced, and the last question could be addressed when students are exploring optimization.

The biologists and the chemist on our team noted that there are many situations in their disciplines where one variable is changing in relation to another and neither variable represents time. In the example above, the population of prey depends on the population of predators and vice versa; the biology models that capture population growth and decline use one population as the dependent variable and the other population as the independent variable. In comparison, for most of the rate of change problems in a calculus course, the independent variable represents time. Thus, the team felt that this predator-prey problem represented a novel approach to exploring derivatives. The team also felt that gaining a broader understanding of these types of problems was important for students to make meaningful connections between calculus, biology, and chemistry.

Example 2: Effusion

Effusion in chemistry is the process in which two or more particles are escaping through a small hole with the lighter particles leaving the container faster than the heavier particles. Heavy particles move slower than light particles with the same kinetic energy. This gives the lighter particles a higher probability of escaping every second. This relationship can be determined by the equation where m represents the mass and $k > 0$ is a constant:

$$\frac{k}{\sqrt{m}} = r_e, \text{the rate of effusion}$$

A ratio can be made of effusion rates by Graham's Law:

$$\frac{rate\ of\ first\ particle}{rate\ of\ second\ particle} = \frac{\sqrt{MM\ of\ second\ particle}}{\sqrt{MM\ of\ first\ particle}}$$

where MM is the particle's molar mass. This formula can be used to find the relative rate of one particle to another. This can be very useful in chemistry when determining how much of one concentration will be present given the concentration of another gas after a certain amount of time.

A chemist has filled a balloon with equal proportions of Argon (MM = 39.948) and Hydrogen (MM = 1.008) he proceeds to poke a hole in the balloon, slowly letting the gas escape.

1. What is the proportion of the rate that Hydrogen escapes to the rate that Argon escapes?
2. If the concentration of Argon in the balloon can be modeled by the equation $A(t) = 20\ln(e^t+1) - 20t$ where time, t, is in seconds and concentration is in grams per liter, then what is the equation for the rate at which Hydrogen is escaping from the balloon?
3. What is the rate of change of the Hydrogen concentration at time $t = 4$ seconds?
4. What is the limit as t approaches infinity of the rate of change of Argon and Hydrogen? (use the equation and answer in part 2).

While the first question for this problem is not directly related to any of the topics in Differential Calculus, proportional reasoning was an important concept included in the partner discipline wish list. The second and third questions can be used when students are learning about derivatives, and the final question can be used when exploring limits.

Step 3: Implementing the Problems in the Calculus Course

With the 25 problems developed over the summer and ready for implementation, during the 2018 – 2019 academic year we asked the coordinator of Differential Calculus to use some of them in the course. The course format is three, 50-minute periods that are taught by an instructor and an 80-minute recitation period led by a graduate teaching assistant. During the recitation, students typically work in groups on activities. The coordinator was asked to incorporate at least two to three of the problems in recitation activities over multiple weeks as new calculus concepts were presented in the course. Beisiegel was on sabbatical during the academic year, which meant that the SUMMIT-P team had fewer meetings, and most of our communication about the project took place via email.

It was at this stage in our process that we experienced a pitfall. Specifically, the coordinator modified the problems in such a way that they no longer attended to the science in a meaningful way. Without regular meetings, these changes went unchecked, unfortunately. We revisit the two examples we provided above to illustrate the modifications that were made to share how this had an impact on the science featured in the problems.

Revisiting Example 1

The modified predator-prey problem that was included in the recitation activity was the following:

Twelve rabbits, some male and some female, escape and begin a wild population. Suppose that population is modeled by:

$$P(t) = 12 + \frac{40000t^2}{t^2 + 1500}$$

where $t \geq 0$ is measured in years. When does the maximum growth rate occur? Does it correspond to a point of inflection in $P(t)$?

The changes degraded both the calculus and the science in the problem in the following ways: The questions about the function can now be answered without calculus skills. The context of the problem is better represented by a logarithmic curve because the rabbit population would level out at a carrying capacity at some time. Finally, rabbit population growth now only depends on time and not on a predator.

Revisiting Example 2

The modified effusion problem used in the recitation activity was the following:

Effusion in chemistry is the process in which two or more particles are escaping through a small hole with the lighter particles leaving the container faster than the heavier particles. Heavy particles move slower than light particles with the same kinetic energy giving lighter particles a higher probability of escaping every second. This relationship can be determined by the equation:

$k\frac{1}{\sqrt{m}} =$ the rate of effusion where m is the mass and $k > 0$ is a constant.

1. Show that the proportion of gas A escaping to gas B escaping (when equal numbers of particles are present) is $\frac{\sqrt{mm_1}}{\sqrt{mm_2}}$ where mm_1 is the mass of one mole of gas A and mm_2 is the mass of one mole of gas B (called "molar masses") measured in the same units.

2. If a balloon is filled with equal numbers of Argon atoms and Hydrogen atoms then immediately after poking a small hole, what is the proportion of the rate of escaping Hydrogen atoms to the rate of escaping Argon atoms?

The changes to the problem that were problematic with respect to the science included: (1) the important phrase "same kinetic energy" which implies that both gases are at the same temperature and that $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2$ is lost; (2) both problems use $\frac{k}{\sqrt{m}}$ for background but the modified problem only uses r_e, which may leave students confused about what that variable represents; (3) the modified problem is about finding the ratio of rates, which is an algebra problem and no longer requires the use of calculus; and (4) the modified problem does not provide all the necessary information because molar masses are not given.

Step 4: Revising the Development and Implementation Process

As a result of the implementation issues we had with the calculus problems, during summer 2019 as well as during the 2019–2020 academic year, we returned to bi-monthly meetings with our entire project team and added two new steps to our process: (a) developing teaching guides for instructors and graduate teaching assistants who are assigned to lecture or run recitations for the course and (b) creating a professional learning community including the calculus coordinator, instructors, and graduate teaching assistants assigned to the course.

After discussions about what the teaching guides should contain, we designed a template (see Figure 5) to ensure that each guide is institutionally relevant and includes the same components.

Figure 5

Template for Teaching Guide for OSU SUMMIT-P

Learning Outcomes for the course

- Syllabus Level
  - Baccalaureate core outcomes*
  - Mathematics department outcomes**
- Problem/Recitation level

Teaching Notes

- What biological and mathematical background information are relevant for this problem. Include web links.

Representations in Biology/Chemistry versus Mathematics

- How are concepts represented in the different disciplines and what are some reasons for the different representations?

Trouble shooting/Common misconceptions

- What might students struggle with? How can you help them?

*Note: OSU has baccalaureate core outcomes defined for each discipline. The rationale for the baccalaureate core in mathematics is stated as: Everyone needs to manipulate numbers, evaluate variability and bias in data (as in advertising claims), and interpret data presented both in numerical and graphical form. Mathematics provides the basis for understanding and analyzing problems of this kind. Mathematics requires careful organization and precise reasoning. It helps develop and strengthen critical thinking skills.
**Every mathematics course at OSU has specific outcomes. These include: (1) identifying situations that can be modeled mathematically; (2) calculating and/or estimating the relevant variables and relations in a mathematical setting; and (3) critiquing the applicability of a mathematical approach or the validity of a mathematical conclusion.

Here we provide some examples from different teaching guides to illustrate what is included in the second half of the guide: “Representations in Biology/Chemistry Versus Mathematics and Trouble Shooting/Common Biology or Chemistry Misconceptions.” We illustrate using a different predator-prey problem than the one shared earlier in the paper.

Example 3

Predator-prey models are usually based on the Lotka-Volterra equations, which are a pair of equations whose solutions cannot be modeled by a single equation. For this problem, an *approximation* was used as a model. Biologists use predator-prey models to predict how increasing or decreasing a predator or prey population will affect the other population. These predictions often lead to changes in fishing or hunting policies. In the Oregon ecosystem, there are a multitude of bears (Ursidae) that can be considered the apex predators of the ecosystem. Salmon (*Oncorhynchus*) are their prey. Below is a graph (see Figure 6) representing the population of bears compared to the population of salmon. Answer the following questions regarding the equations and the graph.

Figure 6

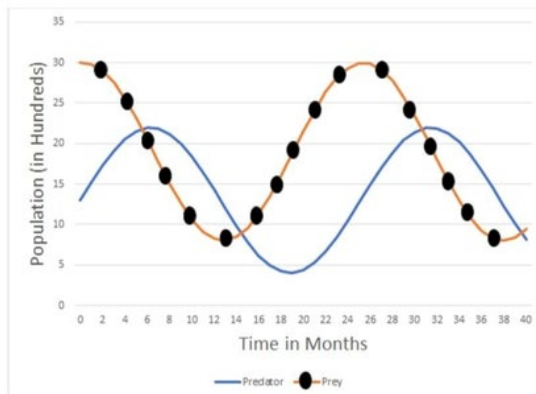
Predator Versus Prey Problem

Predator (bears):

$$B(t) = 9\sin\left(\frac{t}{4}\right) + 15$$

Prey (salmon):

$$S(t) = 11\cos\left(\frac{t - 0.249}{4}\right) + 19$$



1. Find the first derivative for the predator equation and the prey equation.
2. Find the point(s) on the graph that illustrate the greatest number of predators in the ecosystem. How does the prey graph compare at this point? Explain possible reasons for this.
3. Determine $\frac{d}{dt}$ for the prey equation at 14 months, where t = time in months.
4. Determine $\frac{d}{dt}$ for the predator equation at 14 months.
5. What is the equation that models the difference in the rate of change of prey and predator? Think about what it means for the difference to be negative and positive. What does that mean in this ecosystem?
6. When predator populations are at local maxima, what is the sign of the first derivative of the prey equation? Interpret what this means for the ecosystem.
7. When predator populations are at local minima, what is the sign of the first derivative of the prey equation? Interpret what this means for the ecosystem.

The teaching guide includes the following information that we hope will be useful to the instructors and teaching assistants who will use this problem in a recitation activity. We believe the description of the work of biologists is critical information for implementers of the problem. Indeed, the phrasing of the problem itself (i.e., "a pair of equations whose solutions cannot be modeled by a single equation") might be confusing for those unfamiliar with biology. It is useful to mathematics instructors and graduate teaching assistants to be familiar with biology problem solving methods.

Representations in Biology/Chemistry versus Mathematics

- Equations are often represented differently in biology than in mathematics in order to demonstrate the relationship of interest more clearly. See the Lotka-Volterra model (Yorke & Anderson, 1973).
- Using mathematical models in biology requires that biologists make a number of assumptions that may not actually hold true. For example, in predator-prey models assumptions that may not be true in nature include:
  1. The prey population finds ample food at all times.
  2. The food supply of the predator population depends entirely on the size of the prey population.
  3. The rate of change of population is proportional to its size.
  4. During the process, the environment does not change in favor of one species, and genetic adaptation is inconsequential.
  5. Predators have limitless appetites.

Troubleshooting/Common Biology Misconceptions

- Many students think that they do not need mathematics to do biology. In fact, the field of ecology is based on a great deal of mathematical modeling. These models are one example of that.
- Many students view ecology as a study that is not connected with the human species, but ecology can be applied to humans because it is the study of living organisms.
- Students may think that food webs only involve predators and prey but not plants (i.e., producers). Producers are the base of the food web and the source of all energy on earth. These models do not consider the food that is available to the prey.
- Students may incorrectly think that predator and prey populations are similar in size. This is not necessarily true; in fact, prey populations by definition have to be larger than predator populations. Why? We lose energy as we move up the food web due to the maintenance of the organisms at lower levels (i.e., 10% of energy is passed from herbivores to first-order carnivores).
- Students may think that the relative size of one population (predator or prey) has no bearing on the size of the other population. The point of the models is to make predictions about these relationships because they are connected. By looking at specific predators and prey, however, we have to simplify the food web and just look at the relationships between two species. In reality, there is a much more complicated set of interactions between all organisms in the ecosystem.

In our most recent iteration of problem implementation, our goal has been to organize a professional learning community that includes our SUMMIT-P team and the calculus coordinator, instructors, and graduate teaching assistants who are assigned to the course. In spring 2019, the SUMMIT-P team met with the calculus team to discuss the overall goals of the project, the problems that we have developed, and their use in Differential Calculus. In these meetings, we provided opportunities for the partner disciplines to explain the contexts of the problems and the reasons that certain problem features are critical. The group is also discussing the struggles of calculus students with the problems and how to refine the problems, as well as developing further clarifications for the information in the teaching guides. We hope to continue to expand our collaboration with the calculus team and provide ongoing support for them in the future.

Reflecting on Our Process

Our work has been fruitful. We have learned a significant amount about what aspects of our project worked well and how we can continue to improve the steps we take to achieve the overarching goals of the SUMMIT-P project. Here we summarize some of the positive aspects of this experience, the pitfall, and what we aim to do as we move forward.

Meetings with Disciplinary Partners and Users of the Materials

The meetings in which we explored the presence of mathematics content in biology and chemistry were incredibly useful. The learning curve was fairly steep for the SUMMIT-P team, but we have all increased our knowledge and understanding of each other's disciplines significantly, which will help the work going forward. We plan to have meetings with the calculus course coordinator, instructors, and graduate teaching assistants. They can provide feedback on the course materials (i.e., the problems, instructional guides, etc.) and share their students' experiences with the problems. Developing a better understanding of the student experience will help us to continue to improve the problems and determine how we can support the users of the problems. Our goal is to meet with the calculus coordinator, instructors, and graduate teaching assistants after every implementation of problems in Differential Calculus (approximately three meetings per 10-week term).

Employing Students

Employing the graduate student and undergraduate students was a critical part of the process. We would definitely do it again. Their seemingly endless energy and enthusiasm for the work, which is directly related to their fields of study, helped to propel the project forward. We could not have done this work without them. We were lucky to have found students who could make this work a priority. They genuinely wanted to contribute to the project. We were constantly impressed by the amount of time and effort they brought to the project.

Pitfall

The pitfall we experienced was unexpected. In hindsight, however, we should have known that the coordinator and instructors would need support to understand the problems—including why the problems were phrased as they were—and the importance of specifically including certain concepts from chemistry and biology in order to preserve the integrity of the disciplinary context. The breakdown in the process was not providing the calculus coordinator and instructors with the same conversational experiences as the SUMMIT-P team who developed the problems. Moving forward, we hope to minimize this issue with the teaching guides that we have described in this paper. While we expect the teaching guides will be useful to the users of the problems, we also expect that the instructors who teach Differential Calculus will continue to need support in order to use the problems as they are intended. As we mentioned above, we will aim to include the calculus team in our SUMMIT-P meetings more often and also plan more meetings in which they can provide input.

Moving Ahead

As we continue our work on the SUMMIT-P project, we plan to continue working to understand how mathematicians, biologists, and chemists see the same problems from different perspectives. As one example, we revisit the logistic regression problem discussed with the undergraduate students. During the conversation, we talked about scenarios in which the growth of an organism would level off at a certain point; for example, the spread of disease would reach a limit once most people had contracted the disease. In a mathematics class, the function that could be used to model this is:

$$P(t) = \frac{4000e^{-2t}}{1 + 2000e^{-2t}}$$

In contrast, for the same scenario the biologists would use an equation like this:

$$\frac{dN}{dt} = 2N\left(\frac{2000 - N}{2000}\right)$$

Students are likely to struggle to understand how these are related and, as a result, not understand the usefulness of mathematics in biology problems. Thus, our team would like to continue to explore these differences, including addressing questions like: Why are these differences in approach important to the disciplines? How we can support students in understanding the connections between different representations of the same problem? What problems we can design that will help students realize the connections between the disciplines?

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COUNTING ON COLLABORATION: A TRIANGULAR APPROACH IN THE EDUCATOR PREPARATION PROGRAM FOR TEACHERS OF MATHEMATICS

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ABSTRACT
This paper outlines the process of establishing a stronger and more reciprocal partnership for collaboration between an education preparation program and a local education agency. The essential partners identified included the College of Natural Sciences and Mathematics and the College of Education at Lee University and stakeholders in the local school district. First, this paper will discuss a theoretical framework that speaks to the importance of dialogue and a dialogic approach to teaching mathematics. Secondly, the processes and methods of the project involving collaboration through partnerships are described. These partnerships gave rise to the realization that coursework would be more effective if it mirrored the instructional practices of local education agencies. A detailed description of the process of changes to the coursework and initial outcomes of the project are outlined. Included are questions and recommendations for further collaboration.

KEYWORDS
collaboration, elementary mathematics, dialogue, problem solving

This paper addresses the impact of the Collaborative Research: A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) project at Lee University. SUMMIT-P is a multi-institutional project funded by the National Science Foundation that aims at revising lower division mathematics curricula through interdisciplinary collaborations, based on recommendations from the Mathematics Association of America Curriculum Foundations (CF) project (Ganter & Barker, 2004). As one recommendation, CF encourages the creation of Faculty Learning Communities consisting of mathematicians and faculty from other disciplines to help to implement the other CF recommendations in useful and practical revisions to mathematics courses. As a result of preparatory work and ideas gained from the CF Project (Ganter & Barker, 2004), we considered providing "[t]ools for teaching and learning, such as calculators, computers, and physical objects, including manipulatives commonly found in schools […] for problem solving in mathematics courses taken by prospective teachers" (p. 145). Another CF recommendation is that "[m]athematics courses for future teachers should provide opportunities for students to learn mathematics using a variety of instructional methods, including many we would like them to use in their teaching" (p. 145). This report provided a foundation on which to build professional development opportunities, course design, and pedagogical practices at Lee University.

The choice of collaborative partners in this project was based on those who are regularly involved in teaching mathematics educators. Selected participants were those who teach lower level mathematics courses in the Department of Natural Sciences and Mathematics (particularly those who teach mathematics educators), professors from the College of Education who teach courses in pedagogy, and the Coordinator of Professional Development in Mathematics from Bradley County Schools. Over the period of one year (2017 – 2018), discussions took place with all partners. Everyone agreed that, based on multiple data sources, there was a significant need to improve the pedagogical skills of teachers of mathematics. From general observation and research, it was established that improvement might rest on a connection between the theory and practice of dialogue, the importance of collaboration through partnerships, and the pursuit of effective practices such as the use of manipulatives. Out of the project there were initial outcomes and ideas for further collaboration.

The project partners decided that Concepts of Mathematics I and II, the primary courses for preparing pre-service teachers to teach mathematics in the P–8 setting, should be the focus of this project. Before the project began, each class was observed by the Principal Investigator (PI) for mathematics content and pedagogical practices and for the ways mathematics educators engaged in learning. Anecdotal data and the results of a mathematics manipulative tests taken by the students strongly indicated that these courses would benefit from review. Additionally, it was decided that other resources such as local experts involved in professional development in P–8 settings would have valuable input into raising the standard for teaching mathematics. Then during the next year (2018 – 2019), the professors who teach these two courses participated in intensive professional development with the local education agency. Details about our project are provided below.

The Importance of Dialogue

It has long been understood that language and communication are the basis for collaboration, partnerships, associations, and relationships. This is true in interpersonal cooperation but also in developing a cognitive understanding of specific concepts. The value of "talk" cannot be overestimated.

One essential aspect of this project was the realization that a way forward to improvement and enhancement of mathematics course work for elementary teachers was to emphasize "talk" (Meiers, 2010). Words such as talking, discussing, questioning, arguing, chatting, and conferring all conjure up the notion that discourse in the area of problem solving is a necessity. A recognition of the relationship between "talk" and solving problems relating to educator preparation was paramount in the minds of those involved throughout this project.

Problem Solving

The term "problem solving" is in and of itself a mathematical notion, but it does not imply that strict mathematical algorithms are the only methods to finding solutions. In fact, and more importantly, research increasingly suggests that solving "the problem" involves certain essential methods outside the perceived realm of mathematics, not least the idea of the necessity to talk.

The idea of problem solving through the use of discussion is not a new idea and has been used relatively often in the field of mathematics. The ancient Greek philosophers approached problems of mathematics and logic by posing and answering questions based on observation of the real word and on data. Over time it appears that this method was somewhat lost and replaced with rote learning and the memorization of processes. Essential elements of understanding were lost, particularly in P–8 classrooms. This project sought to revive and highlight the approach of collaboration through discussion as a way to solve problems. The courses under consideration were observed, data was gathered from each course, the performance of teachers in the field was investigated, and suggestions for improvement were recommended.

It was recognized, as already discussed, that there should be discourse between the departments involved and with the local education districts. This collaboration should be between all parties involved, with the sole objective of finding more effective ways to deliver instruction in the teaching of mathematics to the P–8 population.

Language and Problem Solving

In the relatively new field of human development, ideas were drawn from the work of theorists Freud, Piaget, Vygotsky, and Bakhtin for ideas related to affective and cognitive development. Considering the topic, "can talking solve problems?" a comment of note that was retrieved from *Psychology Today* (2017) frames a main premise that discourse is not only a way to problem solve but that this approach might also be curative. Conversely, this blog also suggests that the incorrect use of words, discussion, and discourse might also actually cause harm. The recognition that there are more helpful ways to talk through problems is valuable in all areas of life and education and cannot be overstated. Therefore, let us not underestimate the use of words in the realm of teaching mathematics and in the training of mathematics educators at all levels.

Piaget's stage theory of cognitive development has for many years had enormous impact on the teaching of mathematics. It assumes that certain concepts are acquired at certain stages that roughly correspond to age levels. In regards to language and its use in the mathematics classroom, it is generally believed that there is a close correlation to language development and the acquisition of certain mathematical skills. Although, in recent years, certain criticisms have been raised regarding Piaget and against a strict application of developmental stages, it might also be argued that within bands of development there is reason for adjusting and differentiating in the use of language as it relates to mathematical processes (Ojose, 2008).

An acceptance of Vygotsky's social learning theory and his work on language and thought brought to bear a consideration for effective questioning techniques, the increased use of language, the use of manipulatives, the role of the teacher, and collaborative learning within the courses and within the teaching of mathematics in P–8 classrooms (Vygotsky, 1986). The emphasis on the concrete, representational, and abstract steps in problem solving all rely on the essential connection between thought, language, and understanding within a social setting.

More recently, ideas from Bakhtin (1895 – 1975) on the radical importance of dialogue influenced thinking in a new way. In Wegerif's (2011) paper, "Towards a Dialogic Theory of how Children Learn to Think," he informs us that "learning to think" involves a dialogic space that has often been ignored in teaching in general and particularly in mathematics. Wegerif sought to discover why some groups of children were more successful at solving reasoning test problems than others. He observed the dialogue children used in relation to solving problems with seeing patterns, commutativity, and making a graph without instructions. He found that the more successful groups listened more to each other, asked each other for help, and were willing to change their minds as a result of seeing the problem through the eyes of another. Through his observations, Wegerif attributed the more successful activity of some groups to Bakhtin's notions of the ability to connect with a "dynamic continuous emergence of meaning" that depends on previous and succeeding knowledge that is mediated through the effective use of language in dialogue about representations and through posing questions.

It is the premise of this section that emerging educational theories of learning offer sufficient and necessary understanding of the importance of "talk" in understanding important elements in teaching mathematics.

Problem Solving Techniques

Today, there is common acceptance of the idea that all children learn differently and that all learning is a result of: shifts in thought that are properly mediated through language, the use of concrete representation (manipulatives), collaboration, and safe settings.

Advocated strategies such as the use of manipulatives, differentiation, "Accountable Talk," math journaling, math vocabulary, "Think Alouds," community of learners, and students connecting problems to self, others, and the world have all come to the forefront and offer promising results (Kazemi & Hintz, 2014). All these concepts, approaches, strategies, and shifts have emerged from observations of how children develop and learn. They relate directly to the theoretical framework for the Lee University SUMMIT-P project.

In discussion concerning the delivery of our courses, it was recognized that these shifts should receive a greater emphasis in the pedagogical approaches that are taught and modeled to those who will teach mathematics in P–8 classrooms. "Talk" is imperative in all classrooms and at all levels and is conceptually linked to understanding that is gained through active engagement that is brought about by "doing" (Smith & Stein, 2011).

Establishing and Strengthening Partnerships to Enhance Recommended Practices Underlying Rationale

If you make your way into any elementary or middle school, you will find that effective teachers of mathematics appear to have certain practices in common. One of the six Principles for School Mathematics (NCTM, 2013) states, "Research has solidly established the important role of conceptual understanding in the learning of mathematics. By aligning factual knowledge and procedural proficiency with conceptual knowledge, students can become effective learners" (p. 2). The National Council for Teachers of Mathematics also believes that "the foundation for children's mathematical development is established in the early years" (Seefeldt & Wasik, 2006, p. 249). If it is in fact true that conceptual understanding is vitally important in the learning of mathematics, then it seemed relevant to this study to first investigate current practices in teacher preparation that seek to address conceptual learning and second, to seek to improve upon these preparation practices to establish a foundation for the early years of development. We felt that three themes, problem solving, collaboration, and the use of manipulatives, held the keys for improving essential mathematics understanding.

The mathematical education community promotes hands-on learning and manipulatives. Companies such as ETA Hand2Mind, Learning Resources, and EAI Education distribute catalogues to educators advertising a variety of manipulatives. A mathematics educator can purchase products from an extensive list of manipulatives including patty paper, geoboards, counters, algebra tiles, and tangrams. However, if the educator has never learned mathematical concepts using these manipulatives or has never seen them used in mathematics instruction, they are left to wonder about the purpose, necessity, and benefit manipulatives bring to student comprehension. Implementing the manipulatives effectively is also a mystery to the educator that lacks experience and specialized training. Thus, it is imperative that teachers of pre-service teachers incorporate mathematical learning and teaching through manipulatives into course requirements. Recognizing this need, a relationship began between teachers and administrators with Bradley County Schools and the mathematics educators at Lee University to bridge the training gap as it relates to this method of mathematical instruction.

Manipulatives in the Mathematics Classroom

The important role that manipulatives play in the mathematics classroom cannot be overstated. Research shows that mathematics achievement levels increase with the use of manipulatives and learning is enhanced when students are actively engaged in the learning process. Stein and Bovalino (2001) concluded that manipulatives are important tools that can help students to think and reason in more meaningful ways. Sutton and Krueger (2002) found that manipulative use also increased mathematical interest among students. Manipulatives are a common instructional resource found in many mathematics classes. They can be used to model mathematical and often abstract concepts in order to support overall student understanding. Manipulatives can be a variety of objects such as coins, rods, paper clips, pieces of candy, or blocks. However, in recent years some classrooms have switched to using virtual manipulatives on tablets or computers (Uttal, 2003, p. 98). Kennedy (1986) defines manipulatives as "objects that appeal to several senses and that can be touched, moved about, rearranged and otherwise handled by children." He concludes that mathematical lessons should involve a variety of instructional methods. Integrating manipulatives along with other traditional teaching methods increases the likelihood that students will develop a solid understanding of the mathematical concept (p. 55).

According to the National Council of Supervisors of Mathematics in a 2013 statement, "[I]n order to develop every students' mathematical proficiency, leaders and teachers must systematically integrate the use of concrete and virtual manipulatives into classroom instruction at all grade levels" (p.1). NSCM's position statement is based on several research studies that support the practice of using manipulatives throughout classroom instruction. For example, in a study involving 8th-grade math teachers, Raphael and Wahlstrom (1989) concluded that the use of manipulatives along with "successful topic coverage by teachers" (p. 189) had a positive connection with the level of student comprehension. In 2013, a meta-analysis report was compiled involving research studies that had an emphasis on teaching mathematics with concrete manipulatives. Carbonneau (2013) specified that a primary requirement for inclusion in this study required assessment data from "an instructional technique that used manipulatives [and] a comparison group that taught math with only abstract math symbols" (p. 383). Out of 55 studies that were eligible for inclusion in this report, 35 came to the conclusion that students who were taught with manipulatives scored considerably higher on the unit assessment test when compared to those students who did not have access to manipulatives (Carbonneau, 2013).

While the use of manipulatives has been recognized to deliver positive results in many classrooms, it is necessary to highlight the probable explanations behind these results. In 2017, Willingham identified three likely theories for why manipulatives could be directly related to the increased assessment scores. First, manipulatives aid in learning because they require physical movement of the body, which some believe increases cognition. Another reason rests solely on the belief that children are concrete learners and that such learning leads them to understand the abstract. A final theory proposes that manipulatives are simply symbols for innovative mathematical ideas still to be learned in the classroom. However, if used incorrectly, manipulatives can cause difficulties for students to grasp the abstract concept they were intended to represent (Willingham, 2017, p. 26).

The way the teacher introduces and uses manipulatives plays a significant role in how well the mathematical concepts transfer to their students. Because manipulatives can represent abstract concepts, it is necessary that teachers understand how to appropriately use them during a lesson. Unfortunately, many difficulties with using manipulatives stem from a lack of familiarity on the part of the teacher. Kilgo and White (2015) recognized that "providing opportunities for pre-service teachers to use these [manipulatives…] will assist in building their confidence and encourage them to implement the aids in their own classrooms" (p. 217). Teachers have a responsibility to learn how manipulatives can bring about success while attempting to deter any complications that may set their students up to misunderstand a topic. Teachers should seek out opportunities to be trained in the use and functionality of different types of manipulatives. Waiting until days before a high-stakes assessment may result in confusion and frustration, both for the teacher and the students (Cope, 2015, p.17). Those teachers who have received clear directions and strategies for manipulatives are more likely to see positive results in their classrooms.

The Process

Bradley County Schools and Lee University have long been collaborative partners. With the increase of teacher accountability and high-stakes testing, a realization occurred that pre-service teachers enrolled at Lee University should be better equipped to demonstrate the most effective mathematical teaching practices. As a way of strengthening this partnership, an alignment of practices was believed to be essential.

The local school system has invested in curriculum adoptions that include classroom sets of manipulative kits. The district mathematics coordinator was able to utilize some of those sets to offer a hands-on professional development session for Lee University professional mathematics educators on how to effectively use manipulatives in the mathematics classroom. The professors provided the coordinator with course syllabi that included topics students would be learning throughout the courses. Two professional development sessions were designed by the district coordinator based on observational data of effective mathematics instruction with manipulatives in local classrooms, grade level standards analysis for Bradley County Schools from the Tennessee Department of Education, and topics from syllabi provided by the university professors. The district coordinator found natural links between the three pieces of data. Activities were designed to match manipulatives to conceptual understanding of mathematical concepts. Professors participated as learners and experienced learning mathematics with manipulatives, which assured them of the potential of these activities to leave students with long lasting understanding of mathematics at a concrete level. As a result, these professors left the session convinced of the need to incorporate learning mathematics with manipulatives into course requirements.

The development of understanding mathematics concretely is a process that can never be underestimated or overlooked. It is an important and necessary stage of development before a learner attempts to perform mathematics abstractly. In order to develop long-term comprehension, conceptual understanding, and procedural fluency, a mathematical learner must develop initial understanding at the concrete phase (the doing stage) before moving into the representational phase (the seeing stage) and the abstract phase (the symbolic stage). Unfortunately, the concrete understanding of mathematics is oftentimes underestimated and overlooked. Educators are not always equipped with the tools necessary to help students develop understanding at the concrete level, and many secondary mathematics educators do not see the need for it. The “I do, We do, You do” framework supports the belief that if a learner can see a mathematical process performed enough times then the learner will be successful performing the mathematical process alone. However, being fluent in mathematical concepts requires a concrete level of understanding, and learning with manipulatives can provide this type of understanding for students. New educators oftentimes walk into a classroom with cabinets full of manipulatives but with no understanding of how and when to use them. That is why courses for pre-service teachers must include learning and teaching mathematics with manipulatives.

Professors met with the coordinator twice. During the first session, participants explored how to use patty paper (i.e., small square pieces of wax paper) to model multiplication and division. The activities and problems were designed to enhance understanding of multiplication as an area model and division as partitioning. Participants also created hand-made fraction strips, which evoked a deep and specific conversation about the power in a learner creating fractional representations on equal-length strips of paper. Hand-made and store-bought fraction strips were used to create equivalent fractions, adding fractions, and multiplying fractions. Lastly, algebra tiles were introduced to participants as the key to developing number sense and a greater understanding of polynomials. Participants used the algebra tiles to build a foundation for the concrete understanding of the additive inverse property and the distributive property. Participants then modeled the technique of completing the square using the algebra tiles. The Bradley County

Schools Mathematics Coordinator had prepared many more examples and activities with manipulatives than time would allow, so a second professional development session was scheduled.

During the second professional development session, there was great discourse about the importance of concrete-representational-abstract learning. The coordinator shared documents created and produced by Mathematics Coordinators from the Tennessee Department of Education. These documents showed a variety of strategies students could employ in order to demonstrate understanding at each phase of concrete-representational-abstract learning. Participants then explored digital manipulatives such as Geometer's Sketchpad and Geogebra. Publications from Key Curriculum Press and the National Council of Teachers of Mathematics were shared with participants to provide a sample of resources and books with lessons and activities to support the use of Geometer's Sketchpad, an interactive geometry software. Geogebra, a free "dynamic mathematics software for schools that joins geometry, algebra, statistics and calculus through graphing and spreadsheets," was explored to enhance the concrete understanding of fractions, mean, and median (see www.geogebra.org). Participants also studied many pre-made lessons on desmos.com, such as linear functions, parabolas, slope, and graphing stories.

Professors had recently acquired class sets of Cuisenaire Rods and Base 10 Blocks, so a portion of the professional development session was spent using these two manipulatives to support concrete understanding of addition and subtraction, place value, multiplication and division, and the concept of regrouping. Participants finished the professional development session by playing a variety of games that develop and enhance number sense for students of all ages. Students must become comfortable strategizing with numbers.

Initial Outcomes of the Project

Observation Comments

The Principal Investigator (PI) for the SUMMIT-P project at Lee University had the opportunity to observe the two classes that are the subject of the collaboration between the mathematics division and the College of Education (COE): Concepts of Mathematics I and II. These are courses that future elementary and middle-school teachers are required to take for certification. She observed Concepts of Mathematics I in spring 2017 before any of the recommendations by the COE were implemented.

She then observed both courses in spring 2019 after the teaching faculty were provided with professional development opportunities about how to implement the recommendations, with a focus on how to use manipulatives in delivering the course material.

Observations after Professional Development

Concepts of Fractions and Representations

The Concepts of Mathematics I class the PI observed covered the topic Concepts of Fractions and Representations. The manipulatives used were Fraction Towers, consisting of interlocking blocks that indicate different fractions (see Figure 1). The instructor started with explaining the concept of unit fractions, fractions with 1 in the numerator. She then explained a fraction as a collection of equal-sized parts. For example, the 4 in the denominator of ¾ indicates how the whole is divided, and the 3 in the numerator indicates the number of equal parts of the whole we are considering. Students were given some exercises to solidify this concept.

For the in-class activity students were asked to take a ½ tower and find all fractions that are equivalent to it. They then had to represent each equivalent fraction as part of a whole. Thus, they made the connection that 2/4, 4/8, 3/6 and 5/10 were all equivalent to the ½ tower (see right side of Figure 1).

Figure 1

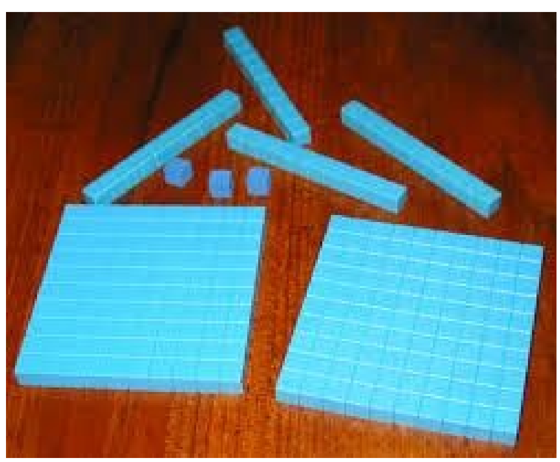
Fraction Tower Manipulative Set Stacked to 1 and to 1/2

Some questions were posed to the students: why can we not use 1/3, or 1/5, creating self-discovery and critical thinking opportunities for students.

In another class in this course students used Base 10 Blocks to model division and multiplication. Base 10 blocks are made up of unit cubes. One unit cube is a 1; 10 unit cubes stacked up together is a rod of 10; 10 adjacent rods make a 100, a flat (see Figure 2).

Figure 2

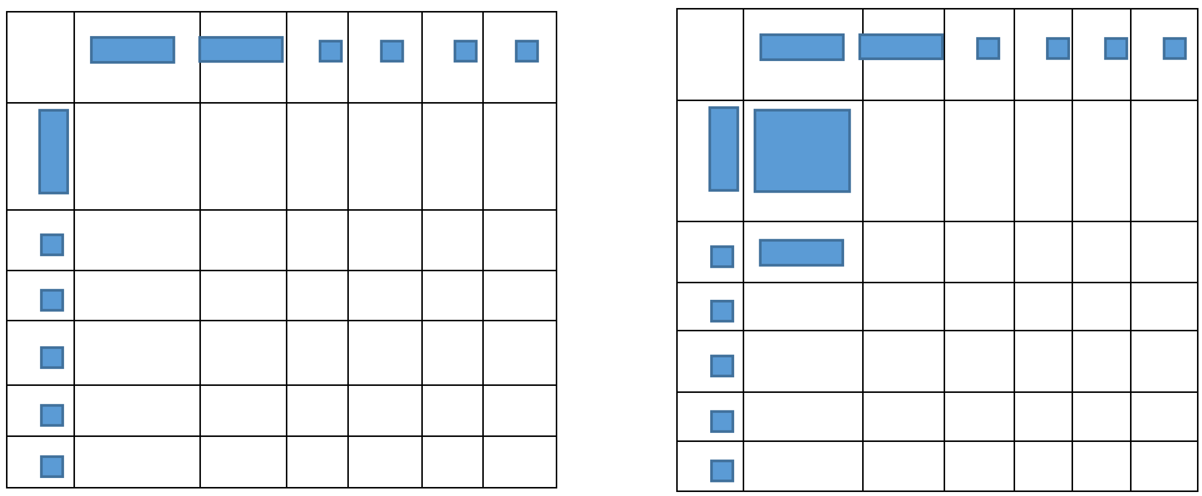
Base 10 Blocks



Activity 1

The students had to model 24 x 15 using Base 10 Blocks. They created a table structure where on the first row they placed two rods and four units, representing 24. On the first column they placed one rod and five units, representing 15 (see left side of Figure 3). They then proceeded to fill the table with appropriate Base 10 Blocks. The space below the first rod in the first row and adjacent to the rod in the first column was filled with a flat (a 100). The space below the first rod in the first row and adjacent to the first unit in the first column was filled with a rod (see right side of Figure 3), and so on.

Figure 3
Illustrations of the Unit Block Practice



After finishing the table, the students added up the Base 10 Blocks to get the answer for the multiplication.

Activity 2

The students had to model 736/3 using the flats, rods, and units. They started with 7 flats (= 700), 3 rods (= 30) and 6 units (= 6). They proceeded by dividing the 7 flats into groups of 3. This resulted in *two groups of 3 flats* each and one flat remaining. The remaining flat was broken to ten rods and added to the original 3 rods, resulting in 13 rods. The division process continued by separating the rods into groups of 3: *four groups of 3 rods* and a remaining rod. The remaining rod was broken then to 10 units and added to the original 6 units. The division process continued to give *5 groups of 3 units* and a *remaining unit*. Students were able to visualize that the result of the division was 245 and a remainder of 1.

Finding Area of Geometrical Figures

The PI observed one class of Concepts of Mathematics II. The day's topic was finding the area of geometrical figures. Each student was given a square and a rectangle cut out from card stock, a pair of scissors, and tape. The objective of the activities done in class was for the students to derive the formulas instead of memorizing them.
The instructor gave the definition of area as the measurement of the surface inside the boundaries of the geometric shape.

Activity 1

Students were asked to trace the square they were handed at the beginning of class on their notebook and find the area. Everyone knew the formula: area is base times height or length times width or $(\text{side})^2$ in this case. Some students had graphing paper, so they traced the square on their paper and were able to count how many squares were inside the boundaries.

Activity 2

The instructor asked the students to cut the square across the diagonal into two triangles and find the area of a triangle. Everyone realized that since the area of the whole square was base times height, and now the area is divided into two parts, then the area of each triangle is ½ base times height.

Activity 3

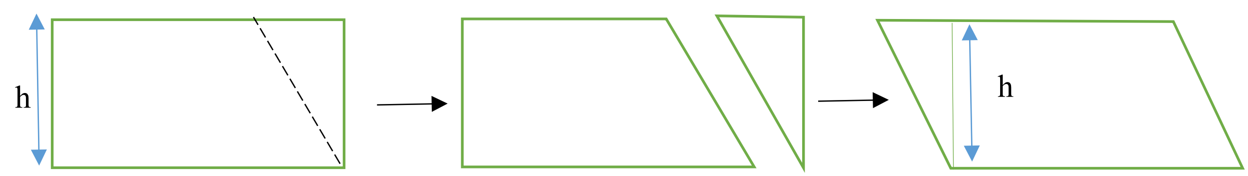
Similarly, students were asked to trace the card stock rectangle onto their paper and find the area. At this point they realized that the base times height, or length times width, formulas are applicable, but not the $(\text{side})^2$ formula.

Activity 4

To derive the area of a parallelogram, students were asked to cut the rectangle starting at any corner and cut off a corner, not necessarily through the diagonal, slide the cut-off triangle to make a parallelogram (see Figure 4) and tape it.

Figure 4

Making a Parallelogram from a Rectangle



In their exploration of finding the area of the parallelogram, they realized it was still base times height, as the area of the original rectangle. They became aware that now the height is not the length of the side of the parallelogram but what was the side of the rectangle.

Similar activities were completed to find the area of trapezoids and circles. These activities gave a deeper understanding of the mathematical concepts covered and revealed the logical reason behind mathematical rules and formulas. They also provided an excellent visual for the students. All students observed were paying attention in class and were rather amused when it came to the activities part. They used the manipulatives with ease, which indicated they have used them before and were comfortable manipulating them. Compared to the class observed before implementing the COE's recommendations, it was clear that the manipulatives kept the students actively engaged in the learning process and that they can create their own ideas for classroom materials they can use when they are in the workforce.

Conclusions and Recommendations

Initial outcomes appear to underline the significance and value of partnerships, close observations of current practices, and a subsequent willingness to dialogue with an eye towards revising approaches to established courses in Mathematics for teacher candidates. The opportunity provided by the SUMMIT-P Project at Lee University was the precipitating step to establishing a triangular approach to one important part of the Educator Preparation Program.

The relationship between Bradley County Schools Mathematics Coordinator and Lee University professors is new, unique, and unprecedented. The mathematical expertise that professors brought to the conversation was invaluable, and the experience from local classrooms that the district coordinator brought was meaningful. Both parties provided a lens through which mathematics education could be enhanced and improved. It is through this partnership that pre-service teachers at Lee University will be better equipped to teach mathematics with manipulatives to help students dialogue about and understand mathematical concepts in the concrete-representational-abstract phases.

Recommendations

The teaching of mathematics in the P–8 setting is often regarded as still "needing improvement." The experience and improvement gained through this project gives some guidance for endeavors for preparing future math educators.

First, the idea of mathematics professor educators learning from teachers in the field is novel. Attendance at trainings facilitated by local education agencies by university professionals appears to be an optimal way of learning the methods and processes that are used by teachers in the P–8 setting. It makes sense to continue this approach as new educators learn to teach in a way that promotes understanding and meets local needs.

Additionally, the consideration of theories of learning, particularly the importance of dialogue, between and within settings might be offered by education preparation programs as perspective for the importance but sometimes gaping nexus between theory and practice.

Consideration should also be given to expanding this process into other courses that address the principles of mathematics instruction and methods for teaching mathematics in clinical experiences.

Finally, observations of Lee University teacher graduates, their continued use of recommended practices in relation to teaching performance scores would strengthen the validity and reliability of this project.

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Integrative and Contextual Learning in College Algebra: An Interdisciplinary Collaboration with Economics

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Abstract

Many students consider mathematics too abstract and useless for their academic and career goals. Meanwhile, instructors in quantitative disciplines such as economics find many students mathematically underprepared for their courses. The disconnect between students' perceptions of the utility of mathematics and their life and career may have contributed to some of the under-performance in learning mathematics. Addressing this problem requires collaboration across disciplines to develop an understanding of each other's needs, more specifically to develop an integrative platform that allows students to apply mathematical skills in interdisciplinary contexts (Ganter & Barker, 2004). We collaboratively designed and implemented an integrative platform that includes creation of assignments and resources that contextualize the course in College Algebra with applications of economics, facilitation of frequent interdisciplinary dialogues among faculty members, creation of a course pair, and expansion of the platform to include 15 sections of College Algebra. This paper describes the process of the design and implementation of the platform.

Keywords

integrative, contextual learning, interdisciplinary, college algebra, economics

LaGuardia Community College (LaGuardia), with an enrollment of about 20,000 students, is part of the City University of New York (CUNY) system. The course discussed here, College Algebra and Trigonometry, hereafter referred to as College Algebra, is a required core course for most majors at LaGuardia. This includes many non-STEM majors such as Business, Tourism and Hospitality, Fine Arts and Design, and Commercial Photography. Approximately 40 sections of College Algebra are offered each semester. Many students consider this course to be too abstract and useless for their academic and career goals.

Meanwhile, instructors in quantitative disciplines such as economics find that many of their students are mathematically underprepared. Instructors find themselves devoting class time to teaching basic mathematics concepts and skills. Students are often late to discover that college algebra concepts and skills are in fact useful for advancing their academic and career goals.

This disconnect between students' perception of the utility of college algebra at different stages of their academic career is not just a problem at LaGuardia Community College. It is a problem observed nationwide, and combating this problem requires meaningful collaboration between partner disciplines (Ganter & Barker, 2004).

The main objective of our project was to make mathematics accessible, useful, and relevant to students while also developing their understanding of economics. Our project included creating mathematics assignments involving contexts in economics, piloting these assignments in a sizable number of College Algebra sections, and developing a course pair (defined below) between College Algebra and Microeconomics to help facilitate the use of these assignments. During the process, we held interdisciplinary conversations between mathematics and economics faculty, used surveys to gather information about the mathematical needs of students in introductory economics courses, developed a curriculum map of economics and mathematics courses at LaGuardia, utilized mathematics instructional resources in economics courses, and conducted a workshop to train mathematics faculty members to use the newly developed instructional platform before scaling up the use of the platform in all sections of College Algebra. These project activities are described in detail below.

Literature Review

Philosophers, educators, and social scientists have long been linking relevance of education to students' motivation to learn (Karabenick & Urdan, 2014; Lazowski & Hulleman, 2016; Albrecht & Karabenick, 2017; Albrecht & Karabenick, 2018). The National Research Council's Committee on Increasing High School Students' Engagement and Motivation to Learn (NRC, 2004) recommended the need for instructional programs to be relevant to and build on students' cultural backgrounds and personal experiences and to provide opportunities for students to engage with authentic tasks that have meaning in the world outside of school. "Relevance interventions" that inject an element of relevance into course curricula have resulted in promising, positive effects on academic achievement and motivation (Karabenick & Urdan, 2014; Lazowski & Hulleman, 2016). Although some recent work has produced null to negative results (Albrecht & Karabenick, 2017), academicians increasingly recognize the importance of contextual learning or situative learning, emphasizing how a person learns a particular set of knowledge and skills and the situation or a context in which a person learns (Lave and Wenger, 1991; Putnam and Borko, 2000). By providing meaningful contexts or situations that are relevant to students, contextual learning or situative learning allows students to construct the meaning of what they learn based on their own experiences.

This conceptual framework leads to integrated, interdisciplinary learning, also called connected learning, that incorporates contexts from different disciplines into a particular subject matter. In our case, the subject is mathematics. Many educators acknowledge the natural overlaps between mathematics and other disciplines in terms of real-world events. "Integrating mathematics and science shows students how applicable mathematics is in the natural world. They are using mathematics to make sense of the world around them," (Frykholm, & Glasson, 2005, p. 132). Our project is rooted in this framework by incorporating economics contexts in mathematics assignments.

Prior studies have found some positive effects of integrative approaches in STEM subjects on students' achievement and motivation (Riskowski et al., 2009; Sanders, 2009). While evidence of the effectiveness of an integrative approach on academic performance in mathematics is not as abundant, Farrior et al. (2007) found that this type of instruction could help students better understand real-world applications of mathematics. The increased intrinsic motivation may improve students' academic performance in the end (Farrior et al., 2007). Research on the effects of integrative and contextual learning on mathematics subjects has been limited, and mathematics is the STEM subject that benefits the least from these approaches (Becker & Park, 2011). The data that will be collected in the later stages of our project will add insight to this emerging field of research.

Creating Assignments

In the early 2000's, the Mathematical Association of America's (MAA) Curriculum Foundation Project (CF) led an effort to achieve the goal of increasing students' perception of the utility of college algebra (Ganter & Barker, 2004). Through a series of disciplinary workshops, participants learned about the mathematical challenges faced by partner disciplines (Ganter & Barker, 2004). The MAA recommended that direct and purposeful connections should be made between lower mathematics courses and introductory courses in other disciplines. With the use of the appropriate technology, mathematical modeling should be used to help students develop the necessary problem solving and communication skills for success in partner discipline fields (Ganter & Barker, 2004).

With the goal implementing the MAA's recommendations to improve lower level mathematics courses, the National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P), a National Science Foundation (NSF) funded project, was formed. As part of the SUMMIT-P project, mathematics faculty in each member institution conducted discussions with partner disciplines (Hofrenning et al., 2020). At LaGuardia, two mathematics faculty members partnered with two economics faculty members to understand how College Algebra could be redesigned to help students studying economics.

Our project started with interdisciplinary conversations to understand the mathematical needs of students enrolled in economics courses. We held multiple face-to-face discussion sessions between the mathematics and economics faculty. Similar to the structure of CF workshops, faculty in economics curated a "wish list" of mathematics topics. We also referred to two reports from the CF project (Ganter & Barker, 2004; Ganter & Haver, 2011) focusing on economics to develop the following list of topics that students need to be successful in introductory economics courses at LaGuardia:

- using basic arithmetic and algebraic skills such as equations and algebra, effects of changing parameters in linear equations,
- calculating areas of relatively simple geometric figures,
- plotting numbers and graphs, interpreting graphs,
- linear data and graphs, exponential data and graphs,
- graphing and interpreting linear equations, including understanding the slope of a line,
- solving simultaneous linear equations,
- total/average/marginal concepts,
- exponential functions.

In addition to the above list, we identified topics specific to the needs of LaGuardia students including absolute value inequalities, rational functions, and trigonometric functions. After the discussions, the economics faculty presented a full list of mathematical topics along with some detailed economics examples that could be used to illustrate the topics. The list can be found in Appendix A.

These discussion sessions helped us collect information on the mathematical needs of students in economics courses at LaGuardia and find connections between mathematics and economics. The conversations led us to a better understanding of the challenges inherent in knowledge transfer across disciplines. For example, presentation styles and notations differ significantly between disciplines. As a result, in order to develop a deeper understanding of the partner discipline's presentation style and how certain concepts appear in both disciplines, as well as other challenges students face when applying mathematics concepts and skills in economics contexts, the mathematics faculty visited the economics courses from time to time.

Input from Economics

In addition to discussions between the mathematics and economics faculty, we also gathered other data to help understand students' mathematical needs in economics courses. We conducted preliminary surveys on students' attitudes toward mathematics and the extent to which they connected mathematics with economics. We also conducted a preliminary assessment in an economics course to identify students' common weaknesses in mathematics.

From the previously mentioned interdisciplinary faculty conversations and class observations, we found that many students in economics courses struggled with quantitative skills. Many had trouble with the basics such as understanding tables, creating and interpreting graphs, and even working with fractions, decimals, and percentages. As a result, many students were not able to develop the ability to explain and apply quantitative concepts and interpret results, which are highly valued skills in economics. Such students needed a variety of basic mathematical knowledge to successfully complete their economics courses. We aimed to further study this issue in two steps. First, we assessed students' attitude towards mathematics and identified weaknesses in mathematical skills that are relevant to economics courses. Second, we intervened in an economics course by incorporating guided mathematics exercises and explicitly making the connection between these mathematical skills and relevant economics topics.

At the beginning of the semester, to identify students' mathematical weaknesses, all students in a microeconomics class took a diagnostic online mathematics pretest (see https://bit.ly/33n3h2m). The test was designed by faculty at Syracuse University as a general mathematics pre-assessment for economics students. No previous knowledge of economics is necessary to take this test. Mathematics skills that are assessed include fractions, decimals, percentages, ratios, proportions, graphs, linear and nonlinear relations, and basic algebra. We then mapped these basic mathematical skills to each microeconomics topic in the course syllabus. For each lesson, we listed both the economics and mathematical goals. We also experimented with an intervention strategy using guided mathematics exercises. A self-tutorial platform, Khan Academy (Khan Academy, 2018), was used for guided exercise to help students work on their weaknesses, as identified in the pre-assessment, and to apply basic mathematical skills, such as algebra, to the analysis and interpretation of quantitative information in the context of microeconomics. Students were required to watch short videos and complete weekly assignments on the Khan Academy website.

Qualitative and quantitative data on students' performance and attitudes towards mathematics were collected at the beginning and at the end of the semester. Students completed pre and post attitude surveys to determine if they were able to make the connections between their mathematical skills and what they were learning in the microeconomics course. A performance comparison was also conducted to ascertain the effectiveness of the intervention, using guided mathematics exercises. In terms of performance, the grade distributions of the class that had not been given guided exercises in spring 2017 (no intervention) and the class that had been given guided exercises in spring 2018 (with intervention) were collected and compared. From the data, there was no significant difference between the two grade distributions. However, the intervention did help some students recognize the relevance of mathematics to microeconomics as evidenced by the following comments by students in the 2018 group:

Comment 1: "The math that was required was not difficult but needed practice to remember. The math tutorials were crucial in equipping me with the necessary math tools for understanding microeconomics."

Comment 2: "I believe that in order to be successful in this course, you would need to have a good understanding of elementary math."

Comment 3: "The class helps me to see the usefulness of math in our life. We are using math to solve the problems in our life, and it is really helpful."

Comment 4: "Microeconomics is totally related to the math, so students should take MATH115 or 120 before this class."

Comment 5: "Being the type of student that strongly despises math, it is being so present in the microeconomics curriculum absolutely deterred me from the subject. My brain simply rejects the field of math. It is obviously up to me to rectify this behavior and put some efforts. But I do wish we were to study the field of economics with math."

Key Elements in the Design of Assignments

Through conversations with other mathematics instructors, we identified the following challenges when designing, implementing, and evaluating assignments that incorporated interdisciplinary contexts: (a) it is very time-consuming to find meaningful real-life problems to embed in mathematics teaching; (b) there is limited class time for students to develop a minimal level of familiarity with the interdisciplinary topics; (c) instructors need some training to design project assignments that address the components of one of LaGuardia's assessed core competencies, namely inquiry and problem-solving.

Noting these challenges faced by mathematics faculty members in general, we designed assignments with the intention that they eventually be used by all sections of College Algebra. In order to ensure a smooth scaling up of our platform of content to more sections in the later stage of the project, the content of the assignments should be accessible to all students and instructors. Very few College Algebra students and instructors have backgrounds in economics. With this in mind, we incorporated three pillars when designing our projects—relevance, scaffolding, and emphasis on written ability.

Relevance

Project assignments should be relevant to students with different academic backgrounds. We should motivate each mathematics and economics concept with real-world examples drawn from student life or subjects they are familiar with or can readily relate to.

Scaffolding

Faculty should be able to use our content with minimum preparation. In addition, we want our project assignments to be self-contained so that students can work independently if they need to. To this end, we scaffold our content. First, each project starts with small group discussions among students. Faculty members can then orient the class with the project topic through a class discussion. Students will have a chance to express their opinions about the key issues addressed in the assignment during the discussion. This sort of personalization will improve student motivation by giving them a sense of ownership over the topic. The group discussions also include technology demonstrations in the form of simple web-apps. Instructors can demonstrate these applications to help students develop intuition about the quantitative aspects of a project. Students can also play with these applications at home. These applications offer students a hands-on introduction to the topic and terminology without requiring them to fully understand the underlying mathematics. An example of one of our web-apps is an activity called Price and Profit (Henshaw, 2019). By varying parameters such as fixed and variable costs, students can see how prices are related to profits. In order to facilitate out-of-classroom learning, for each project assignment, we provide links to a set of videos on an online platform that demonstrate key concepts and techniques of mathematics and economics. Students can practice with the content presented in these videos by doing short sets of accompanying exercises. These sets of short-answer questions are graded automatically so students receive instant feedback.

Emphasis on Written Ability

The final component of each project assignment is an essay prompt which gives students an opportunity to communicate their ideas in writing. The essay is important because it helps students to self-identify areas of confusion or conceptual misunderstanding. It also allows students to meaningfully reflect on the opinions they expressed during the initial class discussion. Written communication makes an assignment memorable and engages students' quantitative reasoning skills.

Sample Assignment

Students apply quadratic functions and vertex formulas to maximize profit and revenue and minimize cost in one assignment we developed (see Appendix B). Topics such as profits, revenues and costs are relevant to many students enrolled in College Algebra as many of our students are business majors or are interested in starting their own business. LaGuardia has a relatively large percentage of non-traditional students, many of whom have experience working in or owning a business. We also attempt to emphasize the relevance of mathematical concepts by incorporating role-playing and using real-life examples. Students are asked to imagine themselves as an executive of a hypothetical smart phone company with a name that reminds students of a very familiar smart phone, the iPhone.

Because students and course instructors may not have rigorous backgrounds in economics, we scaffold the assignments. Students are introduced to economics concepts with question prompts as well as with video clips. They are guided though the activity with problems that require short answers. For example, students are introduced to the concepts of profit, revenue, and cost through the first few question prompts. Students watch video clips that explain these economics concepts and how quadratic functions and vertex formulas can be applied to the problem situation. They complete short online exercises requiring the use of quadratic functions and vertex formulas. By the end of the assignment, students are expected to solve more complex questions. For example, they are asked to solve a couple of maximization problems by applying quadratic functions and vertex formulas in the given scenario.

Finally, students are asked write an essay. Students are expected to include their answers to all of the questions in the assignment in the essay. By communicating their answers in writing rather than merely solving mathematical equations, students learned to integrate mathematics in an interdisciplinary context rather than through seemingly meaningless symbolic manipulations.

Implementation of the Assignments

At LaGuardia, College Algebra is one of the mathematics courses which assesses students' inquiry and problem-solving skills, one of the three core competencies in LaGuardia's general education program (see https://www.laguardia.edu/assessment/). The assessment is based on two to three required project assignments in which students are expected to apply mathematics knowledge to solve real world problems. The assignments we created help the mathematics faculty meet the requirements of this assessment. In part, as a result, the faculty are receptive to using our instructional platform.

As we did not expect all instructors to have an economics background, we offered a workshop to share the philosophy and help with the implementation of the projects. Twenty-six mathematics instructors attended the workshop. Throughout the workshop, the issues and strategies for implementing assignments were fully discussed. Participants cited time constraints as an issue. The additional amount of time needed to guide students through the completion of an interdisciplinary integrative assignment could result in instructors not being able to cover the entire course curriculum as required by the department. Furthermore, participants observed that many students were predisposed to perceive mathematics as a standalone subject and consider mathematics assignments that integrated contexts from another discipline as "extra work". Nevertheless, participants in the workshop agreed that integrative projects help students develop confidence and broaden their awareness. They suggested that these assignments should be assigned early in the semester whenever possible when both instructors and students were less busy and less stressed. They also suggested that instructors should provide students with guidance on the economics context and the available resources for completing the assignments. They also suggested that if these projects were given as group assignments, formal group roles should be assigned and each student should be held accountable.

During the workshop, we briefly described the SUMMIT-P project and LaGuardia's involvement in the project. We outlined the philosophy and goals of the SUMMIT-P project and demonstrated to participants how to import these projects into an online course platform. In order to gain insight into the student experience, workshop participants completed the project assignments in groups. Most participants agreed that these projects were appropriate for students in College Algebra and aligned with LaGuardia's inquiry and problem-solving core competency. In Spring 2019, these project assignments were piloted in 15 out of approximately 40 sections of College Algebra. The workshop was supported by the Center of Teaching and Learning at LaGuardia and CUNY Open Education Resources Initiatives (see https://www.cuny.edu/libraries/open-educational-resources/).

Learning Community: A Course Pair

In order to further expand the context of economics within the College Algebra curriculum, we developed a learning community in the form of a course pair between College Algebra and Microeconomics. The assignments we developed were incorporated into the course pair. Different higher education scholars and practitioners define learning communities differently. LaGuardia's learning communities are essentially curricular learning communities. The same group of students enroll in two or more courses from different disciplines thematically linked in such a way that students find increased coherence and integration across these courses (Lenning & Ebbers, 1999). At LaGuardia, this type of learning community is called a course pair if it consists of two courses and is called a course cluster if there are more than two courses involved. Students enrolled in the pair are to take both courses in the same semester.

A learning community that links courses and enrolls the same cohort of students has been found in previous research to help improve students' experiences in those courses, thus nurturing positive attitudes toward the subject matters (Lenning & Ebbers, 1999). The initial stage of developing our course pair involved reviewing the syllabi of both courses to map topics and timelines (see Table 1). The goal was to incorporate the context of microeconomics in College Algebra whenever possible. After the review, project assignments were designed for five chosen topics in College Algebra (see the Creating Assignments section above and Appendix B). The points in the semester at which the projects were assigned were carefully chosen so that the College Algebra topics aligned with the corresponding topics covered in Microeconomics. This helped to ensure that students were familiar with the related topics in Microeconomics while working on the projects in College Algebra. Students received course grades for the assignments in College Algebra.

The major challenge we faced when launching our course pair was enrollment. Due to low enrollment, we were not able to offer the course pair in Fall 2017 and Spring 2018 as we had initially planned. However, we successfully offered the course pair in Fall 2018 after adapting a strategy we learned from a SUMMIT-P project partner institution, Saint Louis University (SLU).

In April 2018, the LaGuardia SUMMIT-P team conducted a site visit to SLU, where we visited Professor Mike May and his project team. Similar to LaGuardia, SLU was also incorporating business and economics contexts in activities and assignments in College Algebra and Calculus. While visiting SLU, we learned that professional academic advisors promoted the newly-designed, SUMMIT-P-inspired college algebra course to students.

Learning from SLU's example, the LaGuardia team shared our project efforts with program directors and academic advisors. In particular, we promoted the course pair to Accelerated Study in Associate Programs (ASAP) at LaGuardia, a program that provides resources such as advising and financial assistance to help students complete their associate degree programs as soon as possible. This approach resulted in increased advisors' awareness of the pair and its mission and benefits. Conversations with students and advisors at ASAP led to the identification of several problems: (1) students were typically told that these two courses were “very difficult,” thus it was not advisable to take them together, (2) many students took College Algebra several semesters before they took economics courses, and (3) students were advised to take other mathematics or statistics courses instead of College Algebra for the purpose of transfer. After being made aware of these problems, we worked closely with advisors to help ameliorate some of these issues. Our efforts paid off and we were finally able to fill the pair with 29 students in each course section in fall 2018.

Table 1
Curriculum Mapping Between College Algebra and Microeconomics

Topics in College Algebra	Topics in Microeconomics
linear equations	equations of demand and supply curves
system of linear equations	market equilibrium
difference quotient	marginal value (marginal cost, marginal revenue, marginal product, marginal utility); elasticity
operations of polynomials	cost function, production function in polynomial functions
quadratic functions	cost and revenue functions
rational functions	cost function, production function in rational function
inverse function	converting demand/supply functions where the price depends on the quantity demanded/supplied
exponential function	compound interest; growth in macroeconomic variables such as GDP, price level and others. Exponential consumer utility function
exponential equations	consumer preferences
logarithmic functions	compound interest; growth in macroeconomic variables such as GDP, price level; logarithmic production function; logarithmic consumer utility function
logarithmic equations	consumer preferences; production decision

Course Pair Design

There were a few key elements essential to successfully implementing a course pair: (a) developing a sense of community across both courses, (b) maintaining regular communication between faculty members, (c) using similar language across disciplines and sharing resources, and (d) integrating the context in activities and assignments completed inside and outside the classroom. In order to support the sense of community in the pair, instructors in both sections frequently reminded students of the connection between topics and assignments in both courses. In addition, instructors of both courses communicated frequently to detect and discuss issues arising from the implementation so that we could fine-tune quickly. Each instructor used the partner instructor's nomenclature and integrated their course's resources whenever possible. This helped students to transfer contexts from one discipline to another more smoothly.

Also, co-curricular activities took place from time to time to further reinforce the economics context. For example, we organized a trip to the gold vault of the Federal Reserve of New York (FRBNY). FRBNY provided free guided tours for school groups, therefore it was logistically and economically accessible. Students were able to learn about the role of the Federal Reserve System and currency policies. More importantly, students learned about the importance and relevance of quantitative skills in economics policy-making. In addition, we also mentored a team of students in the course pair to compete in Math is Everywhere, a college-wide competition in which participating teams develop a research presentation that integrates mathematical skills with other disciplines. Our team developed the presentation "Quantifying Effects of a Minimum Wage," in which they applied college algebra skills to explain the quantitative implications of having a minimum wage. Last but not least, students in the course pair displayed their work at the annual LaGuardia Learning Community Showcase. These activities were very well-received. These opportunities further emphasized to students the importance and relevance of quantitative skills.

Discussions and Future Work

To date, we have created a series of college algebra assignments with economics contexts, experimented with these assignments in 15 out of approximately 40 sections of College Algebra at LaGuardia and also used the assignments in a course pair of College Algebra and Microeconomics. Throughout the process we learned that knowledge transfer across disciplines is not as straightforward as we had imagined. For example, both faculty members and students had difficulty switching between different notations and definitions used to refer to the same concept. We also observed that many students were not very motivated to complete the interdisciplinary integrative assignments because they tended to be more challenging and, perhaps more importantly, students had traditionally believed that mathematics was a standalone subject and perceived projects that integrated mathematics with another course discipline as unnecessarily burdening. We also speculated that other possible reasons were that students needed to take part in more in-class discussions, needed more guidance from instructors on how to complete the assignments, and needed more information on the economics concepts in each assignment as compared to regular assignments. In other words, they found it difficult to transfer knowledge from one discipline to another.

It was challenging for faculty to devote more time to these assignments as compared to their regular activities and assignments while also fulfilling department-related curriculum requirements. In addition, as most mathematics faculty members did not have much of a knowledge base for economics, it was challenging for faculty members to enrich the economics context with narratives and stimulate dynamic student engagement in class discussions.

We also found that learning communities such as a course pair, while useful in reinforcing the connection between mathematics and other disciplines, could also present challenges. Other than issues surrounding enrollment as described above, the performance of many students in the pair was worse than that of standalone courses. One of the reasons for this could be sample selection bias. As mentioned above, many students at LaGuardia tended to take College Algebra several semesters before they take Microeconomics. Those who waited until later semesters to take College Algebra might have been students who were weaker in mathematics and feared the subject the most.

We have continued to fine-tune our project to address the above challenges. First, we have converted notations and definitions in the assignments to better match what is used in economics. For example, in a demand curve, the Y variable in College Algebra typically denotes the price while the variable P is used in economics. Similarly, the X variable is used to denote the quantity in College Algebra while the variable Q is used in economics. We are encouraging College Algebra instructors to help students translate between the two sets of variables. Second, in order to mitigate issues related to the lack of economics background among students and mathematics faculty members, we are contemplating a crash course in sections of College Algebra taught by a guest economics faculty member twice during the semester. These crash courses may provide opportunities for students to solidify their understanding of the relationships between mathematics concepts and skills and the economics concepts in the assignment. Moreover, the guest lecturer could share narratives with the class that further enrich the context. Third, we are also contemplating making a collection of narratives on the economics concepts covered in the assignments readily available to mathematics faculty who teach College Algebra. Fourth, we plan to include extra-curricular activities such as those mentioned above to further raise awareness among students that mathematics is not just a standalone academic subject but is applicable to many fields and disciplines.

Despite all of the challenges outlined above, we found, through questionnaires and conversations with faculty members and students, that students in our SUMMIT-P college algebra project seemed to have an increased appreciation for the relevance and usefulness of mathematics. In the near future, piloting these assignments in more sections of College Algebra will allow us to quantitatively assess the effectiveness of these integrative projects. In particular, we believe the following research questions are worth investigating:

- What are students’ perspectives on mathematics in relation to their academic and career goals?
- How does the experience of applying mathematics to solve real world problems change students’ attitudes towards mathematics and mathematics learning?
- How has the pilot affected students' performance in current and future mathematics courses?

The data we collect will shed light on these questions. Moreover, we will collect feedback from course instructors and continually update these integrative projects to better accommodate the needs of both instructors and students.

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Appendix A

Economics faculty summarized their mathematical needs in economics courses as the following wish list.

1. Basic arithmetic and algebraic skills—equations and algebra, effects of changing parameters in linear equation.
2. Calculating area of relatively simple geometric figures.
3. Plot numbers, interpret graphs, linear/exponential data/graph.
4. Graphing and interpreting linear equations, slope of line, slope at a point, increasing at an increasing/decreasing rate.
5. Two linear simultaneous equations.
6. Total/average/marginal concepts.
7. Natural log for linearizing data.
8. Compound interest rate.
9. Using multiple variables/parameters, variables names other than x, y parameters can be stated numbers or alphabetical letters, practical limits on the values of variables.
10. Supply-And Demand Analysis:
    - Graphs (P is treated as the dependent variable and graphed on the Y-axis. Quantity Q is regarded as the independent variable and is placed on the X-axis).
    - Systems of Equations (implicit or explicit form) (Solving a system of supply and demand equations algebraically to find the equilibrium point (Price and Quantity).
11. Break-even Analysis:
    The breakeven point (Profit = 0) is found both algebraically and graphically by solving linear or/and quadratic functions (Revenue and Cost curves)
12. Income Determination Models:
    $Y = C + I + G + (X - M)$ where Y= Income; C=Consumption; I = Investment; G = Government Expenditures; X = Export; M = Import. By substituting the information in a model, we solve for the equilibrium level of income. Aggregation of the variables on the right-hand side of the equation allows the equation to be graphed in two-dimensional space.
13. IS-LM Analysis:
    IS-LM analysis seeks to find the level of income and the rate of interest at which both the commodity market and the money market will be in equilibrium. This can be accomplished with the techniques used for solving a system of simultaneous equations. (Matrix Algebra).
14. Optimal Allocation of Scarce resources among Competing Products:
    Optimizing a profit or cost function subject to several inequality constraints by way of linear programing (using graphs).
15. Maximization and Minimization (Cost, Revenue, and Profit) :
    Math Concepts covered: Maxima, Minima, Increasing and Decreasing Functions, Concavity and Convexity, Quadratic and Cubic functions, inflection Points, and Optimization Functions. (Differential Calculus: Uses of the Derivative)
16. Production functions, Economic Growth, Homogeneous Function, Production curve:

Estimating growth rates from data points. Profit or Sales function growing consistently over time, annual growth rates can be measured and a natural exponential function estimated through a system of simultaneous equations.

17. Exponential and Logarithmic Functions.
18. Consumers' and producers' surplus:
    - Shaded areas under/above Demand/Supply curves below/above price line calculated mathematically by way of Integrals Calculus.
    - The shaded areas (triangles…) are used to analyze the market efficiency and the economic welfare of the society.
    - Integral calculus is also used to integrate a marginal cost function to find a total cost function or to integrate marginal revenue function to find a total revenue function.
19. Natural Growth of Income, Investment Theory, Capital Accumulation, and Marginal Efficiency of Capital and Consumer Demand Elasticity
    Math Topics covered: Natural log, Integration

Appendix B

Assignment: Analyzing Cost and Revenue

Introduction
Cost, revenue and profit functions may take parabolic forms. In many business and economics applications, our most important goal is to maximize revenue, profit or minimize cost. We may be able to find the price or the quantity of goods and services that maximizes profit, revenue and minimizes cost by using quadratic formula and vertex formula. The goal of this project is to enhance the understanding of quadratic functions and how to find the maximum/minimum.

Question 1:
If you have the chance to start a business, what business would you choose? Why?

Question 2:
Starting and running a business requires time, effort, hard work and in particular money. What kind of costs do you expect to have to pay in order to start and run your business? Please list them and explain why you need them.

Question 3:
Some costs are fixed, which are called **fixed costs**, such as equipment and buildings. Some cost are variable, which are called **variable costs,** such as labor and material. Please explain what costs in the Question 2 are fixed costs, and what are variable costs.
In general, the total cost consists of variable costs and fixed costs.

Question 4:
In order to keep your business running, you need to make **revenue**. Revenue is the money that comes into the business from customers. Suppose you know the number of products your business sold and the price you sold them at, how can you calculate the revenue? What strategies could you use in order to increase your business' revenue?

The revenue of a business may go up and down depending on many factors. For example, a business that sells ice cream will likely make more money during hot summer months. The profit of your business is the difference of the revenue and the cost. That is,

Profit = Revenue - Cost

If the profit is the positive, your business makes money. If the profit is negative, your business unfortunately makes a loss. If the profit is zero, that is the revenue is equal to the cost, it is called the break-even point.

Suppose that you were the CEO of a giant high technology corporation, Strawberry, Inc, manufacturer of the Strawberry Phone.

Question 5:

This month you have estimated the demand for the Strawberry Phone to be:

$$Q = 220 - 4P$$

where Q is the quantity demanded, and P is the price of a Strawberry Phone.

The cost of producing a phone is constant at $12, which is called marginal cost. The fixed cost that includes the cost spent on the factory, the equipment, among others is $1525. As a result, you have a linear cost function,

$$C = FC + (MC\ Q)$$

Where C is the total cost, FC is the fixed cost, and the MC is the marginal cost, and Q is the quantity as before.

Answer the following questions.

1. What is the price that maximizes the corporation's profit? (Hint: Profit = Revenue - Cost)
2. At what price does the corporation break even?

Question 6:

The other day, as you are burying yourself in the sea of data, you find out that at $500, your corporation sells on average 50,000 Strawberry Phones, the corporation's flagship product, monthly. Additionally, for every $50 increase in the price of a phone, the sales decrease by 1,000 phones. Based on this information, you are very interested in finding

1. the price point that will maximize the revenue, and;
2. what the maximum revenue is

Question 7:

From the two questions above, create a strategy to lower your cost and maximize your profit for the business you chose in question 1.

Essay: Write an essay that discusses the answers to the questions above include a detailed description of your business ideas and how it is possible to use maximization of quadratic functions to find the maximum profit.

PROMOTING PARTNERSHIP, CULTIVATING COLLEAGUESHIP: THE SUMMIT-P PROJECT AT NORFOLK STATE UNIVERSITY

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ABSTRACT
Norfolk State University (NSU) is the only public Historically Black College and University (HBCU) member institution in SUMMIT-P. At NSU, a strong collaboration between the Department of Mathematics and its partner discipline, the Department of Engineering, has been established for the Calculus I and Differential Equations classes as part of the SUMMIT-P project. In this paper, we record a brief history of this collaboration project at NSU, the various structures within the SUMMIT-P Project, the site visit that occurred in Spring 2019, and how recent activities helped guide the direction of the project at NSU.

KEYWORDS
collaboration, partner discipline, fishbowl, site visit, engineering applications

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What does it take to teach mathematics effectively to engineering students in a public Historically Black College/University (HBCU) like Norfolk State University (NSU)? Since a strong foundation in mathematics is necessary to be successful in upper-level engineering courses, how can mathematics faculty help engineering students appreciate and gain core mathematical knowledge? What are some concrete curricular actions that mathematics and engineering faculty can adapt to improve student success and retention? These are some questions that motivated our project, which is part of a National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P). The project team at NSU, which consists of faculty from the mathematics and engineering departments, has the overall goal of helping engineering students become successful in their mathematics courses while also preparing them for future engineering courses. In the spirit of the Mathematical Association of America's Curriculum Renewal Across the First Two Years (CRAFTY) project (Barker & Ganter 2004; Ganter & Haver 2011), we strive to promote partnership and cultivate colleagueship between the mathematics and engineering departments in the College of Science, Engineering, and Technology (CSET), hoping that perspectives and applications from outside the mathematics classrooms will influence vital changes towards a mathematics curriculum that will provide a solid foundation for undertaking upper-level courses.

The Curriculum Foundations reports (Barker & Ganter, 2004; Ganter & Haver, 2011), which were developed as part of CRAFTY, included recommendations that were distilled from the work of several collaborative workshops that took place between 1999 and 2001. Regarding classroom skills, the recommendations for the mathematics community included: emphasize conceptual understanding, problem-solving skills, mathematical modelling, communication skills, and balance between perspectives (i.e., continuous vs. discrete, linear vs. nonlinear, etc.). Regarding instructional techniques, the recommendations include: use a variety of active learning methods, use appropriate technology, and improve interdisciplinary cooperation. Embracing the spirit of these recommendations, the NSU Department of Mathematics aims to provide high quality and relevant mathematics courses to serve our HBCU students.

The SUMMIT-P consortium (see www.summit-p.com for more information) is a collaboration between ten universities nationwide with an overarching goal to revise and improve the curriculum for lower division undergraduate mathematics courses, with a focus on the content and skills needed for student success in partner discipline fields. The collaboration happens in two levels: an intra-collaboration among member institutions and an inter-collaboration between selected departments in each member institution.

This paper is organized as follows: first, we record the history of SUMMIT-P at NSU; second, we narrate the highlights of the site visit; third, we describe the next steps in the SUMMIT-P programmatic structure, starting in Fall 2019; and fourth, we share some reflections on what we have learned thus far, highlighting the effectiveness of integrating classes and team-teaching in achieving our SUMMIT-P goals.

Brief History of SUMMIT-P at NSU

Joint Departmental Summit

The SUMMIT-P project at NSU began with a joint department summit between the faculty in the Departments of Mathematics and Engineering in Fall 2016. All full-time faculty from both departments were invited to attend the summit. The four-hour summit began with the introduction of the project goals and objectives, followed by a fishbowl activity (Hofrenning, et. al, 2020). The main goal for the first semester of the project was to establish a Faculty Teaching and Learning Community between the two departments.

Throughout the summit, the leadership team reiterated the vital role that the partner discipline, engineering, plays in the success of the project. It was, hence, imperative that the partner discipline feel free to express their likes and dislikes as they related to the mathematics curriculum and the prerequisite knowledge their majors develop in courses such as College Algebra, Trigonometry, Calculus, and Differential Equations, which are the required mathematics courses for success in upper-level engineering courses. The team implemented the fishbowl activity to learn about the wants and needs of engineering faculty. In this activity, mathematics faculty served as silent partners. They gathered around a group of seated engineering colleagues while they articulated their concerns with their students' mastery level of certain mathematical concepts. The engineering faculty shared a list of mathematical topics they hoped would be covered in entry-level mathematics courses. The mathematics faculty tried to absorb and understand their colleagues' wish list. Some of the topics in the engineering wish list included conic sections, trigonometric functions and equations, solving for variables in a literal equation, and interpreting graphs. The fishbowl activity proved to be the most productive part of the joint department summit.

Pilot Courses

In Spring 2017, the leadership team decided that sections of Calculus I for engineering majors would be revised through the SUMMIT-P project. Since this would be the first course that the project would focus on, a member of the leadership team assumed the responsibility of teaching the course and implementing some of the recommendations gathered during the joint department summit. This course includes a one-hour session per week for students to engage in problem-solving or attend engineering presentations. The SUMMIT-P team member administered the problem-solving sessions, choosing specific examples that addressed some of the recommendations gathered during the previous semester's summit. For example, when the course content featured finding tangent lines to curves, the instructor drilled the students on finding horizontal tangent lines for trigonometric curves. As a result, during the problem-solving session, students were solving calculus problems using precalculus concepts that were identified as student weaknesses by engineering faculty during summit. During other weekly one-hour sessions, engineering faculty delivered presentations on engineering applications of calculus. Over time, this aspect of the course has evolved into a series of organized presentations delivered by faculty and graduate students from both departments. These presentations are given in either the regularly scheduled classroom or in an engineering laboratory.

In Fall 2017, the focus of the project shifted to the topics in Differential Equations due to their importance in modeling engineering applications in virtually every area of engineering. The leadership team realized the possibility of presenting common applications in both the sections of Calculus I for engineering majors and in Differential Equations. Since many applications modeled using differential equations can be discussed in a calculus course without the need for a detailed discussion of the mathematical modeling, including these applications in both courses helps to make meaningful and useful connections between the courses for students. For example, an engineering-focused modeling application was presented as a topic of discussion in the Differential Equations course, while significant class time was spent developing and analyzing the solution to the same application in sections of Calculus I for engineering majors. To date, this shared application course feature remains as one of the effective program structures implemented as a result of the NSU SUMMIT-P project. We subsequently focused on improving these courses.

New Leadership Team

The departure of one of the members from the initial project leadership team warranted formation of a new leadership team. Maintaining the original team composition, two mathematics faculty and one engineering faculty began their service in February 2019. The new team immediately commenced work to prepare for a site visit in April 2019. The site visit, one of the SUMMIT-P consortium's essential program structures, is where faculty from one member institution visits another member institution (Piercey, et. al, 2020). The new NSU leadership team used this opportunity to re-examine and revitalize the program at NSU, ensuring that the aims of the consortium will continue to be actualized. Without a doubt, the site visit was the impetus that inspired the NSU team to show that effective collaboration can translate ideas into action.

Site Visit at NSU

During the site visit in Spring 2019, the Virginia Commonwealth University (VCU) leadership team and members of the grant management team from Embry-Riddle Aeronautical University (ERAU) and Appalachian State University (ASU) visited NSU for two days. The visitors attended the sessions of courses that had been revised during the first two years of the project, interviewed students taking the courses, met with NSU administrators, held discussions with faculty from both departments, and most importantly, engaged in a deep conversation with the NSU leadership team. Table 1 provides a brief overview of the site visit events:

Table 1
Site Visit Schedule for Spring 2020

Day 1	
	Classroom Visit I – College Algebra
	Discussion with students
	Luncheon with NSU administrators and CSET faculty
	Classroom Visit II – Combined sections of Calculus I and Differential Equations
	Discussion with Mathematics and Engineering Faculty
Day 2	
	SUMMIT-P Meeting between leadership team and site visitors

Teaming Up with Another NSF Grant

At the start of the Spring 2019 semester, even before preparations were made for the site visit, the SUMMIT-P leadership team collaborated with a group of NSU mathematics, biology, and chemistry professors who were working on a National Science Foundation funded grant project, Targeted Infusion Project: Engaging Students for Higher Retention and Building Stronger Foundations in Pre-Calculus Using the Flipped Model (NSU-TIP), to address the mathematics deficiencies of students majoring in biology and chemistry. Poor performance on the university placement test derailed most biology and chemistry majors from graduating within four years. They noticed that, like students majoring in engineering, incoming biology and chemistry majors failed to understand the important connections between mathematics and physical science disciplines. As a result, even those who successfully completed introductory level courses like College Algebra did not perform well in general chemistry courses.

True to the spirit of collaboration and partnership, the NSU SUMMIT-P and NSU-TIP leadership teams decided to work on a course aimed to support the objectives and initiative of both projects. The result of this collaboration was the creation of sections of a three-credit College Algebra course for biology majors and a one-credit College Algebra Lab for biology majors. The laboratory course has been taught by a member of the SUMMIT-P leadership team since it was first introduced. Like the pilot Calculus I sections described above, one of the goals of this laboratory course is to provide a platform for professors in disciplines other than mathematics to present applications of mathematics in biology and chemistry. By the end of the Spring 2019 term, nine different presentations were made by biology and chemistry professors, and five applications were presented by the students enrolled in the laboratory course.

During the site visit, the visitors attended two classes, including a section of College Algebra Lab for biology majors. The session covered an application about creosote exposure of fish and other creatures in the Elizabeth river and was team taught by mathematics and biology faculty. Through this collaboration, the biology faculty demonstrated the use of mathematical concepts in performing biology experiments. For instance, using some laboratory equipment, the biology faculty demonstrated and explained certain issues in the laboratory that can be solved using correct unit conversion procedures and understanding scales of magnitude. The mathematics faculty highlighted the importance of the mathematics concepts during the demonstration. The duo concluded their presentation by posing a question to the students: “Why should a biology major study mathematics?”

The official partner discipline of NSU SUMMIT-P is engineering and the mathematics courses that are the primary focus of the project are Calculus I and Differential Equations. It should be noted, College Algebra Lab for biology majors is not part of the project. However, due to the small size of our mathematics department and the fact that members of the leadership team have connections with faculty that are involved in NSU-TIP grant, we decided to use the site visit as an opportunity to work on and get feedback on both projects. This decision allowed us to accomplish objectives for both projects and show to the site visitors the kind of collaboration that exists at our university. We plan to include biology or chemistry as partner disciplines in future projects.

Two Math Classes, One Engineering Lab

During the site visit, visitors observed another class that gave them the opportunity to interact with students taking Calculus I and Differential Equations. During this class session, each student in Differential Equations, a junior level course at NSU, was paired with a student taking Calculus I to analyze a circuit using the Digilent Analog Discovery Design kit and MATLAB/Simulink computing software. A short lecture by the course instructor on solutions of a second-order differential equations set the stage for a presentation by an engineering faculty member on the design and analyses of certain circuits. What happened next was so promising that the SUMMIT-P leadership team decided to make the activity an integral part of both courses. The synergy between the faculty and students made for such a productive class. The Calculus I students, eager to learn from the upperclassmen, continuously asked questions and received advice about the mathematical concepts that are necessary to succeed in engineering courses. Faculty reported overhearing advice from upperclassmen such as “make sure that you know how to interpret graphs because you will need this in your engineering labs.”

It is well-documented that an active learning experience in mathematics is one of the most effective ways to help address both the learning issue and the diversity issue in a mathematics classroom (Herzig, 2005). Furthermore, in an HBCU mathematics classroom, students learn best when they have the chance to engage socially (Ross, 2014). We are pleased to report that we were able to verify that an active learning experience coupled with social engagement was effective in helping our students appreciate the Calculus I course topics. Moreover, with such a discipline-specific context, we were able to provide a learning experience to the Calculus I students that will hopefully increase their elementary mathematics abilities, enabling them to transfer skills from Calculus I to their engineering courses.

Insights from the Outside

The site visit concluded with a lengthy conversation between the NSU SUMMIT-P leader and visitors about approaches to strengthen the existing mathematics-engineering learning community and the overall quality of the project at NSU. The two-hour brainstorming session that took place proved to be beneficial to the host institution. To engage a greater number of engineering faculty in the SUMMIT-P project, the visitors suggested that faculty pairs be formed, one member of the pair from each department, to collaborate on developing an original engineering application.

The Next Steps

The NSU SUMMIT-P leadership team was encouraged to design two additional program structures in the form of workshops to further strengthen the connection between the mathematics and engineering departments. The first workshop took place at the end of the Spring 2019 semester and a second workshop was scheduled prior to the Fall 2019 semester. Like many other program structures, these workshops are active processes that aim to promote partnership and cultivate colleagueship between the two departments.

Spring Workshop for Consultants

Four faculty from each of the departments were invited to a two-hour workshop to learn about becoming an NSU SUMMIT-P faculty consultant. Consultants are faculty who create application modules that utilize and showcase their academic research expertise. A big part of this workshop followed a modified “speed-dating” activity in the following way: the eight faculty members were divided into four groups, and each group was given ten minutes to talk about their research interests and teaching experiences. The leadership team decided to impose a time limit so that the faculty participants would focus on the salient points of their research and teaching experiences. After ten minutes, the teams were rearranged to allow for meeting other participants. At the end of the speed-dating activity, the mathematics faculty were asked to leave the room while the leadership team gathered opinions from the engineering faculty. In the end, the leadership team decided on which faculty to pair to co-develop activities. With eight faculty consultants, we formed four faculty pairings based on the following guidelines: pair a senior faculty member with a junior faculty member, match the professional personalities and teaching styles, and align the research interests as close as possible. The first guideline had a bonus effect of providing a mentoring opportunity for the pairs, while the third encouraged possible research partnerships. The second guideline, arguably subjective in nature, was introduced to ensure a productive and supportive relationship.

Creating Curriculum Content

Within this faculty consultant program structure, each member of the pair had specific responsibilities. The main task for the engineering faculty member was to suggest various engineering applications in which mathematical concepts play pivotal roles in the modeling process. The main task of the mathematics faculty member was to make sure that the developed lesson modules were written and planned in such a way that they would complement and supplement the existing mathematics curriculum and syllabi of the course the application would be used in. The goal was for each pair to create a lesson module that could be used in one or more of the following courses: Pre-Calculus, Calculus I, or Differential Equations.

Table 2
Timeline for Completion of Intended Tasks During Year 3

Task to be completed	Due Date
Teaching abstract of application module—submitted for review by the NSU SUMMIT-P team	7 June, 2019
Results of the review	14 June, 2019
Lesson plan completed	19 August, 2019
Powerpoint prepared	23 August, 2019
Classroom presentation by engineering faculty	Fall semester

The lesson module had to feature an engineering application of mathematics concepts. The intent was that the application would be related to the engineering faculty member's own research or be an application studied in an upper-level engineering course. Each team was expected to submit an instructional abstract, a lesson plan, a PowerPoint presentation, and a few assessment tools for their proposed lesson module. The abstract requirements were an engineering-based motivation for the project or application, a classroom implementation plan, and expectations for the students. It also needed to specify the mathematics topics that would be covered in the lesson and the level of understanding that students would need to successfully work with the content. The lesson plan included a list of required equipment, tools, or software, the time to complete the activity, and some assessment tools. The assessment tools took the form of a worksheet, quiz, essay prompt, mini-project, etc. The expectation was that the materials should provide all of the information that course instructors would need to be able to deliver the lesson by following the step-by-step instructions to guide students through a 40-minute activity. The PowerPoint instructions were to be student-friendly and contain the salient points of the lesson plan. The lesson plan was intended to serve as a guide for the instructors, while the PowerPoint served as a study guide for students' use.

Table 2 shows the timeline for the completion of the tasks. The NSU SUMMIT-P leadership team made sure that the faculty consultants understood the project expectations from the start. The leadership team accomplished this by establishing solid communication channels during the summer and requiring the teams to submit teaching abstracts before proceeding to the next steps in the module development. The teaching abstracts were carefully reviewed by the leadership team, checking for overlaps between the ideas presented by different pairs and originality of the engineering applications and ensuring that the proposed classroom implementation plan was viable and sustainable. We plan to publish a compilation of the application modules in the form of a teaching manual to be used as a supplementary course material for interested faculty.

The faculty consultants met a couple of times during summer, and members of the leadership team joined one of these meetings. The leadership team recognized that the faculty consultants would not be able to meet and work on the project during the regular semester.

One interesting application module that was approved by the NSU SUMMIT-P leadership team for use in Calculus I and Differential Equations involved a system of differential equations to model and analyze a wireless power transfer. The engineering professor on the faculty consultant team is an expert in wireless communications. Another interesting lesson module that was developed was an activity that featured analyzing the human heart's electrical behavior using data from electrocardiograms (ECGs). The engineering professor on the team who developed the module is an expert in bio-medical engineering. These two modules were incorporated in the section of Calculus I for engineering majors during Fall 2019. Faculty pairs worked together to deliver a 45-minute presentation about the module, including a short motivation video, a brief lecture using the prepared PowerPoint slides, and an in-class, hands-on activity. They made sure to exemplify respectful communication, took turns in explaining their lesson, and supported each other while in front of the class.

By planning and working together during the summer to develop the modules, we hoped to unite the two departments in the purpose of increasing the engineering majors' awareness about the direct connections that exist between the two disciplines. We succeeded in accomplishing this objective.

Summer Workshop on the Syllabus

Before the Fall 2019 semester began, the NSU SUMMIT-P leadership team and all eight faculty consultants met for a one-day follow-up workshop, where the consultants delivered short presentations about their application modules. The leadership team reviewed all the materials and made recommendations for refinements and improvements.

Furthermore, the event was an opportunity for faculty to discuss and revise the existing curriculum and syllabi for Pre-Calculus, Calculus I, and Differential Equations. Since the course coordinators for these courses served as faculty consultants and the chair of the mathematics department served as the NSU SUMMIT-P Principal Investigator, implementing changes to the syllabi was made possible in a more efficient way. Such advantages and logistical efficiencies are possible in a small institution.

What We Learned

This section highlights some insights and realizations that we learned as one of the main proponents of collaborative teaching explorations in our university. In particular, we record some of the benefits and challenges of integrating classes, team-teaching, student assessment, and presentation evaluations.

Integrated Classes

What happens when a laboratory session attended by students in Calculus I and Differential Equations and a specially developed engineering application module is the focus of the lesson? Our experience at NSU revealed many benefits for the students, most notable of which was the increase in peer engagement and classroom participation. Teaching the integrated class required significant preparation and extensive collaboration among the Calculus I and Differential Equations professors and the faculty presenters. Three different topics were covered in a 45-minute session. For example, when we implemented this integrated class structure in Fall 2019, the Calculus I students were learning curve sketching techniques using derivatives, the Differential Equations students were learning second-order differential equations with constant coefficients, and the faculty presenter's topic was simulating solution responses to circuits using differential equations. In order for this one laboratory session to be successful, the faculty invested significant time and effort in preparing the syllabi, students, and laboratory. As a bonus, we had considerable advantage on the scheduling aspect, since the project Principal Investigator, as the department chair, was the responsible party for the scheduling of mathematics classes. Going forward, we intend to explore further the possibility of more regular interaction of freshmen with upperclassmen through combined application sessions.

Team Teaching

Collaborative team-teaching is an important structure of the NSU SUMMIT-P program. Students reported that the presence of an engineering professor in a Calculus I course provided curricular coherence between mathematics and engineering courses. During the site visit, they also reported that hearing a mathematics faculty utter the words "I honestly did not know that algebra has so many concrete applications in biology" while inside a mathematics classroom made them realize that learning is a continual process, even for those with PhDs. This epiphany would likely not have occurred in a traditional lecture course without a team-teaching model.

Aside from the novelty of having another professor deliver lecture materials, students also appreciated the fact that the team-teaching faculty were exemplifying collaboration and respectful classroom communication. From the faculty point-of-view, the NSU SUMMIT-P leadership team hopes that team-teaching will help create stronger bridges of communication and understanding between various departments as well as foster respect of faculty for one another across departments. As an added bonus, the collaboration has also established research relationships between faculty partners. For example, a mathematics faculty and an engineering faculty, as a result of collaborating as faculty consultants on this project, are now working together on a research project to analyze a differential equations model of the propagation potential in the heart.

Assessments and Surveys

It is imperative that students understand that the application module presentations are supplements to their mathematics learning. Faculty presenters were given the freedom to select their own assessment tools that supported their active learning approach to the presentation. To make sure that students put value in the guest faculty presentations, the syllabi in the targeted courses specify that students are required to attend guest faculty presentations and to complete assessments prepared by the faculty presenters.

The students were given a short survey at the end of each presentation to gauge the effectiveness of the delivery. The survey did not measure the students' understanding of the content knowledge of the applications. The survey has a Likert-type scale (see Figure 1):

Figure 1
Student's Survey on the Faculty Presentations

	Strongly Agree	Agree	No Opinion	Disagree	Strongly Disagree
1. The presentation provided practical application.					
2. The presentation was informative.					
3. The presentation was interesting.					
4. The information presented help me see how Mathematics is used in Engineering.					
5. Handouts were valuable.					
6. The facilitator provided clear explanations.					
7. The facilitator presented material in an organized fashion.					
8. The facilitator stimulated interest and discussion.					
9. The facilitator was responsive to participants.					
10. The facilitator was effective in style and delivery.					

The students' feedback through this survey was helpful for faculty as they revised their presentations, and their comments were very encouraging in revealing their interests in future engineering classes and engagement in their discipline. In fact, the presentation made by one of the engineering professors in a laboratory session during the Fall 2019 semester helped him recruit a couple of freshmen engineering majors to work in his optical engineering laboratory. Indeed, we were able to achieve one of our goals at NSU—to establish meaningful academic connections for our HBCU students through mathematics.

A Work in Progress

At NSU, we are continuing to seek ways to improve and enhance the interdisciplinary partnerships through SUMMIT-P. As part of the SUMMIT-P consortium, NSU contributes by offering our HBCU students high-quality training that is comparable to what their peers receive in other institutions, increasing awareness of the challenges and issues that students and faculty encounter in an HBCU, and providing an opportunity to form close teaching, learning, and research connections and opportunities between the students and faculty of mathematics and other departments within the College of Science, Engineering and Technology.

What does it take to teach mathematics effectively to students in a public HBCU? We do not have undisputable answers to this important question. But at NSU, the SUMMIT-P leadership team and the faculty consultant teams recognize that providing quality mathematics education to our HBCU students requires multidisciplinary effort. We strive to promote partnership and cultivate colleagueship between departments in the College of Science, Engineering and Technology (CSET). It is our strong belief that perspectives and applications from outside the mathematics classrooms will influence vital changes towards a mathematics curriculum that will provide a solid and an excellent foundation for success in the upper-level courses.

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DESIGNING A STUDENT EXCHANGE PROGRAM: FACILITATING INTERDISCIPLINARY, MATHEMATICS-FOCUSED COLLABORATION AMONG COLLEGE STUDENTS

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ABSTRACT
Interdisciplinary collaboration is necessary for students' professional preparation (Laird et al., 2014; Repko, 2014) and may promote effective learning transfer of course content. Such collaborations have resulted in enhanced problem-solving skills and conceptual understanding of statistics content (Dierker et al., 2012; Everett, 2016; Hammersley et al., 2019; Woodzicka et al., 2015). As a result of ongoing collaborations between faculty members in different disciplines and at different universities, we created a "Student Exchange Program" to encourage interdisciplinary collaboration between undergraduate students in mathematics and social sciences. In the current paper, we describe past research that informed the design of this program, the specific steps taken to implement the program, preliminary results, and potential challenges to implementing and maintaining such an initiative.

KEYWORDS
interdisciplinary, collaboration, students, statistics

A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Teaching Partnerships (SUMMIT-P) is a large-scale project with the aim of implementing the recommendations from the Mathematical Association of America (MAA) Curriculum Foundations (CF) Project (Ganter & Barker, 2004) to revise lower division mathematics curricula to better meet the needs of students in partner disciplines. A central component of the project is creating Faculty Learning Communities consisting of mathematicians and faculty from other disciplines to determine ways to effectively implement the CF recommendations in the revised mathematics courses. The CF recommendations for developing connections between mathematics and the social science disciplines highlight creating opportunities for graduate student collaborations between mathematics and social science students to facilitate language flow across the disciplines.

Based on these recommendations, faculty members at Lee University designed a novel program to facilitate interdisciplinary collaboration among undergraduate (instead of graduate) students majoring in these two disciplines. A student in the program works together with a student from a different discipline on pre-assigned tasks, with the objective of learning from each other's discipline. Further, the program focuses not only on facilitating language flow between the two disciplines but also on improving students' knowledge of statistics and interdisciplinary knowledge related to the use of statistics. To encourage other institutions to consider creating similar programs, we describe in this paper past research that informed the design of this program, the specific steps taken to implement the program, preliminary outcomes from our program, and some potential pitfalls.

Literature Review

Research in higher education has focused on understanding how to better equip students with the professional skills for academic and vocational success. Students are expected to demonstrate increasingly diverse, complex skills in professional settings, including problem-solving, critical thinking, innovation, teamwork, integrity, ethical decision making, effective communication, and leadership (Everett, 2016; Hart Research Associates, 2013). Due to the high demand for well-rounded professionals, it is imperative to help students develop the detailed knowledge and skills that are expected in various professions (Everett, 2016; Hammersley et al., 2019; Hung, 2013), particularly those that require advanced mathematical or statistical literacy.

Equipping students with such a broad range of necessary skills may be best conducted through collaborative learning opportunities within undergraduate courses. Collaborative learning cannot be described by a single definition, as it is used in a wide variety of fields and is adapted based on different instructional perspectives, but it may be best understood as an interaction, situation, or learning mechanism that manifests a symbiotic relationship of action, status, and knowledge among its actors (Dillenbourg, 1999). This approach not only supports students working together to reach a common objective but also allows students to jointly interact and problem-solve across various contexts, catalyzing effective interactions for learning. In addition, collaborative learning may facilitate an innate, multidisciplinary language among students when students of varying levels of knowledge participate within a group to solve problems and to gain a cohesive understanding of multidimensional concepts (Dillenbourg, 1999; Dillenbourg, Baker, Blaye, & O'Malley, 1995).

An effective platform to foster collaborative learning within statistics courses may be best found through a system that unifies students across disciplines and enhances students' vocational skills within educational settings. Interdisciplinary integration, "the cognitive process of bringing together and blending insights from two or more disciplines," (Everett, 2016, p. 22) may be such a platform; it is especially beneficial for students' professional preparation (Laird et al., 2014; Repko, 2014) and may promote effective learning transfer and knowledge application of course content. Students participating in interdisciplinary work have exhibited improvement in finding novel solutions to complex problems, further facilitating student and faculty expertise, and broadening their conceptual understanding of statistics course material (Dierker et al., 2012; Everett, 2016; Hammersley et al., 2019; Woodzicka et al., 2015).

The publications about the Curriculum Foundations Project (Ganter & Haver, 2011) include recommendations from social science disciplines to foster these types of collaborations. Two of the recommendations are to implement a "graduate student exchange" and to create a venue for social science students to obtain mathematical help from graduate students in mathematics. The graduate student exchange enables graduate students in participating disciplines to work with and support both faculty and students in the other participating disciplines. Similarly, the tutoring lab venue enables social science students to develop their understanding of statistical concepts by working with experienced mathematics graduate students. In this paper we present an alternate version of an exchange program, one that was designed and implemented for undergraduate students.

Knowledge Application

Despite the increasing demand for critical thinking and complex problem-solving skills from twenty-first century employers, adult learners, including college students and workplace trainees, continue to struggle to adequately demonstrate the skills for effectively applying and transferring knowledge (Everett, 2016; Hammersley et al., 2019; Hung, 2013). Notably, many educators may not recognize this deficit in their students or be prepared to help students with the complex cognition that is necessary to correct it. Knowledge application, the basic level of effective learning transfer, is a process that requires higher-order cognitive skills and is one that is not often a natural ability (Hung, 2013). Incorporating discipline-specific language into college-level instruction and including problems with application-based contexts, however, may help students develop the ability to effectively apply their knowledge and transfer skills to other contexts or disciplines.

By introducing students early on to problems with application-based contexts, the instructor establishes the importance of reasoning through problem-solving situations at the very beginning of a course. Then, adapting problems accordingly may help students to expand their familiarity with and understanding of discipline-specific contexts and the relevant connections to mathematics. Students improve their mathematics knowledge application skills when they explore problem-solving situations set in discipline-specific contexts (Hung, 2013). For example, these types of problems help to illustrate the practical utility of applying mathematics concepts to situations in occupations in various disciplines. Hung (2013) maintains that instruction that involves mathematics problems with indirect contexts will, conversely, require students to make an additional cognitive application when faced with similar situations in their future occupations. This type of cognitive reasoning is a cross-context which is not conducive to successful learning transfer and knowledge application across disciplines. Incorporating discipline-focused mathematics problems in instruction will provoke a closer learning transfer and consequently help students to effectively apply mathematical knowledge and skills within their future occupational contexts (Hung, 2013).

Student learning is, furthermore, a multi-dimensional process and dependent upon holistic educational approaches that promote intellectual and personal growth (Baxter Magolda, 2009). To facilitate multi-dimensional student learning and effective application of knowledge and skills in college-level statistics courses, it is important for students to have opportunities to participate in interdisciplinary collaboration. These types of experiences will help students to develop a deeper conceptual understanding of the content which, in turn, may adequately prepare them to excel when working within discipline-specific contexts in occupational settings.

Interdisciplinary Collaboration

Previous research has established that students who engage in interdisciplinary collaboration may more effectively develop the complex skills that twenty-first century employers prioritize, including two high-demand, cognitive skills: innovation and problem-solving (Everett, 2016; Hart Research, 2013). The multifaceted, real-world problems that are conventionally part of interdisciplinary work broaden students' understanding of complex problems (Everett, 2016). Collaboration also often results in novel solutions to intricate problem situations. Work with real-world problems of this type improves students' conceptual understanding of the material being studied. In addition, students who have participated in interdisciplinary team work have demonstrated an even better aptitude for solving complex problems (Hammersley et al., 2019).

However, students are not the only ones who benefit from participating in interdisciplinary team work. This type of collaboration has a significant influence on faculty as well; faculty engaged in interdisciplinary work have reported positive outcomes from the collaborative experience. These types of experiences help faculty better facilitate cross-disciplinary connections with colleagues that would not have materialized otherwise (Goodlad & Leonard, 2018). Faculty have further reported that cross-disciplinary connections in undergraduate settings have challenged faculty intellectually and have subsequently provided a broadened understanding of the value of multidisciplinary work for students. This outcome is similar to the broadened conceptual understanding that students themselves develop from interdisciplinary work (Everett, 2016; Goodlad & Leonard, 2018). Therefore, it is evident that multidisciplinary undergraduate instruction not only supports student development of higher-order cognitive skills that are necessary for effective knowledge application, but also supports the development of cross-disciplinary communication, cognitive flexibility, and a deep understanding of concepts (Dierker et al., 2012; Everett, 2016; Laird et al., 2014), knowledge and skills that are essential for producing erudite, scientific scholars.

Despite the benefits of interdisciplinary work, instruction in undergraduate statistics has become increasingly polarized, with instructors frequently using either discipline-specific statistics curricula or traditional statistics tools which are not useful for making meaningful connections to how different disciplines make use of statistics (Dierker et al., 2012). Such polarization within statistics education appears to also include cross-contexts in student cognition which are not conducive to effective learning transfer and knowledge application (Hung, 2013). This type of polarization can be bridged by integrating cross-field instruction and collaborative opportunities within statistics courses (Everett, 2016).

Past researchers have sought to bridge the divide between traditional statistics instruction and the use of discipline-specific curricula largely by including systems of interdisciplinary, project-based learning in undergraduate introductory statistics courses. In a study conducted by Dierker et al. (2012), students generated scientific questions according to their subjective interests and researched their individual questions while learning about various statistical measures from lectures and with peer tutor support throughout the course. This type of course structure with instruction that is enhanced by team projects and peer mentoring has increasingly been found to encourage student engagement in research (Goodlad & Leonard, 2018; Hammersley et al., 2019). Dierker et al.'s problem-based learning study primarily had participants evaluate the appropriate statistical measures to use for their individual research questions. Through this approach, students developed a broader conceptual understanding of statistics course material, in contrast to more traditional fixed memorization of statistical procedures.

Considering the contributions of prior research on crossing the disciplinary divide that exists within the polarized world of undergraduate statistics education, our program seeks to bridge disciplinary boundaries in ways that are different from the existing and often-used structure of project-based learning. The Student Exchange Program (SEP) instead seeks to introduce a model for interdisciplinary collaboration that promotes effective knowledge application and conceptual understanding in undergraduate students, equipping students with the complex cognitive skills needed for academic and vocational success.

The Student Exchange Program

One purpose of the graduate student exchange as outlined in the CF recommendations is to facilitate language flow between mathematics and the social science disciplines. However, other than stating that "...mathematics graduate students provide statistical and mathematical support to social science students and faculty, while social science graduate students work with mathematics students and faculty" (Ganter, 2011, pp. 34–35), no specific details are provided about this type of program in the CF recommendations. At Lee University we have fleshed out the idea by designing a program that pairs a mathematics undergraduate student with a social science undergraduate student to work collaboratively on pre-assigned tasks. The objective is to create a "student learning community" in the spirit of the faculty learning communities that are quite common in academia.

The Student Exchange Program (SEP) is comprised of undergraduate students who were recruited from a pool of mathematics and social science majors. These students applied to the program after being recommended by various mathematics and social science faculty members who were familiar with SEP. These faculty members, including the first author of this paper, began collaborating in August 2017 to establish and maintain SEP.

The faculty team worked together to select students for the program based on their interests and experiences in mathematics, academic indicators (e.g., grades in mathematics classes, overall GPA), and other relevant skills (e.g., familiarity with statistics software). We also used these factors to determine an hourly pay rate for each student. In addition, we admitted students into the program for one semester and required that they reapply if they wanted to participate again in a subsequent semester. We hired a total of four students (two majoring in mathematics and two majoring in social sciences) per semester. In total, we have hired 16 students (6 men, 12 women) since the program's inception, some of whom participated in the program for more than one semester.

Before participating in the program, students provided informed consent to participate in the collection and analysis of data on their performance in the program and their attitudes about the program. Data collection took place at the beginning and end of each semester. We paired each student majoring in mathematics with one student majoring in the social sciences[1] and required the pair to collaborate on preselected tasks throughout the semester. Faculty members in both disciplines provided input into the tasks before they were assigned. Tasks were chosen to facilitate interdisciplinary collaboration and conversations between students, deepen students' understanding of statistics, and help students understand how statistics are applied in different disciplines. What follows is a detailed description of all of the tasks that were implemented in SEP.

Class Attendance

When the SEP first began, we required each pair of students to attend a section of Introduction to Statistics, a three-credit-hour, freshman-level course. Multiple sections of this general education course are offered every semester. Fall semester enrollment ranges between 300 to 350 students, whereas spring semester enrollment ranges between 200 and 275 students. Class sizes usually range between 16 to 30 students, and all sections are offered in a lecture-based format. This course is required for students majoring in business, nursing, psychology, sociology, anthropology, political science, biology, chemistry, and many of the teacher licensure programs. Students are placed in this course based on their ACT/SAT scores or previous preparations. Although there are no recitation sections for the course, students are encouraged to seek assistance when needed from our Mathematics Tutoring Center, which is run by senior mathematics majors in the evenings during weekdays. Due to the variety of disciplines represented by the student population for this course and the practical statistics applications in these disciplines, the course provides a rich environment in which to pilot SEP. Moreover, all of the aforementioned disciplines have a follow up course for which Introduction to Statistics is a prerequisite or have other situations in which students must apply the concepts covered in the course. For example, future teachers are expected to be able to analyze the data presented in annual course evaluations. This type of analysis requires the knowledge and skills developed in Introduction to Statistics.

We asked all mathematics faculty teaching sections of the course to allow pairs of SEP participants to attend. Participants attended class meetings, took notes on the course content, and occasionally assisted the instructor with class activities and demonstrations. If participants continued working in SEP for more than one semester, they were not required to attend a statistics course in the subsequent semesters.

After SEP had been implemented for one year, we developed new activities for participants to complete and required student pairs to spend more time on these activities. Thus, we eventually phased out the requirement for class attendance to allow time for the new activities which are described below.

[1] Future iterations of SEP will include students from other disciplines, such as nursing, athletic training, or business.

Structured Interviews

To encourage meaningful interdisciplinary dialogue between SEP participants and to help them understand how statistics can be applied in different disciplines, we required pairs to interview each other about one statistical concept of their choice, such as applications of the normal distribution, correlation coefficients, or hypothesis testing. Mathematics and social science faculty developed four questions that the interviewer was to ask about the selected concept:

1. Briefly, could you define the statistical concept in your own words?
2. How do you use the statistical concept in your discipline?
3. What is or was the most challenging part of understanding the statistical concept for you?
4. In your opinion, what is the most important thing to know about using or applying the statistical concept?

Participants recorded their interviews and submitted them to the program developers for review. We did not analyze the participants' conversations, as this was an activity to encourage and facilitate meaningful dialogue.

Resource Website Design

During the first year of SEP, participants collaborated to create a comprehensive website that could be accessed by undergraduate students taking Introduction to Statistics. We encouraged participants to begin by discussing the types of resources that might be helpful for students in the course and which concepts should be emphasized for students completing different majors. With this information and ample guidance, participants found and compiled resources for the website.

For example, participants selected videos for the website for the purpose of providing helpful information about the concepts covered in Introduction to Statistics. For one page of the website (see Figure 1), participants embedded a series of videos covering statistics skills and software navigation (e.g., Excel, SPSS). On another page, participants embedded original videos made by various professionals (e.g., high school teachers, college professors), who each briefly described how they use statistics in their professions.

Beginning in Fall 2017, the URL for the website[2] was made available to all mathematics faculty members teaching statistics courses as well as to all students in the courses. We also asked SEP participants in subsequent years to continue editing the website and adding new content. We review the new content each semester, and revisions to the website are ongoing.

Statistics Tutoring

Since the inception of SEP, pairs of participants have worked in the Mathematics Tutoring Center to assist students who need help with statistics content. We instructed participants to explain or clarify concepts from the perspective of their respective majors, thus helping the students in the tutoring session to consider how the content may be applied to other relevant domains. We solicited feedback from students who attended tutoring sessions, gathering information to help us improve students' experiences and assess SEP participants' performance.

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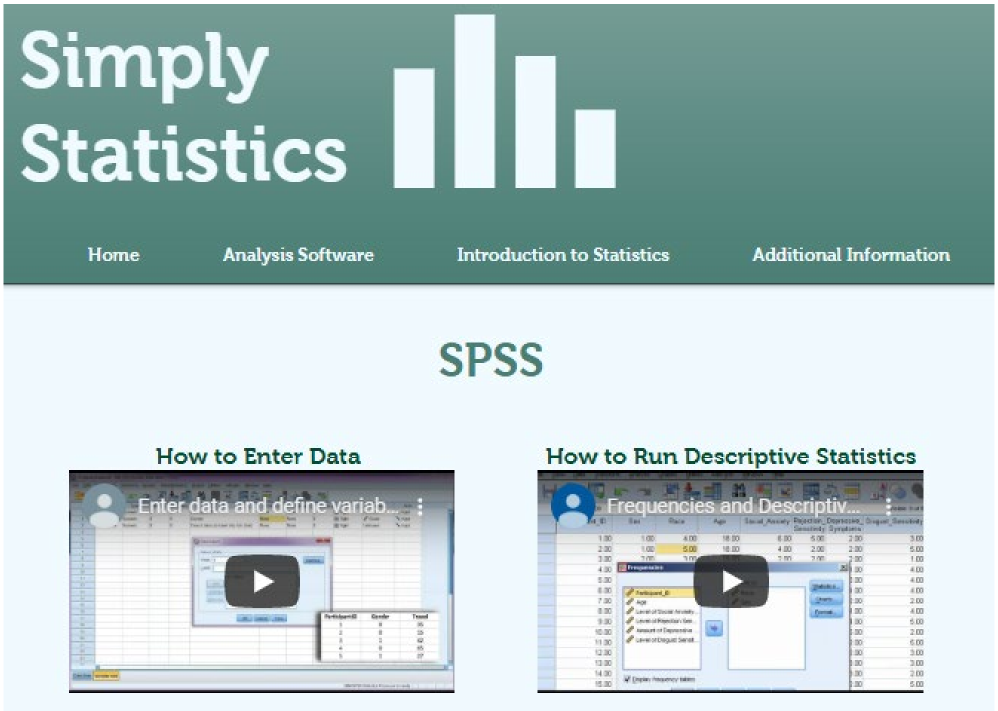
[2] The website URL is available from the first author upon request.

Figure 1
Sample Page from the Resource Website Containing Videos about Analyzing Data with IBM SPSS Statistics Software



Recitation Sections

During the semesters in which participants were not working in the Mathematics Tutoring Center, they hosted weekly recitation sections for all freshman-level statistics students. Past research has shown that recitations, in which facilitators create an environment conducive to review, discussion, and problem-solving (Guetzkow & McKeachie, 1954), are an effective and inexpensive way to improve students' comprehension of course content (Stock et al., 2013). These recitations were established as an additional hour of optional time for students in the course to spend reviewing and practicing class material.

Prior to hosting a recitation, pairs of SEP participants prepared examples of concepts recently covered in the courses and problems for students to solve. They met with attending students for approximately one hour to review course content and work on problems. Students had the opportunity to provide anonymous feedback after the recitation meeting.

Preliminary Outcomes

To determine the effectiveness of SEP, we assessed participants at the end of each semester through a series of targeted questions.[3] Three key questions, which served as the primary source of our qualitative analyses, are listed below:

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[3] We have also been collecting quantitative data on participants' attitudes toward statistics. Currently, however, our small sample size prevents us from making meaningful interpretations of these data. Information about these scales or data is available from the authors upon request.

1. What was the most beneficial aspect of working in the Student Exchange Program?
2. What areas of your knowledge in mathematics or statistics changed as a result of your participation in this program? Be specific.
3. What aspect(s) of the program were most beneficial at impacting your attitudes and/or knowledge about mathematics or statistics? Why?

We also analyzed the feedback provided from students who attended tutoring sessions in the Mathematics Tutoring Center. Our preliminary results are described below.

Qualitative Responses

Currently, we have analyzed participants' qualitative responses to the questions we posed from four semesters (Fall 2017 to Spring 2019) of SEP implementation. We used thematic analysis (Braun & Clarke, 2006) to analyze their responses. Ours was an inductive approach to analysis, coding and developing themes based on all participants' responses from the last four semesters. In the end, we identified three key themes in the responses (see Table 1 below).

Review of Statistical Content

The first theme suggests that SEP provided a valuable opportunity for participants to review statistical content. For some participants, it had been two years since they had taken a freshman-level statistics course. They noted their understanding of content was refreshed or improved by attending class with other students, having access to the textbook used in the freshman-level statistics course, and engaging in conversations with other SEP participants. They also noted that the review was beneficial for refreshing their understanding of specific concepts, such as permutations and combinations, which were not emphasized in some upper-level courses.

Application of Knowledge

The second theme suggests that SEP helped participants understand how statistics can be applied to different domains. For example, participants majoring in the social sciences reported an improved understanding of how statistics is used in mathematics and engineering, whereas participants majoring in mathematics reported an improved understanding of how statistics is used by psychologists.

Benefits of Knowledge Transmission

The final theme suggests that participants felt better equipped to communicate statistical information to others after participating in SEP. By requiring participants to work together to tutor students or host weekly recitation sections, they necessarily reviewed pertinent content, prepared helpful examples in advance, and explained statistical concepts in new ways. Together, these activities enhanced participants' confidence in their understanding and comprehension of statistical information and also provided valuable opportunities for students being tutored or participating in recitations to further develop their understanding of the material.

Table 1
Themes Identified in SEP Participants' Question Responses, Fall 2017 – Spring 2019

Themes	Example quotes
Review of statistical content	"I would say that my knowledge wasn't necessarily changed, but it was refreshed and brought back to the forefront of my mind."
	"A time to re-learn everything again from my previous statistics class."
	"I believe I received a higher score on the GRE this semester than I would have had I not been in the SEP because of all the statistics refreshers."
Application of knowledge	"Interacting with my partner helped me gain appreciation for statistics by seeing how it applies in her life and how important it is to her."
	[Participating] "helped me understand the mathematical side of statistics as well as some of the terminology used in math that is not always used in psychology."
	"My knowledge of how statistics might be used in a more mathematically-driven setting was minimal before participating in this program. Specifically, my partner taught me a lot about how statistics would be used in an engineering context, and how it related to other mathematical concepts. I gained a more holistic view of the usefulness of statistics across broader discipline categories."
Benefits of knowledge transmission	"I felt tutoring was the best area to explore statistics because it was the most hands on and forced the most thought."
	"Tutoring the students was the most beneficial to my own understanding of stats. You never truly learn something until you teach it to something else."
	"Leading the recitation sessions and preparing for them helped me gain more confidence in my knowledge of math/stats. Being placed in a situation that required me to explain/teach the concepts not only helped me further grasp statistics but also helped me gain a newfound appreciation for mathematics as a whole."

Student Tutoring Feedback

We collected feedback from students who were taking Introduction to Statistics who also visited the Mathematics Tutoring Center. After students received help from SEP participants, they responded to three Yes or No questions about (1) whether the help they received was clear and effective, (2) if they would be willing to receive tutoring again, and (3) if they would recommend tutoring to a friend. All students who submitted feedback about SEP participants responded "yes" to all three questions. In addition, when asked if there was anything that could improve the tutoring experience, all students responded "no" to the question. Taken together, these data suggest students benefitted from tutoring by SEP participants.[4]

General Discussion

The Student Exchange Program was designed by mathematics and social science faculty members to provide interdisciplinary, collaborative learning opportunities for undergraduate students, facilitating deeper processing and knowledge transfer of statistical information. Preliminary results from our study of this program suggest that participants benefited by reviewing important statistical concepts and skills, developing an understanding of how statistics are applied in other disciplines, and deepening their knowledge of statistics by helping others better understand the subject.

These initial results suggest SEP may serve as a springboard for deep cognitive processing in participants (Craik & Lockhart, 1972), which often leads to long-term retention of concepts and skills (Bacon & Stewart, 2006). For example, when participants in SEP prepared to lead recitation sections or to work together while tutoring other students, they processed the material on a deeper level by generating novel examples of concepts and refining their explanations of the material. Past research supports this assertion. Fiorella and Mayer (2014) found that participants who learned about an unfamiliar topic by preparing to teach it scored higher on a comprehension test than participants who studied the topic in a different way. In a subsequent study, participants who prepared to teach and presented material in a video-recorded lecture outperformed participants who did not teach the material. Explaining material to others, in real or simulated conditions, is more effective for learning concepts than restudying material and writing down explanations (Hoogerheide & van Gog, 2016). Furthermore, positive feedback from students who attended tutoring session further highlights the benefits of these collaborative learning opportunities for everyone involved. However, due to the preliminary nature of our work, some data are not yet available for analysis and discussion. We believe that our initial conclusions about the program's effectiveness will be supported when we formally analyze the quality of all of the resources posted to the statistics website and gather additional data about the effectiveness of tutoring and recitation sections led by SEP participants.

A Catalyst for Collaboration

In addition to providing valuable opportunities for interdisciplinary collaboration between students, implementing and managing SEP has been a catalyst for collaboration between faculty members in our institution, as well as with faculty at other institutions. For

[4] We also attempted to collect feedback from students attending the recitation sections. However, no students submitted feedback on SEP participants' performance.

example, mathematics and social science faculty members at our institution have created and sustained a faculty learning community that did not exist prior to participation in the SUMMIT-P project. Together we have recruited participants for SEP, exchanged pedagogical ideas and resources, designed and reviewed assignments, and shared access to our classrooms and materials. In sum, collaboration between faculty members of different disciplines has been essential to maintaining and improving SEP.

Moreover, SEP has fit into the collaborative mission of the SUMMIT-P project as a whole. For example, faculty members from other institutions evaluated SEP during a project site visit, reviewing our procedures and interviewing participants. Not only did they affirm our efforts, but they also provided ideas for novel ways to expand the scope of the program (e.g., using SEP participants to tutor both mathematics and social science students). In addition, based on our program's success, other institutions within the SUMMIT-P project, such as Ferris State University, have been inspired to implement similar programs or student-centered workshops. These examples suggest the SEP may provide valuable opportunities for novel student and faculty collaborations within and among institutions.

Challenges and Potential Pitfalls

Creating and managing a program like our SEP presents many challenges. First, the program may consume a significant amount of faculty time. For example, interviewing and hiring students as employees and keeping a record of their hours and tasks can be time-consuming, especially if new students are hired each semester. Designing, compiling, or monitoring activities may also take significant faculty time and attention, especially when being done during the academic year. Based on our experience, we recommend hiring a student worker to help manage some of the simpler tasks required to run the program.

Second, maintaining the program might be a costly endeavor. Providing funding for student participants, student workers, and supervising faculty may be difficult without institutional or external support. However, it may be possible to sustain SEP without excessive financial strain by linking student participation to course credit or some type of extracurricular program.

Third, the success of the SEP is contingent on the engagement of the participants. Students who lack motivation or struggle with time management may inhibit how other students are able to perform in the program or limit the progress of the program itself. Program administrators should adopt effective methods of keeping participants accountable and on task, including meeting with participants on a regular basis.

Finally, the small sample size of SEP at our institution limits the ability to draw some conclusions about the effectiveness of the program. For example, given that we hired 16 students over four semesters, we are currently unable to adequately explore our quantitative data to provide additional supportive evidence of the program's success. However, our qualitative analyses and student feedback have revealed promising benefits from participating in this collaborative program even while it is still being developed and refined.

Conclusion

In the twenty-first century, when students are pressed to develop the skills necessary for occupational and academic performance, programs that provide opportunities for deeper, multidisciplinary learning are important. At our institution, SEP, which is a continually evolving program providing opportunities for engaging undergraduate students in statistics, may be an operative platform for effective knowledge application and learning transfer that can benefit participants in myriad ways. Through the inclusion of details about our activities, assessments, outcomes, and obstacles, our intent has been to guide and challenge readers to consider whether similar programs can be developed and sustained at their own institutions.

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From Creative Idea to Implementation: Borrowing Practices and Problems from Social Science Disciplines

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Abstract

By collaborating with partner disciplines, mathematics educators gain valuable insight into the perspectives and needs of their students. This insight can lead to improved coordination of content and methods between courses in mathematics and the partner disciplines. This curricular coordination not only invites students to apply their mathematical knowledge in their own professional contexts, but also allows students to communicate mathematical mastery in the language of their intended professions. In this paper, we discuss the challenges specific to developing a mathematics course in collaboration with partner disciplines, with particular attention to portability to a wide range of math instructors. We identify three key obstacles to portability: instructor familiarity with application domains, variety of assessment styles, and grading consistency. We describe how each of these obstacles is addressed and discuss strategies that have helped new instructors be successful in teaching the course.

Keywords

course design, interdisciplinary collaboration, inquiry-based learning, mastery grading, faculty learning communities

"Math is everywhere!"—a well-intentioned sentiment to be sure, but one that is often met with skepticism from students. The problem, of course, is never in our ability to impose the language of mathematics upon the world but in the sometimes-lacking motivation for doing so. In this paper, we propose to invert the dilemma: instead of grafting the language of mathematics upon the world, what about teaching mathematics in the language of another discipline? We build on the model of Piercey and Militzer (2017), who showed that beginning and intermediate algebra skills could be taught effectively with business application problems and the use of Excel. This model is also found more broadly throughout the courses designed in the Synergistic Undergraduate Mathematics via Multi-Institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) collaborative research project, which comprises mathematics and partner-discipline faculty members from nine colleges and universities (Beisegel & Dorée, 2020). The stated goal of SUMMIT-P is to promote collaborations with faculty from partner disciplines and create engaging courses that most effectively prepare students to make use of the mathematics that they study in course work in partner disciplines and within the workplace. We also draw heavily on the framework of the Curriculum Foundations Project (Kasube & McCallum, 2001), particularly its emphasis on the role of assessment in student learning.

This project is a collaboration between a mathematics department and colleagues in the disciplines of business, nursing, and social work. The course context of this project is Quantitative Reasoning for Professionals (QRP), a two-semester general education sequence in mathematics that is intended for students who struggled with algebra in high school. We designed this course to help students develop from simply solving problems to forming questions and inferences about the applications of algebra. We apply principles of quantitative reasoning to promote these habits of mind (Karaali et al., 2016). In collaboration with the partner disciplines, we developed a large number of algebra lessons with authentic applications in business, nursing, and social work, and we developed new types of assessments that coordinate with the applications in the lessons. For example, we assess mastery of several algebraic concepts through writing assignments such as funding proposals and budget projections developed in collaboration with colleagues in social work. As a result, QRP is a course that is situated in the language and practices of the partner disciplines and yet also achieves the goal of imparting mathematical skills. In this paper, we describe QRP and its development process and discuss the challenges particular to interdisciplinarity that arise when porting the course to new instructors.

In the first section, we describe the conception and development of QRP and the involvement of the partner disciplines and give a representative sample of lessons that we developed in collaboration with the partner disciplines. In the second section, we describe the structure of a typical lesson, give examples of typical assessment types, and discuss our grading philosophy of holistic or mastery-based grading. In the third section, we outline the challenges we have encountered in preparing and supporting instructors who are new to QRP and discuss strategies that have helped these instructors be successful in teaching the courses.

Course Description and Partner Disciplines

The QRP course sequence was developed by mathematics faculty at Ferris State University (FSU) through conversations with colleagues in several partner disciplines at the same institution over a period of six years. The project was initially developed in collaboration with the College of Business at FSU, which includes students in programs such as business administration, hospitality, and professional golf management. The mathematics focus of these programs is principally in their accounting and statistics. Piercey (2017) found that students in an early version of the course sequence scored higher on an algebra skills assessment than students in the parallel traditional algebra sequence. Motivated by this success, a faculty learning community was formed to broaden the scope of the course sequence to enroll students from the social work and nursing programs and to develop course materials applying algebra to these disciplines (Bishop, Piercey, & Stone, 2020). In order to identify the algebra-related priorities of our partner disciplines, we held a workshop in which partner-discipline faculty participated in a facilitated discussion about the mathematical needs of their students. This so-called “fishbowl” style of workshop was designed according to the SUMMIT-P collaborative research project (Hofrenning et al., 2020). The fishbowl workshop helped us to focus our course design on the following partner-discipline priorities. Our collaborators in social work were interested in preparing students to write profession-specific documents such as budget projections and funding proposals. Our collaborators in nursing gave dosing calculations, unit conversion, and exponential decay of drug concentration as the primary application areas of interest to their program.

Based on the feedback received from our partner disciplines, we developed many lessons that present algebra in the context of authentic applications to business, social work, and nursing. We drew on the recommendations of the Curriculum Foundations Project (Kasube & McCallum, 2001) to encourage students to interact with the instructor and learn from each other. All together, we developed over fifty lessons and over a dozen written and oral projects. The following are a representative sample of these lessons and projects.

- Fry efficiency: students construct a process to measure the efficiency and waste in french fry preparation at a fast food franchise. They have to translate their process into a formula that they program into Excel using the data in a given spreadsheet.
- Dosing calculations: students set up a series of proportions to determine the correct dose of a solution containing a certain concentration of medicine, using dosing guidelines by patient weight.
- Social justice: students analyze real United Nations data on human trafficking, identify trends, formulate recommendations to the UN, and create charts that support their recommendations.
- The credit card problem: given a fixed initial balance, a fixed interest rate, and a fixed monthly payment, students derive a formula for the length of time it will take to pay off a credit card.
- The birth control problem: students use an exponential model to predict the onset of menstruation after hormonal birth control is discontinued.

Implementation

Lesson Structure

The cornerstone of QRP is to help students learn to form questions and inferences about the applications of algebra. We accomplish this by creating lessons in discipline-specific scenarios with deliberate scaffolding to promote student inquiry. While our students do indeed find the inquiry-based applications engaging, this is only a secondary benefit of the lesson design. The primary purpose of these lessons is to provide an environment in which meaningful professional communication can ensue. It is much easier to promote inference in the context of an authentic scenario than in a purely symbolic-algebraic context. Since we view professional communication as an important vehicle of inquiry-based learning in QRP, we conduct the lessons primarily via group work and create frequent opportunities for the groups to discuss and debrief. For an extensive discussion of the use of active learning and inquiry-based methods in mathematics instruction, see Yoshinobu and Jones (2012) and Katz and Thoren (2017a; 2017b).

An active learning technique used by one instructor involves calling on a specific group using a group member's name. While the named student is implicitly encouraged to answer, the whole group is also given space to confer among themselves before offering a solution. Another instructor regularly asks students to rate their understanding with the numbers one through four, where a rating of four means that student feels prepared to teach the lesson to a peer, and a rating of one means the student needs help. This self-rating can help steer the class in a beneficial direction for the students and also helps the instructor form productive student groups.

The Fry Efficiency lesson is as a good example of a learning environment that promotes meaningful professional communication, here in the context of managing an Arby's franchise. The students are placed in groups and told that their "manager" has asked them to come up with a way to measure how efficiently locations are using their french fries and how much waste is occurring on a daily basis. Each group has access to the excel spreadsheets that include nightly data on fry sales and use. The mathematical content of this lesson is the idea that efficiency should be measured as a ratio, not a difference. Students ultimately demonstrate this realization by successfully programming an Excel sheet with the efficiency formula that they derived, as shown in Table 1.

Table 1
Sample Spreadsheet from the Fry Efficiency Lesson

Measurement	Day 1	Day 2	Day 3	Day 4
Small fries sold	98	101	67	76
Med. fries sold	145	87	88	143
Large fries sold	123	96	96	58
Cartons used	4	4	3	2
Bags used	2	1	2	5
Weight of bag	2.3	3.1	1.4	0.23
Fry Efficiency				

However, the pathway to demonstrating this understanding begins with students working in groups of four to brainstorm the various factors that affect the efficiency of business operations in the context of french fry usage at an Arby's franchise. With this population of students, we have found that open-ended lessons may be perceived as intimidating. However, the authenticity of the scenario provides a hook to start conversations, allowing students to make some progress without being told the answers. Furthermore, many instructors assign group roles to each student in the style of Process Oriented Guided Inquiry Learning (POGIL), an active-learning methodology that has been applied in fields such as mathematics, chemistry, and nursing (Hanson, 2006; Moog, 2014; Bénéteau et al., 2017; Roller, 2013). This measure helps focus students on a particular responsibility, such as note-taker, spokesperson, or time manager, which adds some structure to an inquiry-based lesson that may otherwise come across as intimidating. By completing the Fry Efficiency lesson, students develop a working understanding of efficiency in the context of running a business and demonstrate mastery of the underlying mathematics of ratios by correctly programming an Excel spreadsheet.

Assessment

In order to teach mathematics in the language of the partner disciplines, we must welcome students to demonstrate mastery in that language as well. In the Fry Efficiency lesson, students demonstrate mastery in formulating equations by correctly programming an Excel sheet. Later in the course sequence, students might demonstrate mastery of the geometric sum formula by correctly computing the future value of an annuity in a grant proposal report (proposed by colleagues in social work). Similarly, students demonstrate mastery of linear equations by correctly depreciating the value of a business vehicle over ten years. Somewhat surprisingly, we have found that students from one discipline are generally motivated to work problems written for the other disciplines as well. It appears that the mere presence of an authentic application of the mathematics does most of the motivational work, while any connection to a student's particular discipline is viewed as a secondary benefit.

We use a variety of assignment types to help us assess content mastery in the various application domains of our partner disciplines. Excel is used in many lessons and assignments, generally in tandem with data sets drawn from applications in accounting, social justice, and medicine. Written reports and oral presentations are both used in assessment, with particular attention to budget projections and funding proposals.

Our grading philosophy mirrors that of the humanities: revision, iteration, and holistic grading. Just as we present mathematics in the language of the partner disciplines, and just as students demonstrate learning in that same language, we aim to assess that learning according to the norms of the partner disciplines. The upshot is a philosophy of grading that is much more akin to professional evaluations, as is appropriate in the partner disciplines. In the context of mathematics education, this grading philosophy is most akin to varieties of mastery- and standards-based grading that have been practiced for decades in K–12 education and have more recently become popular in college mathematics instruction (Stange, 2018).

For example, we give regular review assignments to prompt students to revisit key topics from the past week. In many cases, questions are drawn verbatim from the lessons. The reviews are graded holistically on a scale of **N**eeds improvement, **P**rogressing, and **M**astered. Students are permitted unlimited revisions but must visit office hours if they fail to make progress. By the end of the semester, all review assignments must be mastered in order to pass the class. This policy communicates the instructor's expectation that every assigned question is important and worth doing correctly. One instructor even lays this out concretely to the students by likening their homework to a professional service to be performed (as opposed to a conventional mathematics assignment with points and partial credit).

In our collaborations with faculty in the English composition program, we have learned of the benefits of holistic grading of writing, as discussed in Piercey and Cullen (2017) and recommended by Nilson (2010). Instead of providing comprehensive feedback on each report, we prioritize two or three main areas of improvement. At the beginning of the first semester, these items may include simple formatting and structural improvements. By the midpoint of the course sequence, it is usually possible to begin focusing feedback on key aspects such as tone and audience awareness. Our approach to grading written work represents a different style of mastery grading when compared to the "revise until mastered" policy with review assignments.

Under a "revise until mastered" grading policy for written work, students would be required to revise their entire written reports repeatedly until they obtained mastery, a workload that would risk student burnout. We avoid this risk by helping students focus on achieving a small list of key improvements to their writing. Based on grading trends, we have observed that students generally finish the second semester with demonstrated proficiency in a type of data-driven executive summary that is consistent with the training requested by our colleagues in social work. Moreover, in composing these reports they also demonstrate proficiency in a variety of key Excel skills that are requested by our colleagues in the College of Business. Crucially, we are able to assess mastery of the underlying mathematical concepts through these non-standard assessment types.

Challenges to Portability

As introduced in the previous section, the underlying philosophy of this course sequence is to promote learning of mathematics through inference and discussion. We approach this goal with lessons in discipline-specific scenarios, but these lessons come with a portability challenge when introducing new instructors to QRP.

Discipline-Specific Knowledge

New instructors in the course sequence must thoroughly understand the mathematical application in each scenario, the discipline-specific vocabulary, and any number of related concepts in the discipline that would motivate the scenario. For example, one lesson on linear functions uses the application of depreciating the value of a business vehicle over ten years; another lesson on logarithms involves the pH scale. While the relative simplicity of the underlying mathematics helps matters considerably, new instructors have still encountered difficulties when confronted with the breadth of application domains in the course. Most of these scenarios have undergone heavy revision of exposition, and supplemental instructor versions have been written to help bridge this gap. Moreover, our collaboration with faculty in partner disciplines has broadened into a formal faculty learning community through which new instructors of the QRP course sequence are able to learn discipline-specific knowledge firsthand from faculty in social work, nursing, and business. In Summer 2018, this faculty learning community held a workshop in which new QRP instructors worked alongside the course designers and members of the partner disciplines on lessons such as Fry Efficiency. The faculty learning community then continued to meet for lunch on a biweekly basis throughout the school year to discuss the course and address any instructional concerns.

Teaching Through Excel

We make extensive use of Excel in our lesson design. The first reason for this is that Excel is a convenient intermediate step between arithmetic and algebraic symbolization that has been shown to be effective with the target student population (Piercey & Militzer, 2017). We also want our students to become familiar with Excel and take that familiarity to their respective professions. However, the prominent use of Excel poses a challenge to porting QRP to new instructors. While some instructors may lack experience in the platform, this obstacle is relatively minor in the course sequence since we rely on basic spreadsheet functionality throughout. The more difficult portability challenge comes when trying to teach algebra *through* Excel, as this use of spreadsheets is novel to essentially every new instructor (Piercey, 2017). Without the proper strategies, an Excel lesson can quickly become bogged down by syntax and formatting errors. Instructors must learn how to quickly diagnose and correct these; further, they must eventually learn ways of teaching the students to diagnose and correct their own errors of this kind.

We have found that it is helpful to insist that students write down their proposed Excel code by hand, and then methodically translate this into standard algebra. Not only does this translation clarify the connection between the two forms, but it also greatly helps when showing that different algebraic forms are equivalent. Moreover, highlighting this particular process also helps resolve parenthesis errors in Excel formulas, which are probably the single most common mistake made by students in the course sequence. By setting standards of Excel work early in the class, we find that students quickly become amenable to checking each other's work for syntax errors as well, further promoting active learning in the classroom.

Grading Consistency

The use of written and oral reports in the course sequence is motivated by the goal of assessing student mastery in the language of the partner disciplines. While this has dramatic benefits in including students whose writing or speaking skills are more developed than their symbolic algebraic skills, new instructors are faced with the obstacle of grading written and spoken mathematics, a new challenge for many in the mathematics community. As discussed in the previous section, the written and oral reports are all assigned in the context of a discipline-specific scenario, such as the Fry Efficiency lesson or the Human Trafficking case study. Instructors must therefore bring their understanding of the vocabulary and context of the scenario together with conventional assessment of algebra, all in the context of a written or oral report. Many new instructors have found this type of grading responsibility to be daunting. In Summer 2018, all instructors in the course sequence met and established a grading rubric for written reports (Bishop, Piercey, & Stone, 2020). We also conducted several grading exercises using exemplars from the previous year, comparing and discussing our findings afterwards.

Beyond the assessment of written and oral reports, even assessing the more traditional review assignments also poses a challenge to portability, due to the use of mastery grading instead of conventional points-based grading with partial credit. We find it highly important to use holistic grading policies not only in the written and oral reports but in the weekly review assignments as well. The main reason is that our population of students is generally accustomed to performing at a minimum passing level; many would not attempt to fully master the concepts in a review assignment if 70% completion were acceptable. In our summer instructor meeting, we also conduct grading exercises of exemplar review assignments to help new instructors become accustomed to this style of grading. A discussion of possible grading heuristics is particularly helpful: “would the student benefit from revising,” “could the student explain this to a classmate,” and similar heuristics can be used by the instructor to determine whether a grade of “mastery” is appropriate.

Conclusion

In collaboration with faculty in business, social work, and nursing, we developed a course sequence that teaches fundamental algebra skills through authentic applications of mathematics in these disciplines. The project has resulted in an approach to mathematics instruction that is fundamentally situated in the language and practices of the partner disciplines, yet still achieves the goal of imparting mathematical skills. Professional communication skills are central to the project, as they help students engage with authentic applications of algebra and demonstrate mastery in multiple contexts. We design lessons to promote discussion and inference through real-world problems and assess student learning through written and oral reports in keeping with the practices and standards of our partner disciplines.

This approach to course design has posed several portability challenges for new instructors. First, new instructors to the course sequence must gain familiarity with partner-discipline content in order to guide students through the variety of application domains in the lessons. We have used a faculty learning community to not only help participating mathematics instructors learn directly from the partner-discipline faculty, but also to revise the existing lessons to more and more clearly express discipline-specific content in elementary terms. This ongoing revision process has helped students and instructors alike to engage with the coursework.

Second, the extensive use of Excel in the course sequence necessitates instructional techniques more often found in a computer lab than a typical mathematics course. Instructors new to the course sequence are not always prepared to facilitate Excel-based algebra lessons. We have developed experience-based practices that help instructors anticipate the single most common student error in Excel based lessons: incorrect or missing parentheses. As an added benefit, these practices also tend to encourage students to check the work of their classmates, promoting collegiality and active learning in the classroom.

Third, the variety of assessment types and the use of holistic and mastery grading represent substantial obstacles to adoption by new instructors in the course sequence, as these factors are rather atypical in mathematics courses. As with the challenge of discipline-specific knowledge, the faculty learning community has proven beneficial in obtaining buy-in from new instructors and establishing grading conventions for written work. By interacting with partner-discipline faculty in this community, new instructors gain insight into the grading practices of other disciplines and the value of assessing our students in these new ways. They also gain experience using grading rubrics for written work that were developed by the faculty learning community.

QRP is a living course whose content and instructors are constantly evolving in conversation with our partner disciplines. By adopting the underlying premise that the content of these courses can and should be taught—at least in part—in the languages of business, nursing, and social work, we invite certain challenges but also reap the benefits of our interdisciplinary collaboration. For anyone considering adopting our approach at a different institution, we offer the following advice. Consider holding a fishbowl workshop early on where colleagues from several partner disciplines come together to discuss the technical skills they would like their students to develop (Hofrenning et al., 2020). Our fishbowl workshop not only laid the groundwork for dozens of new lessons but also helped establish buy-in and general good will from members of the partner disciplines who had perhaps not thought much before about the purpose of algebra in their programs. We also recommend establishing some modest incentive for new instructors to participate in a preparatory workshop and ongoing check-in meetings, as these activities were very helpful in supporting our new instructors in their first year of QRP.

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Good Teachers Borrow, Great Teachers Steal: A Case Study in Borrowing for a Teaching Project

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Abstract

Very few great ideas in teaching are without ancestors or descendants. This paper presents a case study in how one particular pedagogical project, the work at Saint Louis University as part of the National Science Foundation supported SUMMIT-P consortium, borrowed from other sources. The particular project was an interdisciplinary collaboration to make mathematics education more effective for business students. The various borrowings are treated in roughly chronological order from initial inspiration through planned adoption and adaptation of the work of others to the addition of features that only became available mid-project. The kinds of sources include a particular business calculus project, a pedagogical movement, other members of the SUMMIT-P consortium, and independent math technology projects including PreTeXt, WeBWorK, and GeoGebra.

Keywords

borrowing, mathematics, business

Very few great ideas in teaching exist without ancestors or descendants. Instead, most good ideas come about because someone hears of an innovation from someone else and says either “I can do that” or “I can do better than that.” Additionally, good projects generally incorporate features from other projects. This paper explores this practice within A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P), a National Science Foundation supported collaborative project with nine institutions working together. The Consortium is looking to expand the implementation of the recommendations of the Curriculum Foundation Project (CF) (Ganter & Barker, 2004; Ganter & Haver, 2011), developed by the Mathematical Association of America's Curriculum Renewal Across the First Two Years (CRAFTY) committee. Each institution picked at least one partner discipline and asked how to make mathematics instruction more effective for their partner discipline in their local context. The institutions were chosen to have a broad range of sizes and types. This paper will focus on the borrowing connected to the project at Saint Louis University (SLU), the first author’s institution, but will involve borrowings connected to other consortium members. Our project looks at making mathematics instruction more effective for business students, but the practice of borrowing from the work of others easily translates to other contexts.

Phases of Borrowing

The borrowings will be broken down in phases based on SLU’s SUMMIT-P project. As is typical for many grants there is a significant amount of work done before any proposal is written, a pre-project that starts the vision of the project. As with many projects, we started with a first idea from someone else that inspired us. During a talk at a conference, a speaker made a comment on using Excel in calculus for business students. As the project matured, we consciously looked for related pedagogical projects that could be borrowed from to make the implementation of our first idea stronger. This involved a literature search to see what others have tried. We wanted to see what others had done with calculus for business students and what they had done with Excel. As the project became more defined, we looked for other projects that have useful features which we could borrow. We joined SUMMIT-P, a collaborative project, and this involved a concentrated effort to look at the other sites in the consortium to see what could be harvested, either for our local SUMMIT-P project or for other classes on campus. This broadened the project from simply looking at a specific course, Business Calculus, to looking at making the mathematics education of business students more effective. Again, as is typical of any project, there was a shifting of definition and goals of the project, as various ideas were tried and depending on their success, pursued or modified. There were several points where we realized we started with the wrong answer to a question. There were also several points where we realized someone else had a better way to do something we wanted to do.

For all of the SUMMIT-P projects, the pre-project phase included some connection with CRAFTY and specifically with the CF guidelines. For SUMMIT-P there was then the forming of the consortium and bringing the projects together. This allowed borrowing from the other consortium members’ pre-projects. As the implementation has proceeded, the SUMMIT-P members have been meeting with each other at the annual Joint Mathematics Meetings. There have also been a series of site visits among the members of the consortium. Both the meetings of the consortium and the site visits provide an opportunity for sharing ideas and materials.

Phase I: Historical Borrowing

Felkel and Richardson made a presentation at the International Conference on Technology in Collegiate Mathematics (ICTCM) in 2002 with the claim that business students should be taught mathematics mainly using spreadsheets instead of graphing calculators. The seed was planted. They pointed out that a spreadsheet, Excel in particular, was the computational engine of choice in the business world. At the time, incorporating spreadsheets seemed like a good idea that could be added to courses at SLU at some point when time was available. Implementing the idea then would have required using a computer classroom which was not easily accessible at that time. Five years later, the time was right to implement the idea in courses at SLU. The first attempt at implementation of Felkel and Richardson's (2006) ideas at SLU involved simply using their book, *Networked Business Math* (NBM). Our major change to their setup was to run the course with student laptops rather than a computer classroom. This first stage was simply a textbook adoption with an appropriate pedagogical shift. We simply wanted to adopt and adapt the course that Felkel and Richardson developed at Appalachian State University for SLU. We planned to offer feedback from the SLU experience to the textbook authors to be used in the next edition of the book. However, Felkel and Richardson had moved on to the next phase of their careers, which meant there would be no new edition of the book. To continue to use their concept meant creating a new version that used the approach. From the pre-project phase, we moved from borrowing to a phase of borrowing and improving.

Phase II: Turning an Idea into a Project

In the second phase we decided to write a book inspired by NBM. Our initial plan followed the structure of NBM and incorporated several features that differentiated the textbook from other one-semester calculus textbooks. For example, the examples emphasized business applications. Terminology and notational conventions in the book followed the practices used in business and economics. Numeric methods allowed for the development of concepts before the use of symbol manipulation formulas. At the same time, we made some changes that we felt also improved on the work of Felkel and Richardson. For example, instead of incorporating Maple examples and exercises, we incorporated a free online computer algebra system. We adjusted the use of Excel to eliminate macros and instead use it like a spreadsheet with small steps rather than like a graphing calculator with compact formulas. We consulted with the business faculty to supplement the topics covered and determine the order of the topics. For example, the business faculty believe partial derivatives and optimization are more important concepts for students than integration and should therefore be covered earlier in the course (May, 2013).

As the project grew, we desired to see if it could be grant supported. Looking for grant support requires a search of the literature to see what else had been done in the same direction. The search brought up CRAFTY CF and a textbook developed by Lamoureux and Thompson (2003) at the University of Arizona. The search provided reassurance that many of the ideas we had developed and incorporated in our book had also been thought about and implemented by other faculty at other institutions and were ideas that were supported by groups like the MAA. The literature search also led to thinking about why previous projects along the same lines had died out and asking ourselves, "How can we do it better?" The project was transformed from simply developing a new version of NBM to designing and writing a business calculus book, *Business Calculus with Excel* (May, 2019).

Phase III: Becoming Part of a Consortium

As drafts of *Business Calculus with Excel* were being solidified and class tested, an opportunity arose to join the SUMMIT-P consortium. Joining the consortium resulted in a broadening of our focus. Instead of just focusing on business calculus, we decided to make the study of mathematics in general more effective for students majoring in business disciplines. In particular, a central part of the effort would be a faculty seminar in the form of ongoing structured discussions between the faculty in the SLU Department of Mathematics and Statistics and the School of Business to understand which mathematics concepts and skills were important for business majors and what the business faculty wanted to help their students accomplish mathematically in their courses. A group of faculty from both disciplines have a monthly meeting or seminar to discuss how to make mathematics instruction more effective for business students. The business faculty involved in the meetings include representatives of each of their departments. This structure is directly borrowed from the CF recommendations. Our variation on the approach was that instead of having faculty from across many schools and departments at SLU discuss needs in mathematics over a weekend, we would have faculty from one school discuss the matter over several years. The extended discussion has allowed us to look at modifying the instruction in both disciplines. From the mathematics perspective, the discussions have allowed regular review of examples and presentations in mathematics classes to make sure terminology, notation, and applications based on business concepts and scenarios are correctly applied. Early discussions started with the standard issue of the mathematics that the business students did not know even if they had been covered in prerequisite courses. This led to conversations about techniques for identifying students who have gaps in their backgrounds and intervention strategies that could be employed without turning the business faculty into mathematics faculty. This has led to borrowings between the two disciplines related to the way they each operate that will be described later in the paper.

Phase IV: Discussions Do Not Go as Planned, Time for More Borrowing

The SLU project co-PIs started the faculty seminar with what they assumed to be a clear idea of where the business faculty would want to focus. We wanted to extend our work beyond the introductory business calculus course to higher level mathematics courses that are part of business majors and were confident that the business faculty would endorse the plan. We were wrong. Other than the co-PI from finance, the rest of the business faculty who participated in the seminar rarely used mathematics at a higher level than calculus in their undergraduate courses. For them, the bigger problem was student misunderstandings and misuse of lower level mathematics concepts and the hurdles these issues caused in business courses. While the business faculty were happy with the changes we made to Business Calculus, they felt that we should focus our efforts on College Algebra for Business instead of on advanced mathematics courses. This left us looking for examples of CRAFTY CF oriented, college algebra for business courses. We looked to other SUMMIT-P consortium members who had developed successful college algebra courses based on CF recommendations. Virginia Commonwealth University (VCU) had developed such a course before the start of SUMMIT-P (Ellington & Haver, 2011).

The first author arranged a visit to VCU to see how their course was organized and what materials they had developed. The VCU course was designed with different goals and with a different audience in mind than what was envisioned for the SLU course. VCU aimed at a general education student audience with standard technology being used in the course instead of focusing on the particular needs of business majors. The course activities and assignments focused on the use of graphing calculators on a regular basis and only occasionally incorporated the use of Excel during lab activities. The lab classes added an extra contact hour to the course per week. With a larger student population than SLU, VCU offered many sections of College Algebra, so the course was highly coordinated. However, it also was solidly based on a modeling approach to developing concepts and skills and used a textbook that was compatible with the CF recommendations. VCU had developed worksheet materials that were ready to use. In particular, students routinely used functions based on real world data to explore algebra concepts.

We decided to pilot a single section of College Algebra for Business that fit into the standard course schedule time frame, met the learning objectives of the traditional college algebra course, and used Excel instead of the graphing calculator. The following semester offered the course using the textbook that VCU was using. We incorporated some of their organizational ideas and a series of worksheets that they had developed. In subsequent semesters, we have switched textbooks and modified the worksheets. Two particular worksheets were the foundation for developing a modeling-with-Excel thread that extends throughout the entire course. Questions on tests throughout the semester require students to find best fitting curves for a provided set of data. While the course has continued to be redesigned at SLU, borrowing VCU's basic model provided a foundation for developing a modeling based course, College Algebra for Business, at SLU. After observing students at VCU engaged in active learning in small groups, the first author had an idea for what was possible with SLU's course.

Phase V: Borrowing from Planned Site Visits

The design of the SUMMIT-P grant includes two rounds of site visits between members of consortium institutions. The visits serve several roles. They provide an outside perspective for feedback about the institution's project. They also provide an opportunity for the visiting institution to borrow materials, methods, and ideas from the institution being visited. Since each institution's project has a different scope and focus, the borrowing has taken the form of borrowing ideas or materials that were developed before the SUMMIT-P project instead of direct borrowing between ongoing SUMMIT-P projects.

LaGuardia Community College (LCC) visited SLU in the second year of the SUMMIT-P project. They are working with a model of pairing classes offered in the subjects of college algebra, trigonometry, and economics where the students in a particular section of one course also take a particular section of another course, and the two instructors coordinate how content is presented between their respective courses. During the site visit, LCC faculty were introduced to materials from the book *Business Calculus for Excel* (May, 2019), and they adapted these materials for some of their courses. For example, the algebraic review material in *Business Calculus for Excel* is presented through examples with contexts in economics. From this perspective, systems of linear equations are used to understand situations like finding market equilibrium, and solving quadratic equations is used for finding break-even points. LCC faculty met with SLU students taking College Algebra for Business during the site visit. Based on the discussion, they adjusted their approach to getting students to enroll in their courses by enlarging the role of advisors. This led to a more successful implementation of their paired classes idea.

A PI from Ferris State University (FSU) was part of a team visiting SLU in the second year of the SUMMIT-P project, and SLU visited FSU in the project's third year. As a result, FSU has adapted some of the materials and approaches they learned about during the site visits to revamp how they are introducing Excel into quantitative reasoning projects. During the FSU site visit, the SLU team was impressed with the materials FSU had developed for Quantitative Reasoning for Professionals, a two-semester course sequence at the intermediate algebra level. Based on institutional need, the FSU material could not be incorporated in SLU's intermediate algebra course. At FSU, quantitative reasoning fulfills the graduation requirement and is often the last mathematics course taken by many students. At SLU, intermediate algebra is taken by a small student population, most of whom need to take another mathematics course, typically College Algebra. However, we determined that the FSU quantitative reasoning material had great potential for a course offered in SLU's program for inmates at a regional prison. Students in the program are developing their skills in finite mathematics. They do not have internet access to complete online homework. Motivating concepts and skills with applications to real life situations is important for them. The FSU material is worksheet-based and incorporates mathematics concepts and skills into a semester long project. For example, running a shelter for victims of human trafficking or building a Death Star.

In each of these cases of sharing between institutions involved in the SUMMIT-P project, borrowing took the form of using a feature that had matured and been used for several years at one institution and incorporating the feature in a different way in a project at another institution that perhaps involved a different mathematics course or student population.

Phase VI: And the Borrowing Continues

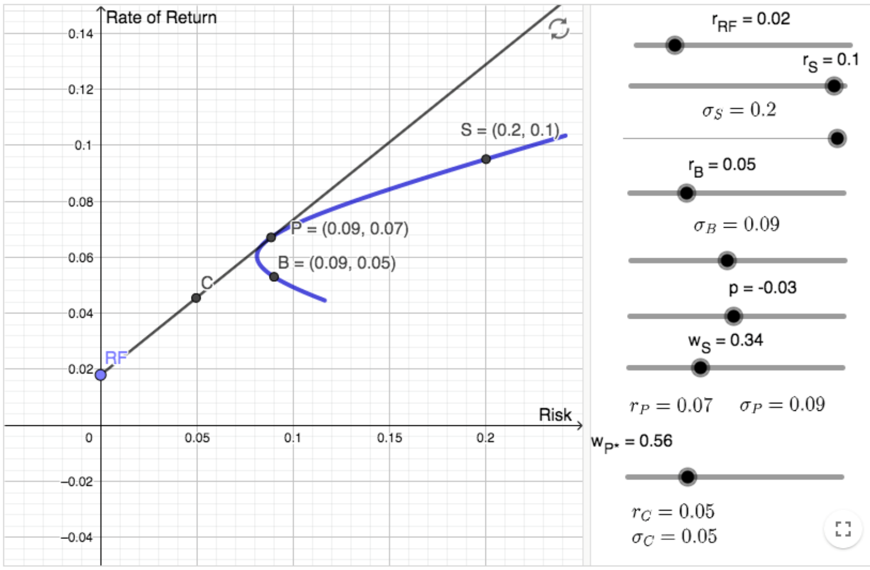
One of the interesting kinds of borrowing for an interdisciplinary project is the borrowing that can be described as taking tools and methods that the mathematics community has been using for years and applying them to another discipline. In mathematics, we have always been concerned with placement level and prerequisite knowledge and have built structures for student success that include placement tests and remediation. SLU's partner discipline of business does not have a history of similar concerns. We used WeBWorK to create a series of online prerequisite skills tests for a number of courses in the SLU School of Business so that business faculty can identify the students who need remediation in material not directly taught in business courses and direct them to online resources designed to cover the specific mathematical skills needed (Bart, 2018).

Mathematics also has a long history of incorporating visualization tools, like GeoGebra, to create dynamic presentations for course content. Our partner discipline does not appear to have a similar history. We have started to develop an online GeoGebra book (i.e., a collection of GeoGebra applets) called *The Mathematics of Finance* (May, 2019) to be used in demonstrations that can be incorporated in business courses. The demonstrations allow the user to adjust the parameters for the function of a graph and then evaluate the function for a given value. This helps distinguish between parameter values for a given curve and the variable values where it is evaluated. For example, in Figure 1 the parameters are the rate of return and risk for two investments, S and B for stocks and bonds, as well as a correlation of the risks. The blue curve represents portfolio performance. The weighting of the investments gives a portfolio performance, which is a point on the curve. Changing the parameters (the right side of Figure 1) changes the shape of the curve, while changing the variable changes the position on the curve.

GeoGebra also allows for the graphing of the locus of points as the parametric values are changed. This may seem mundane from the mathematics perspective, but it is exciting and useful from the business perspective.

Figure 1

GeoGebra Demonstration of Investments



A final kind of borrowing is using new technologies from other projects to make technical details work better. The first version of *Business Calculus with Excel* (May, 2019) was produced as a set of Word files that were posted to the internet. Since then the technology for developing and presenting online textbooks has improved. PreTeXt is an open source technology, developed in part through a National Science Foundation grant, to develop and present mathematics textbooks online. We learned about PreTeXt at a Joint Mathematics Meeting workshop and have converted our textbooks to this presentation technology. We are in the process of converting all the homework problems to a WeBWorK format so that technology can be part of the grading process. With the WeBWorK version the students will get better feedback about their work and will get it instantaneously. The numerical values in the problems are randomized so that each student will solve versions of the problems. It also has a “Show me another problem” feature, which provides students with the opportunity to practice with different variants of a problem. The people behind the WeBWorK project are also working to improve the integration of GeoGebra with WeBWorK, which will allow randomized problems where the solution requires sophisticated numerical methods which are beyond the first author’s ability to code. With the utilization of both GeoGebra and WeBWorK, borrowing involved changing to a different technology when we found a better way to handle some aspect of our project.

Conclusions

As the above narrative illustrates, our pedagogical project, much like pure research projects, builds on work accomplished by others and routinely incorporates techniques devised by others. The project started with an idea we learned about during a talk at a conference that we thought we would get to when we had time. We wanted to use their book. Trying to adapt their idea to our situation meant the project would grow. As adopting a book turned into writing our own book, we consciously started looking at how other people had dealt with similar situations to ours. In our search, we found other innovators had produced technologies we could use, like PreTeXt, WeBWorK, and GeoGebra. We also found a community of people with similar interests in CRAFTY and an effective network to work with in the SUMMIT-P project. Joining the network pushed us to broaden our vision from producing a book to working with the needs of a discipline. It also gave us a structure for better interaction with the faculty of that discipline. The interaction with the business faculty helped us improve the consistency of problems students explore when studying the two disciplines. It also let us share teaching techniques and strategies between the disciplines. At several key points, we found the interaction with the business faculty was leading us in unanticipated directions. We were able to consult with the SUMMIT-P network and find ways that other schools had tried to deal with similar problems. That let us adapt a working product rather than starting from scratch.

Borrowing or stealing (with appropriate attribution, of course) helped us turn an idea from a talk at a conference into a great teaching project.

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EVALUATING A LARGE-SCALE, MULTI-INSTITUTION PROJECT: CHALLENGES FACED AND LESSONS LEARNED

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ABSTRACT
SUMMIT-P consists of nine participating institutions working toward common goals but from unique perspectives. Evaluating such a large-scale project with diverse stakeholders has presented challenges. For one, evaluation on this scale necessitates a team effort rather than a single evaluator. Communication is key among the evaluators as well as among the project players at large. Participation and reliable, timely feedback from participants are perhaps the most important issues while also posing some of our greatest challenges. We present strategies we developed to counteract these challenges. In particular, we discuss the development of an assessment tracking system used to not only monitor responses but to also promote an increase in on-time responses. We conclude with a discussion of some lessons learned about evaluating large-scale, multi-site projects to share with other evaluators and PIs alike.

KEYWORDS
multi-site evaluation, educational research

As educational research projects evolve in the 21st century, evaluation of these projects is evolving as well. Technology allows for patterns and changes to be explored at a greater scale and at distance, which has led to collaborative opportunities to explore change and growth across multiple sites. One such example of this type of project is A National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships (SUMMIT-P).

Background of SUMMIT-P

SUMMIT-P is a curriculum and faculty development project spread across multiple institutions and designed to implement the recommendations from the MAA Curriculum Foundations (CF) Project (Ganter & Barker, 2004). The project is funded by a grant from the National Science Foundation (NSF). The member institutions of SUMMIT-P form a diverse consortium in that they vary by size and type as well as geographic location. Each of nine institutions has formed interdisciplinary teams to organize discussions with local faculty from one or more partner disciplines about how best to implement changes in the lower division undergraduate mathematics courses to reflect the needs of students in those partner disciplines. In addition, these local interdisciplinary teams are expected to:

- organize discussions with local faculty in mathematics and the partner disciplines to make use of insights about interdisciplinary collaboration from the CF reports,
- organize frequent internal project team meetings to discuss course content, development/progress of work, and necessary alterations to the work plan,
- appoint one member of the institution's key personnel to be responsible for working with the central evaluation team to collect institutional data while ensuring that all partners within the institution provide necessary information, including information about faculty members' perceptions of the impact of the intervention on students' attitudes, skills, and vocational interests,
- participate as an interdisciplinary team in regular communications and meetings with the consortium wide project team,
- visit and host several site visits with commonly aligned institutions within the collaborative, and
- contribute to the national impact of the project by reporting on their work in publications and national meetings.

The project aims to create an enduring network of faculty and programs within and across institutions to share experiences and ideas for successfully creating functional interdisciplinary partnerships.

Important Elements of Program Evaluation Relevant to SUMMIT-P

Every NSF-sponsored curriculum reform project is required to have a program evaluation component. In order to support program evaluators in their work, the NSF has produced a useful and clear handbook for conducting program evaluations, *The User-friendly Guide to Program Evaluation* (Frechtling, 2010). In this guide, evaluation is defined as follows:

> A comprehensive definition, as presented by the Joint Committee on Standards for Educational Evaluation (1994), holds that evaluation is "systematic investigation of the

worth or merit of an object." This definition centers on the goal of using evaluation for a purpose. (p.3)

Frechtling (2010) continues to summarize three main purposes of evaluation: (a) to produce information that could help to improve a particular project, (b) to document what has been done on the project, and (c) to potentially gain new insights that were not expected. "What are frequently called 'unanticipated consequences' of a program can be among the most useful outcomes of the assessment enterprise" (Frechtling, 2010, p.3).

Another essential element in conducting a program evaluation is the communication between the evaluators and the stakeholders. Alkin et al., (2006) state the importance of this element by saying, "communication is a part of all program evaluation activities. Indeed, it is probably not an exaggeration to say that evaluation without communication would not be possible," (p.385). In a large-scale project such as SUMMIT-P, the importance of communication is magnified. In our situation, the stakeholders also serve as what Frechtling (2010) defines as "key informants" (p. 71). Key informants are those who have, "unique skills or professional background related to the issue/intervention being evaluated, [are] knowledgeable about the project participants, or [have] access to other information of interest to the evaluator," (p.71). Therefore, communication within this project must be a two-way street. Not only are we, the evaluators, responsible for communicating with the stakeholders, the stakeholders, as key informants, must be in communication with us.

Exploring Large-Scale Program Evaluation

In this paper, we will discuss the process of conducting a large-scale program evaluation, focusing on not just our methodology but also the successes and difficulties we have encountered; in particular, we will present our solution to the communication challenges that occur in a large-scale, multi-site evaluation. In conducting the program evaluation of large-scale projects such as SUMMIT-P, having a diverse project with respect to the types of institutions involved is both a strength and a challenge. While this diversity allows us to examine how the project evolves in many different settings, it has also been the source of many challenges we have faced. The responsibility for the evaluation team is to examine the progress being made towards the SUMMIT-P project goals, as stated below:

- Implement major recommendations from the MAA Curriculum Foundations (CF) Project for the purpose of broadening participation in, and institutional capacity for, STEM learning, especially relative to teaching and learning in undergraduate mathematics courses;
- Foster a network of faculty and programs in order to promote community and institutional transformation, through shared experiences and ideas for successfully creating functional interdisciplinary partnerships within and across institutions;
- Change the undergraduate mathematics curriculum in ways that support improved STEM learning for all students while building the STEM workforce of tomorrow; and,
- Monitor how various aspects of the CF recommendations are being implemented at participating institutions while measuring the impact on faculty and students.

Evaluating progress made toward these ambitious goals would be a challenge to evaluate on even a small scale; considering the magnitude of this project, this task is monumental. A one-size-fits-all evaluation model is not adequate for a project of this scope. In the sections that follow, we will discuss some of the particular challenges faced along with a few strategies we have implemented in our efforts to overcome these challenges.

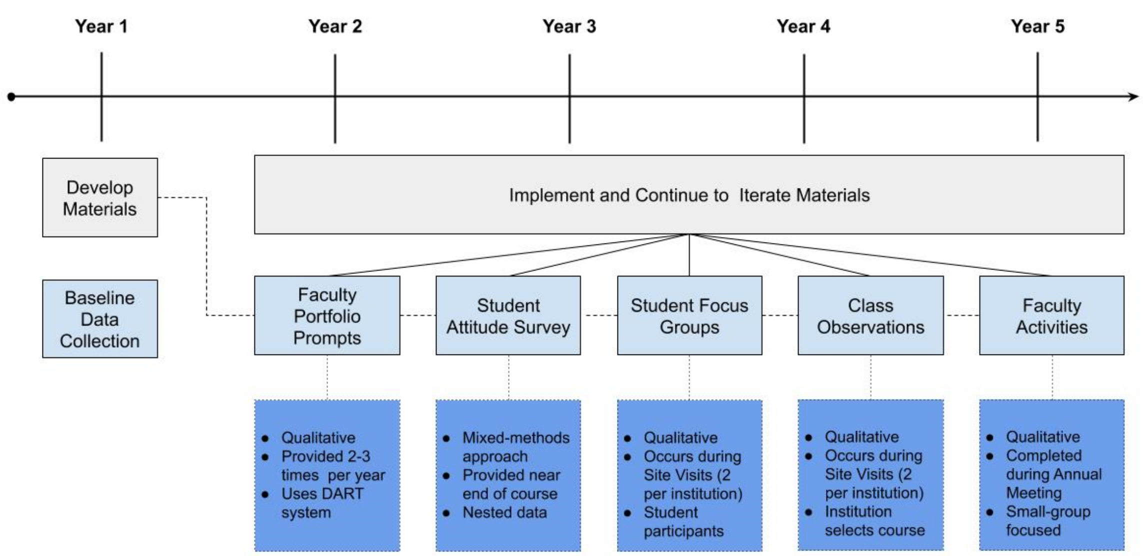
As mentioned above, this project is a collaboration among nine institutions with diverse backgrounds and populations. While all the institutions have a common goal, each institution is implementing its own model of change within the SUMMIT-P framework. This requires the evaluation team to utilize Multisite Evaluation (MSE) methods to conduct the evaluation and research. As stated in Straw and Herrell (2002), "two factors differentiate MSEs from other evaluation activities: the involvement of multiple sites and the conduct of a cross-site evaluation activity" (p. 5). Our evaluation activities are aimed at examining the program as a whole as well as the implementation at each of the sites.

The SUMMIT-P Evaluation Model

The fact that SUMMIT-P is being enacted through a large consortium of institutions creates many logistical challenges for the program evaluation. "The larger the number of sites, the more important are standards for data collection, quality control, and data submission," (Rog, 2010, p. 100). From the start, the plan was for the evaluation to be conducted by a team rather than a single external evaluator. On a project of this scale, the effort of many minds is better than the single perspective of an individual.

Best evaluation practices dictate the use of a mixed-methods design for our research and evaluation efforts due to the nature of the objectives we are assessing (Frechtling & Sharp, 2002). In order to measure the impact the program is having on student outcomes, we are collecting survey data from students enrolled in affected courses, survey data from faculty involved in teaching those courses, and other qualitative measures. Of particular interest to us is the examination of change in this context, one of the main themes of the research questions.

Figure 1
Evaluation Timeline



In addition to baseline survey data from both students and faculty, we used the SUMMIT-P site visits to further triangulate our data collection efforts (see Figure 1). As part of the project, each institution will host two site visits: the first in year two or three of the project, the second in year four or five. At site visits, a team of people travel to the host institution to observe their efforts and progress made toward the SUMMIT-P goals (Piercey & Segal, 2020). This team is composed of Primary Investigators (PIs) and co-PIs from one or two other SUMMIT-P institutions, a representative of the Project Management Team, and a member of the Evaluation Team. These site visits provide us with the opportunity to observe classes where lessons developed as part of the SUMMIT-P project are being taught (SUMMIT-P lessons), participate in both formal and informal conversations with the various stakeholders at the institution, and also conduct focus-group sessions with students whose classes are utilizing SUMMIT-P lessons. We are therefore able to examine the evolution of the project at each site firsthand.

Furthermore, we spend time during the annual SUMMIT-P face-to-face meeting, held in conjunction with the Joint Mathematics Meetings, to conduct focus group sessions with PIs and co-PIs. It should be noted that, although some data is being collected from students enrolled in the courses, our primary subjects are the faculty (i.e., the 39 PIs and co-PIs) involved in the project.

One of our richest sources of data is the Evaluation Portfolio we created in which we list various prompts for the faculty to respond to several times over the course of the year. The first prompt was designed for participants to provide baseline data regarding their prior teaching experiences and teaching philosophies. Participants wrote a "teaching autobiography" wherein they described their teaching experiences over the years, focusing on elements such as: their first teaching experiences, how their philosophy of teaching has changed over time, what teaching methods they employ, how their beliefs about student learning have changed over time (if at all), and what they find most challenging and most rewarding about teaching. Because our aim was to collect baseline data to help us develop a deeper understanding of the participants we are studying, this was a relatively lengthy writing task. The subsequent prompts have been designed for shorter responses and therefore require much less time for participants to complete. Here are examples of some of the other prompts participants have responded to:

- When all is said and done, what would convince you that your project was successful? In other words, how would you define "success" within the context of your specific situation?
- In general, when you are looking for ways to change your teaching, where do your new ideas come from? Tell us about the kinds of sources from which you primarily draw new ideas.
- Recall a recent conversation or interaction with a colleague or a student related to the work of SUMMIT-P. For example, this could be from a discussion in class or office hours, in a planning meeting with colleagues, a conversation with a dean or other administrators, etc. In your response briefly tell us about what was said (just enough to give us the main idea of the interaction/conversation). What did you learn from this interaction?

Crafting prompts that allow participants to share how the project has evolved from their perspective is essential to our research model. This is one of the main benefits of using an Evaluation Portfolio to collect data. Rather than create a list of predefined questions at the beginning of the project to be asked at regular intervals, we designed the prompts to address specific questions that are based on what is relevant to the project at the time. For example, the prompt which asked participants to define "success" within their particular context was based on a discussion between the PIs about the progress being made at their respective institutions. We decided that we needed to hear from all of the participants regarding their definitions of "success" to help us evaluate the progress at each individual institution. We are also interested to see if the definition changes for any of the individuals over the course of the project. We will use the Evaluation Portfolio to ask a follow-up question in the final year.

Since our primary emphasis is on faculty growth, our main artifact for analysis is the Evaluation Portfolio collected from the participating faculty who, as stated above, serve as our *key informants*. We are studying the responses using qualitative content analysis (Mayring, 2000). We will be triangulating our analysis with the data collected through the faculty survey that is being conducted during the site visits and the annual face-to-face meetings. Additionally, in order to develop a more complete picture of the changes taking place within the various institutions, we are collecting student data through surveys, class observations, and focus group sessions conducted during site visits. We will analyze the student surveys using a factor analysis to determine trends in their responses. We will implement a grounded theory approach to coding and analyzing the information collected through class observations and focus group responses.

In typical survey research, as with the student survey we are administering, a 30–50% response rate is considered acceptable. Because of the relatively small number of faculty participants and due to the qualitative nature of our work, we need nearly a 100% response rate to the faculty prompts in order to generate valid results. Considering that the faculty who are being surveyed are also working together on this project and are being partially supported by the grant, we believe this is a reasonable expectation. In order to show growth, we need consistent participation from PIs and co-PIs at all stages of the project: the beginning, the middle, and the end. In other words, we need responses to reach a "critical mass" in order for them to be representative.

Challenges Faced

MSEs inherently come with a set of challenges and our situation is no different. There are a large number of sites, and, by design, each one is unique. We have dealt with unanticipated events such as a change in PI at some institutions. For example, one institution withdrew from the project after two years because the PI accepted a position at a different institution. Also, across all of the project PIs, there is a broad range of prior experience with large-scale funded projects. In qualitative (or quantitative) studies where one is trying to document growth, collecting high quality baseline data is important. This can be a big challenge to project evaluation because of the inevitable unforeseen circumstances, such as changes in project personnel.

We also must consider the significance of individual personalities to program evaluation. Understanding the personalities of the individuals involved is a crucial factor in creating a functional team dynamic. An individual's personality and motivations play a role in project success. This phenomenon has been documented in the literature: "[m]uch also depends on such social factors as political and intellectual alliances, friendships, and institutional loyalties" (Bell, 1998; Godfried, 1999 as cited in Leff & Mulkern, 2002, p. 90).

While being able to examine how the project evolves in many different settings is beneficial, this is simultaneously the source of challenges. With 10 sites, 39 PIs and co-PIs, additional instructors of courses using SUMMIT-P lessons, and over 4,000 students involved, gathering data and tracking responses requires additional oversight and the use of advanced metrics. Moreover, based on the wide range of experience and understanding of project evaluation among the individuals involved in the project, it has been important to spend time explaining and discussing the purposes of program evaluation with the entire group. This has been essential for everyone to understand the significance of evaluation to the SUMMIT-P project. As described in the evaluation model above, the PIs and co-PIs are prompted to submit data including student attitude surveys and responses to evaluation portfolio prompts regularly. Dealing with stakeholders who are non-compliant or demonstrate low levels of participation is a challenge as there has been little by way of repercussions outside of the inconvenience of being asked repeatedly. Closely related to this challenge is the difficulty of finding an effective communication platform. It became apparent early on in the project that email alone was simply not sufficient. Information needs to be communicated among all parties, and, in addition to the evaluation team, PIs need to be able to keep track of the data collection requirements and deadlines. Project evaluators and participants need to know what has been submitted and what is still outstanding.

During the first two years of the project, we attempted to address the problem with email in a number of ways. We looked carefully at semester schedules and deliberately set data submission deadlines to avoid the busiest times of the academic calendar. Initially, we thought that sending email reminders would be sufficient to increase response rates. First, we sent a reminder directly to those individuals who had not yet responded. If that did not produce a response, a second reminder was sent to the participant and the local institution PI was also copied. If necessary, the third email reminder was sent to the participant, the PI, and the lead project director. While multiple reminders did increase the response rates somewhat, they were an inefficient use of the evaluators' time, especially considering that we did not reach our desired response rates.

At the second annual SUMMIT-P face-to-face meeting the evaluation team gave a presentation discussing the importance of the evaluation efforts. We reviewed the project research questions and the goals for the evaluation. We shared the response rates for the various data collection measures and explained how the evaluation depends on timely responses and that it is possible to submit a response that is "too late" to be useful. Our hope was that by educating the participants on the issues underlying the evaluation efforts the response rates would improve. In general, this was not effective. Our response rates improved slightly but were still not at acceptable levels.

While the evaluation team was actively working on a solution to improve response rates, we sought input from the Project Management Team regarding this challenge. Based on their input, we voluntarily participated in a Descriptive Consultancy Protocol during a virtual PI meeting, described in Hobson-Hargraves et al. (2020).

The purpose of a Descriptive Consultancy Protocol is to find a solution to a dilemma during a discussion session with a neutral, skilled facilitator. The person (or persons) with the dilemma poses the problem; then the group, in this instance the PIs at the virtual meeting, restates how they interpret the dilemma. After this, the group brainstorms solutions to the dilemma. "The justification behind this protocol is that framing and reframing a complicated problem is valuable for moving towards a focused solution" (McDonnough & Henschel, 2015, p.147).

We were looking for PI input on what parts of the system would be important to them, and we also wanted to give them some agency over the solution. By giving them some ownership of the process, our hope was that they would be more invested in data collection success. A solution was necessary because, up until that point, response rates were hovering around 25%. Personalized email follow-ups were taking up to six person-hours, per prompt, of evaluation team time, and yet the highest response rate achieved was 70%; this was not a sustainable system. The Descriptive Consultancy Protocol allowed us to work together with the PIs in order to fine-tune a potential solution to the response rate problem. This solution, which we have named the Digital Automated Response Tracking (DART) system, is described below.

Our Solution

We needed a system that would be straightforward for the participants to use and that would collect the data in an organized way. Additionally, we needed to monitor progress simply and efficiently, preferably in real time, and have a way to share that monitoring responsibility with institutional PIs. With Google Forms as a basis for portfolio response submissions, a tracking system was developed in Google Sheets. Using a single, stable web link, participants are able to access all current and previous prompts. They select their name, institution, and which prompt they intend to answer. The prompt question is used for skip-logic branching to lead the participant to the appropriate question set where they submit responses in a "long answer" field. This element alone streamlined the process significantly as participants were previously sending responses via email which the evaluation team then had to compile into a central repository.

The real power of the DART system is the built-in tracking feature on the back end. Within the Google Suite, when a participant completes the response to a particular prompt, that response is automatically logged on the Form Responses tab by Google. A master tracking spreadsheet with the participant names listed as the rows and the status of each prompt (i.e., complete or incomplete) in the columns was created using the Google Sheet linked to the Google Form. This tracking list is a worksheet (also called a tab) that draws the data directly from the Form Responses tab in real-time, using an array formula. That addition is automatically noted by the array formulas on the tracking sheet and the appropriate cell changes from incomplete to complete for that prompt. A corresponding conditional formatting change occurs (i.e., red to green) as well. See Figure 2 for a sample of the tracking form.

Figure 2

Sample of the DART System Tracking Sheet

Institution (Institution names removed to protect privacy)	Person (Individual names removed to protect privacy)	Position	Institution Response Rate	Prompt 1- Spring 2017	Prompt 2- Fall 2017	Prompt 3 - Spring 2018	Prompt 4- Summer 2018	Prompt 5- September 2018	Prompt 6- Spring 2019
Institution A	[Participant 1]	pi	100%	Submitted	Submitted	Submitted	Submitted	Submitted	Submitted
Institution A	[Participant 2]	co		Submitted	Submitted	Submitted	Submitted	Submitted	Submitted
Institution A	[Participant 3]	co		Submitted	Submitted	Submitted	Submitted	Submitted	Submitted
Institution A	[Participant 4]	co		Submitted	Submitted	Submitted	Submitted	Submitted	Submitted
Institution B	[Participant 5]	pi	94%	Submitted	Submitted	Submitted	Submitted	Submitted	Submitted
Institution B	[Participant 6]	co		Submitted	Submitted	Incomplete	Submitted	Submitted	Submitted
Institution B	[Participant 7]	co		Submitted	Submitted	Submitted	Submitted	Submitted	Submitted
Institution C	[Participant 8]	pi	63%	Submitted	Submitted	Submitted	Incomplete	Submitted	Submitted
Institution C	[Participant 9]	co		Submitted	Submitted	Submitted	Incomplete	Submitted	Submitted
Institution C	[Participant 10]	co		Submitted	Incomplete	Incomplete	Submitted	Submitted	Incomplete
Institution C	[Participant 11]	co		Submitted	Incomplete	Incomplete	Incomplete	Incomplete	Submitted

Figure 3
Sample of DART Institution-level Tracking Sheet

Institution C

Person (Individual names removed to protect privacy)	Position	Prompt 1- Spring 2017	Prompt 2- Fall 2017	Prompt 3 - Spring 2018	Prompt 4- Summer 2018	Prompt 5- September 2018	Prompt 6- Spring 2019
[Participant 8]	pi	Submitted	Submitted	Submitted	Incomplete	Submitted	Submitted
[Participant 9]	co	Submitted	Submitted	Submitted	Incomplete	Submitted	Submitted
[Participant 10]	co	Submitted	Incomplete	Incomplete	Submitted	Submitted	Incomplete
[Participant 11]	co	Submitted	Incomplete	Incomplete	Incomplete	Incomplete	Submitted

Team response rate: 63%

From this master tracking document, there are separate tabs for each institution that import the data via an index function. Separate Google Sheets for each institution were then created that use an IMPORTRANGE function to link each school's data to the new sheet; this allows for real-time updates to be visible in the new Google Sheets without access to other institution submission records. Institutional PIs were then granted "view only" shared rights to the sheet for their institution, allowing PIs to view the information for their institution without having the ability to modify it. Each of the lead PIs has access to his or her institution's sheet, therefore allowing them to track in real-time who has submitted responses and to which prompts. See Figure 3 for a sample.

Reflections on the Effectiveness of the New System

Thus far, the DART system has been an effective tool. The first submission collected via email, prior to implementing the DART system, had a response rate of 36% before reminder emails and 71% after two rounds of reminder emails; the most recent prompt had an 87% submission rate with no individual email reminders sent out. Response rates are much improved and, equally importantly, the time required to monitor the submissions has drastically decreased. Because the lead PI at each institution can see their own response data (including who has and who has not yet responded) they are in a better position to encourage and monitor participation, removing this burden from the evaluation team. Another benefit of the DART system is that it allows a "one-stop shop," so to speak, for participants to catch up on prompts they have not yet completed. They do not need to dig through their email inbox to find and respond to overdue prompts; instead, they are able to complete any of the evaluation portfolio prompts within DART using a single web link.

Another advantage of this system is that it houses all responses in one spreadsheet, allowing the evaluation team to easily read through responses to a particular prompt. Responses can be exported to another Google Sheet or Google Doc effectively and efficiently by using Google Suite add-ons which allow us to annotate documents with our comments and conduct a content analysis.

Many of the challenges we are facing in this MSE are par for the course in examining a complex system such as the SUMMIT-P consortium project. Analyzing data from multiple institutions will always be a challenge for evaluators, especially when there exists significant variation from one institution to another. There will always be challenges associated with unexpected events, such as a change of the PI at an institution, and there is no way to control for the "human element" inherent in the personalities, expertise, and priorities of the people involved in the project. However, we are pleased to have found a solution to one significant challenge faced in this MSE through the implementation of the DART system.

Lessons Learned

In this section, we will summarize the lessons we have learned thus far in conducting this MSE. We have organized our thoughts into two sections—advice to other evaluators and advice to future PIs.

Advice to Other Evaluators

One of the most important pieces of advice we can pass along to other evaluators, in particular those conducting MSEs, is to always focus on the people. Whether they are your subjects of analysis, those facilitating the project, or, as in our case, serving in both roles, the individual personalities, the group dynamics, and the institutional cultures will all play a large role in your work. It is important to take all of these elements into account when planning for and then carrying out the evaluation activities.

Also, there is never a "good time" for participants. Time is a precious commodity for everyone, especially in academia; everyone is busy. It is important to set clear expectations for participation up front but also to be prepared to be flexible as needed. Participants need to be willing to make time for evaluation activities, and evaluators need to be willing to adjust evaluation plans when warranted. We have found clear, effective, and efficient communication to be helpful here, a lesson we learned at times the hard way. By providing a scaffolded data collection system like DART from the beginning, investigators could set clear expectations while also providing scaffolding for successful data collection rates. It is recommended that a response system like DART be included in the data collection plan of a proposal to ensure its use from the beginning. By anticipating and preparing for the complications that come with an MSE in the planning process, evaluators can focus more on the evaluation content.

Advice to Future PIs

Communication plays a key role in the advice we offer to both future evaluators and PIs. One important role of the PI regarding the evaluation activities involved in a large-scale project is to establish a communication plan at the start of the project. The role of a local PI in a multi-site consortium, such as SUMMIT-P, is different from that of a PI of a single-site project. In a multi-site grant, the local PIs play the role of "middle management." They communicate with not only the local project participants but also with the project leadership and other PIs as well. Anticipate that email alone might not be sufficient.

Additionally, ensure you have adequately budgeted time and resources for the evaluation at both the institutional and consortium levels. In a large-scale MSE, the evaluation cannot be an afterthought; it must be an integrated part of the project. There should be clear expectations for how things will be done as well as the required levels of participation in evaluation activities. This is crucial if there are qualitative aspects of the MSE, which there very likely will be. A single evaluator will likely not be adequate; an evaluation team is probably needed. In addition to the budget for the project-level evaluators, consider adding a percentage to each local budget to support a PI or co-PI who will be responsible for overseeing project evaluation and research efforts at each site, who would then report to the central evaluation team.

Our final piece of advice is to be flexible. Expect and be prepared for unanticipated events. Have a clearly laid out plan, but know that just as projects evolve as time goes on, evaluation plans must evolve, too.

Conclusions

Project evaluation conducted by external evaluators is more than just a required element in grant work; it is a crucial piece of the project being implemented. External evaluators provide a birds-eye view of the project that few others involved in the project are able to see. We are entrenched in the work being done. We are able to see all of the various parts of the project and how they are (or are not) working together. In a large-scale consortium like SUMMIT-P, this birds-eye view is even more important. We hope that the lessons we are learning as part of our work on this project will help others who may be involved in MSEs—both participants and other evaluators. Each project comes with its own unique set of challenges. Likewise, each project needs a customized evaluation plan that can be responsive to those unique challenges; however, the lessons we have shared in this paper can be broadly applied.

Our parting thoughts are as follows: projects are composed of many elements—the “products”, the people doing the work, the data being collected, the students being taught, etc. However, in all of this, the importance of the personalities of the people involved cannot be overstated. With the right groups of committed, dedicated people working together, great things can get done.

Overall, the main lesson we have learned from this project can best be encapsulated in a quote from Holley (1982), “Those being evaluated often feel threatened by the evaluation. Evaluators need to accept the behaviors of evaluation subjects. They must be patient, persistent, and persuasive” (p.1). It all comes down to this: in the end, it is always about the humans.

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CURRICULAR CHANGE IN INSTITUTIONAL CONTEXT: A PROFILE OF THE SUMMIT-P INSTITUTIONS

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ABSTRACT
There is a national call to improve the mathematics curricula in the first two undergraduate years to improve student success and engagement. But curricular change happens in an institutional context: Who are the students, and what do they need to succeed? What is the climate for change? Does the department regularly revise its courses and curriculum? Is it common for different departments to collaborate on curricular change? What supports or obstacles does the department, college, or university have for changing the curriculum? Who are the institutional stakeholders, and what practices build their buy-in? In the SUMMIT-P project, nine different institutions ranging from small private colleges to mid-sized state universities to large public universities and a community college worked on changing the undergraduate mathematics curricula in the first two years. This paper examines the context at each institution in the project. We hope that other institutions looking to follow in our collaboration with the partner disciplines on revising the introductory mathematics curriculum at their institution will find a familiar context in one (or more) of these institutions. We include a list of questions that programs can use to examine their own institutional context.

KEYWORDS
curriculum, interdisciplinary, mathematics, partner disciplines, SUMMIT-P

In the National Consortium for Synergistic Undergraduate Mathematics via Multi-Institutional Interdisciplinary Teaching Partnerships (SUMMIT-P) collaborative research project, teams of mathematics and partner-discipline faculty members from nine colleges and universities are revising introductory mathematics courses to better meet the needs of students in the partner disciplines. The project builds on the findings of the Curriculum Foundations (CF) project, which was a collaborative effort of mathematics and partner-discipline faculty members to determine the important mathematics concepts students need to understand in order to be successful in courses in Science, Technology, Engineering, and Mathematics (STEM) disciplines. The findings from the CF project appeared in two seminal publications (Ganter & Barker, 2004; Ganter & Haver, 2011).

In this paper we describe each of the participating institutions and the project that each institution's collaborative team is working on. Then we share some of the lessons we are learning about how curricular change is, for better or for worse, influenced by the local institutional context. We end with a list of questions for faculty members to consider when planning a new curriculum revision project. We invite faculty members at other colleges and universities interested starting similar work to peruse the list of projects presented below for a familiar or useful context and to reach out to the team leader for guidance or support.

The Nine Colleges and Universities

Augsburg University is a private university in Minneapolis with around 2,000 undergraduates. The institution is known for experiential and active learning and for programs for students with disabilities or who are in recovery. Augsburg University is a popular transfer destination for students from local community colleges. The leadership of the Augsburg University SUMMIT-P team includes five faculty members: a chemist, an economist, and three mathematicians.

The Augsburg University SUMMIT-P team is revising their three-semester calculus sequence to align with the CF recommendations. Specifically, they are working to focus instruction on students' conceptual understanding and to strengthen students' ability to transfer knowledge to new contexts. The team is revising existing lab activities and creating new weekly lab investigations for Calculus I & II. These investigations allow students to explore applications of mathematics content. Through the lab experiences, they recognize different concepts and apply their mathematical knowledge and skills in STEM-related in contexts.

They are increasing active and collaborative learning in Calculus I & II by creating a three-part structure for the three weekly class sessions. Each class session now includes group work on an exploratory activity, a brief lecture, and exercises worked by pairs of students on the classroom white boards. Both the daily exploratory activities and the weekly labs are inquiry-based. The materials developed throughout this project are suitable for use as supplementary activities for calculus courses at other institutions.

The team is also re-examining and re-ordering the content of the three-course sequence, including: bringing some differential equations and multivariable calculus into Calculus I; creating a pathway from Calculus I directly into Calculus III; moving the study of limits, continuity, and formal definition of the derivative and integral from Calculus I to Calculus II; and increasing the depth and breadth of vector calculus concepts in Calculus III.

Ferris State University is a public university in Michigan serving 13,000 undergraduates. This institution is a popular transfer destination for students from local community colleges. The leadership of the Ferris State University SUMMIT-P team includes three faculty members: a social worker, a nurse, and a mathematician.

In an earlier course design project, faculty members teaching mathematics and business collaboratively developed a course, Quantitative Reasoning for Business. This new project is focused on revising and scaling the course to reach more students under a new name, Quantitative Reasoning for Professionals. The revised course now serves students majoring in business, social work, and nursing. The updated course expands the use of inquiry-based pedagogy in applied contexts to develop students' problem-solving skills. It incorporates CF recommendations to prioritize depth over breadth of knowledge, emphasizes problem solving, uses active learning pedagogy, and incorporates the appropriate technology. Project faculty members are creating additional instructional materials to focus on the mathematical concepts used in health science and social work professions. These new materials also supplement the existing inquiry-based "explorations" designed for business contexts. Health science and social work faculty members are identifying the mathematical concepts needed for contexts associated with their disciplines, reviewing exploration-based activities written by mathematics faculty members, and assessing the impact of the curriculum materials on the students in their programs. The materials that have been developed for this project are suitable for use as a textbook for a quantitative reasoning pathway course at other institutions.

Located in New York City, **LaGuardia Community College** (LAGCC) is a public, Hispanic-serving institution with over 50,000 students. A large percentage of the students are first-generation college students or are from low-income families. One of LAGCC's central goals is to improve students' quantitative reasoning and digital literacy. The leadership of the LAGCC team includes faculty members who teach mathematics and economics.

The project team is revising College Algebra to include interesting, authentic applications from business and the social sciences, to increase the emphasis on students' conceptual understanding through hands-on explorations using applets and to improve students' ability to transfer mathematical knowledge to other contexts. They are incorporating real-world problems from economics, business, and the social sciences in the course, including carefully incorporating the notation and vocabulary used in these disciplines.

The LAGCC team is also creating open-source, interactive web applets that demonstrate different quantitative relationships graphically. Students use the applets to explore concepts and to collaborate on questions about the mathematical relationships and properties that they encounter. These applets help students to deepen their conceptual understanding before they do computations and follow other procedures. The applets are suitable for use at other institutions.

Lee University is a private, four-year liberal arts college in east Tennessee serving 4,000 undergraduates. The leadership of the Lee University SUMMIT-P team consists of faculty members who teach mathematics, biology, chemistry, education, psychology, and sociology. The team from Lee University originally planned to work on three courses for SUMMIT-P, but their work expanded to five courses.

Before the project, Lee students in a wide range of majors took College Algebra. Project faculty recognized that future STEM majors, pre-service teachers, and other students would be better served by a trio of courses, each focused on the needs of the partner disciplines. They created Algebra for Calculus for students intending to major in a STEM field, revamped the existing College Algebra course to address content necessary for pre-service teachers, and added Introduction to Statistics as the course of choice for students majoring in disciplines outside of STEM and education. All three courses have been informed and enhanced by the conversations with the partner discipline faculty members participating in the project and the CF recommendations. In discussions with education faculty about College Algebra, the project team recognized some changes were also needed to Concepts of Mathematics I & II, the mathematics content courses that pre-service elementary school teachers take after College Algebra. This example of a "pathways" structure could be useful to other institutions. For a discussion of pathways models and their connection to student success see Charles A. Dana Center (2019).

In addition, the Lee University team is implementing CF recommendations for the social sciences by developing a "student exchange" program. They have recruited and prepared a team of advanced social science and mathematics majors to provide peer support for students enrolled in the Introduction to Statistics course. Faculty-supervised seminars and mentoring sessions are part of the support efforts for students participating in the exchange. This type of exchange model could be adopted by other institutions.

Norfolk State University is a public, historically black university (HBCU) in Virginia serving 5,500 undergraduates. The Norfolk State University SUMMIT-P team includes faculty members from the mathematics and engineering departments.

Project faculty partnered to redesign Calculus I & II and Differential Equations, which are courses that, due to high drop, fail, or withdraw (DFW) rates, had previously been identified as roadblocks for science and engineering majors. The engineering department teaches a course called Engineering Problem Solving that introduces engineering applications requiring the use of algebra and trigonometry concepts to strengthen students' readiness for calculus. Through this project, faculty members incorporated application-based activities into Differential Equations and Calculus I & II in a manner similar to what is used in Engineering Problem Solving. The team developed problems for key applied topics and created video lectures for procedural skills for the students to watch before class. These changes to the course also freed up class time for the study of other concepts. The activities that have been developed are suitable for use as supplementary materials in similar courses at other institutions.

Oregon State University is a public, land-grant institution serving 23,000 undergraduates. The leadership of the Oregon State University SUMMIT-P team includes faculty members who teach mathematics, biology, and chemistry courses, as well as graduate and undergraduate serving as curriculum development assistants.

The team is creating activities for Differential Calculus based on CF guidelines for biology and chemistry. Differential Calculus is a 10-week, first trimester calculus course, as compared to the 15-week, first semester Calculus I courses which also includes concepts from integral calculus. These activities help students develop connections between mathematics concepts and the content covered in General Chemistry and Principles of Biology. These activities are being implemented in recitation sections of Differential Calculus that are taught by graduate teaching assistants. Course coordination includes a shared calendar and weekly activities that are used across all sections. In addition to connecting mathematics concepts to partner discipline content, the activities also include questions with a conceptual focus to encourage students to participate in pair or small group conversations. The activities developed through this project are suitable for use in first semester calculus courses at other institutions.

The team is also designing and offering professional development sessions for Differential Calculus instructors and teaching assistants. The purpose is to increase the consistency in the content that is covered and the pedagogy that is used across all course sections, with the goal of improving students' abilities to transfer knowledge across the different disciplinary contexts.

Saint Louis University is a private university in Missouri serving 8,000 undergraduates. The Saint Louis University SUMMIT-P team consists of faculty who teach mathematics and business courses.

Through prior collaborative efforts, mathematics and business faculty wrote a textbook for Survey of Calculus that featured computing with spreadsheets (i.e., Microsoft Excel) in the course. While the course helped students to understand the important connections between mathematics and business concepts, students continued to have difficulty transferring their knowledge between courses in the two disciplines.

The current SUMMIT-P project at Saint Louis University extends that work to College Algebra. The team also developed curriculum units for two upper division finance courses, Fixed Income Securities and Markets and Derivative Securities and Markets. To help students see that the algebra and finance courses were connected, examples from the finance courses were incorporated into College Algebra with nearly identical notation and terminology. For example, in College Algebra a constant revenue stream is used as an example of exponential growth, and in the finance courses, a constant revenue stream is the focus of a curricular unit. The materials developed for this project are suitable for use as a textbook for a business calculus course or as supplemental materials for coordinated business and mathematics courses at other institutions.

San Diego State University (SDSU) is a public, Hispanic-serving, research university serving over 30,000 undergraduates, with over half of the student population being from underrepresented groups. SDSU is committed to increasing student retention in STEM courses and has conducted research on the factors leading to high failure rates in bottleneck courses such as Precalculus. The leadership of the SDSU SUMMIT-P team includes biology, physics, and mathematics education faculty members.

In this project, the SUMMIT-P team is revising the content and pedagogy for Precalculus based on CF recommendations to emphasize conceptual understanding while also maintaining a focus on procedural fluency and increasing the number of mathematical applications from other sciences. The team is also revamping recitation sections of the course to incorporate active learning strategies. They are also working to improve the training provided for Precalculus teaching assistants. The materials developed through this project can be used as supplementary activities for Precalculus at other institutions.

Virginia Commonwealth University (VCU) is a large, urban research university serving 22,000 undergraduates. Each year, approximately 7,000 students take a lower-division mathematics course ranging from College Algebra through Differential Equations. Almost 80% of these students are STEM majors. Helping students develop procedural fluency, conceptual understanding, and reasoning skills in mathematics is critical to students' future success in STEM, and so these lower-division mathematics courses are a priority for STEM partners, including mathematics. The leadership of the VCU team includes engineering and mathematics faculty members.

Through an earlier collaboration based on CF recommendations, VCU substantially reformed College Algebra to a modeling-oriented course that includes a variety of applications from the partner disciplines. In the SUMMIT-P project, faculty members are working to better align other lower-level mathematics courses with engineering and the sciences. The team decided to start with Differential Equations. Changes to the course include incorporating collaborative learning experiences for students, increasing the use of technology in the classroom, and presenting examples from engineering fields. The materials that have been developed for this project include engineering examples that can be used as supplementary activities in Differential Equations courses at other institutions.

A Few Lessons Learned

In this section, we mention a few ways in which the institutional context had an impact on the development and implementation of the SUMMIT-P projects. We hope these examples might illustrate some less-obvious considerations for institutions considering similar curricular work.

Each team started by considering the student population for the courses that were being redesigned and the reasons students were enrolling in the courses. Rather than designing a mathematics course linked to one major (e.g. College Algebra for Biology), most institutions revised a mathematics course to address students from a range of related majors. For example, Lee University's College Algebra for STEM serves students from the STEM disciplines; VCU and Norfolk State University's focus on calculus applications from engineering in Calculus I & II and Differential Equations serves students from all of the available majors in engineering and physics; Ferris State University's Quantitative Reasoning for Professionals serves students majoring in business, nursing, and social work. Such collections of academic majors that have related course content or career goals, and often overlapping sets of introductory courses, are commonly known as "meta-majors." Focusing curricular reform on meta-majors appears to have a positive impact on college completion rates (Waugh, 2016). Waugh explains:

> Sometimes also referred to as 'career clusters' or 'communities of interest,' meta-majors [as an approach to curriculum reform] refers to the creation of broad program streams such as allied health or business as a key component of guided pathways reforms. (p. 2)

A critical step for each institution was building support for the project among mathematics faculty members beyond the project. The type of support varied considerably between different institutions. For example, at Augsburg University, one of the smallest institutions in the collaboration, there are only five tenured mathematicians in the department. Three of these faculty members are directly supported by the project. It became clear early on that the team needed to find ways to include the other two tenured mathematicians in the work. The institution supported the project by providing internal support funding for each of them to participate in the curriculum reform process for one summer. In addition, teaching schedules were revised so that all tenured faculty members in mathematics were able to teach a section of the course that was being redesigned.

In contrast, teams from larger departments often had only one mathematician who was being supported by project funding. In these cases, rather than working towards consensus across the entire mathematics faculty, the teams identified a smaller, critical group of faculty members who would directly interact with the courses being revised. The instructors that were chosen to participate depended on the type of work being done. The team at VCU worked closely with all mathematics faculty members who were teaching the courses under revision. The Oregon State University project brought together mathematics course coordinators and instructors. At SDSU, the collaborative efforts involved the Math and Stats Learning Center coordinator and teaching assistants for mathematics courses.

Registration issues can throw a project off-track. For example, after mathematics and business faculty members at Saint Louis University developed sections of College Algebra specifically designed for business majors, they found out that the registration system would not separately identify the special sections. As a result, both business and non-business majors registered for the section that was intended to focus on business applications. While this problem was resolved for future terms, it resulted in a delay of the full implementation of the course.

A defining feature of the SUMMIT-P project is the close collaboration between mathematics and partner discipline faculty members. Sometimes even the course under consideration can change based on those conversations. At Oregon State University, an early outcome of this collaboration was that the faculty members changed which mathematics course would be the focus of their project. Originally, they planned to work on College Algebra. In conversations with chemistry and biology faculty, the derivative as a rate of change emerged as a pivotal concept. Based on this information, the team switched to work on Differential Calculus where derivative rate of change could be addressed at multiple points using problems from biology and chemistry.

Similarly, when project faculty at Augsburg University were working on finding chemistry examples for Calculus II, they quickly realized that multivariable calculus and vector calculus concepts in Calculus III were much more commonly used in Physical Chemistry I & II. As a result, they created a pathway from Calculus I to Calculus III for chemistry majors.

Mathematics faculty member's level of knowledge of disciplines beyond mathematics and prior experience working in interdisciplinary contexts contributed to their ability to work in collaboration with faculty in the partner disciplines. This finding aligns with the research; for example, Bouman-Gearheart et al., (2014) found that “[s]uccessful collaborations recognize the value of others’ expertise and that those involved in postsecondary improvement activities are at different points in their appreciation of interdisciplinary knowledge and work” (p. 42).

At Saint Louis University, the lead professor had participated in an earlier project to develop a business-rich curriculum for Survey of Calculus. This expertise helped to jump-start the new partnership through the SUMMIT-P project work on College Algebra. VCU selected Differential Equations as the first course to revise in part because the faculty members teaching that course were more familiar with modeling and applications in science and engineering contexts than faculty members teaching other lower-level mathematics courses. Increasing the focus on modeling in Differential Equations was a natural fit for their project.

Early in the project, each team held “fishbowl” format listening sessions to hear partner discipline faculty members discuss the mathematics their majors needed to know. Follow-up discussions often began by trying to translate the skills and concepts identified by the partner disciplines into mathematical skills and concepts. For example, partner disciplines might refer to “variation” or “dependence,” which mathematics faculty members would call “functions” or “covariation.” Both instructors and students have trouble navigating across the linguistic differences between disciplines. In creating curricular materials, faculty members paid attention to need for a glossary to help with the translation between mathematics and the other disciplines.

This ability to speak across disciplines echoes broader research findings showing that teams can benefit from having members who "bridge the knowledge and experience gaps of those struggling with content or practices, often in informal ways during everyday situations that do not necessarily have this bridging as an explicit focus" (Vasquez et al., 1994, p. 41). The authors dubbed these collaborators "brokers," who they defined as individuals who provide "recourse to multiple sources of linguistic and cultural knowledge in order to create meaning, negotiate a task, or solve a problem" (Vasquez et al., p. 96).

For example, in a previous project at Ferris State University, the lead mathematics faculty member had worked closely with members of the business faculty to design a course. He was able to leverage that expertise when working with members of the social work and nursing faculty to transform the mathematics courses for business majors into Quantitative Reasoning for Professionals, which serves students majoring in business, social work, nursing, and other pre-professional majors.

At Augsburg University, applied mathematicians with expertise in biology, chemistry, and physics are participating in the project, and another mathematics faculty member had studied economics in college. While not experts in these partner disciplines, the additional knowledge base has been very helpful when engaging in in-depth conversations with the partner discipline faculty members.

Questions to Ask

Here are some questions the institutions considered, or in some cases wish they had considered, early in the project planning process. The reader who is interested in embarking on a similar curricular journey should ask themselves questions like these.

1. What are the goals of the interdisciplinary collaboration at your institution?
   a. What issues or problems are you trying to address through the work?
   b. How does the work align with departmental, college, or institution-level initiatives?
   c. How will you disseminate the work within your institution? Are there campus leaders who should be engaged early in the project?
   d. How will you determine the impact of the project? How will you get feedback during the initial phases? How will you know if it was successful?
2. What course or courses are the focus of your project?
   a. What is the scale of your project—completely revising a course or course sequence (large scale), or adding some activities labs to an existing course (small scale), or something in between?
   b. Does the project have a curricular focus, pedagogical focus, or both? If the focus is on the curriculum, what level of support do you need from the department in which the course is being offered? If the focus is on pedagogy, will professional development opportunities be needed for instructors or teaching assistants?
   c. What partner disciplines are most interested in or affected by the course?
3. Who are the possible team members (including yourself)?
   a. What key interests and expertise should be represented in the project?
   b. How many members of your department should be on the team?

   c. How will members of the partner disciplines faculty contribute to the project? What will members of the partner disciplines faculty gain from the project?
   d. What are individuals' interests and commitment levels?
   e. How does the work support what individuals are currently doing or align with work they want to be doing?
   f. Will the work be valued in decisions about an individual's tenure, promotion, or re-appointment?
   g. Realistically, are individuals already too busy to commit to another project?
4. How and when will the work be accomplished?
   a. What resources will you need to complete the project? How much time does each team member need to do the work?
   b. How often will the team meet? Will there be intensive working times (e.g. summer or travel to workshops)?
   c. Where will you travel to gather information for the project (e.g. workshops or site visits to similar institutions)? Where will you present your findings (e.g. conferences)?
   d. What is the project timeline? How soon will you be able to implement some or all of the changes?
   e. Have you considered any obstacles to the work? How will obstacles be addressed should they arise?
5. How might you build a collaborative network that extends beyond your institution? Could you, for example, join a SUMMIT-P project?
   a. Which SUMMIT-P projects have similar aspects to your intended project?
   b. Have you reached out to a SUMMIT-P team as a possible collaborator?
   c. Can you use some of the methods from the SUMMIT-P project to start or accelerate your work?

An Invitation

A key purpose of this paper is to describe the SUMMIT-P institutions and provide examples of how each institutional context has influenced the work with the hope that faculty members at other colleges and universities interested in starting a similar project might recognize a familiar context. Such faculty members are welcome to contact the SUMMIT-P project team leader for possible support which might take the form of video conversations, in-person meetings at mathematics conferences, or potential on-site visits.

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ANALYZING THE COGNITIVE DEMAND OF ENACTED EXAMPLES IN PRECALCULUS: A COMPARATIVE CASE STUDY OF GRADUATE STUDENT INSTRUCTORS

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ABSTRACT
The cognitive demand of mathematical tasks is an important aspect of analyzing the impact of instruction on student learning. The purpose of this study was to examine the instructional examples enacted by graduate student precalculus instructors in order to answer the following questions: What is the cognitive demand of the enacted examples? What does a high cognitive demand example look like when an instructor uses direct instruction? And how are examples drawn from the written curriculum enacted in different ways? Using both random and purposeful sampling of precalculus lessons, I conducted classroom observations as well as pre- and post-observation interviews with the instructors. A modified version of the Task Analysis Guide (Smith & Stein, 1998) was then used to categorize the cognitive demand of the instructional examples. As a result, I found that 25 out of the 93 examples (27%) I observed were enacted at a high level of cognitive demand. I also present vignettes that illustrate how three different instructors chose to enact the same example type at differing levels of cognitive demand.

KEYWORDS
cognitive demand, example, precalculus, graduate student instructor

The cognitive demand of mathematical tasks is something that has been widely studied in the literature (Boston & Smith, 2009; Jackson, Shahan, Gibbons, & Cobb, 2012; Kisa & Stein, 2015; Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996). Studies have found that high cognitive demand tasks provide students with more opportunities to learn (Floden, 2002; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Smith & Stein, 1998; Stein, Remillard, & Smith, 2007) but are difficult for instructors to enact (Charalambous, 2010; Henningsen & Stein, 1997; Rogers & Steele, 2016). However, much of the literature on this topic has focused on analyzing the cognitive demand of tasks where students are the primary *doers* of mathematics. This lens makes sense, since reforms in mathematics education have called for more student-centered instruction and engaging students in authentic problem solving (National Council of Teachers of Mathematics, 2000; Mathematical Association of America, 2018). However, this lens makes it difficult to analyze the cognitive demand of instructional examples, which are presented through direct instruction.

Purpose

The purpose of this collective case study is to examine the instructional examples enacted by graduate student instructors in precalculus courses at a large public university. First, I examine existing literature on the cognitive demand of mathematical tasks and the use of examples. Next, I address some common concerns that often come up from the assertion that instructional examples presented by the instructor can be presented at a high level of cognitive demand. Third, I explain the methods that I used to analyze the cognitive demand of instructional examples used by graduate student instructors teaching a precalculus course. I present a modified version of Smith and Stein's (1998) Task Analysis Guide that disentangles the *who* from the *what*. Finally, I present the results of my analysis and use three vignettes to illustrate what low-level and high-level instructional examples might look like.

Framework and Research Questions

The framework that I used in this study was Stein, Remillard, and Smith's (2007) temporal phases of curriculum use. Building on the concepts of *formal* or *planned* curriculum, *institutional* or *intended* curriculum, *enacted* curriculum, and *experienced* or *attained* curriculum (Doyle, 1992; Gehrke, Knapp, & Sirotnik, 1992; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002), Stein et al. identified three temporal stages of curriculum unfolding: *written*, *intended*, and *enacted*. The authors define the *written* curriculum as "the printed page" in textbooks or teacher materials, the *intended* curriculum as "the teachers' plans for instruction," and the *enacted* curriculum as "the actual implementation of curricular-based tasks in the classroom" (p. 321). These three phases are viewed as unfolding in a temporal sequence, and all phases have an impact on student learning. However, studies have shown that the final stage, the *enacted* curriculum, is the phase that has the greatest impact on student learning (Carpenter & Fennema, 1991). At each stage in the process, factors such as teachers' beliefs and knowledge, orientations towards curriculum, professional identity, and professional communities, in addition to the organizational and policy contexts as well as the classroom structures and norms, all impact the unfolding of the curriculum.

The research questions that guided this study were:

- What is the cognitive demand of the *enacted* examples in precalculus courses taught by graduate student instructors?
- What might a high cognitive demand example look like if an instructor chooses to *enact* an example using direct instruction?
- What similarities and differences are there between the examples that graduate student instructors *enact* when they use the same *written* curriculum materials?

Literature Review

Cognitive Demand

The framework for analyzing the cognitive demand of mathematical tasks was developed by Stein and Smith (1998). In their framework, they defined lower-level demand tasks as "tasks that ask students to perform a memorized procedure in a routine manner" and higher-level demand tasks as "tasks that require students to think conceptually and that stimulate students to make connections" (p. 269). Each of these categories was then broken down into two subcategories: memorization and procedures without connections (lower-level demands) and procedures with connections and doing mathematics (higher-level demands). Smith and Stein differentiated procedures with and without connections as representing different levels of cognitive demand. They separated these two types of tasks in order to categorize mathematical tasks that "use procedures, but in a way that builds connections to the mathematical meaning" (p. 270) of the underlying concept as a higher-level demand task. Tasks which require doing mathematics are categorized as higher-level demand tasks that require "students to explore and understand the nature of relationships" (Smith & Stein, 1998, p. 347). To aid in differentiating between the different types of tasks, Smith and Stein developed the Task Analysis Guide (p. 348), which lists the characteristics of the four types of mathematical tasks.

Stein et al. (1996) used the Task Analysis Guide to analyze a sample of 144 tasks that were implemented in reform-oriented classrooms. They found "the higher the cognitive demands of tasks at the set-up phase, the lower the percentage of tasks that actually remained that way during implementation" (p. 476). This finding provides confirming evidence for the claim that tasks with high cognitive demand are difficult to enact (National Council of Teachers of Mathematics, 2014, p. 17). To facilitate the design of lesson plans that would support high cognitive demand tasks, Smith, Bill, and Hughes (2008) developed the "Thinking Through a Lesson Protocol" (TTLP). Moving from the intended phase to the enactment phase, Jackson et al. (2012) examined four crucial elements for launching complex tasks: discussing the key contextual features, discussing the key mathematical ideas, developing a common language to describe the key features, and maintaining the cognitive demand. Similar to the TTLP, the authors provide teachers with a set of planning questions to reflect on what to do to launch a complex task effectively. In another paper, Jackson et al. (2013) examined how the launch of tasks correlated with opportunities to learn mathematics during the whole-class discussion. They found that by attending to the crucial elements for developing a common language to describe the key task features and maintaining the cognitive demand of the task during the launch, students had opportunities for higher quality learning during the concluding mathematics discussion.

Instructional Examples

Bills et al., (2006) highlighted the importance of studying examples and exemplification in mathematics. First, examples play a central role in the development of mathematics as a discipline and in the teaching and learning of mathematics. Second, “examples offer insight into the nature of mathematics through their use in complex tasks to demonstrate methods, in concept development to indicate relationships, and in explanations and proofs” (pp. 126 – 127). While examples can be presented in a variety of ways, Bills et al. emphasized that “providing worked-out examples with no further explanations or other conceptual support is usually insufficient,” as “learners often regard such examples as specific (restricted) patterns which do not seem applicable to them when solving problems that require a slight deviation from the solution presented in the worked-out example” (p. 140). Therefore, the authors emphasize that it is important for worked-out examples to include explanations and reasoning.

By studying the purpose, design, and use of mathematical examples in elementary classrooms, Rowland (2008) found that teachers need to attend to variables, sequencing, representations, and develop learning objectives when choosing which examples to use in the classroom. Similarly, Muir (2007) found that teachers need to attend carefully to the examples that they choose to use when teaching numeracy in order to “avoid the likelihood of students developing common misconceptions about important mathematical concepts” (p. 513). Zodik and Zaslavsky (2008) examined the different characteristics of how teachers choose mathematics examples. They developed a framework that captures the type of examples teachers choose and how the examples are generated. Finally, Mesa et al., (2012) looked at the opportunities to learn through the examples included in college algebra textbooks. In particular, the authors examined the cognitive demand of the examples by coding them according to the categories in Smith and Stein’s (1998) framework (i.e., memorization, procedures without connections, procedures with connections, and doing mathematics). They found that of the 488 textbook examples that they analyzed, 445 (91%) of them could be described as procedures without connections. Looking at individual textbooks, 75% – 100% of the examples included fell into this category. Of the remaining examples, 41 (8%) were determined to be procedures with connections, two (<1%) were described as doing mathematics, and none of the examples were coded as memorization tasks.

Methods

Overall Approach and Rationale

I chose to use a case study methodology, since I was interested in developing in-depth descriptions of what low-level and high-level cognitive demand enacted examples can look like in precalculus. According to Creswell (2013), “case study research involves the study of a case within a real-life, contemporary context or setting” (p. 97). Case study methodology is rooted in medicine and law, but is also a common methodology in educational research (Yazan, 2015). Because I examined multiple instructors, this project was designed as a *collective* case study (Yin, 2009). Also, since I was interested in examining similarities and differences between the examples that graduate student instructors enacted when they used the same written curriculum materials, I framed this as a *comparative* case study. The main *bounded system* that defined a case in this study was the individual examples. However, I grouped examples by instructor and lesson in order to conduct a cross-case analysis.

Site Description

The Mathematics Department

This study was conducted at a large, public university in the Midwest. In order to improve student experience and success in lower-level courses, the mathematics department had successfully transformed their precalculus courses by incorporating active learning and course coordination (raising pass rates from 60% to 80%). To oversee this transformation, the department hired a director of first-year mathematics, who was a term faculty member, and formed a faculty committee to help lead a research project to study the department's changes in instruction and to provide formative evaluation to inform and improve the initiative. The department defined active learning as involving teaching methods and classroom norms that promoted student engagement in mathematical reasoning, peer-to-peer interactions, and instructors inquiring into student thinking. Class sizes were capped at 35, and first-time instructors were provided with undergraduate learning assistants who provided additional support during class. In an effort to provide a more uniform experience for students and support for instructors (who were primarily mathematics graduate students), all precalculus courses were heavily coordinated. Each course had an experienced graduate student who served as the primary coordinator and worked closely with a faculty course coordinator. Instructors were expected to use active learning and attended weekly course meetings. There was a common course schedule, homework assignments on WeBWorK, group quizzes (often written by the instructors), individual exams (written by the course coordinators), and shared grading of exams (by the instructors).

Precalculus Courses

Students could choose to take College Algebra (3 credits) and Trigonometry (2 credits) over two semesters or a combined course, College Algebra + Trigonometry (5 credits), during one semester. Graduate students and faculty members were involved in the development of the departmental precalculus curriculum materials, which focused on students making sense of mathematics and developing procedural fluency. Although the structure and content of the materials stayed the same, the coordinators and instructors would collaboratively make changes and improvements throughout the summer and academic year. The curriculum materials included student workbooks and instructor lesson guides, which promoted student engagement and built on student thinking. During class, students were expected to propose questions, communicate their reasoning, and work in groups to complete workbook problems. Instructors were expected to dedicate the majority of class time to group work and student presentations and limit periods of direct instruction to at most 15 – 20 minutes at a time.

Instructor Population

As mentioned previously, the majority of the instructors for precalculus courses were mathematics graduate students in the department's doctoral program. After serving as recitation instructors for Calculus I or II, graduate students were typically assigned to teach College Algebra or Trigonometry in their second year. For many of the graduate students, this was their first experience serving as the instructor of record for a course, so the department ran an intensive three-day workshop before the Fall semester and required graduate student instructors to take a year-long course on teaching and learning mathematics at the post-secondary level. During their third year of doctoral studies, graduate students were typically assigned to teach the combined College Algebra + Trigonometry course for one semester.

Sampling Techniques

Participants

For my study, I chose to interview and observe instructors who were graduate students in their third year or higher and were teaching a precalculus course for at least the third semester (see Table 1). The reason I chose to study more experienced graduate student instructors instead of novice second-year instructors was twofold. First, while active learning is becoming more common in mathematics education, many of the graduate student instructors had only experienced lecturing in mathematics courses. So not only were they teaching their own course for the first time, but they were also being asked to use classroom practices that were new to them. Second, while the departmental lesson guides were beneficial in that they provided the instructors with suggested sequencing, examples, and timing, the second-year graduate student instructors were still teaching the course for the first time. Therefore, they may have struggled with not understanding some of the content, spotting common student conceptions and misconceptions, or applying different approaches to teaching procedures and concepts. (As a note, instructors chose their own pseudonyms in order to preserve anonymity.)

Table 1
Instructors' Year in Graduate School and Course Assignment

Instructor	Year	Course
Juno	3	Trigonometry
Emma	3	College Algebra + Trigonometry
Kelly	3	College Algebra + Trigonometry
Alex	4	College Algebra + Trigonometry
Dan	4	College Algebra + Trigonometry
Greg	5	Trigonometry
Selrach	5	College Algebra + Trigonometry

Lessons

During the first semester, I used random sampling for classroom observations. In particular, I asked instructors to pick one date in September, October, and November for me to observe their class. Since I only observed one class, there were times where I only observed part of a lesson because it was spread out over multiple days. During the second semester, I implemented purposeful sampling. First, I analyzed the lesson guides to identify lessons that were more procedural in nature, because I thought that this would provide me with the opportunity to see how a procedural example could be enacted at either a high or low level of cognitive demand. Then I verified that I could observe each instructor teach the lessons I had identified, although I still only observed each instructor teaching three times. Table 2 contains a list of the lessons I observed during the second semester. Several of the lessons were spread out over two days, so I visited the classroom on both days.

Table 2
Purposeful Sampling of Lessons

Title	Course
The Vertex of a Parabola	College Algebra
Function Compositions	College Algebra
Logarithms and Their Properties	College Algebra
Properties of Inverse Functions	College Algebra
Tangent and Reciprocal Trigonometric Functions	Trigonometry
Trigonometric Equations and Inverse Functions	Trigonometry
End-of-Semester Review	Trigonometry

Data Collection Methods

There were three primary forms of data that I collected for the study: curriculum materials, video recordings of my observations, and recordings of my pre/post-observation interviews with the instructors. The curriculum materials that I collected included the departmental lesson guides and student worksheets for each lesson that I observed. Since the lesson guides had undergone multiple revisions throughout the years, instructors tended to use the version that they were given the first time they taught a precalculus course. So, while I observed several instructors teach the same lesson, some of them used different versions of the lesson guides. Also, there were slight differences between the lesson guides for the Trigonometry course and the lesson guides for the trigonometry unit of the College Algebra + Trigonometry course. However, all instructors who taught the same course used the same student workbooks. In addition, I asked instructors to provide me with a copy of their lesson plan, if they made one. Some instructors would create a separate lesson plan that followed the lesson guide, while other instructors used the lesson guide as a lesson plan.

Before each classroom observation, I met with the instructor for approximately 30 minutes to discuss their plan for the lesson. During the interview, I focused on the examples that they planned to present during the class. In some of the lesson guides, exact examples were given, while others provided example parameters or a description of the type of example that should be used. Typically, I conducted the pre-observation interview the morning before I observed the lesson, but occasionally schedules required that we meet the day before. The full semi-structured, pre-observation interview protocol can be found in Miller (2018). During the classroom observations, I videotaped each example the instructor presented and took detailed field notes. After each observation and before the post-observation interview, I analyzed the video recordings to determine the level of cognitive demand of each example. During the post-observation interview, I asked questions about the instructors' decision-making process, which was the focus of another part of the study that is not reported in this paper.

Data Analysis Procedures

In order to analyze the cognitive demand of the enacted examples, I used Smith and Stein's (1998) Task Analysis Guide. However, the language used in the original framework specified both *who* was doing the mathematics (students) and *what* mathematical work was being done. Since in my study, examples were often presented primarily by the instructor, I found the original framework difficult for me to apply. For example, the original framework includes phrases such as “students need to engage,” “require students to explore and understand,” “require students to access,” and “require students to analyze” (p. 348). While these are all desirable undertakings for students, I realized that many instructors viewed examples as mathematical tasks for the instructor to present through direct instruction and worksheet problems as mathematical tasks to engage students. However, other aspects of the cognitive demand framework focused more on the mathematical work being done, with phrases like “reproducing previously learned facts,” “have no connection to the concepts or meaning,” “represent in multiple ways,” and “analyze the task and actively examine task constraints” (p. 348). Therefore, in order to analyze the cognitive demand of the examples that were presented by the instructor, I felt it was necessary to modify some parts of the original Task Analysis Guide.

Since the Memorization category of the Task Analysis Guide is primarily described in terms of the mathematical work inherent in the task, this first low-level category did not require any modifications. The second low-level category, Procedures without Connections, only required slight modifications. While the descriptors do not reference *who* is completing the task, Smith and Stein (1998) claim that Procedure without Connections tasks will “require limited cognitive demand for successful completion” (p. 348). Since my purpose for using the Task Analysis Guide was to determine the cognitive demand of a task, I found this recursive definition to be problematic. Therefore, I chose to remove this language from both the Procedures without Connections and Procedures with Connections descriptions.

In addition to removing the phrase “require some degree of cognitive effort” (Smith & Stein, 1998, p. 348) from the Procedure with Connections category, I also removed any reference to who was completing the task. In particular, the original description stated that when working on Procedure with Connections tasks, “students need to engage with conceptual ideas that underlie the procedure to complete the task successfully and that develop understanding” (p. 348). While this language could possibly be applied to moments when the teacher is using direct instruction, I thought that the language used earlier in the description for this category (“focus students’ attention on” p. 348) more clearly applied to any situation, so I decided to use this language in two places (see bullets 1 and 4 in Table 3) to describe how students might (directly or indirectly) engage with the task. The final category, Doing Mathematics, required the most modification to remove descriptions for referencing *who* is doing the mathematical work and instead ensure that the statements focus on *what* mathematical work is entailed in the task. In total, I used the phrase “focus students’ attention on” in three places as a replacement for the phrase “require students to.” I also removed the reference to requiring “considerable cognitive effort.” My complete modification of the Task Analysis Guide appears in Table 3.

Table 3
Modification of Smith and Stein’s (1998) Task Analysis Guide

Low-Level Demands
Memorization • Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.

- Are not ambiguous. Such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meanings that underlie the facts, rules, formulas, or definitions being learned or reproduced.

Procedures without Connections

- Are algorithmic. Use of the procedure is either specifically called for or is evident from prior instruction, experience, or placement of the task.
- Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.

High-Level Demands

Procedures with Connections

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Although general procedures may be followed, they cannot be followed mindlessly. They focus students' attention on engaging with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Doing Mathematics

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or worked-out example.
- Focus students' attention on exploring and understanding the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulations of one's own cognitive processes.
- Focus students' attention on accessing relevant knowledge and experiences and making appropriate use of them in working through the task.
- Focus students' attention on analyzing the task and actively examining task constraints that may limit possible solution strategies and solutions.
- May involve some level of anxiety for the students because of the unpredictable nature of the solution process required.

Results

Over the two semesters, I observed a total of 24 lessons presented by the instructors, which spanned 33 days and included 93 different examples (see Table 4). Twenty-five of the examples were presented at a high level of cognitive demand (which I refer to as HCD examples). As a note, Greg was the graduate student course coordinator for Trigonometry, so I was able to observe him during both semesters. All of the other graduate student instructors only taught a precalculus course for one semester.

Table 4
Distribution of Observed Examples

Instructor	Lessons	Days	Examples	HCD Examples
		Fall Semester		
Emma	3	3	9	1
Kelly	3	3	7	4
Alex	3	3	5	3
Greg	3	3	6	1
Subtotal	*12*	*12*	*27*	*9*
		Spring Semester		
Juno	3	5	14	4
Dan	3	6	18	3
Greg	3	5	19	9
Selrach	3	5	15	0
Subtotal	*12*	*21*	*66*	*18*
Total	24	33	93	25

In the Fall semester when I used random sampling for the observations, I observed an average of 2.25 examples per day, 33% of which were enacted at a high level of cognitive demand. Emma had the lowest percentage of HCD examples (11%) but the highest number of observed examples, and Alex had the highest percentage of HCD examples (60%) but the lowest number of observed examples. In the Spring semester when I used purposeful sampling for the observations, I observed an average of 3.14 examples per day, 24% of which were enacted at a high level of cognitive demand. Selrach had the lowest percentage of HCD examples (0%) and the second lowest number of observed examples, while Greg had the highest percentage of HCD examples (47%) and the highest number of observed examples. Over all of my observations, 27% of the examples were enacted at a high level of cognitive demand.

Vignettes

In order to gain a better understanding of what low-level versus high-level cognitive demand examples look like, I have included three vignettes below. These three vignettes were selected because they demonstrate how different instructors enacted the same type of example at differing levels of cognitive demand. In the Spring semester, I observed Juno, Greg, and Dan each teach the lesson entitled *Trigonometric Equations and Inverse Functions*. Juno and Greg each taught this lesson in their Trigonometry course in March, while Dan taught this lesson in his College Algebra + Trigonometry course in April. In all three cases the lesson was spread over two consecutive days, and I observed both days. I chose to observe this lesson because through random sampling I had observed Greg teach the second half of this lesson in the Fall semester and realized that it involved a lot of procedures. Within the lesson I chose to focus on a particular example that involved finding all solutions to a trigonometric equation that correspond to a non-standard unit circle angle. I did this because Juno and Greg enacted their examples at a high level of cognitive demand while Dan enacted his example at a low level of cognitive demand.

Written Lesson Guide Description

These vignettes feature the final example that was included in the written curriculum for the first day of the lesson entitled Trigonometric Equations and Inverse Functions. The learning objectives for this day were that students should be able to (i) graphically represent solutions to an equation, (ii) understand the process of finding all θ that satisfy the equation $f(\theta) = a$, where a is fixed and f is a periodic function, and (iii) use the unit circle to solve equations in part (ii) for f, a trigonometric function. The lesson guide provided the following outline for the lesson.

I. Demonstrate how solutions to equations of the form $f(x) = a$ can be represented graphically as the intersection points of two equations, $y = f(x)$ and $y = a$.
II. Give students time to work in small groups on a Ferris wheel word problem in the workbook and identify periodic patterns in their solutions.
III. Work through an example of graphing two functions, $y = \cos(\theta)$ and $y = \sqrt{3}/2$, and prompt students to notice the pattern in the occurrence of the intersection points.
IV. Introduce the procedure for solving trigonometric equations by first finding the "initial" or "core" solutions that occur in one period and then adding integer multiples of the period (e.g. 2π).
V. Give students 10 minutes to work in small groups on two similar worksheet problems (i.e. solve $\sin(\theta) = -\sqrt{3}/2$ and $\cos(\theta) = -1/2$)
VI. Bring the class together to discuss the final example (described below).

Even though there were slight differences in the lesson guides used by the instructors (which I describe below), the general lesson outline was the same.

Lesson Guide Example Descriptions

Juno and Greg drew from the same version of the lesson guide. Figure 1 contains the example description that Juno and Greg used for this lesson. This example is procedural, explicitly stating that "the strategy for finding θ is still a two-part process: find initial solutions and then translate them." I categorized this example as Procedures without Connections (see Table 3) because it does not explicitly make connections to the concepts or meaning that underlie the procedure being used (i.e., there is no explanation for why there are two initial solutions or why it is necessary to add the period to an initial solution to determine an infinite family of solutions). In addition, the example, as written, is more focused on students producing correct answers (e.g., "use inverse trigonometric functions on our calculators," "add or subtract copies of 2π in order to find additional solutions," and "letting $\theta = 0.84$ and calculating $\cos(\theta)$ on the calculator should give a value very close to $2/3$") instead of students developing mathematical understanding of the concepts. However, as I will demonstrate, both Juno and Greg transformed this example in ways that raised the level of cognitive demand.

Figure 1

The Example Description in the Version of the Lesson Guide Used by June and Greg

Example. Solve the trigonometric equation $\cos(\theta) = 2/3$.
Notice that $\cos(\theta) = 2/3$ is not satisfied by any standard angle present on the unit circle. However, the strategy for finding θ is still a two-part process: find initial solutions and then translate them.

(i) Since $\cos(\theta) = \frac{2}{3}$ is not satisfied by any standard angle present on the unit circle, we need to use inverse trigonometric functions on our calculators. This produces one solution of:

$$\theta = \cos^{-1}(2/3) \approx 0.84$$

The second solution we obtain by symmetry, finding $\theta \approx -0.84$.

(ii) Recall now that, since $\cos(\theta)$ is periodic, we can take our two initial solutions to $\theta = 0.84$ and $\theta = -0.84$, and add or subtract copies of 2π in order to find additional solutions. For example, two additional solutions are given by:

$$\theta = -0.84 + 2\pi \quad \text{and} \quad \theta = 0.84 + 2\pi$$

To account all possible solutions, we write:

$$\theta = 0.84 + 2\pi k \quad \text{and} \quad \theta = -0.84 + 2\pi k$$

Again, k can be **any** integer!

Important: We can check our solutions! For example, letting $\theta = -0.84$ and calculating $\cos(\theta)$ on the calculator should produce a value very close to $2/3$ if we have the correct solution. Note that we should have our calculators in radian mode for this task.

Important: Solving a trigonometric equation involves finding all of the solutions within a single repeated segment. Once all initial solutions are found, we use the fact that the trigonometric function is periodic to find the other solutions (by adding copies of the period to the solutions). It will not always be the case that we find two initial solutions or that we add 2π. This will be explored in Exercises 4 and 5 of the worksheet and will be seen many times in the future.

Figure 2

The Example Description in the Version of the Lesson Guide Dan Used and the Problem Dan Chose from the Student Workbooks

Example. Discuss the case in which the initial solutions are not angles on their unit circles. You can make up your own example here, or do Problem 3(a) as a class. Remind students that they can check their solutions, and walk them through the process of doing so in this scenario.

Problem 3(a) Solve the trigonometric equation $\sin(\theta) = -2/3$.

Dan, however, used a different version of the lesson guide and therefore the description he based his example on was different (see Figure 2). In particular, the version of the lesson guide that Dan used provided much less detail regarding how to present the example and what to emphasize, but the equation used in the example was similar to the one that Juno and Greg used. The lesson guide stated that the instructor could make up their own example or use a problem from the student workbook. Dan decided to do the latter. Because the lesson guide was so vague, it was difficult to determine the cognitive demand of the example. However, I chose to code it as Procedures without Connections (see Table 3) because the problem statement only referenced solving the trigonometric equation and the lesson guide prompted Dan to "remind students that they can check their solution and walk them through the process of doing so in this scenario." So this example also seemed to primarily be focused on students producing correct answers. Dan's presentation did not change the cognitive demand of the task.

Juno. During the observation, Juno followed the lesson guide closely (see Figure 1). At the end of the first day, Juno began the final example by directing students' attention to a graph she had drawn on the board at the beginning of class (see Figure 3). In describing how this example was similar to previous problems they had completed (e.g. solve $\sin(\theta) = -\sqrt{3}/2$ and $\cos(\theta) = -1/2$), she emphasized that "the fact that it's not on the unit circle doesn't change anything on the graph…. We still have two solutions in one period, so we still want to find the base solutions, and then add $2\pi k$." Juno proceeded by explaining how to use the inverse cosine function on the calculator to find one base solution, and then she asked her class, "Do you remember how we find the other solution?" Student 1 responded with "add π," so Juno drew Figure 4.

Figure 3
The Graph Juno Drew on the Board at the Beginning of Her Lesson

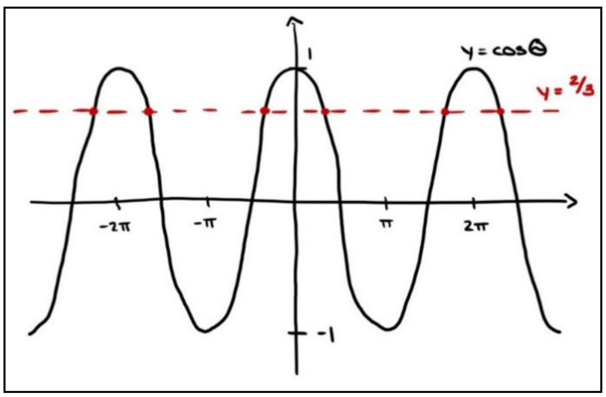
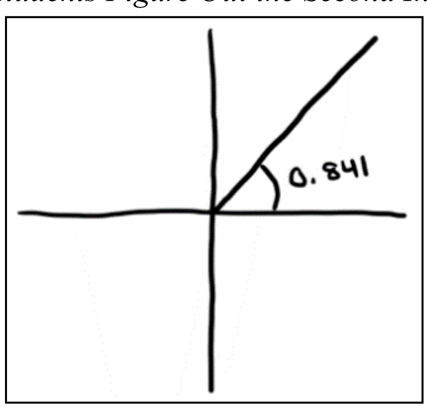


Figure 4
The Graph Juno Drew to Help Students Figure Out the Second Initial Base Solution.



The discussion that ensued is presented here as an excerpt of the transcript of the observation video.

Juno: So, here we got to know what quadrant we are in. So, this is 0.841 *[draws Figure 4]*. Which other quadrant is cosine positive in?

Student 2: Four.

Juno: Yeah, fourth quadrant. *[Adds the corresponding fourth quadrant angle to Figure 4.]* So, that's the angle we want. You can either do $2\pi - 0.841$, or you can just do -0.841. Ok, so if I do $2\pi - 0.841$ I'm getting this solution *[pointing to the intersection point immediately to the left of $x = 2\pi$ in Figure 3]*. If I do -0.841, I'm getting that solution *[pointing to the intersection point immediately to the left of $x = 0$ in Figure 3]*. But it doesn't really matter, because we are just going to be shifting them by multiples of the period anyway. So, then our general solution is going to be

$$\theta = 0.841 + 2\pi k$$
$$\theta = -0.841 + 2\pi k$$

for any integer k.

In this exchange, Juno referenced graphical representations from earlier in the lesson in order to help her students understand how to find the second initial solution based on the value given by calculating with inverse cosine. She also made connections to the original graph she had drawn with intersection points for the two functions (see Figure 3) in order to help students understand which intersection point corresponded with $\theta = -0.841$ and which intersection point corresponded with $\theta = 2\pi - 0.841$. Juno wrapped up the example by asking if any students had questions. One student asked if they could just write $\theta = 0.841 + \pi k$ to capture all the solutions to the equation. Juno explained that shifting one intersection point by only half a period (π) would not result in landing (graphically) on another intersection point. Another student asked if it would be possible to turn $2\pi - 0.841$ into a decimal. Juno explained that the number 0.841 was already a rounded approximation, so converting $2\pi - 0.841$ to a decimal would also be an approximation and not an exact solution. Finally, Juno wrapped up the example by explaining how students could check their work by plugging in a few values from their solution families for θ into the expression $\cos(\theta)$ to make sure the results were close to $2/3$.

Even though Juno was doing much of the mathematical work, the mathematical focus of the example justified it being coded as Procedures with Connections (see Table 3). While Juno used a procedure for finding initial base solutions and then translated them, she focused students' attention on the use of the procedure for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. In particular, Juno emphasized that, regardless of whether or not the trigonometric equation corresponds with a standard unit circle angle, the process still involves finding intersection points. Also, she consistently referenced the graphical representation of the problem (Figure 3) in order to help her students understand that, while there are an infinite number of intersection points, they actually correspond with two initial solutions that repeat in a periodic fashion. Second, Juno did not represent the procedure for solving the problem in an algorithmic way. [1]

Juno also used multiple representations to help her students understand the example. Throughout her presentation, she consistently referenced the graphical representation of the problem (Figure 3) to help her students develop an understanding of why there were infinitely

[1] In contrast, I observed Selrach present a similar example where he listed a five-step algorithm on the board and instructed students to follow the algorithm in order to solve trigonometric equations.

many solutions. When her students struggled to identify how to find the second initial base solution, Juno referenced another representation (Figure 4) to help her students reason with the symmetry of the unit circle to find the solution instead of memorizing and implementing a formula like $2\pi - \cos^{-1}(x)$. Finally, although Juno followed the general procedure for finding initial base solutions and then translating them, she challenged her students to engage with the conceptual ideas that underlie the procedure. In particular, Juno focused students' attention on the periodic nature of the cosine function, the symmetry of corresponding angles on the unit circle, and the graphical consequence of choosing different equivalent representations for the initial base angles (e.g. -0.841 vs. $2\pi - 0.841$).

Greg. For the beginning of class, Greg followed the lesson guide directly. However, Greg decided to use a different final example than the one provided. Instead of illustrating how to solve $\cos(\theta) = 2/3$, Greg chose to use the equation $1 + 2\sin(\theta) = 4/3$.[2] During the pre-observation interview, Greg explained that he changed the example because he wanted his students to understand that even when there is a lot of other "stuff" going on (i.e., transformations of the trigonometric function), they should still follow the same procedure. He also wanted to connect students' prior experiences with solving linear equations with what they were learning about solving trigonometric equations. Moving forward, Greg knew that his students would need to continue to combine ideas (i.e., solving linear and trigonometric equations) when working through problems. In selecting his example, he knew that he wanted to incorporate a constant multiple of the sine function plus a constant, and he didn't want the solution to involve a unit circle angle because the prior example did so. He wanted to introduce the students to the general procedure for solving trigonometric equations regardless of whether or not the problem required analyzing standard unit circle angles.

When Greg first introduced the final example, he acknowledged that they were "stepping it up a little bit." However, he emphasized that, although the example may look intimidating, "this is really just combining two ideas that you already know." In order to help his students recognize that part of the problem involved solving a linear equation, Greg decided to substitute $X = \sin(\theta)$ into the equation. Greg explained that he used a capital X, in order to "remind myself that I'm not solving for X, I'm solving for θ." However, making this temporary substitution would make it clear what the next step should be to work through the problem. Greg then rewrote the original equation as $1 + 2X = 4/3$ and worked through the steps for solving the linear equation to get $X = 1/6$. Greg labeled this as Step 1 and said "for Step 2, remember we didn't want to solve our equation initially for X, we wanted to solve it for θ." So, he substituted $X = \sin(\theta)$ which resulted in the equation $\sin(\theta) = 1/6$.

With the trigonometric equation isolated, Greg explained that they couldn't use the unit circle, because $1/6$ is not a standard unit circle angle. So instead, he asked his class "What do we want to use to move the sine to the other side?" The discussion that ensued is presented here as an excerpt of the transcript of the observation video.

Student 1: Arcsine.
Greg: Yeah, we'll use an arcsine. Ok? I'll write it as sine inverse. So, I get $\theta = \sin^{-1}(1/6)$. So that's one solution. Where does the second solution come from? I gave a chart.
Student 2: I just have a quick question. How did we know to use arcsine?

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[2] As a note, solving more complicated trigonometric equations was the first learning objective for the second day of the lesson, so Greg was leading into what they would be doing next.

Greg: [*To Student 1*] How did you know to do that?
Student 1: Because you're trying to find θ, and to get rid of sine you have to move it to the other side using sine inverse.
Student 2: Oh, yeah.

As a quick aside, Greg mentions that it is not acceptable to divide both sides by sin to isolate θ and arrive at $\theta = \frac{1}{6\sin}$. Greg identifies $\theta = \sin^{-1}(1/6)$ as an initial solution and asks, "Someone, remind me where the other solution comes from whenever you use arcsine? The other solution in $[0, 2\pi]$." One student responds with, "$2\pi - \sin^{-1}(1/6)$," and another asks, "Isn't it just $\pi -$?"

Greg: Why do you think it's $\pi -$? I mean that's the chart, right? But…the $2\pi -$ has the effect of flipping over the x-axis, and that works for cosine. But for sine values, we want to flip over the y-axis. And to do that, we use $\pi -$.

Greg labels $\theta = \pi - \sin^{-1}(1/6)$ as the second initial solution and then reminds the class that in the earlier graphed examples, they saw that sine and cosine usually have two initial solutions. He then wrote the following on the board: $(\text{init}){+}(\text{per})k$ where init stands for initial solution and per represents the period. Greg first introduced this notation for the general form of an infinite solution family of a trigonometric equation during the second example when they solved the equation $\cos(\theta) = \sqrt{3}/2$. When I observed Greg teach the second half of this lesson the prior Fall, he did not use this notation. However, Greg consistently used this notation during both lessons in the second semester as a way to help students see that the structure of the solutions was consistent regardless of the differences in the trigonometric equations.

Next, Greg asked, "What's the period that I'm looking for in this case?" He paused for approximately 6 seconds, but no one responded, so he answered his own question and explained that it was possible to find the period by looking at the equation $\sin(\theta) = 1/6$. Greg wrote the two solution families on the board, using the general form $(\text{init}){+}(\text{per})k$, and summarized that "It's really the same pattern that we were using before. Now it's just two steps." At the end, Greg asked if any students had questions. One student asked what type of equation would require a change in the period in the general form. Greg explained that an equation like $1 + 2\sin(3\theta) = 4/3$ would have period $2\pi/3$. In fact, the change from θ to 3θ in the equation would mean that both the initial solutions and the period would have different values in the general form. Another student asked if the final answer was simply the two initial solutions, or the "longer equations." Greg explained that the final answer must include all solutions, not just the initial solutions. Finally, Greg wrapped up the example by explaining how students could check their work by plugging in some of the solutions to the original equation into make sure they get $4/3$.

In this example, Greg involved students in the process of finding the solutions more than Juno did, but he still worked through most of the mathematics himself. Greg listed some procedures and referred to Step 1 (i.e., isolate the trigonometric function using algebraic manipulations) and Step 2 (i.e., solve the trigonometric equation), but he mainly focused on the broad, general procedures and on developing student understanding. So I coded this example as Procedures with Connections (see Table 3). In particular, Greg consistently focused students' attention on the general procedure for finding initial solutions and then adding on integer multiples of the period. He introduced the general form $(\text{init}){+}(\text{per})k$ in the previous example, brought it up again in this example, made connections between the general form and the graphical representation of the trigonometric equation, and emphasized repeatedly that the general procedures for finding initial solutions and identifying the period were core features of the problem. Like Juno, he did not present the procedure algorithmically but rather emphasized how the procedure was connected to the periodic nature of trigonometric functions. Although he did not graph $1 + 2\sin(\theta) = 4/3$, he did make references to the graphs he had drawn previously when explaining why there are two initial solutions. He also referenced the graph when explaining why the second initial solution was $\pi - \sin^{-1}(1/6)$ and not $2\pi - \sin^{-1}(1/6)$. Finally, Greg often paused and asked his students questions, which helped them do more than just mindlessly follow the procedure. Rather, he consistently focused their attention on engaging with the concept that solution families of trigonometric equations have the general form $(\text{init})+(\text{per})k$.

Dan. While the example in the lesson guide that Dan used was not as detailed as the one followed by Greg and Juno, he ended up using an example similar to Juno's example: Solve the trigonometric equation $\sin(\theta) = -2/3$. In the pre-observation interview, Dan explained that he picked this equation because he wanted an example that didn't have a lot of clutter so that the students could focus on how to solve trigonometric equations with a "not nice" (i.e., non-standard) unit circle coordinate. He planned to walk his students through five steps: (1) draw the unit circle, (2) draw $y = -2/3$, (3) note which quadrant(s) the two solutions are in, (4) find the actual angles using arcsine, and (5) given that the period is 2π, write the equations representing infinitely many solutions. Although he didn't list these steps explicitly on the board, he did reference them verbally in the example presented here and his previous example (solve $\cos(\theta) = \sqrt{3}/2$).

Dan started the example by mentioning that although the equation $\sin(\theta) = -2/3$ does not correspond to a "nice" unit circle coordinate, "we shouldn't throw the baby out with the bath water." In particular, Dan emphasized that all of the work was going to be almost exactly the same as with the previous example. But instead of using the unit circle to find the initial solutions, they would have to use inverse trigonometric functions. Here is an excerpt from the observation video transcript where Dan begins to work through the example:

> *Dan:* Again, the overall analysis is exactly the same. We draw our picture [*draws Figure 5 while talking*], we turn our equation into a label. Sine tells me I'm looking at y-coordinates. $-2/3$ tells me I want $y = -2/3$. That gives me two points, one in the third quadrant and one in the fourth quadrant.

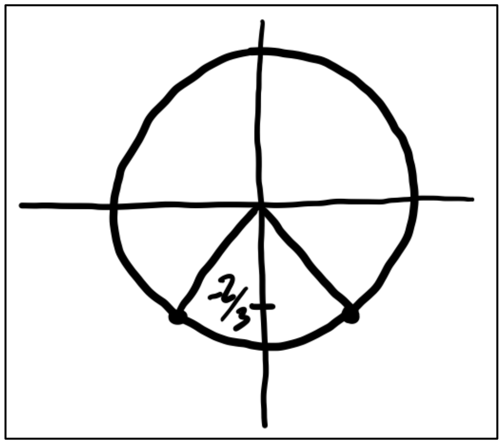
Dan reminded the class that they should use arcsine on their calculators but that the value provided, $\theta \approx -0.73$, is an angle in either the first or fourth quadrant. He explained that the fourth quadrant angle represents starting at 0 and subtracting 0.73 to get -0.73, while the third quadrant angle is found by starting at π and adding 0.73 to get $\pi + 0.73$. Once Dan had the two initial solutions, he skipped immediately to writing the solution families (shown below) and then asked if anyone had questions.

$$\theta = (-0.73) + 2\pi k$$
$$\theta = (\pi + 0.73) + 2\pi k$$
$$k \text{ any whole number}$$

One student asked, "So you know how if you take $\arcsin(-2/3)$ it's -0.73? So, for the second one, you did $\pi + 0.73$. What about the negative?" Dan explained that the two points in the third and fourth quadrant are based on the symmetry of the unit circle, and the -0.73 is the result of moving clockwise while the $+0.73$ is the result of moving counter-clockwise. Another student asked if there are four ways to write the solution, since each initial solution could start at 0 and move either clockwise or counter-clockwise. Dan explained that while it is possible to come up with different initial solutions, all solutions to the equation are equivalent and "it's sort of all taken care of in this $+2\pi k$ business."

Figure 5

Figure Dan Drew to Demonstrate which Quadrants Contain the Angles Corresponding with the Equation $\sin(\theta) = -2/3$.



In this vignette, Dan worked through the same procedure as Juno and Greg. However, he emphasized the procedure itself instead of using the procedure to highlight the underlying conceptual ideas. So, I categorized this example as Procedures without Connections (see Table 3). While Dan included a graphical representation in his explanation, he mainly used it to determine which quadrant the solutions are in. So, his representation was used primarily to produce correct answers instead of to develop mathematical meaning or understanding. He did reference the conceptual structure of the infinite families of solutions for trigonometric equations, but his references were brief and mainly focused on writing down the solution (i.e., he skipped immediately from finding the initial solutions to writing the family of solutions without discussing why it is necessary to add constant multiples of the period or why the period is 2π). When the student asked if there were four different ways to write the initial solutions (based on working clockwise or counter-clockwise around the unit circle), Dan could have explained how different initial solutions correspond with different intersection points of the graphs of $y = \sin(\theta)$ and $y = -2/3$ (like Juno did), but instead he said it did not matter which approach was used and "it's sort of all taken care of in this $+2\pi k$ business."

Discussion

In this study, I examined the cognitive demand of the examples that graduate student instructors chose to enact in precalculus courses. At first, I attempted to use the original Task Analysis Guide developed by Smith and Stein (1998) to code the cognitive demand of the enacted examples. However, the original version of the Guide includes language that specifies that students are the ones doing the mathematical work (e.g., "require students to explore" and "students need to engage"). While some instructors did involve students explicitly in working out examples, others chose to present examples using direct instruction. Therefore, I created a

Modified Task Analysis Guide for analyzing the cognitive demand of examples (Table 3) that removed any language concerning *who* is doing the mathematical work. Using this Modified Task Analysis Guide, I found that of the 93 examples that I observed over two semesters, 25 (27%) of them were enacted at a high level of cognitive demand.

Next, I conducted a cross-case analysis in order to illustrate what high cognitive demand precalculus examples might look like when instructors use direct instruction and to identify similarities and differences between the examples that different instructors enacted when using the same written curriculum materials. In the vignettes of Juno, Greg, and Dan, I illustrated how instructors can enact the same type of example at a high level of cognitive demand but emphasize different concepts (i.e., connections between algebraic manipulations and graphical representations versus the underlying structure of solution families). In addition, I found that while high and low cognitive demand examples might use similar representations (i.e., algebraic and graphical), focusing on finding the answer instead of on developing student understanding can lower the cognitive demand.

Implications

One implication of this study is that even through moments of direct instruction (i.e., during the enactment of an example), instructors can present mathematical tasks with higher levels of cognitive demand. A precalculus instructor who was not a participant in this study but taught in the department where the study was conducted told me that all of the content he covered was easy and did not require deep thinking. However, Juno, Greg, and several of the other graduate student instructors demonstrated that it is possible to teach concepts in precalculus that focus on developing deeper understandings of the underlying mathematics and make connections between multiple representations.

The Modified Task Analysis Guide that I developed for analyzing the cognitive demand of examples is useful for both researchers and practitioners. First, this framework gives researchers a way to analyze the cognitive demand of tasks independent of *who* is doing the mathematical work. This is especially important for examples, since instructors can present them in a variety of ways. While it is similar in many ways to the original Task Analysis Guide (Smith & Stein, 1998), I modified the categories by removing any reference to *who* is doing the mathematical work. The Modified Task Analysis Guide is also useful for practitioners as a planning and reflection tool. As teachers plan and reflect on their teaching, they can use this framework to assess the cognitive demand of the examples they use.

Limitations

One limitation of this study is that while the instructors were using the department lesson guides, the different versions they were using had some clear and distinct differences. In the example that I highlighted in my vignettes, the structure of the two lesson guides was the same; however, the amount of descriptive text that accompanied the example varied greatly. The version with more descriptive text still focused mostly on producing the correct answer, so it was still categorized as a Procedures with Connections task. However, it is difficult to determine what effect the different versions may have had on the cognitive demand of the enacted examples.

Another limitation of this study is that not only did the instructors know when I was observing them teach, but they also knew I was there to examine the cognitive demand of the examples they presented. Therefore, they may have spent more time thinking about the cognitive demand of the examples that they enacted on the days that I observed, which may have influenced the results. In conversations with Greg, who I observed both semesters, he mentioned that our interviews made him think more deeply about the content that he was teaching. So, while these conversations may have impacted the results, if the outcome was that he presented more examples at a high level of cognitive demand, then I view that as a positive impact of this work.

Finally, another limitation of my study is that I cannot make any claims related to student learning. Since the focus of this study was the graduate student instructors and the choices that they made when planning and enacting examples in their precalculus classrooms, I did not collect any student data. Therefore, I cannot make any claims about whether high cognitive demand examples presented through direct instruction have a positive impact on student learning or understanding. Rather, enacting these types of examples provides students with *opportunities to engage* with mathematical tasks that require higher levels of cognitive demand. I was not able to determine whether or not students actually took advantage of these opportunities.

Future Directions

There are some aspects of my modifications to the Task Analysis Guide that may need further attention. I chose to use the phrase "focus students' attention on" instead of "require students to" in order to remove references to *who* is doing the mathematical work of the task and to allow the framework to be used to analyze instructional examples that are presented using direct instruction. However, it is possible that a teacher could lecture for 50 minutes and claim their aim is to "focus students' attention on" high-cognitive demand tasks, but the students may never truly engage with the mathematics. Therefore, I think it would be beneficial to analyze student engagement, or perhaps students' opportunity to struggle, in addition to analyzing cognitive demand. This would provide a clearer picture of the type of mathematical work that students actually engage in during class. Finally, a follow up study that I could complete from my data set is to analyze the cognitive demand of the written and planned examples and then examine whether the cognitive demand of the examples increased, decreased, or stayed constant as the lesson unfolded.

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THE INFLUENCE OF OBSERVATIONAL EXPERIENCE AND METACONCEPTUAL TEACHING ACTIVITIES ON SECONDARY SCIENCE TEACHER CANDIDATES' CONCEPTUAL UNDERSTANDINGS OF THE PRACTICES OF SCIENCE

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ABSTRACT

Research supports that the use of intentional, metaconceptual teaching practices enhances teacher candidates' conceptual change, thus affecting teacher candidates' accumulation of new knowledge and understanding. With the increased focus on and need to accommodate the calls for enhanced science teacher education of the Next Generation Science Standards and the related practices of science, it is important to consider how teacher preparation programs enact conceptual change among teacher candidates that align with the current calls. This study examines intentional, metaconceptual teaching activities coupled with observational field experiences with mentor scientists to determine whether engagement in these activities affects teacher candidates' understandings of the practices of science. Results indicate that teacher candidates did enhance their understandings of the practices of science after participation in one science methods course incorporating metaconceptual teaching activities and observational field experiences. Individual and overall results are discussed along with implications for future research and practice.

KEYWORDS

conceptual change, metaconceptual teaching, practices of science, secondary science, teacher education

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Theory and research support that learning results from the interaction between an individual's current understanding of concepts and new information that is introduced through experience and instruction (Posner et al., 1982). When an individual purposefully examines conceptual understandings in light of new information and new experiences, the choice is made to either adopt new understandings that are robust and rational or to reject new understandings that fail to meet these criteria. This is understood to be the process of conceptual change (Posner et al., 1982). While learning is dependent on myriad factors, conceptual change is the process that allows learning to occur and, importantly, allows for individuals to advance in their understanding of the world around them.

Recently, a renewed focus has been given to learning in the field of science education and, more specifically, the practices of science and the applicability of such practices to day-to-day life. Therefore, education reform in the area of science has highlighted the need to identify conceptual understandings that transcend mere content and, rather, embrace the process of "doing" science (NRC, 2012). The *Framework for K-12 Science Education* (NRC, 2012) was developed to highlight the desired outcomes of effective science learning and thus served as the foundation for the development of the *Next Generation Science Standards* (NGSS) (NGSS Lead States, 2013). The framework moves beyond the prescription of what science content students must learn and calls for the conceptual understanding and active engagement of students in the practices of science. To achieve this, the framework is built on the foundational ability to ask fundamental science questions and conduct scientific investigations in an effort to develop students who can think, question, communicate, and act like actual scientists in practice. The framework states that students' knowledge should be guided "toward a more scientifically based and coherent view of science and engineering, as well as of the ways in which they are pursued and their results can be used" (NRC, 2012, p. 11). Emphasis is placed on understanding "the practices needed to engage in scientific inquiry" (NRC, 2012, p. 11); thus, science education should support not only the attainment of conceptual understanding of science content but also conceptual understanding of scientific practices.

What seems to have fallen by the wayside, however, is the need to first educate teachers in ways that support both the understanding of science practices and, more importantly, how to integrate such understandings into K–12 science teaching. Loughran (2014) argues that "the very language of science teacher learning purposefully links expectations that it is not just students but also teachers who are active learners of science, congruent with the ideas of constructivism" (p. 811). As many science teachers teach science in a way that mimics how they were taught—often following approaches that are no longer considered evidence-based best practices in the field—the challenge has become how to "help student teachers see beyond their own experiences of teaching and find new ways to engage…in conceptualizing practice as something more than how they themselves were taught" (Loughran, 2014, p. 812). With this need in mind, it is suggested that teachers examine their conceptual understandings of science and of science as a practice and participate in the active engagement of the practices of science. Thus, teachers can not only learn how to think, question, communicate, and act like scientists but also learn how to adapt and integrate such practices in the context of the K–12 science classroom.

A robust body of literature exists that examines conceptual understandings held by students (Burgoon et al., 2011). However, a need has been documented in the literature to study conceptual change as it relates to teaching and learning among science teacher educators as little research currently exists (Russell & Martin, 2014). While some research supports that conceptual change frameworks may assist science teachers in developing scientifically acceptable and accurate views of science concepts (Treagust & Duit, 2008), little research exists that examines conceptual understandings as they relate to science teachers' knowledge of the practices of science. This exploratory study, therefore, seeks to examine the experiences of teacher candidates enrolled in a graduate-level science methods course that was designed to enact conceptual change through participation in an observational field experience component coupled with intentional participation in metaconceptual teaching activities.

The Practices of Science

Recent educational reform in the area of science has centered on the "power of integrating understanding the ideas of science with the engagement in the practices of science" (NRC, 2012, p. x). Reform efforts support the idea that a deficit exists in students' understanding of science as a practice—that is, how to engage in inquiry and investigation as a scientist would—as a matter of the ability to engage as adults in informed decision making and socially responsible practices. In the *Framework for K–12 Science Education*, the National Research Council (NRC, 2012) explains the importance of this renewed focus:

> Science, engineering, and technology permeate nearly every facet of modern life, and they also hold the key to meeting many of humanity's most pressing current and future challenges. Yet too few U.S. workers have strong backgrounds in these fields, and many people lack even fundamental knowledge of them. This national trend has created a widespread call for a new approach to K–12 science education in the United States. (p. 1)

While the NGSS (NGSS Lead States, 2013) is not without controversy and is, in the opinion of some science educators and science education researchers in the field, in need of further revision and refinement (Cunningham & Carlsen, 2014), the three dimensions set forth in the *Framework for K–12 Science Education* (NRC, 2012) and the NGSS serve as a foundation for science curriculum development and implementation to meet the existing call for a fresh approach to educating today's youth. The three dimensions are scientific and engineering practices, crosscutting concepts, and disciplinary core ideas. This study will focus on scientific and engineering practices, defined as:

1. Asking questions (for science) and defining problems (engineering)
2. Developing and using models
3. Planning and carrying out investigations
4. Analyzing and interpreting data
5. Using mathematics and computational thinking
6. Constructing explanations (for science) and designing solutions (for engineering)
7. Engaging in argument from evidence
8. Obtaining, evaluating, and communicating information (NRC, 2012, p. 3)

Considering that active engagement in the practices of science enables learners to experience science in new ways, thus challenging learners' pre-conceived notions and assumptions, the field experience component in this study served as one method by which the practices of science were presented to produce the cognitive conflict necessary to begin the process of conceptual change (Posner et al., 1982). Likewise, metaconceptual teaching activities were utilized to encourage teacher candidates to consider how their conceptual understandings of the practices of science aligned with scientifically accepted understandings of the practices of science and, if necessary, to make the necessary conceptual changes.

Theory of Conceptual Change

It is commonly accepted that all learners hold conceptual understandings based on prior experience when entering the classroom (Treagust & Duit, 2008). Teacher candidates are no exception. Of specific importance are secondary science teacher candidates, who may hold preconceptions that do not align with scientifically acceptable understandings of science content and science practices (Fulmer, 2013; Sadler et al., 2010; Treagust & Duit, 2008). These conceptions are termed “alternative conceptions” (Posner et al., 1982) and are more formally defined as logical ideas that are different from accepted scientific knowledge (Burgoon et al., 2011), denoting that they are not inherently incorrect as they rely on an individual’s past observations and experiences but are incomplete or inaccurate understandings of science. As Yürük et al. (2011) says, “changing these conceptions with scientifically accepted ones is not an easy and straightforward process…as it requires learners to recognize and evaluate their existing and new conceptions[,] associated commitments, everyday experiences, and contextual factors” (p. 459). While teachers’ content knowledge is important to their ability to teach science content in ways that align with accepted scientific understandings (Burgoon et al., 2011), teachers’ conceptual understandings of science as a practice are essential to the ability to teach science to secondary students in ways that align with *The Framework for K–12 Science Education* (NRC, 2012) and the NGSS (NGSS Lead States, 2013). Importantly, if the conceptual understandings held by teachers are inaccurate, teachers may inadvertently reinforce or promulgate alternative conceptions among students (Burgoon et al., 2011). Thus, considering ways in which to enact conceptual change that align with calls for action and reform in science education (NGSS Lead States, 2013; NRC, 2012) is timely.

Several models of conceptual change exist within the educational literature (Treagust & Duit, 2008). This study, however, will focus on the epistemological and intentional perspectives of conceptual change. The epistemological perspective is grounded in the theory that, when learners experience dissatisfaction with a prior conception and an acceptable replacement conception is introduced, learners may accommodate the new conception and, thus, initiate conceptual change (Posner et al., 1982; Treagust & Duit, 2008). It should be noted, however, that the new conception must first be considered plausible, intelligible, and fruitful. In this study, new conceptions were introduced through the observational field experience component as well as instructional components related to the science methods course, such as lectures, discussions, homework, and assigned readings.

The intentional perspective is grounded in the theory that learners must “be active and…have a certain intention to learn” (Treagust & Duit, 2008, p. 301). Thus, the intentional perspective supports that conceptual change is dependent on learners’ metacognition as well as their motivational and affective processes. In this study, metaconceptual teaching activities served to support participants’ metacognition as they related to conceptual change, and, further, engagement in the observational field experience component supported participants’ active engagement in conceptual change.

Metacognition and Metaconceptual Teaching Activities

Metacognition, defined as thinking about one’s own thought processes (Brown, 1987), has been identified as an important factor in the conceptual change process (Yürük et al., 2011). Metacognition allows the learner to examine their own thoughts, prior knowledge, conceptions, and misconceptions and to adjust their conceptual understandings based on new experiences and new information, thus leading to enhanced learning and meaningful understanding. Metaconceptual teaching activities, therefore, encourage learners to consider their own thought processes as well as their current conceptual understandings and, importantly, to consider alternative perspectives that may or may not be adopted. Metaconceptual teaching activities in educational research have included the use of concept maps (Kaya, 2008), discussions, self-reflections, and other activities that encourage the thoughtful consideration of one's own conceptual beliefs (Yürük et al., 2011) in an effort to provide "a rational basis for a conceptual change" (Posner et al., 1982, p. 223).

While metaconceptual teaching activities have been shown to support the learning process (Treagust & Duit, 2008), little research currently exists that examines teacher candidates' conceptual understandings of the practices of science. While some preliminary research exists that examines the conceptual change process among teachers and pre-service teachers in general (Treagust & Duit, 2008; Yürük et al., 2011; Kaya 2008) and the role of metaconceptual activities in altering alternative scientific conceptions (Pintrich et al., 1993; Vosniadou, 2003; Georghiades, 2004; Yürük, 2007; Yürük et al., 2009; Yürük et al., 2017), research is needed that examines conceptual change as it relates to understanding science as a practice. Additionally, no research currently examines the conceptual change process among teacher candidates engaged in observational field experience coupled with intentional engagement in metaconceptual teaching activities.

One preliminary study on conceptual change and metaconceptual teaching activities examined 47 pre-service teachers enrolled in a general chemistry laboratory class in Turkey (Kaya, 2008). Participants were provided instruction regarding how to construct concept maps and how to use concept maps as a tool for assessing conceptual understandings and were encouraged to use concept maps to examine their own conceptual understandings. Using qualitative interviews, the results of the study demonstrated that concept maps were a valid tool for assessing conceptual understandings and, importantly, assisted students in revising their current understandings to align with those that are scientifically accepted.

Yürük et al. (2011) examined pre-service biology teachers' conceptual change related to understanding of flowering plants after engagement in metaconceptual activities. The metaconceptual activities included poster construction, journal writing, concept mapping, and class discussion and were focused on enacting conceptual change around alternative conceptions of flowering plants. The findings indicated that the majority of pre-service teachers' alternative conceptions were replaced by scientifically accurate conceptions after engaging in the metaconceptual activities.

Yürük, Selvi, and Yakisan (2017) further examined the metaconceptual processes of pre-service biology teachers as they engaged in metaconceptual teaching activities surrounding understanding of seed plants, including alternative conceptions. The metaconceptual activities included poster construction, concept mapping, discussion, and journal writing. Journal entries of five teacher candidates were analyzed. Results indicated that metaconceptual awareness, metaconceptual monitoring, and metaconceptual evaluation processes were activated through engagement in the metaconceptual activities. Furthermore, the results supported the idea that metaconceptual processes are multifaceted and that different metaconceptual processes can be observed in different settings and contexts. Importantly, metaconceptual processes may differ depending on teacher candidate ability and the specific content being studied. The authors further clarify that some metaconceptual processes require teacher candidates to engage in more than one process at a time. That is, in order to enact conceptual change, teacher candidates must first be aware of their conceptions and ideas, compare and contrast their current ideas with their past ideas, and monitor the consistency between new ideas and existing ideas from a variety of sources.

The current study aims to add to the literature and increase understanding of practices that support conceptual change and the formation of scientifically acceptable conceptions of the practices of science among teacher candidates. Enhancing current understanding may serve to inform future curriculum design and implementation as well as the design and implementation of professional development opportunities that assist teachers in attaining accurate conceptions of the practices of science and how to apply such practices in instruction in the secondary science classroom.

Purpose

The purpose of this exploratory case study was to understand the impact that participation in an observational field experience component paired with metaconceptual teaching activities had on students' conceptual understandings of the practices of science. Conceptual change was measured by the degree to which the students demonstrated a move from a broad and general understanding of the practices of science to a complex understanding of "the practices needed to engage in scientific inquiry" (NRC, 2012, p. 11) and, importantly, to demonstrate an understanding of how to adjust and integrate such science practices for middle school science instruction.

Methodology

Research Design

The study followed a case study design as it examined "an issue explored through one or more cases within a bounded system" (Creswell, 2007, p. 73). In this study, multiple cases or participants were included. Participation in one graduate level science methods course was considered the bounded system. Since individual participants may have experienced the course differently or may have different preconceptions of science content or the practices of science, a collective case study approach was utilized to provide a more comprehensive understanding of the impact of participating in the course on students' conceptual understandings of science and science as a practice.

Participants and Setting

Archival data was collected for teacher candidates enrolled in the residential graduate level science methods course—one course in the Master of Arts in Teaching program at a public, historically black university (HBCU) in the mid-Atlantic region of the United States, geared towards preparing urban educators. A total of five teacher candidates were enrolled in the course. One teacher candidate did not complete all assignments and, thus, was removed from the data analysis. The sample included in the data analysis consisted of one man and three women (N = 4). Candidates ranged in age from 29 years old to 60 years old. 50% of participants were African American, and 50% were Caucasian. All participants were career switchers who had some level of prior teaching experience. Three participants were not currently teaching in the K–12 classroom, and one participant was currently teaching in the K–12 classroom but as a provisionally licensed teacher that had not completed a formal teacher education program.

The science methods course was held during the Fall 2015 semester for a total of 15 weeks. The course was taught by the primary researcher of the study, and data was analyzed as archival data upon the completion of the course. The course consisted of in-class lectures and discussions, homework assignments, projects, and a required observational field experience component with faculty members at the university. For the observational field experience component, students were required to observe faculty members conducting laboratory practices (bench science) for a minimum of 25 hours over the course of the semester. The faculty members who allowed students to observe their practices were completing laboratory research in the fields of biology and chemistry. In order to allow for an accurate portrayal of how scientists conduct their work, no parameters were provided to the faculty. That is, students observed the faculty members conducting their research holistically. Students observed faculty members doing a range of activities in laboratories, including conducting fractional distillation, extracting DNA from planaria, and researching the replication of cancer cells found in human breast tissue. Faculty members were selected by the researchers due to their levels of scholarship in their respective fields, timing of current research activities (i.e., experiments) with the science methods course, and willingness to allow students to observe.

The course itself was designed to identify and affect change on students' conceptual understandings of the practices of science. Thus, metaconceptual teaching activities were provided as interventions as part of the normal curriculum of the course in addition to the required observational field experience component. Metaconceptual teaching activities included concept mapping, research experience analyses, in-class group discussions, reflections, and a group presentation, aligning with the research literature.

Procedures and Data Collection

Students participated in the normal curriculum of the course, with the major components of the course being the required observational field experience component and inclusion of purposeful, metaconceptual teaching activities. The rationale for including these components in the initial design of the course was to provide students with an opportunity to actively engage in the practices of science in order to enact conceptual change and to provide sufficient opportunities for reflection and application of conceptual understandings related to the practices of science. The class met once per week for one hour to participate in lecture and discussions, and students were required to complete weekly readings, assignments, and in-class discussions to provide sufficient instructional opportunities and assessments to demonstrate an understanding of science as a practice.

More specifically, the *Framework for K–12 Science Teaching* was introduced, including the practices of science, the NGSS, and the three-dimensional structure of the NGSS (e.g., cross-cutting concepts [CCs], disciplinary core ideas [DCIs], and science and engineering practices [SEPs]). Lectures, discussions, and activities were provided to unpack the CCs, DCIs, and SEPs, such as the activities made available online from the California Academy of Sciences (2020). Instructional activities were aligned with what each of the practices of sciences looks like, both in the field and from an educator's perspective of modeling and communicating such practices to middle school students. For instance, students read and deconstructed research articles to identify examples of each of the practices of science. Students were then asked to translate the articles in ways that would be developmentally appropriate for and understood by middle school students, while retaining the components modeling the practices of science. Debriefings on the observational field experiences occurred on a regular basis. Students also constructed and critically analyzed lesson plans to ensure accurate identification and application of the practices of science in the context of the middle school classroom. Students engaged in writing activities to reflect on how the observational field experiences and in-class instruction were informing and shaping their perspectives and understanding of effective teaching practices.

Note that the observational field experience hours were completed throughout the course, with the requirement that a minimum of 25 hours would be completed prior to the end of the 15^{th} week. Completion of the observational field experience hours was verified by a log signed by both the student and the faculty member observed as well as follow-up email verification with each of the respective faculty members.

Following Creswell's (2007) and Saldaña's (2016) recommendations, multiple artifacts were collected and analyzed using Dedoose. A total of six artifacts were collected and analyzed for each participant: pre- and post-instructional concept maps, pre- and post-nature of science and inquiry reflections, and pre- and post-instructional research analysis reports.

Pre- and Post-Instructional Concept Maps

For the Instructional Concept Maps, students were asked to construct a concept map (e.g., Venn diagram, logic model) that articulated the characteristics of science research, the characteristics of effective science teaching, and how the two disciplines are similar and different, with a specific focus on the practices of science. Students were asked to consider what a researcher/scientist does in daily practice and how a science teacher would communicate and model those practices. Students were asked to consider the following types of questions: What are the various activities in which each engage? Within each map, how are these various activities related to each other? Is there overlap between the map for teaching and the map for research?

Pre- and Post-Nature of Science and Inquiry Reflections

For the Nature of Science and Inquiry Reflections, students were asked to write a two- to four-page reflection of their personal view, supported by research, of the nature of science and scientific inquiry and how students learn science, with a specific focus on the practices of science.

Pre- and Post-Instructional Research Analysis Reports

For the Instructional Research Analyses Reports, students were asked to write a two- to four-page analysis of their observational field experience, describing specific examples of where they identified the practices of science. Students were asked to make connections between faculty members' research activities and elements of teaching and learning in order to facilitate the effective implementation of NGSS-aligned science instruction.

Analysis

As conceptual understandings of science as a practice was the core aspect under consideration in this study, an embedded analysis of each artifact was conducted (Creswell, 2007). Specifically, following the design of Yürük et al.'s (2011) metaconceptual study, artifacts were coded in terms of the presence of meaningful application of the NRC's (NRC, 2012) Practices for K–12 Science Classrooms. Meaningful application was defined as a purposeful mention of a specific practice or an easily identifiable example of a specific practice. For instance, use of the term "inquiry" or "questioning" was counted as the presence of application of the practice "Asking questions (for science) and defining questioning problems (engineering)" (NRC, 2012, p. 3). Use of the term "argue," "discuss," or "debate" was counted as the presence of the application of the practice, "Engaging in argument from evidence" (NRC, 2012, p.3). Redundant occurrences were not coded. For ease of coding, descriptors were created for each of the Practices for K–12 Science Classrooms, as shown in Table 1.

Table 1
Descriptors Used in Data Analysis for Each of the Practices for K-12 Science Teaching

Practice	Descriptor used in Coding
Asking questions (science) and defining problems (engineering)	Questioning
Developing and using models	Modeling
Planning and carrying out investigations	Investigating
Analyzing and interpreting data	Analyzing
Using mathematics and computational thinking	Thinking
constructing explanations (science) and designing solutions (engineering)	Constructing
Engaging in argument from evidence	Arguing
Obtaining, evaluating, and communicating information	Informing

A second round of coding was conducted to identify the magnitude of accuracy for each occurrence of the NRC's (NRC, 2012) practices. A code of 1 was applied for an inaccurate conception of the practice, a code of 2 was applied for a somewhat accurate conception of the practice, and a code of 3 was applied for an accurate conception of the practice. An example of a passage coded as 1 (inaccurate) was: "A scientific model is a testable idea created by the human mind that tells a story about what happens in nature. An example of this observed during the observation was the utilization of the incubator to store the cells." This passage was coded as a 1, as the use of a scientific tool—in this case, an incubator—is not an accurate example of the use of a model.

An example of a passage coded as 2 (somewhat accurate) was: "Because of the massive volume of researchers, it is almost impossible to find a completely novel idea, however, every researcher has to come up with a different way of thinking and looking at topics in order to find an innovative way of looking at a similar issue." This passage was coded as somewhat accurate as the student indicated some understanding of questioning, but did not display an entirely accurate view of questioning (i.e., perceiving that a vast number of researchers stifles the ability to construct novel ideas or to pose new questions).

An example of a passage coded as 3 (accurate) was: "After staining and photographing them [planarian], Dr. [Sanvi] analyzes photographs for ocelli (eye) regeneration, growth and location, and lastly use[s] analysis of variance (ANOVA) statistical tests to analyze variance or differences in group means i.e. ensures that her measurements are statistically significant." This passage was coded as accurate given that the student correctly identified the process of analyzing.

An accuracy score was computed by averaging the codes applied. Thus, a higher score indicates higher accuracy. A table was created for each practice to document the magnitude of accuracy of the NRC's (2012) practices from pre-instruction to post-instruction. Throughout each step of the process, inter-rater reliability was ensured by each researcher independently coding each item, then comparison of coding to ensure alignment, and discussion, agreement, and resolution of any differing codes. Frequencies were calculated by tallying the number of times a descriptor was noted divided by the total of all occurrences.

Results

When analyzed as a group, the results demonstrated that the frequency of questioning, investigating, arguing, and informing increased, and the frequency of modeling, analyzing, thinking, and constructing decreased over time, as shown in Table 2. When analyzed as a group, the results demonstrated that the accuracy of all practices (questioning, modeling, investigating, analyzing, thinking, constructing, arguing, and informing) increased over time, as shown in Table 3. When considering accuracy of the practices, no particular misconceptions or alternative conceptions were noted as common across the teacher candidates.

Table 2
Collective Results of Frequency Analysis of Artifacts

Practice	Pre-Artifact Frequency	Post-Artifact Frequency
Questioning	44.1%	55.9%
Modeling	55.6%	44.4%
Investigating	42.6%	57.4%
Analyzing	54.5%	45.5%
Thinking	53.3%	46.7%
Constructing	52.0%	48.0%
Arguing	45.5%	54.5%
Informing	49.3%	50.7%

Table 3
Collective Results of Accuracy Analysis of Artifacts

Practice	Pre-Artifact Frequency*	Post-Artifact Frequency
Questioning	2.5	2.6
Modeling	2.7	2.8
Investigating	2.5	2.8
Analyzing	2.1	2.8
Thinking	2.0	2.3
Constructing	2.7	2.8
Arguing	2.5	2.9
Informing	2.5	2.6

*A code of 1 was applied for an inaccurate conception of the practice, a code of 2 was applied for a somewhat accurate conception of the practice, and a code of 3 was applied for an accurate conception of the practice.

Discussion

While the frequency of direct indication of modeling, analyzing, thinking, and constructing decreased over time, overall, teacher candidates were able to more accurately identify all components of the practice of science as outlined by the NRC after participation in the course. These findings are important as they indicate that the combination of metaconceptual teaching activities and observational field experiences may have contributed to enhanced understanding of the practices of science. While other factors may have also influenced students' understandings of the practices of science, the findings are promising as they provide preliminary indication that purposeful incorporation of metaconceptual teaching activities and observational field experiences are beneficial in one science teacher preparation program. These findings are supported by previous research that demonstrated that metaconceptual teaching activities enable conceptual change from alternative science conceptions to more accurate science conceptions (Kaya, 2008; Yürük et al., 2011; Yürük et al., 2017).

It is important to note, however, that the field of science that each respective teacher candidate chose to observe was not included in the archival data that was analyzed and could have played a role in the results. Previous research has indicated that content and context may yield activation of different metaconceptual processes (Yürük et al., 2017) and, thus, may impact conceptual change. Some aspects of the NGSS may be stressed more in some science disciplines than others, which also indicates that some practices of science may be stressed more in some science disciplines than others. However, the findings support that interventions specifically focused on modeling, analyzing, thinking, and constructing as practices may need to be implemented in order to enact conceptual change that supports accurate understanding across all practices of science among the sample population studied.

The findings of the current study are further supported by previously published research that has shown the positive effects of conceptual change using metaconceptual activities (Yürük et al., 2011; Yürük et al., 2017). The current study additionally builds on the existing body of research by adding observational field experiences to the existing framework, which has not been done in previous research to date. While further study and replication is needed, these findings indicate that adding the components of metaconceptual activities and observational field experiences may enhance understanding of the practices of science among science teacher educators.

The findings have implications for how we prepare science teachers and what resources are provided to teachers as they seek out training and experiences to remain current in the field. Following current initiatives within science research fields to include a "learning science by doing science" approach (Labouta et al., 2018), it may be advisable to include observational field experience components with scientists in teacher training as well as professional development regimes to allow teachers opportunities to re-engage with the practices of science.

Teacher educators and teacher education programs are, therefore, encouraged to consider the implementation of observational field experiences in science teacher preparation courses and programs in order to provide teacher candidates the opportunity to see the practices of science in action. That is, observational field experiences may allow teacher candidates the opportunity to learn how science practices are conducted in the field, enhancing their understanding of how scientists do science and thus enhancing their understanding of how to practically integrate the practices of science in the K–12 science classroom. Furthermore, it is suggested that intentional metaconceptual activities be integrated in science teacher preparation courses and programs in order to enact conceptual change and to allow teacher candidates the opportunity to engage in metaconceptual processes that enhance the frequency of accurate conceptions of science while simultaneously dispelling alternative science conceptions.

Limitations

Although the results of this preliminary study show that overall intentional metaconceptual activities combined with observational field experiences has a positive effect on conceptual change, the authors are cognizant of several limitations presented by the current research design. The sample size used in this study was limited to four teacher candidates. While appropriate for a case study design, the use of a larger sample size in tandem with quantitative measures could yield a more robust understanding of the impact on teacher candidates' conceptual change.

Research design and analysis was limited by use of archival documents. Repeating this study and engaging in interviews and focus groups with teacher candidates would enhance the data collected and allow for a more substantive evaluation of the teacher candidates' understanding of the practice of science. As it stands, the authors were not able to interact with the teacher candidates to gather interview and focus group data for this study as the study utilized archival data. Thus, some points of understanding may not have been accurately or adequately accounted for in the data analysis. Incorporation of face-to-face interviews and/or focus groups would allow researchers to better incorporate new information from teacher candidates that allows for a better analysis of conceptual change as well as the effectiveness of the approach used in this study design. Future explorations into this topic could utilize a more diverse method of data collection employing meta-analysis, incorporating various data collection methods, most notably interviews (structured and unstructured), focus groups, and questionnaires.

Teacher candidates engaged in observational field experiences in only two fields of science: biology and chemistry. Further exploration is needed to determine if similar results are found when engaging in other fields of science. Additionally, all teacher candidates that participated in the current study had some experience teaching in the middle school classroom. Results of the current study may not be generalizable to teacher candidates with no prior experience in teaching, suggesting the need for further research. Teacher candidates that participated in the current study were also enrolled in a teacher preparation program that focuses on preparing teachers for urban classrooms. Thus, the generalizability to teacher preparation programs that seek to prepare teachers for non-urban classrooms may be limited. The limitations of generalizability, however, are inherent in the case study design.

Another important limitation that should be noted is that teacher candidates' application of the practices of science in their own teaching practices was not examined in this study. Future study should examine not only teacher candidates' understanding of the practices of science but their ability to apply and integrate such understandings in the K–12 classroom.

Conclusion

From the first step of inquiry to the final step of defending the importance of scientific findings and communicating that importance to the scientific world, ultimately, the goal is to improve application of learned scientific concepts. However, in accomplishing this goal, it is crucial to ensure that the facilitators of this transition from concept to action, classroom teachers, understand the actual process of science in a meaningful way. To date, there is a dearth of published data examining conceptual understandings as they relate to science teachers' understandings of the practices of science, although research has supported the use of metaconceptual activities in enacting conceptual change of teacher candidates within specific science content. The results of the current study demonstrate that the combination of metaconceptual activities and observation of faculty members conducting laboratory practices had a positive impact on teacher candidates' understandings of the practice of science.

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