ONLINE VERSUS IN-PERSON MATHEMATICS INSTRUCTION: A COMPARISON OF TWO INSTRUCTIONAL MODELS

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ABSTRACT

Our paper explores the differences between online and traditional, in-person teaching and learning modalities, looking specifically at courses preparing teachers to be mathematics teacher leaders. In the context of current research on the teaching and learning of mathematics in an online setting, we share our own experiences. We describe the preparation for and teaching of online mathematics, focusing on establishing norms and the use of technology. The changing teaching and learning opportunities of the 21st century require discussion of these vital issues. We include stories of interactions between candidates and teachers and among groups of candidates in mathematics courses, detailing not just the discursive and work-sharing tools but the nature and nuance of these interactions and how they mediate mathematics learning. We share our online teaching and learning experiences, drawing on research to frame our impressions. By identifying key similarities and differences between instructional modalities and by reflecting critically on our own successes and challenges, we present a vision of online teaching and learning for mathematics courses, in particular those for mathematics specialists, that can be effective, inclusive, and relational.

KEYWORDS

online mathematics instruction, mathematics leadership

In the summer of 2017, a group of 30 elementary and middle school educators across Virginia logged into a virtual mathematics classroom. It was the first of several Virginia Commonwealth University (VCU) courses designed to prepare in-service teachers to serve as mathematics specialists. Mathematics courses were designed to broaden candidates' understanding of both content and pedagogy, and leadership classes helped cultivate the professional dispositions unique to instructional coaches. This cohort represented a landmark change in the evolution of VCU's program: their courses would be delivered entirely online through a Learning Management System (LMS) with video conferencing capabilities. Candidates were provided headsets and a drawing tablet for creating digital drawings. Throughout this paper we refer to technology tools in a general way, focusing on functionality, and we believe our findings to be relevant to online mathematics instruction overall regardless of specific tools.

This mathematics specialist preparation program consisted of six mathematics courses, three mathematics education leadership courses, and a capstone project spanning two semesters. All courses were taught using a flipped classroom model. Candidates were provided with prerequisite work through the LMS for each class session including readings, mathematics activities, reflections, and small group activities. For all courses in the program, prerequisite work guided the activities and discussions during the following synchronous class sessions.

Literature Review

Our framework for reflecting on mathematics courses, from both an instructor and candidate point of view, is informed by the concept of a "community of inquiry," developed by Garrison, Anderson, & Archer (2001; 2010). Features of this framework include the Deweyan notion of inquiry as a social activity, which despite the qualifier "social" also hinges on the private, reflective actions of the individual learner. In other words, we recognize that meaningful online mathematics learning happens when "students move repeatedly between reflection and discourse" (Garrison et al., 2001, p. 9). We also draw on their notion of "social presence," which is related to *group cohesion*—a feature we believe develops from a strong sense of relationship and trust—and *shared social identity*, established early on by virtue of the common goal of state certification but strengthened over time by philosophical and methodological consensus (namely, a commitment to teaching mathematics for understanding).

In comparing in-person to online teaching and learning modalities, we draw on the work of Claire Howell Major, who, in 2015, published a long overdue guide to the praxis of teaching online. Her prescient opening essay examines how teaching and learning are mediated by technology itself, how technology shapes both interactions and products, and indeed how virtual educational reality is experienced by all participants. When she states that the online instructor "interpret(s) the instructional experience *with* the technology," (p. 11) she does not limit the "experience" to comments, solutions, models, and other such empirical products. Educational experience includes "feelings, thoughts and relationships," which are no less refracted through the lens of technology. Pauses during class discourse, verbal and textual exchanges among students, fully-formed (rather than inchoate) posted solutions: these are interpretable "through and with technology" and hence influence the instructor's judgment not just of student learning but of the overall affective domain of the online classroom.

Our aim is to share both learner and instructor experiences in online mathematics courses, illuminating key differences between in-person and virtual classroom spaces. We emphasize the need for nurturing strong interpersonal connections within the peculiar space of online learning, perhaps especially in contemporary mathematics courses where intellectual risk-taking and open discourse are generative forces.

Comparing Online and In-person Mathematics Teaching and Learning

Teachers transitioning from in-person to online mathematics instruction can benefit from reading testimonials about this paradigm shift, as the foray into online education can be uneasy or even unsettling. With synchronous class meetings, the virtual classroom is neither room-like nor entirely cold or inhuman. For instructors and candidates alike, the environment borders between the familiar and unfamiliar. Some hallmarks of in-person learning environments are reproduced in online instruction. For example, there are typically many students and few teachers, there may be a front-and-center “whiteboard,” and so on. Yet despite these traditions, virtual classrooms cultivate a markedly different classroom ecology. This section will compare online instruction to in-person instruction to help readers make connections between the styles of teaching and make suggestions for intentional change. We describe some of the struggles of online instruction and will include a discussion of strategies that can be used to address the challenges of online instruction.

As a teacher new to online instruction said recently, “In an online class, it’s very hard to take the temperature of the room” (personal communication). In the context of an online *mathematics* class, this difficulty is especially problematic. Although physical and affective cues are not under complete erasure, they are surely less apparent. As Claire Howell-Major says, “We cannot see a student’s happiness at answering a question well or puzzlement over another students’ response” (2015, p. 12). Our ability to read the room—to know whether or not candidates understand a given mathematics problem, to sense when they are persisting or capitulating—is based less on interpersonal skills and more on the technology itself. For example, after we posed a mathematics problem and broke candidates into virtual small groups, instructors were able to “visit” these small groups to check in with students. On several occasions, when we entered a small group session, a candidate would immediately ask a clarifying question about what they were “supposed to do.” To the seasoned mathematics instructor, this might sound familiar. Indeed, appealing to the instructor for clarification and support is not uncommon in an in-person mathematics class. But in an online format, we are entirely dependent on the affordances of the technology (here, the “join group” feature that allowed instructors to enter small group forums) to support learning.

Similarly, the chat feature became a means of clarifying questions and responses and even arguing for or against ideas. We recognize these actions as critical mathematical habits of mind. In a chat forum they are usually textual (we say “usually” because emojis were also featured prominently in whole group chats) and appear in rapid succession. But as a candidate in an online course said recently, “What you say in a chat—it’s just *out there* in print for everyone to see.” Might it have seemed riskier for candidates to contribute to a chat forum? Alternatively, some candidates may have sensed *less* risk in contributing to the chat forum, a forum that is not restricted to the “one-at-a-time” formality of group discourse. Since robust mathematical discourse requires some degree of risk-taking, the textual chat function may have mediated the quantity and quality of discourse.

Building Classroom Culture

As with in-person courses, teachers of online mathematics courses spend time establishing norms and expectations and building a culture of collaboration through a community of inquiry. Fortunately, many online platforms incorporate tools to help establish norms, including "raise hand," "thumbs up/thumbs down," typing into a chat box, and sharing pictures (all experienced through on-screen notifications for the instructor). In our online cohort, candidates were expected to participate using these tools as a quick check for understanding. For establishing relationships, the use of breakout groups provided the opportunity for small group work before and during class.

Camera use varied by instructor: some required candidates to have cameras on during whole group discussions, and others left it up to the individual. Most candidates preferred to use the cameras during small group interactions, even if their cameras were off in the large group. Through the use of the camera and other aforementioned platform tools, instructors *could* document student participation in a variety of ways. It is important for instructors to be specific with students about how participation will be evaluated, as "active participation" can look quite different online than in person.

The norm of using physical manipulatives helped develop the mathematical content of our courses. When planning lessons, instructors made a list of required manipulatives for each class. Candidates were expected to have those manipulatives available for use during class. Virtual manipulatives were used at times but *were not used in place of physical manipulatives*. Frequently, instructors utilized discussion boards where candidates could contribute a photo of their manipulatives with a description of their work. These images were then used to guide conversation about the mathematical activities.

Instructors provided some form of agenda document to drive instruction. Many professors opted for a slide presentation that included directions for activities and live links to external tools used for those activities. Frequently this agenda document would be shared with candidates prior to class, and it was always made available after class, along with a video recording of the synchronous session, through the online platform.

Finally, attendance at synchronous sessions was mandatory for all candidates. As a graduate level mathematics specialist cohort, attendance overall was not an issue. On the rare occasion that a candidate missed a synchronous class, the video recording could be used to fill in learning gaps. However, *watching the recorded session was not viewed as a substitute for in-class learning*.

Candidate Experiences

Patrick

The VCU mathematics specialist preparation program was my first experience with an online class. I was nervous the first day because I knew the other candidates were also strong mathematics instructors. I wanted to do well not just for myself, but also because I was representing my district. I found that the strength of the other students in the program helped push me to perform beyond what I thought I was capable of accomplishing.

I had a preconceived idea that we would not be doing any group work since everyone was online. I found out otherwise on the first day of class. I was surprised the LMS had breakout rooms where we could meet in small groups for discussion. The small group discussions were beneficial because I was able to hear strategies and ideas from mathematics teachers across the state. One strategy I learned from another candidate incorporated numberless word problems to build understanding of practical problems. I researched the concept of numberless word problems, used it in the classroom, and also facilitated a professional learning experience on the subject. That one strategy helped hundreds of students in my district.

We also had group work outside of class. At times, that became problematic due to short turnarounds and work obligations. It was difficult to find meeting times that worked for all group members. The use of a collaborative document improved our asynchronous communication. We would share the work and provide feedback for everyone to see and respond. As the classes progressed, this became the normal way we would complete the weekly group work.

I prefer to "see" a concept to understand it. I wondered if I would have difficulties learning new concepts online. I found it was much easier than I expected. All of the candidates were instructed to have access to certain manipulatives for each lesson. We would build a model, take pictures of what we built, and upload to share and explain with the class. Seeing everyone's pictures as they explained their model helped me learn a concept I was struggling to understand.

In an in-person classroom setting, I am an active participant and enjoy engaging with the instructor and class members. It was the same in an online setting. I had the ability to use the raise hand tool to ask any questions that were pertinent. But in a classroom, I can view other students' faces and body language to gauge their understanding of a concept and compare it to my own understanding. In an online setting, that was difficult to accomplish. I did not know if I was the only one having difficulty grasping a new concept, or if I was one of the few who immediately understood it. During the first few classes, the isolation made me question my ability at times. Through whole group and small group discussions, I found my understanding mirrored the majority of the cohort most of the time, and I was able to feel more comfortable learning new concepts and asking questions without fear of ridicule.

Allison

Prior to beginning the VCU program, I had taken an online mathematics course at another university. The class was asynchronous, so lessons were posted in the form of videos, digital presentations, textbook reading, and homework problems. I never met my professor or had any interaction with other students in the class. I found learning in this environment to be extremely challenging.

VCU's online mathematics specialist preparation program was vastly different from my previous experience. One of the aspects that made the biggest difference for me was the cohort of candidates. We spent two and half years learning together virtually, only meeting each other in person one time at the beginning of our program. Through the small group work, breakout rooms, and synchronous class time, we were able to build relationships and develop trust with one another. This allowed the learning experience to be authentic, for candidates to ask questions without fear of judgment, and for candidates to take risks. Additionally, the cohort represented a group with diverse backgrounds in teaching mathematics. I learned so much from candidates who were from the elementary world, giving me a window into how students learn before coming to middle school. The relationships I developed during this program continue to play an important role in my life, both professionally and personally.

Instructor Experiences

In planning our mathematics instruction, we took activities that had previously been used for in-person classes and adapted them to an online format. Specifically, we used materials from the Developing Mathematical Ideas (DMI) professional development program[1] and altered them to meet our online needs. It is important to note that the DMI materials were not originally designed for an online environment, but the program's unique blend of classroom case studies and rich mathematics tasks generated, as we had hoped, strong mathematical and discursive engagement among the candidates. What follows are the testimonials of two instructors whose courses may be regarded as bookends of sorts: Numbers and Operations was the first course in the program, and Algebra and Functions II was the last.

Cat

When my colleagues and I began preparing for the course we taught in the summer of 2017, Numbers and Operations, our discussions included familiar topics: What multi-base activities would help enrich candidates' understanding of base ten numeration? What kind of models for fraction multiplication should we emphasize? Mathematics content and pedagogy were certainly in our wheelhouse, but when our discussions turned to the subject of technology, I was in new territory. There were tools both tangible (headphones, electronic personal slates, and pens) and intangible (tabs, links, menus, pages, and buttons) to contend with. In fairness, I was no stranger to digital technology. After all, I had used digital technology capably enough in an in-person classroom setting. This time, however, what was daunting was not the presence of digital technology but its primacy. In the online version of Numbers and Operations, the quality of mathematics teaching and learning would be tied inextricably to the capability of the tools and, of course, to user fluency. As instructors, we also felt strongly that the key to promoting a true "community of inquiry" was in using the candidates' own responses and solutions to help move through mathematical content in a meaningful way. It was therefore crucial that we quickly adapt our digital presentations to reflect the mathematical thinking of candidates.

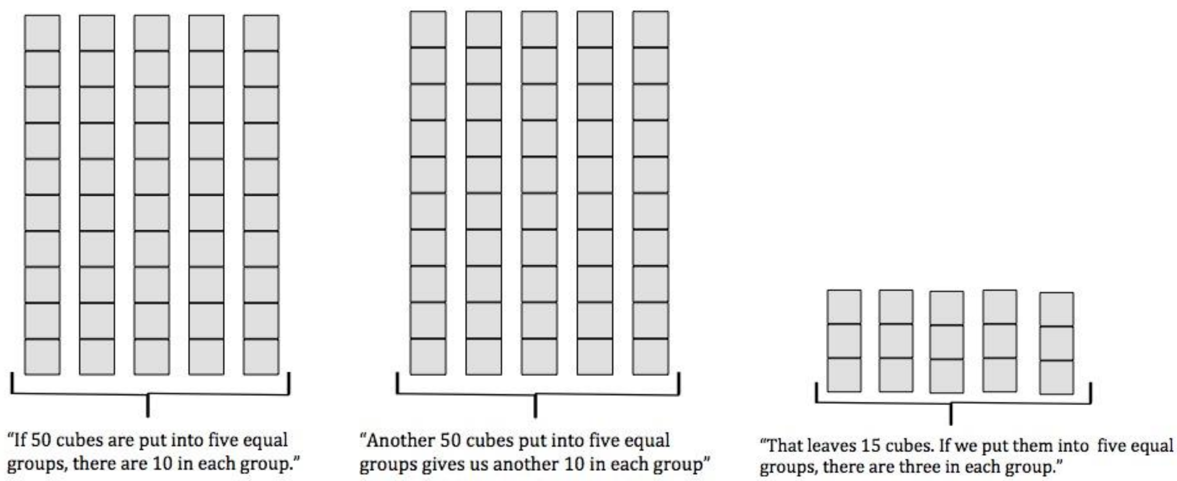
For each class session, my two co-instructors and I created a digital presentation using images of children's work featured in the DMI case studies. For example, we devoted a significant portion of a class session to a whole number division strategy involving an intentional decomposition of the dividend. Connected to what is formally called the "right distributive property of division," this invented strategy is one we asked candidates to explore by (1) creating a contextual division problem, (2) using and modeling the targeted strategy, and (3) stating and defending whether or not the strategy would always work. Figure 1 represents how a candidate modeled a division strategy using snap cubes. The model demonstrates how $115 \div 5$ is equal to $(50 + 50 + 15) \div 5$.

To answer mathematical focus questions such as the one related to this division strategy, candidates frequently photographed, annotated, and uploaded their responses to discussion forums. After reviewing all submissions, instructors selected a few samples, with an eye towards diversity of mathematical representation, and embedded them in the presentation for the following session. Candidates' solutions therefore did not function merely as assessments or as

[1] DMI was a project originally spearheaded by researchers Deborah Schifter, Virginia Bastable, and Susan Jo Russell and was an outgrowth of the *Teaching to Big Ideas* project funded by the National Science Foundation. (See, for example, Schifter, D., Bastable, V., & Russell, S., (2016)).

punctuation marks ending a particular mathematics topic. Rather, they drove group discussions focusing on the similarities and differences among models/solutions.

Figure 1
Division Modeling using Snap Cubes



Note. The dividend, 115, is partitioned into "chunks" that are divisible by 5. To find the quotient, the partial quotients, 10, 10, and 3, are added together.

The prompt, "Will the strategy always work? Why or why not?" was designed to deepen thinking and, when shared publicly, drive consensus or disagreement. Candidates used everyday language to defend their position on the generalizability of the division strategy. Some candidates indicated that this strategy would always work, while one in particular argued that the appropriateness of the strategy is entirely context-bound. In other words, if the problem were instead $116 \div 5$ and the context involved putting people into equal groups, how would we wrestle with the remaining $1 \div 5$? The tension among conflicting responses, and the conversation it generated, is one of the ways mathematical understanding was negotiated within the community of inquiry.

The use of candidates' own work helped the group build what Garrison et al. (2010) calls a "shared social identity," that of teachers taking a deeper dive into the complexity of elementary mathematics. However, doing so was not without its challenges. We knew there was an element of risk in putting certain candidates' work "on the spot," so to speak. It is one thing for a candidate to speak up voluntarily in an online discussion or contribute to a chat forum, but it may have been awkward for candidates to find themselves involuntarily at the center of discussion. Periodically, it was even instructive to use examples that were mathematically incorrect, leaving it to the group to analyze. As stated earlier, it is far more difficult to read affective cues in an online course, so whether or not this was productive for all remains unclear.

Chelsea

When I taught Algebra and Functions II as the last class for the cohort, it was my first foray into teaching at the collegiate level, as well as my first time teaching online. After attending a previous in-person cohort, I was very familiar with the material, but completely unfamiliar with the LMS. I co-taught the class with two other instructors who had more experience than I did, both in collegiate level teaching and online instruction.

The teaching team took turns planning different parts of the lessons. Most classes began with a discussion of reading material, sometimes a discussion of homework assignments, at least one mathematics activity that corresponded to the readings or homework, and at least one summary closure activity. For many tasks, we divided candidates into small groups using the online platform. In doing so, the instructors all remained in the main "room," and candidates moved to breakout rooms with two to three people in each. With three instructors, we were each able to visit two to three rooms to offer assistance to candidates and listen to their conversations. At times, I got caught in deep discussions with one room and did not get a chance to visit with all of the candidates. On the rare occasion that only two instructors could be present, attending to several rooms was much more challenging, as we were unable to spend significant time in any one discussion. My natural desire to reach every candidate was complicated by the need to move quickly between virtual rooms, a technology skill I am still developing.

Another difficult aspect of teaching mathematics online is the required use of wait time, especially while in small groups. When teaching in person, I use wait time to allow students to process directions and gather their thoughts before discussing an answer to a question. During that time, I walk around and observe students working and *see their thinking in action*. In an online setting, I have to trust that students are engaging with the learning, and the amount of wait time required becomes guesswork. When I only see what is on camera (often not showing the 'work' that students are completing), I have to fight the urge to continue talking just to fill the silence. To me, the wait time silence in an online format is excruciating compared to wait time when teaching in person. I find it best to explain the directions, answer clarifying questions, and then shut off my microphone entirely until students use virtual tools to signal they are ready to discuss.

Gauging student understanding in a virtual environment can be challenging. During in-person classes, I scan the room and watch students' body language as an indication of understanding. I regularly make decisions about my next move by observing students nodding, shrugging, tilting their heads, etc. Some online tools are useful for that type of feedback. I regularly use the thumbs up/thumbs down tool to gauge student understanding, but I find it more time consuming to gather and interpret that quantitative feedback online than in person. Observing and engaging in small group discussions provides a crucial structure for connecting with students, without which it would be impossible to gauge student understanding. By building relationships, we are able to gather qualitative feedback that provides more insight into each student's experience.

Conclusion

The interactions among instructors and candidates in the classroom are, importantly, both verbal and non-verbal. In an online setting, half of that interaction is missing. The challenge is that neither instructors nor candidates have the ability to read one another's non-verbal cues, thus limiting a mathematics instructor's ability to assess whether a candidate's struggle is productive or unproductive. Furthermore, in an online course, instructors can only view the end product (perhaps a solution to a mathematics problem), and it is more difficult to gauge how deep the candidate's understanding of a concept is without non-verbal cues or without seeing problem solving in action. With experience comes intuition: teachers and candidates alike can "read the room" in an in-person setting—sensing understanding, confusion, frustration, and even revelation. This intuitive ability is somewhat lost in online instruction.

Unquestionably, a major success of the online course was the small group interactions. The candidates in the program were mathematics teachers brought together from across the state, making it unlikely that relationships had been established prior to the onset of the program. Instead, candidates built relationships during small group interactions, which increased mutual trust and mathematical understanding, helping candidates feel more comfortable in asking questions of each other and developing an authentic community of inquiry (Garrison, 2001; Anderson & Archer, 2010).

A timely byproduct of the online coursework was that it prepared candidates for distance learning, which would prove invaluable during the COVID-19 pandemic. This preparation extends far beyond merely developing fluency with online tools, which are themselves rapidly changing and quickly rendered obsolete. Rather, candidates came to understand the unique ecology of online learning settings. After experiencing online learning, one candidate described empathizing with his students' fears of the unknown as they made the unexpected transition to remote learning. Teaching and learning online "is a form of change that involves our instructional realities, forms, and attitudes" (Major, 2015, p. 15). While online learning has its communicative challenges, we strongly believe that the success of the virtual mathematics class is deeply rooted in human relationships.

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