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PART I: SPECIAL ISSUE

Mathematics teacher leadership preparation, mentorship, and service in Virginia featuring the use of online modalities for instruction or support.

PART II: REGULAR JOURNAL FEATURES

Virginia Mathematics and Science Coalition

Journal of Mathematics and Science: Collaborative Explorations

The *Journal of Mathematics and Science: Collaborative Explorations* is the official journal of the Virginia Mathematics and Science Coalition (https://www.vamsc.org/), a non-profit organization comprised of education, scientific, corporate, and public policy leaders committed to the sustained elevation of mathematics and science education to ensure that all Virginia's students and citizens have the foundation required for life-long success in their daily lives, careers, and society.

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The *Journal of Mathematics and Science: Collaborative Explorations* is a forum which focuses on the exchange of ideas, primarily among higher education faculty from mathematics, science, and education, while also incorporating the perspectives of elementary and secondary school teachers. Articles are solicited that address the preparation of prospective teachers of mathematics and science in grades K–12, the preparation of mathematics and science teacher leaders for grades K–12, and innovative programs for undergraduate STEM majors.

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Part I: Special Issue

Mathematics teacher leadership preparation, mentorship, and service in Virginia featuring the use of online modalities for instruction or support.

MATHEMATICS TEACHER LEADERS PREPARATION, MENTORSHIP, AND SERVICE: COMMUNITIES OF PRACTICE THROUGH ONLINE MODALITIES

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ABSTRACT
This article is a preface to a special issue of the *Journal of Mathematics and Science: Collaborative Explorations* which features articles that describe (a) online components of mathematics specialist preparation and mentoring programs, (b) the mentoring and support of teachers preparing to serve as mathematics teacher leaders, and (c) the subsequent service of mathematics specialists in leadership roles. This preface describes the context within which the described online activities took place, provides a common glossary of terms that will be used consistently across all the articles, and briefly introduces each of the fourteen papers that constitute the special issue.

KEYWORDS
mathematics specialist, mathematics teacher leaders, online professional development, online mentoring

This special issue of the *Journal of Mathematics and Science: Collaborative Explorations* features articles that describe (a) online components of mathematics specialist preparation and mentoring programs, (b) the mentoring and support of teachers preparing to serve as mathematics teacher leaders, and (c) the subsequent service of mathematics specialists in leadership roles. This preface describes the context within which the described online activities took place, provides a common glossary of terms that will be used consistently across all the articles, and briefly introduces each of the fourteen papers that constitute the special issue.

Mathematics Specialists

Similar to the approach taken by McGatha and Rigelman (2017), throughout this special issue, the term *mathematics specialist* will refer to individuals in PK–12 schools who serve as mathematics coaches or program leaders. Indeed, the preparation programs described in this issue are primarily designed to prepare teachers to serve as mathematics coaches or program leaders. However, program graduates may and do, at the discretion of their school systems, serve in one or more of the following roles: mathematics teacher, mathematics interventionist, and mathematics coach or program leader.

Research on the Impact of Mathematics Specialists

While a large portion of the research on mathematics specialists has focused on the various mathematics specialist roles and responsibilities (e.g., Mudzimiri et al., 2014), researchers have investigated several areas in which the work of the mathematics specialist is important including (a) the exploration of the mathematics specialist's relationship with and work with teachers (Chval et al. 2010; Gibbons & Cobb, 2012; Marsh et al., 2010; Polly, 2012; Race et al., 2002); (b) the knowledge and ongoing support required of the mathematics specialist (Baldinger, 2014; Bitto, 2015; de Araujo et al., 2017; Fennell et al., 2017; Shaughnessey et al., 2017; Sutton et al., 2011); (c) the preparation of teachers to serve in these roles (Baker et al., 2018; Myers et al., 2020); and (d) the design of the online learning environment (Baker & Hjalmarson, 2019). Furthermore, research has also explored the impact of mathematics specialists on teachers and students (Baker et al., 2017; Balfanz et al., 2006; Campbell et al., 2017; Markworth, 2017; Polly, 2012; Race et al., 2002).

In Virginia, several large-scale studies have been conducted on the impact of mathematics specialists in K–8 schools. In a three-year, randomized, control study, Campbell and Malkus (2010; 2011) found that over time mathematics specialists had a significant positive influence on student achievement in third, fourth, and fifth grades. The impact was evident after two years on the job with the increase in scores for students in schools with a trained mathematics specialist on average ten or more points higher on mathematics achievement tests when compared to students in schools without a specialist. Additionally, a two-year study explored the relationships among mathematics specialists, classroom teachers, and the building administrators in order to develop a deeper understanding of the processes that influence continuous improvement of K–8 mathematics achievement and effective mathematics teaching in school settings. Ellington et al. (2017) found that being highly engaged with the mathematics specialist had a significant impact on middle school teacher beliefs about how students learn mathematics and on student achievement. Over time, highly engaged teachers developed an understanding that students should work through ideas to make sense of them in order to develop a deep understanding of mathematical concepts. With respect to student achievement, in grades six and seven, students of teachers who were highly engaged with the specialist performed significantly better on achievement tests than students of teachers who were not highly engaged.

Online Professional Development for Teachers

Online learning is an effective mode for delivering graduate and professional education. The benefits of online learning are well documented with respect to accessibility, efficacy, cost effectiveness, learner flexibility, and interactivity (e.g., Sinclair et al., 2016). Online communities of practice promote and deepen teacher reflective practice (Hough et al., 2004; Stiler & Philleo, 2003), afford opportunities for teachers to share their expertise and develop collegial, long lasting relationships (Hanson-Smith, 2006; Paulus & Scherff, 2008), enable educators to collaborate and integrate educational theory into their practice (Dibbon & Stevens, 2008), and increase teachers' self-efficacy (Vavasseur & MacGreor, 2008). Dede et al. (2009) assert that teachers need access to professional development experiences that capitalize on "powerful resources often not available locally, and that can create an evolutionary path toward providing real-time, ongoing, work-embedded support" (p. 9).

Virginia Mathematics Specialist Add-On Endorsement

The K–8 Mathematics Specialist endorsement in Virginia requires that individuals be fully licensed teachers with at least three years' experience. To meet these requirements, candidates complete a master's degree that aligns with the national preparation standards for mathematics specialists (Association of Mathematics Teacher Educators, 2013; National Council of Teachers of Mathematics, 2012) and includes opportunities for candidates to develop a deep understanding of (a) K–8 mathematics in areas such as number and operations, rational numbers, geometry, algebra and functions, and probability and statistics; (b) pedagogical content knowledge across the K–8 curriculum; and (c) leadership and coaching skills needed for working with teachers. Master's level programs have been developed at a number of different universities across the state and are approved by the Virginia Department of Education. At most institutions, teachers who have previously earned a masters' degree in a related area can earn the add-on endorsement by completing only the mathematics and leadership courses necessary to meet the endorsement requirements that were not satisfied by prior course work.

Transition to Online Mathematics Specialist Preparation Programs

In Virginia, the initial mathematics specialist preparation programs were taught in a face-to-face format and fully engaged candidates in in-depth discussions of mathematical ideas to prepare them for work as mathematics coaches. Virginia Commonwealth University began to offer a blended program in 2010 and then, with support from a National Science Foundation Noyce grant that is also funding this special issue, offered fully online courses to its first cohort beginning in 2017. Many of the instructors for courses in this cohort had participated in or been candidates in face-to-face mathematics specialist preparation programs. George Mason University's program has also offered an online hybrid format since 2010 and added a fully synchronous online format in 2017, so the instructors in that program have significant experience with online instruction. The Longwood University program has been offered through a hybrid format for approximately 10 years.

Synchronous Online Learning Environment

Each of the courses described in this special issue have significant synchronous learning components, during which all candidates are online concurrently. The online synchronous format permits lively whole group discussions. Candidates are also regularly placed in virtual breakout rooms where they can discuss the mathematical ideas under study in small groups.

Asynchronous Online Learning Environment

Some components of the courses are offered in an asynchronous format during which candidates complete assignments on their own time away from the synchronous learning environment. An example of this type of assignment is an assigned reading in which candidates respond to prompts in a discussion board or some other virtual medium, and their responses are subsequently read and responded to by other candidates as well as the course instructors. Select portions of the mathematics specialist courses described in this special issue involve this format.

Co-Authors of Papers

All of the papers in this special issue are co-authored. These teams typically consist of individuals who are endorsed mathematics specialists, school administrators, full-time university mathematics educators, or university mathematicians.

Terminology

- ***Instructor*** — This term refers to any individual who teaches or co-teaches a mathematics or leadership course in a mathematics specialist preparation program.
- ***Candidate*** — This term refers to any K–12 teacher who is or was enrolled in a mathematics specialist preparation program.
- ***Teacher*** — This term describes any PK–12 school personnel who is primarily responsible for educating PK–12 students.
- ***Student*** — This term refers to any PK–12 student in both public and private settings.

JMSCE Special Issue Articles

The papers in the special issue describe transitioning from face-to-face to online learning environments, challenges and benefits of an online mathematics specialist preparation program, mentoring new mathematics coaches or on-going mentoring programs, using online tools to coach teachers; and specialists developing partnerships with principals.

Online versus In-person Mathematics Instruction: A Comparison of Two Instructional Models explores the differences between online and traditional in-person teaching and learning modalities. The authors describe the preparation for and teaching of online mathematics, focusing on establishing norms and the use of technology. By identifying key similarities and differences between instructional modalities and by reflecting on successes and challenges, a vision of online teaching and learning for mathematics courses emerges that can be effective, inclusive, and relational.

Connected at a Distance: Experiences and Efforts Within a Synchronous, Online Mathematics Specialist Program describes the purposeful opportunities that were provided throughout a mathematics specialist preparation program for candidates to make ongoing personal and professional connections with each other. Based on the idea that learning is a social construct, instructors and candidates worked to form and sustain an online learning community. The authors share ways that intentional connectedness can be extended to other educational contexts.

Instructor Perspectives: Transitioning from Face-to-Face to an Online or Hybrid Graduate Level Mathematics Education Course shares reflections and lessons learned from instructors at three different institutions as they made the transition from face-to-face to online or hybrid instructional models. The authors share their experiences in constructing lessons and facilitating class sessions. They describe their personal and professional growth through the experience and share takeaways for institutions planning to develop online professional development programs for teachers.

Transitioning a Mathematics Specialist Preparation Program into an Interactive Online Program: Insights from the Developer and Candidate Perspectives describes how an entire preparation program transitioned from a set of face-to-face courses to an entirely online instructional format. Both the instructor and the candidate perspectives on the changes that were made are shared. The paper states that the goal was to use online tools and remote instruction for all aspects of the program while at the same time maintaining the highly interactive nature and the rigorous instruction that the face-to-face preparation program was known for.

Developing Equity-Centered Leadership Knowledge and Skills via Lesson Study in an Online Mathematics Specialist Program describes coursework within a synchronous online mathematics specialist program that enhanced candidates' leadership knowledge and provided structures that addressed issues of equity and access. The paper focuses on an online assignment grounded in Lesson Study that played a pivotal role in helping candidates develop equity-centered leadership and instructional practices. The experiences shared by course instructors and recent program alumni support the broader goal of achieving a cohesive vision for the teaching and learning of K–8 mathematics, while promoting equitable practices in school-based work.

Learning to Anticipate in an Online Class: Perspectives of an Instructor and a Mathematics Specialist Candidate features the practice of anticipating how a learner will approach an activity from two perspectives: a course instructor and a mathematics specialist candidate. The authors note that learning to anticipate was one skill that helped to develop a rich community of learners that provided opportunities for everyone to grow through their interactions with and reflections on course content.

Mathematical Representations in a Synchronous Online Mathematics Specialist Preparation Program addresses the possible concerns of compromising quality pedagogy for convenience when designing synchronous online courses. In addition to maintaining rich discussion and student collaboration in an online environment, mathematics content courses include the additional challenge of incorporating problem-solving with multiple representations. This paper focuses on how these mathematical representations emerge and develop during a synchronous online course for mathematics specialists.

Team Teaching for Discourse: Perspectives of Instructors and a Student in an Online Probability and Statistics Course for Preparing Mathematics Specialists describes the interactions of and reflections from three course instructors and a mathematics specialist candidate during the planning and enactment of a Probability and Statistics course for mathematics specialists. The authors discuss the strengths of discourse in the planning stage as a way to create and sustain a sense of community and share multiple perspectives in an online course. They share how the experiences of a diverse team were crucial to successful implementation of a team-teaching approach to instruction.

Equity and Access: Empowering Change Agents shares how mathematics specialists are uniquely situated to contribute to the creation of access and equity for all learners by addressing three target areas with their mathematics teachers and administrators. Three possible obstacles to access and equity are: beliefs and expectations, curriculum and instruction, and intervention. Mathematics specialists can be prepared to address these obstacles through their preparation in leadership courses that are intentionally designed to help them practice negotiating the role of change agent.

The Role of a Mathematics Content-Focused Coaching Project in Preparing Mathematics Specialist Candidates to Coach describes the mathematics content-focused coaching process from the perspective of mathematics specialists in their work with teachers. In particular, the paper outlines effective strategies and techniques used by the mathematics specialists as they work with teachers and focus on mathematics and student learning. The authors share an activity that provides novice mathematics specialists with the opportunity to reflect on all aspects of the coaching cycle. They share ways in which this reflective activity can be used to support learning.

Online Education: Transferring Personal Experiences to Professional Development describes how participation in a mathematics specialist preparation program helped prepare one mathematics teacher leader to develop and offer online professional development for teachers. The paper highlights the importance of building relationships and using high-quality mathematical tasks in online professional development. This case study provides evidence that exposure to online learning environments as a learner can help lower the barrier of entry for planning and providing online learning experiences as a mathematics specialist.

Virtual Mentorship of Teacher Leaders: The Ripple Effect describes a monthly online mentoring program for novice mathematics specialists. Two mathematics specialists serving as mentors and two candidates participating in the mentoring program share their thoughts and ideas on the support provided through online mentoring. The authors discuss the benefits and constraints of mentoring in an online environment and ways this particular program can be a model for other virtual mentoring programs.

Providing Job-Embedded Professional Development for Mathematics Specialists highlights the importance of providing job-embedded professional development for mathematics specialists. Just as mathematics specialists provide coaching to teachers to help with their professional growth, similar opportunities for the growth must be identified for mathematics specialists. This paper identifies purposes of professional development for coaching to include supporting growth in content knowledge, pedagogical expertise, coaching skills, professionalism, and leadership. The authors advocate for a virtual network of mathematics specialists in similar positions.

A Relationship Built to Impact Instruction: Developing and Sustaining Productive Partnerships between Mathematics Specialists and Principals emphasizes the importance of a strong relationship between the mathematics specialist and school administrators to the success of the mathematics specialist's work in the school building. The authors share examples of partnerships that are based on a shared vision for mathematics instruction and describe the impact on student mathematics achievement. The authors share examples from prior to the COVID-19 pandemic as well as examples of successful partnerships during the pandemic that required brainstorming and creativity on the part of teachers, mathematics specialists, and administrators.

Acknowledgment

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ONLINE VERSUS IN-PERSON MATHEMATICS INSTRUCTION: A COMPARISON OF TWO INSTRUCTIONAL MODELS

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ABSTRACT
Our paper explores the differences between online and traditional, in-person teaching and learning modalities, looking specifically at courses preparing teachers to be mathematics teacher leaders. In the context of current research on the teaching and learning of mathematics in an online setting, we share our own experiences. We describe the preparation for and teaching of online mathematics, focusing on establishing norms and the use of technology. The changing teaching and learning opportunities of the 21st century require discussion of these vital issues. We include stories of interactions between candidates and teachers and among groups of candidates in mathematics courses, detailing not just the discursive and work-sharing tools but the nature and nuance of these interactions and how they mediate mathematics learning. We share our online teaching and learning experiences, drawing on research to frame our impressions. By identifying key similarities and differences between instructional modalities and by reflecting critically on our own successes and challenges, we present a vision of online teaching and learning for mathematics courses, in particular those for mathematics specialists, that can be effective, inclusive, and relational.

KEYWORDS
online mathematics instruction, mathematics leadership

In the summer of 2017, a group of 30 elementary and middle school educators across Virginia logged into a virtual mathematics classroom. It was the first of several Virginia Commonwealth University (VCU) courses designed to prepare in-service teachers to serve as mathematics specialists. Mathematics courses were designed to broaden candidates' understanding of both content and pedagogy, and leadership classes helped cultivate the professional dispositions unique to instructional coaches. This cohort represented a landmark change in the evolution of VCU's program: their courses would be delivered entirely online through a Learning Management System (LMS) with video conferencing capabilities. Candidates were provided headsets and a drawing tablet for creating digital drawings. Throughout this paper we refer to technology tools in a general way, focusing on functionality, and we believe our findings to be relevant to online mathematics instruction overall regardless of specific tools.

This mathematics specialist preparation program consisted of six mathematics courses, three mathematics education leadership courses, and a capstone project spanning two semesters. All courses were taught using a flipped classroom model. Candidates were provided with prerequisite work through the LMS for each class session including readings, mathematics activities, reflections, and small group activities. For all courses in the program, prerequisite work guided the activities and discussions during the following synchronous class sessions.

Literature Review

Our framework for reflecting on mathematics courses, from both an instructor and candidate point of view, is informed by the concept of a "community of inquiry," developed by Garrison, Anderson, & Archer (2001; 2010). Features of this framework include the Deweyan notion of inquiry as a social activity, which despite the qualifier "social" also hinges on the private, reflective actions of the individual learner. In other words, we recognize that meaningful online mathematics learning happens when "students move repeatedly between reflection and discourse" (Garrison et al., 2001, p. 9). We also draw on their notion of "social presence," which is related to *group cohesion*—a feature we believe develops from a strong sense of relationship and trust—and *shared social identity*, established early on by virtue of the common goal of state certification but strengthened over time by philosophical and methodological consensus (namely, a commitment to teaching mathematics for understanding).

In comparing in-person to online teaching and learning modalities, we draw on the work of Claire Howell Major, who, in 2015, published a long overdue guide to the praxis of teaching online. Her prescient opening essay examines how teaching and learning are mediated by technology itself, how technology shapes both interactions and products, and indeed how virtual educational reality is experienced by all participants. When she states that the online instructor "interpret(s) the instructional experience *with* the technology," (p. 11) she does not limit the "experience" to comments, solutions, models, and other such empirical products. Educational experience includes "feelings, thoughts and relationships," which are no less refracted through the lens of technology. Pauses during class discourse, verbal and textual exchanges among students, fully-formed (rather than inchoate) posted solutions: these are interpretable "through and with technology" and hence influence the instructor's judgment not just of student learning but of the overall affective domain of the online classroom.

Our aim is to share both learner and instructor experiences in online mathematics courses, illuminating key differences between in-person and virtual classroom spaces. We emphasize the need for nurturing strong interpersonal connections within the peculiar space of online learning, perhaps especially in contemporary mathematics courses where intellectual risk-taking and open discourse are generative forces.

Comparing Online and In-person Mathematics Teaching and Learning

Teachers transitioning from in-person to online mathematics instruction can benefit from reading testimonials about this paradigm shift, as the foray into online education can be uneasy or even unsettling. With synchronous class meetings, the virtual classroom is neither room-like nor entirely cold or inhuman. For instructors and candidates alike, the environment borders between the familiar and unfamiliar. Some hallmarks of in-person learning environments are reproduced in online instruction. For example, there are typically many students and few teachers, there may be a front-and-center "whiteboard," and so on. Yet despite these traditions, virtual classrooms cultivate a markedly different classroom ecology. This section will compare online instruction to in-person instruction to help readers make connections between the styles of teaching and make suggestions for intentional change. We describe some of the struggles of online instruction and will include a discussion of strategies that can be used to address the challenges of online instruction.

As a teacher new to online instruction said recently, "In an online class, it's very hard to take the temperature of the room" (personal communication). In the context of an online *mathematics* class, this difficulty is especially problematic. Although physical and affective cues are not under complete erasure, they are surely less apparent. As Claire Howell-Major says, "We cannot see a student's happiness at answering a question well or puzzlement over another students' response" (2015, p. 12). Our ability to read the room—to know whether or not candidates understand a given mathematics problem, to sense when they are persisting or capitulating—is based less on interpersonal skills and more on the technology itself. For example, after we posed a mathematics problem and broke candidates into virtual small groups, instructors were able to "visit" these small groups to check in with students. On several occasions, when we entered a small group session, a candidate would immediately ask a clarifying question about what they were "supposed to do." To the seasoned mathematics instructor, this might sound familiar. Indeed, appealing to the instructor for clarification and support is not uncommon in an in-person mathematics class. But in an online format, we are entirely dependent on the affordances of the technology (here, the "join group" feature that allowed instructors to enter small group forums) to support learning.

Similarly, the chat feature became a means of clarifying questions and responses and even arguing for or against ideas. We recognize these actions as critical mathematical habits of mind. In a chat forum they are usually textual (we say "usually" because emojis were also featured prominently in whole group chats) and appear in rapid succession. But as a candidate in an online course said recently, "What you say in a chat—it's just *out there* in print for everyone to see." Might it have seemed riskier for candidates to contribute to a chat forum? Alternatively, some candidates may have sensed *less* risk in contributing to the chat forum, a forum that is not restricted to the "one-at-a-time" formality of group discourse. Since robust mathematical discourse requires some degree of risk-taking, the textual chat function may have mediated the quantity and quality of discourse.

Building Classroom Culture

As with in-person courses, teachers of online mathematics courses spend time establishing norms and expectations and building a culture of collaboration through a community of inquiry. Fortunately, many online platforms incorporate tools to help establish norms, including "raise hand," "thumbs up/thumbs down," typing into a chat box, and sharing pictures (all experienced through on-screen notifications for the instructor). In our online cohort, candidates were expected to participate using these tools as a quick check for understanding. For establishing relationships, the use of breakout groups provided the opportunity for small group work before and during class.

Camera use varied by instructor: some required candidates to have cameras on during whole group discussions, and others left it up to the individual. Most candidates preferred to use the cameras during small group interactions, even if their cameras were off in the large group. Through the use of the camera and other aforementioned platform tools, instructors *could* document student participation in a variety of ways. It is important for instructors to be specific with students about how participation will be evaluated, as "active participation" can look quite different online than in person.

The norm of using physical manipulatives helped develop the mathematical content of our courses. When planning lessons, instructors made a list of required manipulatives for each class. Candidates were expected to have those manipulatives available for use during class. Virtual manipulatives were used at times but *were not used in place of physical manipulatives*. Frequently, instructors utilized discussion boards where candidates could contribute a photo of their manipulatives with a description of their work. These images were then used to guide conversation about the mathematical activities.

Instructors provided some form of agenda document to drive instruction. Many professors opted for a slide presentation that included directions for activities and live links to external tools used for those activities. Frequently this agenda document would be shared with candidates prior to class, and it was always made available after class, along with a video recording of the synchronous session, through the online platform.

Finally, attendance at synchronous sessions was mandatory for all candidates. As a graduate level mathematics specialist cohort, attendance overall was not an issue. On the rare occasion that a candidate missed a synchronous class, the video recording could be used to fill in learning gaps. However, *watching the recorded session was not viewed as a substitute for in-class learning*.

Candidate Experiences

Patrick

The VCU mathematics specialist preparation program was my first experience with an online class. I was nervous the first day because I knew the other candidates were also strong mathematics instructors. I wanted to do well not just for myself, but also because I was representing my district. I found that the strength of the other students in the program helped push me to perform beyond what I thought I was capable of accomplishing.

I had a preconceived idea that we would not be doing any group work since everyone was online. I found out otherwise on the first day of class. I was surprised the LMS had breakout rooms where we could meet in small groups for discussion. The small group discussions were beneficial because I was able to hear strategies and ideas from mathematics teachers across the state. One strategy I learned from another candidate incorporated numberless word problems to build understanding of practical problems. I researched the concept of numberless word problems, used it in the classroom, and also facilitated a professional learning experience on the subject. That one strategy helped hundreds of students in my district.

We also had group work outside of class. At times, that became problematic due to short turnarounds and work obligations. It was difficult to find meeting times that worked for all group members. The use of a collaborative document improved our asynchronous communication. We would share the work and provide feedback for everyone to see and respond. As the classes progressed, this became the normal way we would complete the weekly group work.

I prefer to “see” a concept to understand it. I wondered if I would have difficulties learning new concepts online. I found it was much easier than I expected. All of the candidates were instructed to have access to certain manipulatives for each lesson. We would build a model, take pictures of what we built, and upload to share and explain with the class. Seeing everyone’s pictures as they explained their model helped me learn a concept I was struggling to understand.

In an in-person classroom setting, I am an active participant and enjoy engaging with the instructor and class members. It was the same in an online setting. I had the ability to use the raise hand tool to ask any questions that were pertinent. But in a classroom, I can view other students' faces and body language to gauge their understanding of a concept and compare it to my own understanding. In an online setting, that was difficult to accomplish. I did not know if I was the only one having difficulty grasping a new concept, or if I was one of the few who immediately understood it. During the first few classes, the isolation made me question my ability at times. Through whole group and small group discussions, I found my understanding mirrored the majority of the cohort most of the time, and I was able to feel more comfortable learning new concepts and asking questions without fear of ridicule.

Allison

Prior to beginning the VCU program, I had taken an online mathematics course at another university. The class was asynchronous, so lessons were posted in the form of videos, digital presentations, textbook reading, and homework problems. I never met my professor or had any interaction with other students in the class. I found learning in this environment to be extremely challenging.

VCU’s online mathematics specialist preparation program was vastly different from my previous experience. One of the aspects that made the biggest difference for me was the cohort of candidates. We spent two and half years learning together virtually, only meeting each other in person one time at the beginning of our program. Through the small group work, breakout rooms, and synchronous class time, we were able to build relationships and develop trust with one another. This allowed the learning experience to be authentic, for candidates to ask questions without fear of judgment, and for candidates to take risks. Additionally, the cohort represented a group with diverse backgrounds in teaching mathematics. I learned so much from candidates who were from the elementary world, giving me a window into how students learn before coming to middle school. The relationships I developed during this program continue to play an important role in my life, both professionally and personally.

Instructor Experiences

In planning our mathematics instruction, we took activities that had previously been used for in-person classes and adapted them to an online format. Specifically, we used materials from the Developing Mathematical Ideas (DMI) professional development program[1] and altered them to meet our online needs. It is important to note that the DMI materials were not originally designed for an online environment, but the program's unique blend of classroom case studies and rich mathematics tasks generated, as we had hoped, strong mathematical and discursive engagement among the candidates. What follows are the testimonials of two instructors whose courses may be regarded as bookends of sorts: Numbers and Operations was the first course in the program, and Algebra and Functions II was the last.

Cat

When my colleagues and I began preparing for the course we taught in the summer of 2017, Numbers and Operations, our discussions included familiar topics: What multi-base activities would help enrich candidates' understanding of base ten numeration? What kind of models for fraction multiplication should we emphasize? Mathematics content and pedagogy were certainly in our wheelhouse, but when our discussions turned to the subject of technology, I was in new territory. There were tools both tangible (headphones, electronic personal slates, and pens) and intangible (tabs, links, menus, pages, and buttons) to contend with. In fairness, I was no stranger to digital technology. After all, I had used digital technology capably enough in an in-person classroom setting. This time, however, what was daunting was not the presence of digital technology but its primacy. In the online version of Numbers and Operations, the quality of mathematics teaching and learning would be tied inextricably to the capability of the tools and, of course, to user fluency. As instructors, we also felt strongly that the key to promoting a true "community of inquiry" was in using the candidates' own responses and solutions to help move through mathematical content in a meaningful way. It was therefore crucial that we quickly adapt our digital presentations to reflect the mathematical thinking of candidates.

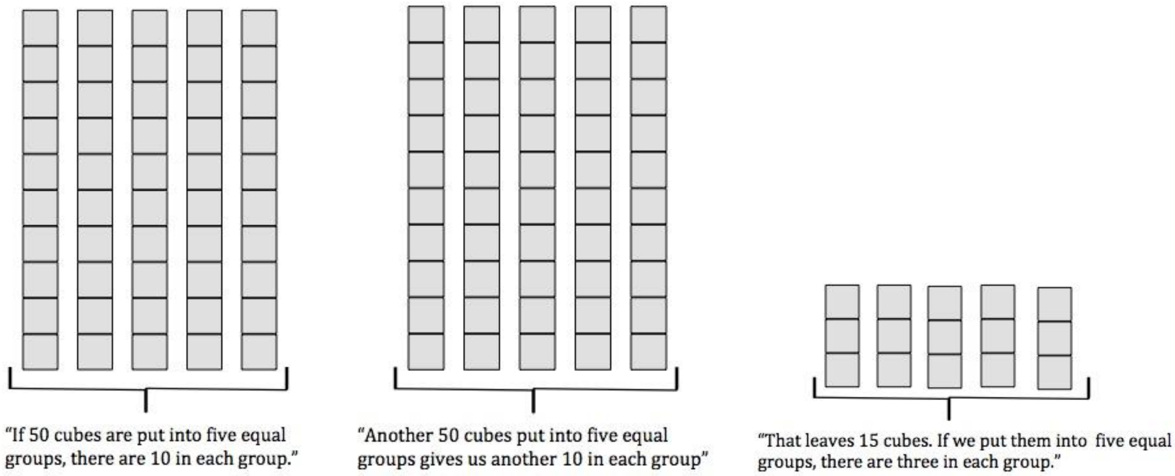
For each class session, my two co-instructors and I created a digital presentation using images of children's work featured in the DMI case studies. For example, we devoted a significant portion of a class session to a whole number division strategy involving an intentional decomposition of the dividend. Connected to what is formally called the "right distributive property of division," this invented strategy is one we asked candidates to explore by (1) creating a contextual division problem, (2) using and modeling the targeted strategy, and (3) stating and defending whether or not the strategy would always work. Figure 1 represents how a candidate modeled a division strategy using snap cubes. The model demonstrates how $115 \div 5$ is equal to $(50 + 50 + 15) \div 5$.

To answer mathematical focus questions such as the one related to this division strategy, candidates frequently photographed, annotated, and uploaded their responses to discussion forums. After reviewing all submissions, instructors selected a few samples, with an eye towards diversity of mathematical representation, and embedded them in the presentation for the following session. Candidates' solutions therefore did not function merely as assessments or as

[1] DMI was a project originally spearheaded by researchers Deborah Schifter, Virginia Bastable, and Susan Jo Russell and was an outgrowth of the *Teaching to Big Ideas* project funded by the National Science Foundation. (See, for example, Schifter, D., Bastable, V., & Russell, S., (2016)).

punctuation marks ending a particular mathematics topic. Rather, they drove group discussions focusing on the similarities and differences among models/solutions.

Figure 1
Division Modeling using Snap Cubes



Note. The dividend, 115, is partitioned into "chunks" that are divisible by 5. To find the quotient, the partial quotients, 10, 10, and 3, are added together.

The prompt, "Will the strategy always work? Why or why not?" was designed to deepen thinking and, when shared publicly, drive consensus or disagreement. Candidates used everyday language to defend their position on the generalizability of the division strategy. Some candidates indicated that this strategy would always work, while one in particular argued that the appropriateness of the strategy is entirely context-bound. In other words, if the problem were instead 116 ÷ 5 and the context involved putting people into equal groups, how would we wrestle with the remaining 1 ÷ 5? The tension among conflicting responses, and the conversation it generated, is one of the ways mathematical understanding was negotiated within the community of inquiry.

The use of candidates' own work helped the group build what Garrison et al. (2010) calls a "shared social identity," that of teachers taking a deeper dive into the complexity of elementary mathematics. However, doing so was not without its challenges. We knew there was an element of risk in putting certain candidates' work "on the spot," so to speak. It is one thing for a candidate to speak up voluntarily in an online discussion or contribute to a chat forum, but it may have been awkward for candidates to find themselves involuntarily at the center of discussion. Periodically, it was even instructive to use examples that were mathematically incorrect, leaving it to the group to analyze. As stated earlier, it is far more difficult to read affective cues in an online course, so whether or not this was productive for all remains unclear.

Chelsea

When I taught Algebra and Functions II as the last class for the cohort, it was my first foray into teaching at the collegiate level, as well as my first time teaching online. After attending a previous in-person cohort, I was very familiar with the material, but completely unfamiliar with the LMS. I co-taught the class with two other instructors who had more experience than I did, both in collegiate level teaching and online instruction.

The teaching team took turns planning different parts of the lessons. Most classes began with a discussion of reading material, sometimes a discussion of homework assignments, at least one mathematics activity that corresponded to the readings or homework, and at least one summary closure activity. For many tasks, we divided candidates into small groups using the online platform. In doing so, the instructors all remained in the main "room," and candidates moved to breakout rooms with two to three people in each. With three instructors, we were each able to visit two to three rooms to offer assistance to candidates and listen to their conversations. At times, I got caught in deep discussions with one room and did not get a chance to visit with all of the candidates. On the rare occasion that only two instructors could be present, attending to several rooms was much more challenging, as we were unable to spend significant time in any one discussion. My natural desire to reach every candidate was complicated by the need to move quickly between virtual rooms, a technology skill I am still developing.

Another difficult aspect of teaching mathematics online is the required use of wait time, especially while in small groups. When teaching in person, I use wait time to allow students to process directions and gather their thoughts before discussing an answer to a question. During that time, I walk around and observe students working and *see their thinking in action*. In an online setting, I have to trust that students are engaging with the learning, and the amount of wait time required becomes guesswork. When I only see what is on camera (often not showing the 'work' that students are completing), I have to fight the urge to continue talking just to fill the silence. To me, the wait time silence in an online format is excruciating compared to wait time when teaching in person. I find it best to explain the directions, answer clarifying questions, and then shut off my microphone entirely until students use virtual tools to signal they are ready to discuss.

Gauging student understanding in a virtual environment can be challenging. During in-person classes, I scan the room and watch students' body language as an indication of understanding. I regularly make decisions about my next move by observing students nodding, shrugging, tilting their heads, etc. Some online tools are useful for that type of feedback. I regularly use the thumbs up/thumbs down tool to gauge student understanding, but I find it more time consuming to gather and interpret that quantitative feedback online than in person. Observing and engaging in small group discussions provides a crucial structure for connecting with students, without which it would be impossible to gauge student understanding. By building relationships, we are able to gather qualitative feedback that provides more insight into each student's experience.

Conclusion

The interactions among instructors and candidates in the classroom are, importantly, both verbal and non-verbal. In an online setting, half of that interaction is missing. The challenge is that neither instructors nor candidates have the ability to read one another's non-verbal cues, thus limiting a mathematics instructor's ability to assess whether a candidate's struggle is productive or unproductive. Furthermore, in an online course, instructors can only view the end product (perhaps a solution to a mathematics problem), and it is more difficult to gauge how deep the candidate's understanding of a concept is without non-verbal cues or without seeing problem solving in action. With experience comes intuition: teachers and candidates alike can "read the room" in an in-person setting—sensing understanding, confusion, frustration, and even revelation. This intuitive ability is somewhat lost in online instruction.

Unquestionably, a major success of the online course was the small group interactions. The candidates in the program were mathematics teachers brought together from across the state, making it unlikely that relationships had been established prior to the onset of the program. Instead, candidates built relationships during small group interactions, which increased mutual trust and mathematical understanding, helping candidates feel more comfortable in asking questions of each other and developing an authentic community of inquiry (Garrison, 2001; Anderson & Archer, 2010).

A timely byproduct of the online coursework was that it prepared candidates for distance learning, which would prove invaluable during the COVID-19 pandemic. This preparation extends far beyond merely developing fluency with online tools, which are themselves rapidly changing and quickly rendered obsolete. Rather, candidates came to understand the unique ecology of online learning settings. After experiencing online learning, one candidate described empathizing with his students' fears of the unknown as they made the unexpected transition to remote learning. Teaching and learning online "is a form of change that involves our instructional realities, forms, and attitudes" (Major, 2015, p. 15). While online learning has its communicative challenges, we strongly believe that the success of the virtual mathematics class is deeply rooted in human relationships.

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CONNECTED AT A DISTANCE: EXPERIENCES AND EFFORTS WITHIN A SYNCHRONOUS, ONLINE MATHEMATICS SPECIALIST PROGRAM

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ABSTRACT

Online learning offers flexibility and convenience to students regardless of their proximity to a traditional campus. However, online programs can also feel isolating. Beth, a mathematics specialist candidate, completed a graduate program while living 7000 miles and seven time zones away from her instructor and peers. Through intentional planning by instructors, Beth found community by making personal connections, celebrating life experiences, and sharing a passion for mathematics education with her peers. Furthermore, Beth felt empowered to take academic risks and expose professional vulnerabilities in the learning community. The instructors within the program valued learning as a social construct and therefore designed opportunities for candidates to make ongoing personal and professional connections. In this mathematics specialist program, participants and instructors each took responsibility for forming and sustaining the online community. Although the examples shared in this manuscript are one student's experiences in a specific context, mathematics leaders may be able to extend the idea of forging connections into other virtual contexts. Specifically, we value and highlight the importance of creating an environment that recognizes the learner as a whole person with competing personal and professional priorities.

KEYWORDS

mathematics specialist, online learning, online community

Learning is an inherently social endeavor because knowledge is socially constructed (Vygotsky, 1978) and takes place within communities of practice (Wenger, 1998). These beliefs have long guided teachers in face-to-face settings and have more recently been considered in regard to online learning environments. As we examine community and connectedness in our synchronous virtual learning experiences, we draw from Swan's (2001) model that joins Community of Inquiry in online learning (Garrison et al., 1999; Rourke et al., 2001) with Moore's (1989) theory of interactions between learners, content, and the instructor. This model posits that online learners' experiences are products of interactions between learners, content, and the instructor, and that those interactions are influenced by teaching presence, cognitive presence, and social presence. Early research into online Communities of Inquiry focused on asynchronous learning experiences, and our work adds to the expanding literature base about synchronous learning (e.g., McDaniels et al., 2016; Brown, et al., 2016; Hjalmarson, 2017; Hoffman, 2019). In our references to community, we use Conrad's (2005) definition, which describes community as "a general sense of connection, belonging, and comfort that develops over time among members of a group who share purpose or commitment to a common goal" (p. 2).

Though students in synchronous courses meet and interact online in real-time, merely attending class sessions together does not ensure that students will have meaningful interactions with the content, instructor, or peers. Rather, research from asynchronous settings suggests that instructors and students both have agency in developing a sense of community within online courses (Arasaratnam-Smith & Northcote, 2017; Conrad, 2005). In synchronous online learning contexts, strong teaching presence can increase student engagement and sense of community. For example, to facilitate effective communication and connections between students, instructors can utilize and manage multiple modes of online communication, including the microphone and chat messages (Hoffman, 2019; McDaniels, et al., 2016). Also, Hjalmarson (2017) linked collaborative, authentic projects to students' sense of community. To date, there is a dearth of research from the student perspective about forming community in synchronous online learning.

Working loosely in the tradition of narrative inquiry, this paper captures a recent graduate's reflections on being a candidate in a synchronous online master's degree program for mathematics specialists. We highlight links between her experiences and instructors' intentional efforts to help students feel connected in a distance learning environment. The intent of our paper is for readers to experience "a vicarious testing of life possibilities…[and] a new sense of meaning and significance" (Clandinin & Connelly, 2000, p. 42) around creating community in online courses. Our narrative contributes to the literature base by exploring a student's perspective about forming community in synchronous online learning while illuminating her instructors' intentionality to facilitate such community.

Context

Mathematics specialists are professionals "with an advanced certification as a mathematics instructional leader or who works in such a leadership role" (McGatha & Rigelman, 2017, p. xiv). Their titles, roles, and responsibilities vary, but nonetheless, they consistently act as leaders within their unique contexts, advocating for productive mathematics teaching and learning (Fennell, et al., 2013; National Council of Teachers of Mathematics, 2014).

The Mathematics Specialist Leader (K–8) program at George Mason University was first established in 2005 as a traditional, face-to-face program, then, a few years later, transitioned to a hybrid model (virtual and face-to-face courses) as well as offering fully online programs through synchronous classes. Upon completion of this 10-course program, candidates obtain state licensure as a K–8 mathematics specialist in addition to a master's degree in Education Leadership with a concentration in Mathematics Specialist Leadership (K–8). The synchronous online courses are conducted through Blackboard Collaborate Ultra, which offers video, audio, screen, and text sharing, as well as small group formations in "breakout rooms" within a class session. During class, students also interact via shared Google presentation slides that function as media for both presentations and collaborative work, much like a whiteboard. Occasionally, candidates utilize their camera function to share work such as modeling with manipulatives, but the camera is not usually on during discussions.

Specialist candidates complete the ten courses in loosely formed cohorts. Based on course offerings each semester, most students follow a typical progression of course completion. However, students are not required to take courses in a lockstep order or begin and end the program on a rigid schedule. Though this is not a true cohort program, because of limited course offerings, many students complete most of the courses together.

A Recent Graduate: Beth

Beth attended the online synchronous classes from Bahrain, seven time zones ahead of her colleagues and instructors in Virginia. Because of the flexibility of being online and knowing colleagues who spoke highly of the program, Beth decided it would be worth the effort of completing the program from afar. She graduated in December 2019 and subsequently worked as a mathematics coach at an international school in the Middle East.

Beth's Reflections

To explore feelings of connection while in the program, Beth retrospectively wrote a series of reflective memos for this paper. In these memos she freely wrote but allowed the following questions to guide her:

- How, if at all, did I feel connected to my class peers?
- What experiences stick out to me most, across the coursework, as a time that I felt connected to my class peers?
- Why did it matter that I felt or wanted to feel connected to my class peers?
- How else did I feel connected to learning within this program?

Next, the first two authors holistically analyzed the memos for overall themes that best captured Beth's experiences. In reading her memos, we saw how connections with peers allowed Beth to take risks in her own learning, which supported her knowledge development. We share purposeful selections of key recollections from her memos, as well as perspectives of intentional design behind the learning experiences that Beth highlights across several different courses. We understand that Beth's experiences are unique, and we do not claim that they are representative of all program participants. Rather, we examine her experiences in the hope that "in-depth exploration of an individual life-in-context brings us that much closer to understanding the complexities of lives in communities" (Cole & Knowles, 2001, p. 11).

Feeling Connected to Class Peers

> I very clearly remember the first day (morning...2am in the morning) when I was sipping peppermint tea and nervously introducing myself to others via a Google Slide...It was my first time creating a Google Slide and using the various tools. I was nervous, but willing to take a risk. The community was supportive, and I was reminded that we are all there for the same reasons and the same goals. (B. Terry, personal communication)

Beth sat in front of her laptop, 7000 miles away from her instructor and peers. While the flexibility and relative convenience of the online program were appealing, the idea of isolation was a worry. Beth wondered, "Will I be successful in a program where I never see my class peers face-to-face? How will the instructors be able to support me from afar? Will I be motivated to attend class if I feel lonely?" Delightfully, from the onset of her first class, Beth began to connect to her class peers.

> I learned on that day that there were teachers who knew some of my former co-workers, that a cohort member was expecting her first child, that there were members who were new to coaching and some who have never coached, that others were tuning in from outside of Virginia and that we ALL had a passion for mathematics education. This initial assignment and opportunity to work with Google Slides was day 1 of our learning community and I couldn't wait for more. (B. Terry, personal communication)

Not only did the instructors intentionally design opportunities for candidates to connect initially within a course, but we planned for ongoing personal and professional connections. Similar to how a face-to-face class session may have small talk or informal discussions as learners enter a space, we created an intentional space and time for our candidates to share and connect each time they entered our virtual classrooms. Within our interactive class slides, we provided space for candidates to use a textbox and/or upload pictures to share updates and celebrations.

> Throughout each course, we shared personal stories about our families, our health, our fears, our successes and were also encouraged to share about professional moments. It was this intentional invitation to reach out to each other that allowed us to feel recognized as individual humans with life beyond Blackboard and Google Slides. (B. Terry, personal communication)

Through making such personal and professional connections, we believed that the candidates felt supported by one another. This support and trust enabled Beth and the other candidates to connect not only in a social sense but also as they developed their mathematical knowledge and leadership skills.

> With each small group discussion and collaborative assignment, I continued to grow in my own professional understanding while the personal connections continued to strengthen as well. As we continued to grow together and as our community was fostered by each other and the instructors, we became more comfortable in our abilities as leaders. By the end of our first two courses, we were also connecting, personally and professionally, with one another on social media. (B. Terry, personal communication)

Connections Allowed for Risk Taking

Because Beth felt connected to others in the class, she was willing to take risks in her learning. In instances when she lacked confidence to perform the mathematical task, she leapt towards opportunities to learn the mathematical perspectives alongside her trusted peers.

> In one particular course, we were asked to form small groups to learn about and develop materials to demonstrate a progression of a particular content area. Not always a risk-taker but always a learner, I chose to assign myself to a group focusing on proportional reasoning, an unfamiliar content area. (B. Terry, personal communication)

Candidates are often given choice when forming small groups within our learning community and work in both grade level groups and among peers with a broad range of backgrounds. In the example above, Beth chose to partner with peers for a learning project that had a different grade-level focus than her own. She was uncomfortable due to her unfamiliarity with the mathematical topics related to the middle grades but trusted her peers to include her in what their experiences had been. She knew that in taking this risk, even within her distanced community, her own learning would increase. By trusting her connections with her peers, she was able to safely explore a mathematical topic out of her comfort zone.

Even though the community provided trust through strong connections, there were times that Beth lacked confidence around her peers.

> In the third semester of the program, I was required to interview a student and select video clips to share with the whole class. I am admittedly nervous in situations like this and was not looking forward to sharing my video clip to my peers. (B. Terry, personal communication)

After Beth presented her video-clip, the connections between her and her class peers allowed for meaningful feedback and mutual respect through the vulnerability.

> Immediately upon receiving peer feedback and suggestions for improvement, I was reminded of the support and network related to this community of adults. (B. Terry, personal communication)

To align to practical experiences of mathematics specialists, instructors carefully planned assignments that often require shared video or a synchronous presentation among peers. Through such interactions, giving and receiving peer feedback—a practice common for mathematics specialists (Fennell et al., 2013)—was repeated and refined. Throughout the program, Beth was afforded the opportunity to learn from her trusted peers, working through feelings of vulnerability while also building confidence in a common practice of mathematics specialists.

Discussion

Isolated from her fellow mathematics specialist candidate peers through distance and time zones, Beth found community in an online synchronous program. By making personal connections, celebrating life experiences, and sharing a passion for mathematics education with her peers, Beth felt empowered to take risks and expose vulnerabilities in the learning community. The instructors within the program valued learning as a social construct and therefore designed opportunities for candidates to make ongoing personal and professional connections. In this program, participants and instructors each took responsibility for forming and sustaining the online community.

Returning to the online Communities of Inquiry model (Garrison et al., 1999), Beth's meaningful learning experiences can be framed by the productive interactions she had with peers, instructors, and the content. Building on initial social and interpersonal connections during the five semesters of coursework, Beth deepened her cognitive presence to build knowledge about mathematics content and best practices for coaches. This included risk-taking when working with course content and reflecting on her ongoing work as a coach.

The lessons learned in the program about the importance and potential benefits of connections between professionals can be expanded outward from university coursework. Mathematics specialists often try to form a learning community or Community of Practice with stakeholders in their school and local contexts. It is our hope that Beth and other students in our program will understand that beneficial risk-taking and meaningful knowledge building can occur when participants feel socially and academically connected to colleagues. We hope that mathematics specialists who complete our programs value the community we intentionally built and see the potential for building such community within their own practices.

Although the examples shared in this paper are one student's experiences in a specific context, we feel that mathematics leaders may be able to extend the idea of forging connections into other virtual contexts. Specifically, we value and highlight the importance of creating an environment that values the learner as a whole person with competing personal and professional priorities. We ask our candidates to share and celebrate throughout each of the courses, and we ask instructors also do the same. For Beth, the social presence of instructors and peers ultimately allowed her to take risks in her learning, thereby further developing our online community of inquiry.

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Instructor Perspectives: Transitioning from Face-to-Face to an Online or Hybrid Graduate Level Mathematics Education Course

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Abstract

In this paper, the authors reflect on their transitions from teaching a face-to-face mathematics education course for teachers to teaching using an online or hybrid model. As three veteran educators at two different universities, we share lessons learned in constructing and implementing an online or hybrid learning environment. For us, learning to be flexible in how students completed assignments was important. Although we faced many challenges, we looked at the experience through a novice learner's lens, and recognized that each of us grew from teaching these classes. We found that the instructors' experiences in working with mathematics specialist candidates in graduate courses are similar to the experiences of instructors teaching undergraduate level courses. Instructors' perceptions are important as universities seek to provide more online and hybrid programs.

Keywords

online learning, hybrid course, online course, mathematics education, mathematics teacher education

Transitioning from a face-to-face course to an online or hybrid course is challenging for instructors especially when they only have face-to-face teaching experiences to draw upon for course development and instruction. For the purposes of this paper, an online course will refer to a learning environment where the candidate and the instructor interact completely online. A hybrid course will refer to an environment where the candidate and instructor interact through a combination of online and face-to-face environments. It is important for online and hybrid courses to be comparable to their traditional counterparts in quality and content. The authors will share their personal learning experiences as they each designed and implemented a graduate course with the goal of maintaining the integrity of original face-to-face course designed prepare candidates to serve as mathematics specialists.

Literature Review

Due to their flexibility and accessibility, online programs have become common for in-service teachers. As they become more common, instructors need guidance on how to design and implement courses online. Bailey and Card (2006) draw upon Chickering and Gamerson's (1987) seven principles for high-quality post-secondary instruction and apply them to online learning. The principles are communicating high expectations, incorporating active learning, providing cooperative learning opportunities, emphasizing time on task, ensuring prompt feedback, maintaining frequent faculty and student interaction, and differentiating for diverse learning styles. Based on the personal experiences and perceptions of 15 online instructors, Bailey and Card (2006) identified eight high-quality online teaching principles. They are fostering relationships with students and between students, creating opportunities for student engagement, providing timely feedback and communication, being attentive to communication style and tone, creating a structured course organization, advocating for the use of technology, being flexible with students, and promoting high expectations. These principles are foundational for educators who are developing and implementing online courses.

Experienced online instructors have found that providing prompt feedback supports high levels of student motivation (Adelstein & Barbour, 2016; Grant & Thornton, 2007, Martin et al., 2019) and encourages student engagement both individually and in small groups (Grant & Thornton, 2007; Palloff & Pratt, 2005). Online learning instructors should provide "students the opportunity to work together to create knowledge and meaning, rather than providing facts and information that they memorize and retain in some fashion" (Palloff & Pratt, 2005, p. 126). Anderson (2004) states that motivation and engagement of students supports deep learning.

Developing a high quality online course that implements the best principles described by Bailey and Card (2006) can affect teacher motivation and perseverance. The time it takes to communicate and provide feedback can lead to instructor dissatisfaction with online environments (Bollinger & Wasilik, 2009; Cavanaugh, 2005). Bollinger and Wasilik (2009) connected many issues like these to the use of technology in an online class. Online instructors felt overwhelmed with getting all of the necessary work accomplished (Hogan & McKnight, 2007). The work of instructors outside of an online class can become overwhelming due to the pressure to be available to provide feedback and to answer student questions promptly. We provide insights to help future online graduate level instructors to persevere and find the ideal balance when teaching an online class.

Program Descriptions

Virginia Commonwealth University: Transitioning to an Online Synchronous Program

In 2017, Virginia Commonwealth University (VCU) transitioned from a three year, face-to-face mathematics specialist preparation program to a two-year, online program. Each course in the revised program consisted of pre-class work that was completed individually and in small groups prior to the bi-weekly, synchronous class meetings. Three courses in a sequence of mathematics teacher leadership courses were each taught and developed by a team of three instructors. Each instructional team consisted of faculty from the School of Education, the mathematics department, and a Virginia school division. The team developed the course syllabi, all course assignments, and weekly pre-class and in-class content. The leadership course sequence addressed mathematics content pedagogy, learning progressions, instructional design, student learning, and effective school-based mathematics leadership. The semester-long courses ran concurrently with the K-8 school year to allow for the implementation of course activities and assignments in the candidates' classrooms or schools. The experiences described by Instructor A and Instructor B below took place in the first and last courses in the leadership course sequence, Leadership I and Leadership III.

Longwood University: Transitioning to a Hybrid Course

In the spring of 2019, Longwood transitioned their mathematics teacher leadership program from a face-to-face format to a hybrid model of learning. The experiences described by Instructor C below took place during an eight-week Instructional Design course which included asynchronous, synchronous, and face-to-face instruction. The first four weeks consisted of weekly face-to-face meetings with participants meeting in two different locations. The instructor simultaneously taught 14 candidates in a face-to-face setting and seven candidates via teleconference. A practicing mathematics specialist supported the remote location. During the last four weeks, the candidates met twice synchronously. Coursework focused on problem solving and mathematics pedagogical content knowledge. Prior to each class meeting, candidates prepared for the meeting by completing reading assignments and participating in online discussions.

Instructor Reflections

Virginia Commonwealth University: Online Synchronous Reflections

Instructor A

As a twenty-plus year mathematics educator, I had taught several mathematics content courses in the mathematics specialist program before teaching Leadership I online. The objectives, goals, and assignments for Leadership I were developed as part of VCU's original face-to-face program, but the instructional team worked over the summer to modify the course to meet the online structure. A learning management system course shell had been created for the program to help standardize the class formats. Although everything appeared to be ready before the first class meeting, there were still many things that I felt I needed to learn about online teaching.

I believe learners should be actively engaged both individually and in collaboration with others by listening, observing, and talking with others, and by using manipulatives. I had a level of doubt in my ability to actively engage the candidates in an online class. By developing the pre-course and course work activities, I learned the importance of implementing different activities using online learning tools. These tools permitted active learning in a manner similar to a face-to-face course. The instructional team integrated voice recordings, virtual bulletin boards, discussion posts, virtual manipulatives, breakout groups, and videos into the pre-course and in-course activities. For example, a mathematics education theorist's project in Leadership I required groups of three candidates to research the learning theories associated with a given theorist and create a five-minute, narrated presentation to share with the other candidates. Participants posted a reflection on a course blog as to which theorists closely aligned with their mathematical teaching and learning beliefs. Previously this project culminated in a class-led presentation. The implementation of these different technology tools into the online classroom gave me access to new teaching modalities for active student engagement.

The necessity of regular communication with candidates and frequent feedback to candidates were components of the course that I reflected upon many times. The development of mathematical content and pedagogical skills was a set of building blocks that required frequent feedback for candidate growth. Several course assignments were embedded components of major course projects. A regular cycle of feedback allowed me to engage and monitor candidates as they grew in their pedagogical knowledge. Many times my feedback would receive a comment or a question from the candidate prompting a deeper conversation about their knowledge base.

For me, there never seemed to be enough time to plan, teach, reflect, and provide feedback in an online course. A statement in an email that I wrote to program coordinators expressing my concern for the time candidates would spend on course work stated, "I think the hardest thing for me to wrap my brain around is the amount [of work] they will need to do outside of meeting with us." If only I had known that the "they" in the email should have been a "we." Classroom discussions and activities that we had anticipated would take twenty minutes often took forty minutes. Questioning whether to end a rich discussion was a problem when considering all of the content that needed to be covered during a class session. Our instructional team developed a detailed structure for course meetings, but very rarely did we cover all content we intended to cover. Outside of class, I felt obligated to respond to emails promptly at all times even outside of work hours. With the candidates completing most of their studies in the evenings after the end of the school day, this meant many interactions were late at night or on weekends. Finding work, life, and family balance was difficult.

In a face-to-face class, I use a person's body language to inform pedagogical decisions. A student's gestures, posture, and facial expressions support me in knowing when to spend more time on a topic or move on. The lack of these visual cues in an online learning environment was a struggle for me. The inability to read body language meant many times I was not aware of a misconception until it was voiced in class or shared in an email. I provided time for questions during each class, but I felt that I missed other concerns or questions that may have been more evident in a face-to-face environment.

Instructor B

While I had taught many courses for in-service teachers in the past, I had never taught an online course. Team teaching and using technology were not difficult for me, but interacting with the candidates who I had only met once, briefly, was an adjustment for me as an instructor. My first experience with the candidates was as an instructor for Leadership III. While I took cues from a co-instructor who had taught the candidates in the prior courses, I had to figure out how to lead activities and encourage online discussions. I found it challenging to adapt the aspects of face-to-face assignments that required candidates to be in close proximity to an online environment.

Transferring a face-to-face course to an online format has its challenges. It can be difficult to decide whether assignments and projects will be effective in an online environment. For two assignments, a school-based data project and a lesson study project, I believe our instructional team had achieved success through the revisions we made to these projects as we developed the course. By integrating the appropriate technology, providing clear expectations, and utilizing collaborative learning, the projects were incredibly worthwhile and successful learning experiences for the candidates.

The school-based data project required students to simulate engagement with data to identify and investigate a learning problem in their school. The project consisted of four clearly described steps completed over four weeks. Each week we checked in with the candidates as they progressed through the project. For many candidates this was their first time of looking carefully at testing data and it was overwhelming. The online environment made this project easier to facilitate since we were able to use online tools explore the data together. The candidates were comfortable using technology to gather and present data because of their previous online course experiences. After the course was over, the candidates identified the school-based data project as being beneficial in empowering them to collect, present, and discuss testing data with their administration.

The lesson study project required a small group of candidates to complete a Lesson Study Cycle (Wang-Iverson & Yoshida, 2005) including planning, delivering, and reflecting to achieve the goal of perfecting a single mathematics lesson. The logistics of implementation were challenging since the candidates lived in diverse geographical locations. This made it difficult for a group to travel to different schools to observe a lesson. To make this project a success, flexibility on my part, as the course instructor, was essential. I facilitated candidates in creating lesson study groups that were either cohort-based or school-based. In the latter case, groups included teachers in a candidate's school or district who were not taking Leadership III.

Everyone was successful in completing the lesson study project in a way that worked best for their particular circumstances. Interestingly, groups formed exclusively from cohort members were the most successful. I believe this was because they were all aware of every aspect of the project and clearly understood their roles in the Lesson Study Cycle. Those that chose to do the project in school-based groups had to do more organizational work than those that worked in cohort-based groups. In school-based groups, the candidates had to find teachers willing to participate, explain the Lesson Study Cycle, and make sure that everyone provided feedback to synthesize and analyze the success of the lesson. While the school-based lesson study groups reflected more on logistical issues, the cohort-based groups reflected more on the lesson study process. Though the two different types of groups had varying degrees of success in the lesson study process, providing flexibility in implementation of the project and recognizing the differences in group structures was a successful learning experience for me.

At VCU, one instructor led each instructional team for all three courses in the mathematics teacher leadership course sequence. Through their prior work in mathematics and leadership courses, a community of learners had been created in the online learning environment. Being the new instructor during the last leadership course in the sequence meant I had to figure out my role within the cohort. I struggled with learning the class "climate" quickly enough to not have to be dependent on another instructor's opinion of how to maximize everyone's strengths. I struggled to find my niche within the learning community. In a face-to-face class, I could have walked around during class to gauge how students worked and interacted with each other. This was not possible in the online class.

Longwood University: Hybrid Course Reflection

Instructor C

Though I have taught for almost 30 years with 15 of those teaching pre-service and practicing teachers, this was my first experience teaching a hybrid course. The course I taught was developed using a face-to-face design, but there were issues for me with transitioning to the hybrid format. My struggles and successes focused on the implementation of the course.

I knew that switching to a hybrid learning environment would be a challenge for me. To help with this transition I implemented the ideas shared with me in a university program for switching to online course modalities. Due to the flexibility of course delivery with a hybrid model, I had three times the number of candidates when compared to prior face-to-face cohorts. As the only instructor for the course, I was concerned with how I could provide the necessary feedback for candidates in a condensed eight-week semester.

Being able to balance the amount of feedback I wanted to provide the candidates with the number of hours it took to read and grade assignments was very difficult. I wanted to assign frequent activities for candidates to assimilate their learning but this would create an increase in the number of hours I would spend grading. I recognized that feedback and assessment could take on many different forms in an online learning environment. For me this meant that I did not need to grade every activity; instead, I needed to use the online tools that were available to me. Feedback could come from more than just me. Feedback would also come from the candidates themselves.

Discussion boards were an important component of weekly feedback from the instructor and candidate peers. The integration of discussion posts and feedback from peers can empower candidates to realize that their thoughts are valuable as leaders in mathematics education. Initially, candidates were limited to 300 words per post, but after reflection, I began to require candidates to post a two-minute video discussion. I found that candidates were more organized and concise in their video posts. I believe the videos played to the candidates' strengths as versatile presenters due to their teaching backgrounds. Video discussion posts were easy for me to view to assess a candidate's content knowledge each week. They were also easily accessible so discussions could occur either in or out of class. The video discussion posts were a successful way to balance the time needed to assess student understanding with the number of assignments candidates completed. Even with incorporating the changes to the discussion post, finding balance in the number and type of assignments was important to being able to provide useful feedback on every assignment.

Being a hybrid course, creating an online community with candidates in two different locations was difficult due to connectivity issues with technology. The frustration felt by everyone in the class, including me, made it hard for us all to develop relationships with each other. While teaching, I was often unaware that the remote location technology was not working since I only had one computer screen and I was unable to see the remote site when I was presenting. Overcoming these issues required switching online meeting platforms from WebEx to Zoom and incorporating a remote site facilitator to increase student engagement and to address technical issues. Our re-envisioned program did not initially include the remote facilitator, but this role was deemed necessary for future courses. I added a second screen at my teaching location so everyone could see the slides and I could observe the remote location. A willingness to adapt and try new things was important for me when transitioning to a hybrid course.

Conclusion

Our experiences varied as we each transitioned to an online learning platform. We were each given a course template and goals from a prior face-to-face program, but making everything work within the confines of our different online situations was tough. Maintaining the integrity of the courses was important when transferring them online. Finding the perfect balance for each instructor was difficult.

As instructors transitioning to an online learning environment, our lessons learned were tied to the effective practices for implementing and teaching online courses as described by Bailey and Card (2006). We each recognize the importance of setting clear goals and building a community of learners, but these things can be difficult when time is a factor. Managing our time and the time of the candidates meant that we had to be deliberate and flexible in the opportunities provided for candidate engagement. Learning how to fit the assignments into the new online format required careful planning. We each had success with using technology in course design and the implementation of online modalities. Incorporating these modalities did not change our instructional beliefs or practices, but allowed us to consider different ways to modify our teaching and the candidates' learning experiences.

There are things, such as the number or format of assignments, which can be issues when transitioning a course to an online format. Successful online instruction requires flexibility or the implementation of new technologies. Finding the time to plan, provide feedback, and engage candidates in their learning is imperative for student success. Careful consideration of best practices for online instruction is essential as one develops and implements an online course. In each of our reflections, we recognize that what works can also be what challenges us. Our experiences as instructors at the graduate level parallel the literature research at the undergraduate level in how best practices can affect instructor effectiveness and motivation (Anderson, 2004; Bailey & Card, 2006; Hogan & McKnight, 2007).

Learning to adapt to an online learning environment and finding ways to integrate the technology can greatly influence the development and implementation of any course. Comparing and contrasting our situations, one thing that stands out to us is that having more than one instructor for a course and having the support of fellow online instructors is valuable. Talking to others provides a sounding board for personal growth and reflection. Teaching online is a cycle of planning, teaching, reflecting, and revising. This process is about growing as an educator and recognizing that not all online situations are the same. For us, teaching online did not mean that we had to change our pedagogical beliefs but instead demonstrated that perseverance was important in each of our situations and will be important as we continue to revise how we teach online.

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TRANSITIONING A MATHEMATICS SPECIALIST PREPARATION PROGRAM INTO AN INTERACTIVE ONLINE PROGRAM: INSIGHTS FROM THE DEVELOPER AND CANDIDATE PERSPECTIVES

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ABSTRACT
This paper will describe how the Virginia Commonwealth University mathematics specialist preparation program transitioned from a face-to-face format to a fully online format. We will describe the technology and instructional methods that are used for course meetings, activities, and assignments. We will describe the development and implementation of a mathematics activity from instructors designing the activity, participants completing the activity, and instructors providing feedback on the activity. One mathematics activity will be shared that demonstrates the process of the program model that includes independent work, small group work, and in-class discussions. We will describe one participant's experience as she developed and implemented a capstone externship project in the preparation program and how activities in this program inspired her classroom practice more broadly.

KEYWORDS
mathematics teacher leaders, mathematics specialists, professional development, online learning, technology enhanced instruction

Professional development and career advancement programs for teachers come in many different forms. In the 21st century, online digital platforms play a significant role in the delivery and implementation of these kinds of experiences. In addition to providing a wider variety of opportunities, online learning also allows participants to learn in the environment in which they are most comfortable and at times that are convenient for them, which are important factors when designing programs for adult learners (Knowles et al., 2015). A flipped classroom is an instructional model that provides participants with opportunities to work on course content on their own time prior to engaging in student-centered, in-class experiences (Stapleton, 2020), allowing for more dynamic interaction through small group and whole class activities. A community of inquiry (Garrison et al., 1999) is a social constructivist learning experience model for online instruction that emphasizes the importance of establishing cognitive, social, and teaching presence to ensure that all participants have a satisfying and meaningful learning experience.

This paper describes the transition of a mathematics specialist preparation program from a face-to-face format into an online format that incorporates a flipped classroom model and utilizes a community of inquiry. We share information about the program including a sample mathematics activity and the program's capstone experience. We describe the experiences of one candidate in the program including her in-school externship project. We begin by briefly describing the history of the mathematics specialist movement in Virginia and the origins of the preparation program featured in this paper.

Early Statewide Work on Mathematics Specialist Preparation

Under the leadership of the Virginia Mathematics and Science Coalition (VMSC), in 2002 a new approach to K–8 mathematics teacher leadership began to emerge in Virginia and was soon followed by recommendations for the specialized preparation individuals should complete before assuming the leadership role. Over several years through the work of two different statewide working groups comprised of school district mathematics supervisors, K–8 teachers, and higher education faculty, the role of the elementary mathematics specialist was first defined and then refined to also include the unique demands of middle school mathematics education. During this time, the Virginia Commonwealth University (VCU) Mathematics Outreach office under the leadership of Dr. Bill Haver and Dr. Reuben Farley received a series of four large-scale National Science Foundation (NSF) grants to develop a mathematics specialist preparation program and study the impact of mathematics teacher leadership in Virginia's K–8 schools. The courses were developed and offered in face-to-face formats through several state grants and the first in the series of NSF grants. This work was a collaborative effort of four institutions of higher education and 45 urban and rural school districts in Virginia. With the support of the next three NSF grants, the courses were refined and adapted as the program was completed by several cohorts of teachers across the state. Almost all of these initial course offerings took place in face-to-face formats including 5-week summer residency programs, 2-week intensive summer courses, and semester courses. There was one notable exception in which a cohort of teachers in rural districts completed the program in a blended format including both online and in-person components. More information about the foundational work that led to the mathematics specialist movement in Virginia and early efforts to prepare mathematics specialists can be found in VMSC (2016).

This work culminated in a teaching license endorsement for mathematics specialists and a rich and rigorous program to prepare generalist teachers for this leadership role. In these early years, twelve state universities established master's degree programs to prepare K–8 teachers to be mathematics specialists. The Virginia Board of Education also realized the importance of the mathematics specialist role to K–8 education and recommended one specialist for every 1,000 students.

In 2017, the VCU Mathematics Outreach office received an NSF Noyce Teacher Scholarship Program grant to modify the existing VCU face-to-face mathematics specialist preparation program into a fully online professional development and certification program and to enroll a cohort of teachers serving in high-need school districts across the state. In addition, program graduates served for three years as mathematics teacher leaders in their school districts.

VCU Mathematics Specialist Preparation Program

Through the statewide work described above, VCU developed a 36-hour master's degree program consisting of (a) six core mathematics courses designed so candidates develop a deep understanding of the K–8 mathematics content; (b) three mathematics education leadership courses in which candidates develop the skills necessary to work with all members of the educational team (i.e., teachers, principals, parents, children, central office personnel, members of the community, etc.) and, most especially, work with adults; and (c) a capstone experience in the form of a two semester externship during which candidates design and implement a research-based, in-school project using the knowledge and skills they acquired through the prior course work. Activities and assignments throughout the program target specific areas of need for mathematics specialists including (a) advanced middle school mathematics content; (b) methods for helping teachers work with diverse populations of students (i.e., English language learners, gifted students, students with learning disabilities, etc.); and (c) analysis and implementation of the current trends in mathematics education research.

The courses in the program directly align with the standards set forth by the Conference Board of Mathematical Sciences (CBMS, 2010), the Association of Mathematics Teacher Educators (AMTE, 2013), the National Council of Teachers of Mathematics (NCTM, 2012), and the Teacher Leader Exploratory Consortium (TLEC, 2008). The mathematics courses provide an in-depth study of the content covered by the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Specific details about each of the courses in the program can be found in VMSC (2016).

The instructional team for each mathematics course consists of a VCU mathematician or mathematics educator and an experienced mathematics specialist. Candidates conduct an in-depth study of K–8 mathematics and also connect the concepts and skills to teacher practice. Assignments and activities also allow candidates to make connections to higher-level mathematics. Throughout the courses, candidates engage in making conjectures, developing generalizations, and making mathematical arguments in order to deepen their understanding of the content. In particular, they build a working knowledge of the properties of arithmetic, proportional relationships, geometry, algebra, probability, and statistics. Class time is spent in small group and whole group discussions anchored in written and video case studies of children's mathematics thinking; cooperative group work around mathematics content and pedagogy; and analyzing children's work, including case studies from candidates' practice.

The instructional team for each mathematics education leadership course consists of a mathematics educator and a district mathematics supervisor or an experienced mathematics specialist. Candidates conduct an in-depth analysis of their own teaching and learn to coach one-on-one and in small groups. The leadership courses develop candidates' skills related to working with adult learners and deepen their mathematics content and pedagogical content knowledge as they refine their philosophy about mathematics teaching and learning. The courses are interactive and project based. Discussions often begin with individual reflection, followed by pair conversations, expanding to small group and then whole group sharing.

Transition from Face-to-Face to Online

Our goal was to convert the existing mathematics specialist preparation program into an online program while maintaining the rigorous content and interactive nature of the activities and assignments in each of the courses. The online version of the program consists of technology-enhanced active learning mathematics and mathematics education leadership courses and employs a modified flipped classroom model to provide an accessible and interactive learning environment for candidates. The instructional model is described in more detail below.

The transition process took place over three summers and two academic years. Two separate course redesign teams, one team for mathematics courses and one for mathematics education leadership courses, worked to transition the courses to an online format. The team evaluated the existing content and pedagogical strategies; explored online learning technologies for delivering material, facilitating discussions, and completing activities; and made the necessary revisions to all aspects of the courses being taught during that year. The conversion was an iterative process. In addition to receiving guidance from course designers with extensive experience in making this type of transition and who specialize in online program development, the redesign teams considered the feedback gathered from candidates and instructors during the first year when selecting instructional tools and developing activities and assignments for the courses taking place during the second year.

Online Program Structure

While each course covers different mathematics and mathematics education content and has different requirements, all of the courses in the program use similar methods for content delivery and student preparation for whole class synchronous meetings. Each course has synchronous and asynchronous components. The amount of time spent in synchronous whole class meetings is significantly less than a traditional face-to-face class. Information is organized sequentially in the online course management system according to each synchronous whole class meeting, called course sessions. Each session contains 4–8 hours of prerequisite work for candidates to complete prior to the synchronous meeting.

Prerequisite activities are carefully sequenced so that candidates can complete the activities independently or in small groups without instructor support. Activities are grounded in the principles of a community of inquiry (Garrison et al., 1999). A significant aspect of teaching presence is designing and facilitating educational experiences. While facilitation is primarily a role for course instructors, each course includes opportunities for candidates to assume the role of facilitator, with increased responsibility in later courses in the program. All readings and activities include prompts to help candidates initiate cognitive presence to explore ideas and concepts. Social presence is purposefully integrated in all sessions through small group collaboration and providing an open, judgment-free environment for small group and whole group discussions.

They begin their preparation by completing individual activities including reading case studies about children doing mathematics, writing responses to posed questions, and completing mathematics activities. The class is divided into small groups each one consisting of 3–4 cohort candidates. Each group meets online (through Blackboard Collaborate, Zoom, etc.) at least once a week at a time that fits the schedules of all group members to discuss activities, share ideas, and complete additional small group activities. Individual and group responses to all prerequisite work are uploaded into an online course management tool (e.g., Blackboard or Padlet) for easy access by everyone in the course. Before each synchronous session, the instructional team reads through the candidates' responses to prerequisite assignments and activities to gain insight into candidates' understanding of the concepts and to find ideas that should be reinforced or misconceptions that need to be addressed. In addition to addressing any pressing issues, time during the whole class meeting is spent working on activities that reinforce the most important concepts studied during that course session. An outline of prerequisite work for a sample Rational Numbers and Proportional Reasoning course session appears in Figure 1 below.

Figure 1
Rational Numbers and Proportional Reasoning Prerequisite Work Outline

1. Revisit Math Activity 1.3 **Interpretations of 3/4** in the Concurrent Work: Session 1 folder. Make note of any adjustments you would make to your sort based on the 5 ways to interpret a/b: (1) part-whole comparison, (2) quotient, (3) measure, (4) operator, (5) ratio/rate.
2. For Math Activity 2.1, read the handout, *Some Thoughts on the History of Mathematics* and individually answer the questions for addition and subtraction and solve the problems for multiplication and division.
3. In small groups, complete Math Activity 2.2, Lamon, Chapter 6, p. 143 problem 5 parts a, b, c & d. For each part, give a brief explanation for what happens when the fraction is changed using the specified conditions. Provide an example to support your reasoning.
4. Read YMW Chapter 6, pp. 92 - 107. Individually, develop responses to the Focus Questions. In small groups, discuss your responses to the questions. Be prepared to share your ideas during our next class meeting.
5. On YMW page 95, Fosnot and Dolk paraphrase a statement from Liping Ma "One could argue that if we taught the algorithms conceptually, more understanding would develop." The authors then pose several questions. In your small groups, in 3-4 sentences, craft a statement that addresses one of the authors' questions (see below).
   Groups Un & Deux: Should the algorithm be the goal of computational instruction?
   Groups Trois & Quatre: In today's world, do we want learners to rely on paper and pencil?
   Groups Cinq & Six: Is the algorithm the fastest, most efficient way to compute?
   Groups Sept & Huit: When are the algorithms helpful? When does one pull out a calculator?

Note: Lamon refers to the book *Teaching Fractions and Ratios for Understanding*, 3rd Ed. YMW refers to the book *Young Mathematicians at Work: Constructing Fractions, Decimals, and Percents*.

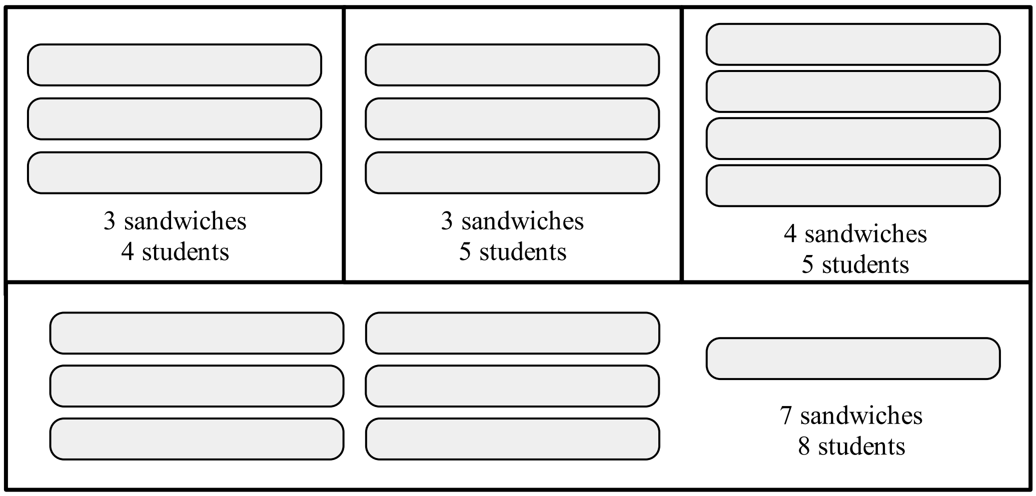
To illustrate the course session structure used throughout the program, we share a mathematics activity from one of the courses and one candidate's experience with completing the activity as part of course session prerequisite work and working with a small group of candidates to develop a deep understanding of the mathematics under study.

Sample Mathematics Activity

Rational Numbers and Proportional Reasoning is typically the third mathematics course candidates complete in the preparation program. The course begins with the following rich task:

> A class of fifth graders go on a field trip. They split into four groups and go to four different locations. Each group takes a number of submarine sandwiches with them. In the picture below, you can see the number of sandwiches each group received and the number of students in the group. Is this distribution of sandwiches fair? Why or why not? Solve the problem using a representation. Explain the reasoning behind your solution strategy without using a standard algorithm.

Figure 2
Fifth-Grade Field Trip Submarine Sandwich Distribution



Purpose of the activity

This problem, adapted from Fosnot and Dolk (2002), provides candidates with the opportunity to explore fair sharing, equivalence, and other proportional relationships. Candidates begin by solving the problem individually. They are encouraged to try multiple approaches to find a solution and to draw pictures as they explore different solution strategies. Then, in small groups, they share their strategies and develop one solution to post in an online course management tool. After they have reasoned through the problem for themselves, they read a chapter in Fosnot and Dolk which presents a case study of children exploring fair sharing and equivalence as they also solve the problem in small groups and then share their ideas and strategies with the whole class.

This problem was chosen to start this course because it is a rich task based on concepts that are frequently revisited throughout the course. For example, in addition to fair shares and equivalence, other concepts that are explored are common denominators, common fractions, and the connections between fractions, division, and multiplication. Also, by delving into the Fosnot and Dolk (2002) case study, candidates have the opportunity to discuss children's thinking about proportional relationships and ways to engage children in working through and discussing rich tasks like this problem. One candidate's experience with this activity is described in Figure 3.

To many, this problem may be a simple one exploring ideas of fair sharing or division. This is how some program candidates saw the problem at first glance. But like many rich mathematics tasks, the exploration can go much deeper than the concepts that are obvious on the surface. By studying the work that other groups posted to the course management system, candidates saw new ways to solve the problem that they would not have thought of otherwise. During the whole class discussion, the class explored other big ideas like fraction equivalence and comparison. This problem also provided a strong foundation for exploring other rational numbers concepts and proportional reasoning strategies in subsequent course sessions.

Figure 3
A Candidate's Experience with Completing the Submarine Sandwich Activity

As a middle school teacher, I often solved the mathematics activities throughout the courses with strategies I use with my students. My approach to this problem was no different. I started with a visual representation of each set of sandwiches (see Figure 2) and divided each sandwich into pieces based on the total number of students in the group. Every student could be given a piece from each sandwich and I could determine how much each student received by combining the unit fractions. Lastly, by comparing each fraction, I determined that across the groups, students received different fractional parts of a sandwich, thus it was not a fair distribution. After solving the problem one way, I would explore other ways to solve the problem, asking myself, "how might other students, younger or older, try to solve this problem?" This was often a challenge for me, thus working with a small group of candidates was essential to explore different ways of thinking and to deepen my understanding. Solving this problem involves ideas of fair share, division, equivalent fractions, and proportional relationships. It could be solved with manipulatives, models, or traditional algorithms. The rich and meaningful connections became more clear through the small group and whole class discussions.

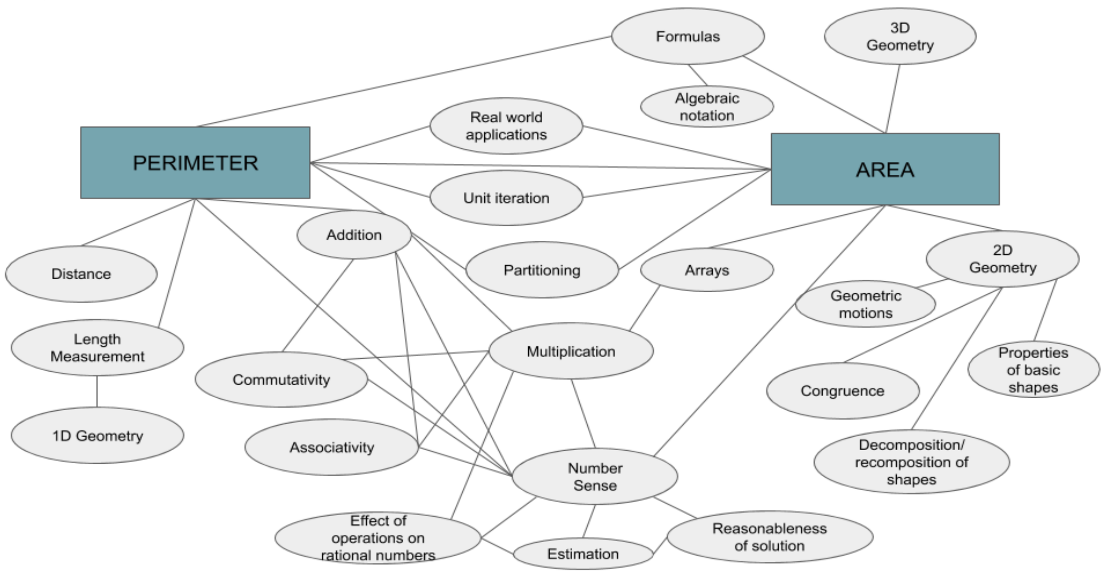
Externship

The externship is a two-semester capstone experience in which candidates have the opportunity to integrate and apply what they have learned throughout the program through a practical, school-based project. An overarching goal is for candidates to further develop the skills and practices of a reflective practitioner by grounding their project goals in the appropriate mathematics education research, revising the project as needed, and using data they collected during project implementation to analyze and reflect on the entire process, including the project's impact on the participating teachers or students. During the first semester, through a series of course sessions consisting of prerequisite work and small-group and whole-class discussions, as well as individual consultations with a course instructor, candidates develop an action research project and write a detailed proposal for the project. They then implement the project during the second semester and write a detailed final report. Information about the various components of the externship and one candidate's externship experience are presented below.

Proposal Writing

The first semester begins with a prerequisite activity in which candidates develop a knowledge package for one of the mathematics concepts presented in Liping Ma's (2010) book detailing differences between elementary school teachers' understandings of mathematics in the United States and China. A knowledge package is a way of thinking about a mathematical topic as "group-by-group rather than piece-by-piece" (Ma, 2010, p. 18). Creating a knowledge package requires an understanding of what ideas and procedures support the mathematical topic of focus. Figure 4 shows an example of a knowledge package. The rectangles contain the mathematical topics of focus, and the supporting ideas and procedures are shown in ovals with lines representing the connections between concepts.

Figure 4
Knowledge Package for Linear and Area Measurement



As the semester progresses, candidates spend time developing a knowledge package for the mathematics concepts featured in their project as well as conducting an in-depth review of the content and pedagogy literature for their topic. Based on this foundational work, candidates develop a detailed project proposal. They develop professional and pedagogical goals as well as a set of guiding questions to be answered by completing the project. The goals, supported by the literature review, provide a framework for both the mathematical and pedagogical work to occur during the implementation. The detailed implementation plan includes a description of the setting of where the project will occur as well as a list of daily learning objectives and activities. Candidates spend at least 120 hours preparing lessons and activities, analyzing data, and developing the final report. The proposal includes a timeline which outlines how each hour will be spent, spanning several months from preparation to final reflection. The plan includes research-based tools for evaluating the project outcomes and data analysis. Sample lesson plans are included as appendices to the proposal. The complete proposal is approved by the externship supervisor before the project is implemented.

Proposal Implementation

Early in the second semester, candidates make any final revisions to the proposal and prepare for implementation. They share the proposal with the school building principal or district supervisor and receive feedback on the plan. Each candidate is assigned a university supervisor who oversees and evaluates the externship. The candidates meet in an online whole group session to share their proposal goals and project questions and briefly outline their plans for implementation. Throughout the implementation and data analysis phases, each candidate participates in a series of online check-in meetings with their university supervisor to discuss their progress and get feedback. The number and frequency of meetings depends on the nature of the project. At the end of the semester, candidates meet online as a whole group to share their project results through a 20-minute presentation and answer questions about the project and receive feedback from the group. One candidate's experience developing and implementing the externship is described in Figure 5.

Figure 5
A Candidate's Externship Experience

The focus of my externship project was on teaching concepts of linear and area measurement through the math workshop model of instruction. Math workshop consists of three main components: number sense routines, mathematical tasks, and small group learning. This project took place in my sixth grade classroom with a high population of English Learners. The use of math workshop provides an opportunity for greater differentiation and the ability to support all students at their diverse levels of mathematical understanding. I first developed the knowledge package for the key area and perimeter concepts (see Figure 4). This helped me to better understand the necessary background knowledge for students to deeply understand these concepts. Using the knowledge package as a resource, I created a pre and posttest to help me determine the students' current level of understanding and to show their growth after instruction.

Students who struggle mathematically often rely heavily on formulas in the study of geometry and do not develop a deep understanding of the concepts. My goals for the project were to develop number sense routines, mathematical tasks, and small group learning activities that would assist students in building a conceptual understanding of area and perimeter without emphasizing the traditional algorithms. Using the results of the pretest, I created small group mathematics activities to help students progress through the knowledge package.

The instruction took place over seven days and covered three main concepts: perimeter, area of squares and rectangles, and area of triangles. The routines for each day were largely the same. Each class started with a number sense routine focused on foundational ideas of the day's work. A mathematical task was used to introduce the big ideas of each new concept, followed by small group learning and independent and partner practice. The results of this project revealed that math workshop is an effective way for a diverse group of students to learn the concepts of area and perimeter without relying on the use of formulas when problem solving.

This externship experience was a culmination of all I had learned through the preparation program. Each mathematics course pushed me to dig deeper to understand how different mathematical concepts build and connect with one another. I put my learning into action as I developed rich mathematical tasks, number sense routines, and assessments rooted in the knowledge package for linear and area measurement. Through what I learned in our education leadership courses I was prepared to support a diverse group of students, determining where they were in the knowledge package and providing them with appropriate and meaningful work to help them further develop their understanding. In addition to what I learned in this program, the relationships I developed with both candidates and professors were essential to my professional growth.

Conclusion

The VCU mathematics specialist preparation program was successfully transformed to an online format utilizing interactive and collaborative learning experiences. All 26 members of the cohort who participated in the initial implementation of the online version of the program successfully completed the program. Candidates expressed satisfaction with their experiences in completing the course work and their preparation to serve as mathematics teacher leaders. Many individuals stated that the online learning experience was better than they could have ever imagined.

The online model included all of the activities and assignments that had been developed for the face-to-face mathematics specialist preparation model and met all of the requirements for Virginia's K–8 mathematics specialist add-on endorsement. Based on the principles of a community of inquiry (Garrison et al., 1999) but, most notably, social presence (e.g., open communication, group collaboration, and bonding), technology virtually connected a group of teachers from across the state and helped the cohort to develop and grow into a tight-knit professional learning community. The flipped classroom model including independent work, small group activities, and whole class discussions helped candidates to explore concepts and ideas in a variety of meaningful ways. The externship allowed the candidates to put what they learned into practice. The result of this experience was a new cohort of strong mathematics teacher leaders across the state, who are prepared to coach and mentor other teachers in mathematics content and pedagogical best practices. While online best practices were not explicitly taught during this program, they were constantly modeled during each course. Therefore, an unexpected but important by-product of this experience is that the candidates were fully prepared to transition to online instruction in their schools when schools unexpectedly had to close in March of 2020 due to the COVID-19 pandemic. This model of professional development was extremely successful and is replicable with other programs and in other contexts.

Acknowledgment

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DEVELOPING EQUITY-CENTERED LEADERSHIP KNOWLEDGE AND SKILLS VIA LESSON STUDY IN AN ONLINE MATHEMATICS SPECIALIST PROGRAM

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ABSTRACT
This paper highlights how coursework within a synchronous online mathematics specialist program enhanced candidates' leadership knowledge and provided structures that addressed issues of equity and access. A focus on one online assignment grounded in Lesson Study played a pivotal role in developing equity-centered leadership and instructional practices. Program instructors and recent alumni illuminate how designing, implementing, and reflecting on the Lesson Study experience served as a cornerstone for advancing their mathematics instruction in the following ways: (a) as instructors designing an online leadership course, (b) as learners within an online environment, and (c) as educators within their K–8 school settings. The description of these experiences supports the broader mathematics education community's goal of achieving a cohesive vision for the teaching and learning of mathematics, while promoting equitable practices in school-based work.

KEYWORDS
mathematics specialist, teacher leader, online learning, Lesson Study, equity, agency

Educators across the nation are working towards the creation of equitable mathematics instruction in which every child has access to a "powerful learning environment" (National Council of Teachers of Mathematics [NCTM], 2020, p. 1). Equitable mathematics instruction should be designed in a manner that promotes "access and attainment for all students" (NCTM, 2000, p.12). It should be responsive to individual student needs and integrate student background knowledge and experiences (NCTM, 2020; NCTM, 2014) so that students can develop mathematical agency and actively demonstrate the knowledge and skills they possess instead of passively receiving information (Berry, 2016; Lawler, 2012). However, to achieve this vision, a transformation of our current instructional practices is essential, and systematic support is required (National Council of Supervisors of Mathematics & TODOS, 2016). Mathematics specialists are one way in which this reconceptualization can occur.

The Mason Mathematics Specialist Program

Over the past 15 years, it has been the goal of George Mason University's (Mason's) Mathematics Specialist Program to support the initial preparation and professional development of mathematics specialists. As the program transitioned to a fully synchronous online program, multiple benefits have emerged. For instance, as Mason's program has expanded beyond Virginia into other states along the East Coast and across international boundaries, the candidate population has diversified, leading to an increase in divergent thinking. Additionally, the online platform has amplified shared ownership in our learning communities and has become a model for enhancing instructional opportunities via technology (Baker & Hjalmarson, 2019). However, most importantly, candidates have implemented equity-centered instructional shifts within their K–8 school settings. This paper speaks to one key assignment, which serves as such a model, the Online Lesson Study Assignment.

The Online Lesson Study Assignment: Instructors' Perspective

While there are many variations of lesson study throughout the United States, when designing the original Lesson Study Assignment, Mason instructors drew upon the Lewis and Hurd (2011) model. This model allowed for developing a professional learning community that values participants, emphasizing research-informed teaching and responding to individual student needs by integrating student background knowledge and experiences. In the transition to a synchronous online format, instructors were able to uphold and enhance the lesson study components we valued while allowing the candidates to implement these lessons in a face-to-face context. Candidates continued to create task-based lesson plans that promoted engaging students in learning and doing mathematics aligned with national and state standards (NCTM, 2000; National Governor's Association Center & Council of Chief State School Officers, 2010; Virginia Department of Education, 2016). However, the work of examining learning progressions and research-informed curricular resources now occurred in virtual breakout rooms using interactive tools such as Google Docs and Google Slides. Instructors integrated video recordings of the implemented lessons into the assignment so that lesson study teams could collaboratively reflect on and adapt the lesson for future implementations. Furthermore, Courtney and Spencer [instructors/authors] increased the emphasis on equitable instruction (NCTM, 2014, 2020) by asking candidates to utilize resources and strategies to promote students' mathematical agency. Designing online, collaborative, research-informed lessons centered on equitable teaching afforded candidates multiple opportunities to ensure all students have equitable access to powerful mathematical learning while developing valuable leadership skills required to shift mathematics instructional practices.

Candidates' Online Experiences & Perspectives: Adrienne, Alyson, and Scarlett's Stories

In the following sections, three Mathematics Specialist Program alumni, Adrienne, Alyson, and Scarlett, describe their experiences with the Online Lesson Study Assignment and how the assignment influenced their teaching and leadership practice. The themes common across their learning and design experiences are presented first, followed by stories of their unique implementation experiences (see Table 1).

Table 1
Candidates' Online Lesson Study Assignment Implementation Summaries

Candidate	Lesson title	Grade(s)	Mathematics topic	Research goals
Adrienne	"Array-bow of Color"	4th & 5th	Multiplication within 1000	Explore strategies and representations
Alyson	Promoting problem solving	4th & 5th	Decimals and decimal operations	Integrate multiple representations
Scarlett	Inventive strategies for subtraction	4th	Subtraction word problems	Examine students' conceptualization

Designing Equitable In-Person Learning Experiences in a Synchronous Online Cohort

Even though we (Adrienne, Alyson, and Scarlett [alumni/authors]) participated in different lesson study groups, we each worked "to challenge and build one another's knowledge of subject matter and of student thinking" (Lewis & Hurd, 2011, p. 3). We participated in several preliminary discussions and activities to develop respectful and collaborative relationships. We established group norms and roles, shared our individual mathematics instructional goals, listened to each other's project ideas, and wrestled with how we could address our individual goals while also meeting the group's needs.

Following the requirements of the Online Lesson Study Assignment, we discussed our different instructional styles and classroom experiences, which provided insight into who we are as educators and how to meet students' instructional needs during the collaborative lesson. We ensured each group member felt heard and valued through our lesson study log, to which we all had online editing access. This afforded us the ability to record our thoughts, questions, sources, and lesson ideas in real time. By identifying commonalities and sharing strengths and weaknesses, we established a level of accountability and commitment to the lesson study cycle process.

Our prior knowledge and pedagogical strategies began to coalesce during the lesson creation and development stage. Each of our lesson study groups began by having common, broad goals in mind: (a) to probe students through problem-solving activities, (b) to ask purposeful questions to deepen their learning, and (c) to help students connect mathematical tasks to real-world contexts. These goals guided the creation of student-centered lessons and built on our collective knowledge of mathematics content and pedagogy. Because we were separated by physical distance, the online learning platform allowed us to stay connected, learn from one another's contexts, and discuss the required research readings. It became apparent how imperative it was to listen to and honor each of our voices to make this experience purposeful and meaningful for all.

While the lesson study exposed our pedagogical practices and how students responded to our carefully planned lesson, we felt the greatest learning opportunities came from the intense reflection process in which we engaged independently and as a group. The process of reviewing artifacts, such as student work, built systemic knowledge about and for teaching and learning (Lewis & Hurd, 2011). Through the collaborative examination of student work, we experienced a joint investment and insight that led to further refinement of teaching pedagogies that influenced our broader understandings of teaching and learning mathematics.

One of the most valuable parts of our online lesson study experience was the recorded video component. The videos generated discussion, illuminated missed instructional opportunities, and prompted reflection that helped the teams make lesson improvements for subsequent implementations. Watching a video of one's teaching and sharing it for discussion with colleagues is an incredibly vulnerable act. For example, Alyson's reflection on her examination of the lesson video speaks to the importance of this element of the assignment.

> I found that watching and sharing the video of my own teaching was scary, humbling, and empowering. It's one thing to be observed in real-time. It's another thing to examine a video of your teaching. However, I knew that my team and I had created the plan together and it was our lesson even though I was the one teaching it. (A. Eaglen, personal communication)

Alyson's reflection further accentuates the need to build trust and develop community with lesson study team members prior to the lesson's execution. The honest feedback we received from our lesson study teams provided us with the opportunity to reflect on and grow our pedagogical practice.

Developing an Equitable Leadership Practice in an Online Environment

Our graduate program's online platform allowed us to seamlessly connect, create, and collaborate, which was pivotal to our mathematics leadership journey. Instead of basing our lesson study self-reflections on memory, we reviewed the lessons in their entirety using video recordings. We benefited from the diverse perspectives of our group members, unbounded by geography. The online platform of our graduate program provided consistent access to multiple educators from various grade levels and differing roles throughout the East Coast and international settings.

Because we were never in the same room together, collaborating online required professionalism and collegiality when providing honest and constructive criticism. We achieved these goals in three ways. First, the online format allowed us to develop better time management due to candidate locations across multiple time zones. Second, we learned to communicate more succinctly because online communication prevented us from relying on facial expressions or body language. Having candid conversations enhanced and invigorated our leadership skills because we learned to courteously, yet frankly, critique our peers, which in turn provided invaluable practice for us aspiring mathematics teacher-leaders. Lastly, the online lesson study format was executed without the need for substitute coverage. During live lesson study experiences, teachers often have to get substitute coverage in order to be able to view and debrief the lesson. Using the online format, we were able to meet and discuss our lesson outside of the time constraints of a school day. This format's flexibility makes it an interesting option for schools and districts looking to normalize in-house professional development.

Our K–8 Students' Experiences with Equitable Mathematics Instruction

Students possessing mathematical agency participate in meaningful mathematics that connects to their background knowledge and experiences as well as those of their peers (NCTM, 2020). The K–8 students who participated in each of our lesson studies engaged in rich mathematical tasks which were intentionally designed with multiple entry points so all students could access the problems. Holding true to our lessons' student-centered design, our learning goals were driven by students learning from one another as they listened, questioned, and explored each other's ideas and made mathematical connections independent of teacher input. In this way we embraced *Catalyzing Change*'s (NCTM, 2020) intent to ensure all students' voices and ideas were welcomed into our classrooms and fostered others' learning. Below are the stories from each of our lesson study implementations and how we cultivated an equitable mathematics practice that emphasized students' mathematical agency.

Adrienne's Implementation: Equity in Exploration and Discovery

At the core of Three-Act Tasks is student-driven engagement and participation. The tasks cannot be solved without student input. Student input is rarely categorized as "correct" or "incorrect"; rather, it is considered integral to progressing along a solution path. Because the teacher serves as a facilitator, student input leads to understanding with pivotal observations, questions, exploration, risk-taking and decision-making. My (Adrienne's) lesson study team's plan was structured to maintain these critical aspects of the Three-Act Task. During the process, we designed my instructional role to shift from teacher to facilitator by explicitly contemplating how we could launch the task.

In the first act of the task, I asked students to brainstorm the focus question that would guide their inquiry. Each student voiced an opinion that I recorded on our chart paper, which was posted for all to view. As each student actively contributed and listened to their classmates, they individually and collectively determined whether an idea could be further considered, developed, or eliminated. Every student played a leadership and collaborative role by sharing ideas, attempting to justify their thoughts, and critiquing others' suggestions. During a 40-second turn-and-talk, students shared their thinking and considered the ideas of one or more classmates. The students spent almost eight minutes discussing the possibilities of the main question, during which time they combined questions, eliminated unnecessary ones, and eventually realized that they would answer subsequent questions with an exploration of their selected focus question.

This dynamic aspect of the task on which we focused in our lesson study helped students take ownership of their learning. They could not progress to the next act without making decisions collaboratively around the first act. At some point in the discussion, each student played both a follower and a leader, further developing a sense of mathematical confidence and agency.

Alyson's Implementation: Equity in Access

A productive belief about children's mathematical ability in *Catalyzing Change* (NCTM, 2020) is that access to high-quality mathematics instruction is impacted by the labels we place on children. Similarly, teachers must leverage student differences by considering how we can invite all students to participate by encouraging representations that support sense-making at all levels (NCTM, 2020). In my (Alyson's) lesson study project, my group strove to create an accessible mathematical opportunity through which all students could engage in the same rich task. We did this by selecting a problem with multiple entry points, providing students with a variety of manipulatives, and encouraging student groups to investigate a solution strategy that made sense to them.

The task we selected was centered on a real-word activity with which most students in the classroom had personal experience: raking leaves. While launching the task, students were shown different manipulatives that they could use to help them explore and solve the problem, including play money, snap cubes, pattern blocks, counters, and color tiles. The variety of materials sent a clear message to students that varied representations and creative strategies were welcomed and invited students to construct their own meaning. Student groups were also given chart paper and markers to record their thinking as they collaborated to make sense of the task. While groups collaborated, I circulated from group to group, checking in on student thinking. I supported sense-making by asking purposeful questions that probed their mathematical understandings and encouraged divergent solution strategies. The freedom to develop their own strategies, along with verbal encouragement to represent strategies with manipulatives, pictures, words, and numbers encouraged all students in the class to be *doers* of mathematics. This belief that all students are capable of doing mathematics and the practice of giving them the means to access the problem and materials are huge steps towards creating equitable mathematics instruction.

During the post-lesson reflection, our group realized that our purposefully planned lesson had positively impacted student learning. By creating the conditions whereby students had both access to the problem and the materials that enabled them to collectively reach a solution, students demonstrated a deeper understanding of the content and feelings of being valued as mathematical thinkers.

Scarlett's Implementation: Equity in Opportunity

Equity does not mean that everyone gets the same instruction; it means that every student receives quality instruction and the opportunities that they individually need to find success in mathematics. One of the benefits of a lesson study is that it allows educators to plan and consider how to provide quality instruction for all learners. Lesson study becomes extremely important when planning effective and meaningful mathematics instruction, especially because students bring with them varied experiences and readiness for learning.

In my (Scarlett's) lesson study experience, students were given opportunities to discuss and evaluate strategies and consider strengths and weaknesses of their computation. Students were given time to analyze strategies based on their experiences with numbers and to decide why and how their strategies worked. Students identified and made connections between new strategies and ones they had used previously. Through this practice, students developed deeper conceptual understandings for how to decompose numbers into friendlier numbers to use when calculating. Through this lesson, students were able to build on the idea that there are multiple pathways to solving a problem in mathematics. Once students made a meaningful connection to a concept they understood, they experienced a "lightbulb" moment. As students analyzed the strategies more closely, their conceptual understanding and confidence increased, enabling them to move forward with problem-solving because they were able to find a point of familiarity.

The mathematical insights students gained through making a mathematical connection during this lesson will support them as they continue to deepen their mathematical thinking. The lesson created in our Online Lesson Study Assignment offered a lens into how developing leadership skills fosters instructional practices which lead to equitable learning experiences for students.

Final Thoughts

Whether in a formal or informal leadership position (McGatha & Rigelman, 2017), mathematics specialists can be positioned as powerful change agents who support the transformation of mathematics instruction into collaborative spaces: spaces in which students are encouraged to take new approaches, advance their learning, and foster mathematical agency (NCTM, 2014). To accomplish these goals, mathematics specialists require targeted leadership knowledge and skills so that they can help transform mathematics instruction in their schools (AMTE, 2013; NCTM, 2012; Sutton et al., 2011). The Online Lesson Study Assignment in George Mason University's Mathematics Specialist Program provided candidates with opportunities to intentionally design and facilitate lessons that provided multiple access points to ensure all students meaningful engagement with rich mathematical learning experiences. Ultimately, engaging in this experience allowed candidates the opportunity to transform their classrooms into mathematically powerful spaces in which teachers facilitated equitable learning opportunities and students increased their mathematical agency.

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Learning to Anticipate in an Online Class: Perspectives of an Instructor and a Mathematics Specialist Candidate

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Abstract

This paper will highlight two perspectives, a course instructor and a mathematics specialist candidate working together in an online course, on the practice of anticipating how a learner will approach a task or assignment. The candidate shares her experiences in developing an understanding of what it means to anticipate student responses and implement mathematical practice in the classroom. She also shares how learning to anticipate has impacted her teaching. The instructor reflects on her experiences (or lack thereof) in anticipating how students would engage in the online environment. From the instructor and the candidate perspectives, learning to anticipate helped to develop a rich community of learners that allowed everyone to grow through their interactions and reflections on course content.

Keywords

online learning, mathematics education, mathematics specialist, teacher education

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A mathematics specialist candidate was considering asking her middle school students to utilize mathematical symbols and order of operations to derive the numbers 1–20 using only four 4s. Anticipating student responses prior to the activity, the candidate wrote, “I perceive students will have difficulty with solving an expression for 13, 17, 18, and 19.” The instructor responded, “Why? Can you give me a reason?” This simple interaction between an instructor and a mathematics specialist candidate in an online course sparked a relationship that would have a lasting impact on both of them. The candidate recognized the importance of anticipating student mathematical answers. The practice of anticipation provided the candidate with a means to integrate the content knowledge from her online mathematics courses with the pedagogical content knowledge in her mathematics leadership course. Learning to anticipate not only had an impact on the teaching practices of the candidate but also on the practices and perceptions of the instructor. For both individuals, the importance of learning to anticipate student responses in an online graduate course and a face-to-face middle school class supported the instructor and candidate in revising their pedagogical beliefs.

Literature Review

Smith and Stein (2011) provide five practices for supporting teachers in leading more purposeful mathematical discussions. In the first step, anticipation, the teacher solves the problem and reflects on possible student strategies and misconceptions. Monitoring, the second step, requires the teacher to observe student thinking. In the third step, selecting, the teacher must purposefully identify student solutions to highlight in the whole group discussion. Sequencing, the fourth step, asks the teacher to make “purposeful choices about the order in which students' work is shared, [so] teachers can maximize the chances that their mathematical goals for the discussion are achieved” (Stein et al., 2008, p. 329). In the last step, connecting, teachers must pose questions that support students in finding connections between the different student strategies to develop the key mathematical ideas for students.

Embedding the practices into a class can change the way a teacher develops their mathematical understandings to support the learning of their students and improves their ability to lead productive discussions (Stein et al., 2008). Implementing the step of anticipation into the lesson planning process provides teachers a means to recognize different concepts, procedures, and practices that students can use to solve a mathematical task. Before a lesson, reflection on student responses supports a teacher in being prepared to address student misconceptions and solutions (Schoenfield, 1998). Anticipating allows the teacher to develop questions to assess and advance students' thinking. The process of anticipation not only promotes student-centered mathematical discussions that move beyond "show and tell discussions" (Stein et al., 2008, p. 316), but it also provides a chance for teachers to reflect on mathematical content and pedagogical strategies needed for high-quality mathematics instruction.

In any classroom, understanding effective teaching practices is imperative for student success. Research on online learning has focused on the importance of building a sense of community that includes and supports learners as they interact with the content (Barry, 2019; Swan, 2003). The instructor has to be visible and engaged in the online learning community. The benefit of the teacher and the student interactions can have a positive impact on student learning (Serdyukov & Sisteck-Chandler, 2015). Similar to interactions in a face-to-face classroom, in an online class "the quantity and quality of teacher interaction with students are linked to student learning" (Swan, 2003, p.25). Online teachers must be present in the online learning environment, but the question is how does an instructor anticipate the level of engagement and the types of questions that will arise during the online interaction?

Setting the Stage

The Class

Mathematics Education Leadership I is a course designed to develop effective school-based mathematics teachers and leaders. Course readings, discussions, and assignments support the development of mathematical content knowledge and mathematical content pedagogical knowledge. Course objectives mirror the effective teaching practices and guiding principles presented in *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics [NCTM], 2014). Careful attention is given to the designing, teaching, and evaluating lessons and assignments that supported inquiry-based learning in the classroom.

The Candidate

I (Melody) had been a fifth-grade teacher for thirteen years and a middle school mathematics teacher for four years. I considered myself to be a successful, knowledgeable teacher when I enrolled in an online professional development program. I had completed one mathematics course in the program and, from that experience, knew that my interactions with the Leadership I course instructors would take place through email, phone calls, and online course meetings. During each course session, I was busy taking notes and digesting the new information that I learned through our class discussions. Initially, I was afraid to ask questions in the online class out of my fear of not having the skill set to be successful in this program. If the class met face-to-face, I would ask the instructor, Kristina, any questions or request clarification at the end of a class meeting. At times, I felt isolated in the class due to the geographical distance between program participants and the online nature of the program.

The Instructor

I (Kristina) had been a K–8 mathematics teacher and university instructor for over twenty years before teaching Leadership I. I thought of myself as a knowledgeable instructor, but I had concerns about teaching an online class on pedagogy when most of my prior work as an instructor had focused on mathematical content knowledge in a face-to-face setting. I had completed variations of the Leadership I course assignments when I completed a similar professional development program a few years ago. Through the process of anticipating the questions and misconceptions that could arise for candidates, I reflected on my own prior face-to-face experiences.

The Assignments

The Task-Based Assignment

One task-based assignment in the course began with each candidate selecting a cognitively demanding task to implement in a K–8 classroom. The purpose of the assignment was to help candidates in their development of listening, observing, and questioning skills. The assignment consisted of two components. In the first component, candidates addressed the goals for the task, the purpose of the task, and the implementation of Smith and Stein's (2011) five practices for orchestrating a productive mathematical discussion. Implementing the five practices in the development stage enabled the candidates to make meaningful connections between their mathematical knowledge and pedagogical knowledge to reflect on anticipated student mathematical ideas. The second component of the assignment was for candidates to write an analysis of the implementation of the task including providing a description of the mathematical thinking of several students as they worked through the task, the future instructional needs of the class, and a personal reflection on the entire process. Candidates received feedback on the first component before completing the second component of the assignment. This assignment afforded candidates the opportunity to reflect on their students' mathematical thought process before, during, and after the task.

The Candidate

I had questions before the assignment even began. This was my first time doing an assignment like this, and I needed support. I reached out to other students, but they were not always able to help me. After feedback from other students and a conversation with Kristina, I chose the task "The Four 4s" (see youcubed.org). The task required my students to utilize mathematical symbols and order of operations to derive the numbers 1–20 using only four 4s. As part of the assignment, I anticipated student solutions. This was the first time I had chosen such an open-ended task to implement in my classroom, and I struggled to think like a middle school student. I had a (one) method for using fours to come up with each of the numbers 1–20 but had difficulty thinking of others.

For the first component of the assignment, I stated that certain solutions would be a challenge for students because they were challenging for me. In her feedback to me, Kristina asked me why I thought the students might struggle with these solutions. I had to admit that the task was hard for me. When I implemented the lesson in my classroom, my students did not have difficulty with the same numbers that I did. This experience helped me to reflect on how I could anticipate student solutions and the approaches my students would utilize to complete the task. I had gone through the process of anticipating as part of the assignment, but I had not anticipated as thoroughly as I should have. For example, I had not thought through the possible misconceptions about the order of operations or misuse of grouping symbols. Connecting the learning experiences in the mathematics content course with what I was learning in the leadership course was important if I was to become a mathematics specialist. I needed to think about how there was more than one way to solve a problem. I had to challenge myself before I could challenge my students.

The Instructor

In the first component of assignment, candidates addressed three questions about the learning goals, the task description, and anticipated student strategies for the task they had chosen. The candidates uploaded their chosen task and question responses to a discussion thread that was used for providing feedback to each other. Melody initially picked a task on integer operations. Her classmates suggested reflecting on the open-ended nature of the task or developing context for the problems she had chosen. I agreed with the suggestions but also noticed that in her response to the question about anticipated student strategies she had provided some general misconceptions instead of possible solutions.

Melody and I communicated several times about her concerns and questions about the initial task assignment. In our discussions, Melody shared that the task "Four 4s" was more open-ended and provided multiple entry points for her students. As I graded her assignment, I read her statement about several of the numbers in the 1–20 range being hard for her represent with only 4s and her conclusion that they would be hard for her students. I asked Melody about this. My question led to a conversation about what it meant to anticipate in a mathematics lesson. For me, I had not thought that anticipation would be an issue within the assignment; I assumed that her prior work in her previous mathematics content course had given her a foundation for exploring strategies and misconceptions to support her in learning to anticipate. Reflecting, I believe my prior experience in teaching mathematics and my lack of experience teaching a mathematics pedagogy course led to my inability to anticipate these types of issues.

The Lesson Planning Project

The Lesson Planning Project in Leadership I required candidates to revise and refine a lesson plan for their K–8 class. Candidates used their prior knowledge from course discussions and the task-based assignment to plan, teach, and analyze a student-centered mathematics lesson. Similar to the task assignment, this project was broken into two components. First, a current lesson plan or school division lesson plan had to be revised using backward design and the Smith and Stein (2011) five practices. After the lesson was completed, the second component of the assignment was for candidates to analyze student work and develop an instructional plan to meet student needs.

The Candidate

This project stood out to me because of its use of backward design. I had written many types of lesson plans, but I struggled to anticipate what the instructors wanted for this specific type of lesson plan. Connecting the mathematics content standards to students' prior knowledge and to their post-lesson knowledge to rewrite a lesson plan was a new experience for me. My lessons tended to focus on the mathematics content my students needed to understand, with little consideration about what they needed to know after they left my classroom. This was a new form of anticipation that I needed to incorporate in my teaching practice. I had to anticipate where my students had been, where they needed to be, and how to help them bridge any gaps to aid their mathematical understanding. I was just beginning to learn to anticipate my students' responses, but this added a new twist.

I choose to do a lesson on order of operations and mathematical properties. I remember the lesson plan template said, "Let go!" But I was not ready to let go. I was working on handing over more responsibility to my students, learning to anticipate their strategies, and then I needed to let go so they could think about mathematics. This was all new for me. I knew I needed to anticipate strategies and misconceptions, but I would never have all the possible solutions. I knew I needed to anticipate their strategies to guide their thinking. I knew I could do this, but I needed support.

I contacted Kristina. I asked her: What does "understand" mean in backward design? In the assignment, it said "understand," but are these the "big ideas?" We would communicate when I needed support. I think Kristina was learning to know when I needed clarity or just a probing question to help me reflect on what I was thinking. Sometimes it would just take a simple question of "what would you do if this happened?" or "how would you do this?" Through our communication, I realized that I could do the assignment. In my reflection on the lesson plan project, I shared how I had been narrowly focused on the skills at hand and had never thought about how my students would solve a problem. In our communication, I realized that I did know where my students would make mistakes from my prior experiences with them and with other students and I could be prepared to address them. As a teacher, when I can anticipate a student's mistakes, I can have strategies prepared to address my student's needs.

The Instructor

In anticipation of the lesson plan project, the candidates worked in small groups on the steps of backward design planning. The instructors moved among the groups (in online breakout rooms) and supported the candidates as they worked. Again, I felt that the students were prepared for the assignment. I believed the assignment was written clearly and prior activities in the course had prepared the candidates to successfully complete the assignment. It was not long before I heard from Melody about her struggles in thinking about the "big ideas" and the meaning of "understand" in backwards design (Wiggins, 2005). As I talked with Melody, it appeared that the problem was not with the assignment itself but was related to her ability to communicate her thinking. Our conversations centered on what a concept meant for her and how it could be transferred to her classroom.

I was not anticipating her questions, but what I was beginning to understand was that it was not about my helping Melody directly. Instead, she needed me to ask a probing question to support her in her understanding. Melody was learning to make sense of the pedagogical knowledge that she was gaining. Melody shared with me that she was allowing her students to take more chances in the classroom, and she was taking more chances as well. I was beginning to see a change in her, but at the same time, I was seeing a change in how I anticipated Melody's needs. My concerns about teaching an online class made me unsure about how to anticipate and address student needs. Melody's needs were no different than any other student learning to make sense of new material, and what she needed was a place to feel comfortable asking questions. I needed to pose questions that allowed her to reflect on her thinking.

Conclusion: Learning to Anticipate Together

We each used the idea of learning to anticipate in different ways to inform our practice in our respective classrooms. Our takeaways from this experience are presented below.

The Candidate

A big takeaway from the Leadership I course was that I can prepare for student answers. I will never have all the possible solutions, but that is alright. Mathematical learning is not about the correct answer but is instead about guiding student thinking. Schoenfield (1998) stated that "having a deeper understanding of teaching should have real payoffs in the long run" (p. 92). I have learned the importance of building strategies and filling my student's mathematical "tool-box." I need to anticipate how students could solve a problem and what misconceptions they might have as I guide their instruction. In my final written reflection during the course, I think I said it best:

> Anticipating was new for me. I had never really thought about how students might solve a problem. I would work the problems and I knew what the answer was, but here was this new concept of analyzing what I thought they might do. I always chose a student with the right answer to go up to the board and share. I didn't look for different ways to solve the problem. If you didn't solve it the same way as me, then it wasn't correct. You had to do it the ONE and ONLY way. I sometimes wish I could go back and apologize to those early classes. (M. O'Quinn, personal communication)

Learning to anticipate allowed me to connect the ideas from my mathematical content classes with the pedagogical ideas in the leadership courses. I now share the idea of anticipating students' work with others in my school building to help them see the importance of this step before teaching.

The Instructor

Learning to anticipate in an online class helped me to recognize that teaching in an online setting does not mean I have to be a different teacher. Instead, the mode that I use to communicate with my students needed to change. Just like in a face-to-face class, I cannot always anticipate all of the misconceptions that may arise, but what I can do is ask questions that make the student reflect on what they know and where they want to go. This is also true in a mathematics course or a mathematics education (leadership) course. Prior learning experiences had an impact on how I anticipated what took place during the class. I needed to remember that my experiences in any classroom are not the same as others. I need to take time to reflect on how others may interpret assignments based on their own classroom experiences to improve my teaching (Ball & Bass, 2003). Focusing on the interaction between my prior experiences, beliefs, and knowledge when anticipating will support my learning and the learning of my students (Schoenfield, 1998). Recognizing that this does not change in an online learning environment is important.

Learning Together

Together, the instructor and student became learners in this online class. It was a new setting for both of us, but in learning to anticipate in our respective classrooms, we formed a community of inquiry. This community allowed both of us to reflect on our teaching practices and beliefs. Swan (2003) described the importance of engaging with the content and with each other in an online learning environment. Instructors cannot "give a sense of community to learners" (Conrad, 2003, p.17). Instead, the sense has to grow out of members being present and active in the community.

Through our interactions during the Leadership I course, we both learned the importance of anticipating student strategies and misconceptions. The true mathematical and pedagogical learning did not emerge from the correct answers but developed through being reflective as part of the learning experiences that took place. Our interactions in the online learning environment were high quality (Swan, 2003). These interactions supported both of us as we worked to develop deep connections between content and pedagogy (Ball & Bass, 2000).

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MATHEMATICAL REPRESENTATIONS IN A SYNCHRONOUS ONLINE MATHEMATICS SPECIALIST PREPARATION PROGRAM

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ABSTRACT
Universities are implementing more online courses (Yamagata-Lynch, 2014). However, instructors may feel a sense of trepidation in transitioning a mathematics class to a synchronous online platform because they do not want to compromise quality pedagogy (Herrington et al., 2001) for the convenience of an online environment (Wills, 2021). Some courses have successfully transitioned to a synchronous online environment while maintaining rich discussion and student collaboration (Baker & Hjalmarson, 2019); however, mathematics content courses include the additional challenge of incorporating problem solving with multiple representations. This paper focuses on how mathematical representations emerge in a synchronous online course for mathematics specialists.

KEYWORDS
synchronous instruction, multiple representations, discourse, distance learning, face-to-face instruction, rich tasks

The purpose of this paper is to show how students recorded their representations in both face-to-face (F2F) and synchronous online mathematics content courses in a mathematics specialist preparation program at George Mason University and to show the intentional instructional planning that encouraged students' use of multiple representations. We will guide readers through various mathematical representations (concrete, pictorial, and abstract) created in both F2F and online classrooms. Examples of the representations include pictures of student work and group posters presented in the F2F class and the student work visible on collaborative slides (e.g., Google Slides) in the online class. We will address the successes and challenges of implementing a mathematics education online course through the eyes of multiple stakeholders. Theresa Wills and Deborah Crawford are university instructors who have taught multiple mathematics courses in both F2F and online settings, and Deborah is also a district leader in Virginia. Shruti Sanghavi and Kate Roscioli are K–12 educators and alumni of George Mason University's Mathematics Educational Leadership (MEL) program. Shruti experienced a 100% online program, and Kate participated in a hybrid program with four mathematics courses taught in a F2F format and one taught online.

The National Council of Teachers of Mathematics (NCTM) (2014) states that "effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures as tools for problem solving" (p. 10). Representations come alive in F2F and online classrooms in many different formats, including drawings, physical manipulatives, formulas, tables, graphs, virtual manipulatives, and digital tools.

Representations

Mathematical representations are essential components in mathematics classrooms. Representations such as drawings, concrete models, and abstract symbols are necessary components to help students build deep conceptual understanding (Berry & Thunder, 2017). Comparing representations through discussion helps make connections to the mathematical goals (Smith & Stein, 2011). Lesh et al. (1987) emphasized the importance of students moving flexibly between representations to understand the mathematical concepts fully.

Discourse and Rich Tasks

Mathematical discourse involves the student to student discussion of models, representations, and strategies used in problem solving (Smith & Stein, 2011). Students must communicate and collaborate as they solve problems to develop a deep mathematical understanding (Steele, 1999; Walshaw & Anthony, 2008). Facilitating meaningful mathematical discourse is challenging because of the intricacies involved in the process (Stein, 2007). It requires student engagement with multiple, student-created representations and a teacher that possesses content knowledge, conceptual understanding, and a mindset to commit to changing their instruction (Smith & Stein, 2011; Firmender et al., 2014). It also requires the teacher to act as a facilitator to guide students' thinking and understanding in the classroom (Steele, 1998) as students discuss *how they arrive* at a solution, not just the solution (Stein, 2007). Discourse about rich tasks serves as a tool for equity as students can access the tasks through multiple entry points (Sealey, 2016) and the voices of all students are valued through their different representations of the problem situation.

Rich tasks serve as the vehicle through which students' mathematical thinking becomes visible. However, through mathematical discourse, students create a shared understanding of the big mathematical ideas in focus of the lesson (NCTM, 2014). Through discussion, students can compare and contrast multiple representations of different strategies used to solve a task and connect different representations to the underlying mathematical ideas and relationships (NCTM, 2014) which are intertwined with other mathematics teaching practices (Smith et al., 2017).

Representations, Discourse, and Synchronous Online Classrooms

A synchronous online classroom setting is a live experience that takes place via a video conference tool at a specified time. "In synchronous online courses in higher education, there is a tremendous pressure to ensure our students are engaged in their online learning environments." (Baker & Hjalmarson, 2019, p. 12). Rich tasks are a catalyst for engagement in mathematics education courses because they are designed to be accessible to all learners, are solved using various representations and strategies, and relate to students' lived experiences (Wolf, 2015). Regardless of the classroom format, students must have the ability to create and compare mathematical representations to fully explore and transmit conceptual understanding (Wills, 2019), which brings additional challenges for the planning and implementation of tasks in an online environment.

When teaching in a synchronous online format, instructors must anticipate student representations and strategies which may be shared using the available technological tools such as virtual manipulatives and collaborative slides (Wills, 2021). In order to ensure that these representations are accessible to everyone involved, instructors need to consider Technological Pedagogical Content Knowledge (TPACK). TPACK (Mishra & Koehler, 2006) describes the intersectionality of technological knowledge (creating the digital representation), pedagogical knowledge (knowing a variety of representations and when to use them), and content knowledge (mathematical knowledge and skills). Deficits in any of these three pieces of knowledge will result in incomplete or incorrect representations in the synchronous online classroom.

Additional challenges and opportunities arise in the types of representations used in the online classroom. Wills (2019) found various representations in synchronous online classes, including abstract, concrete, pictorial, and dynamic-pictorial (see Figure 1). Dynamic-pictorial representations are "pictorial models that use the advantages of technology to move representations on the screen in a way that could not be reasonably replicated using hand-held manipulatives" (p. 1). In other words, it moves during the discussion. Dynamic-pictorial representations (see Figure 2) are unique to synchronous online learning. They allow students to easily work with large quantities (e.g., candidates can copy and paste hundreds of squares efficiently) and easily make visual connections between models (e.g., candidates can duplicate a representation to show both a before and after manipulation efficiently).

Situation

The MEL masters degree program, described in this paper, is offered in various formats. In one format all courses are offered 100% synchronously online, while another provides a hybrid experience for students including a mix of F2F and synchronous online courses.

Figure 1

Multiple Representations in Multiple Formats

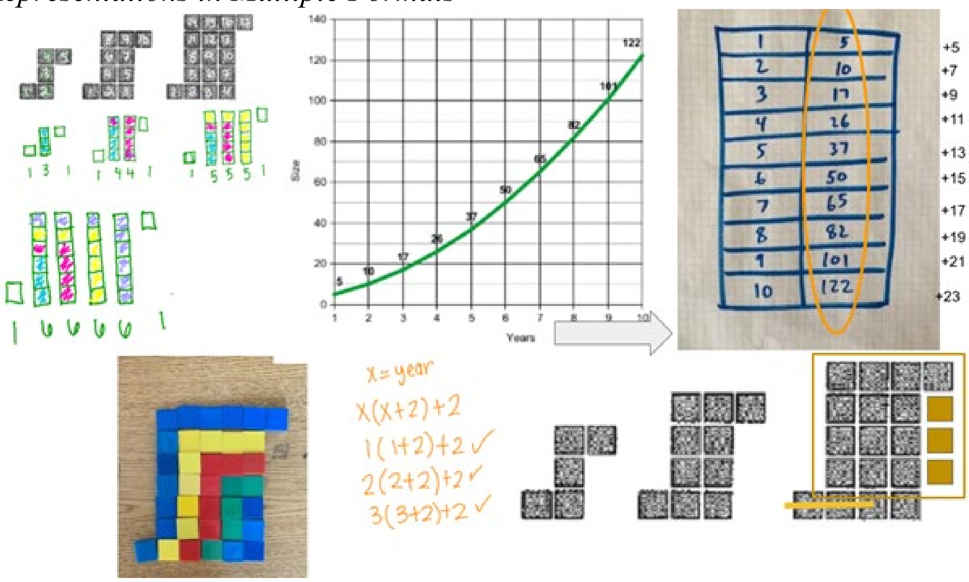
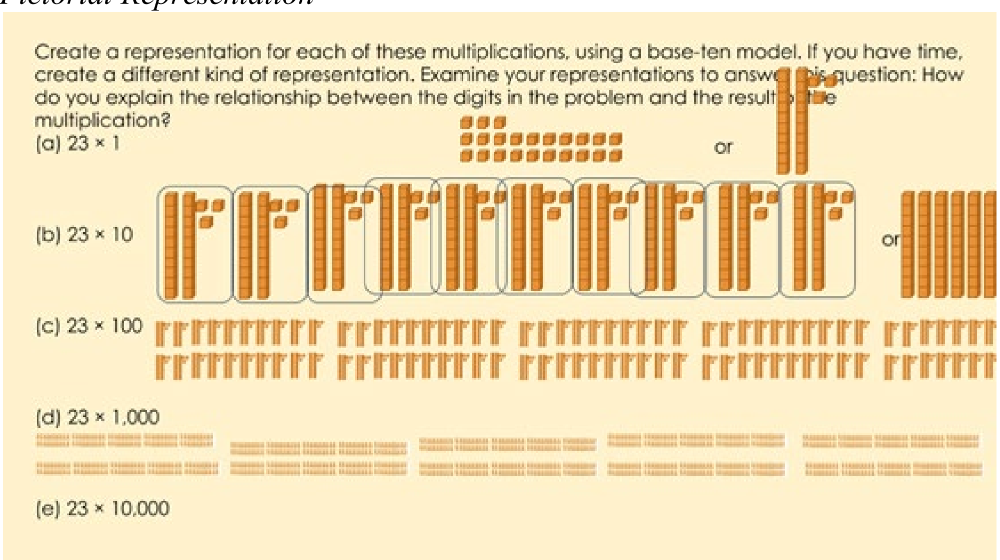


Figure 2

Dynamic Pictorial Representation



Stakeholders and Diverse Perspectives

The authors of this paper have various experiences in online and F2F mathematics content courses and describe these unique perspectives throughout the paper to explain the complexities of planning, implementing, and participating in online mathematics courses.

University Instructors

Deborah and Theresa, both university instructors, taught both F2F and synchronous online sections of the same course to prepare mathematics specialists. They co-planned their classes to ensure that the online sections incorporated the same tasks and activities as the F2F section. This planning ensured that the content and pedagogy remained consistent and that the rich tasks, representations, and mathematical discussions were not compromised in the online course.

District Supervisor

Deborah is also a school division mathematics supervisor who hires mathematics specialists as classroom mathematics teachers, coaches, Title 1 mathematics teachers, STEM specialists, and other locally defined roles. Since teachers tend to teach or coach in the same way they were taught (Wiliam, 2011), she wants to ensure that the candidates learn mathematics content that models the Mathematical Teaching Practices (NCTM, 2014) and mathematics teacher leadership attributes such as coaching the Process Standards (NCTM, 2000).

Students

Kate and Shruti were both candidates in MEL masters degree program. Shruti was part of a 100% synchronous online cohort, while Kate experienced a hybrid instructional model with only one content course taught in the synchronous online setting and the other four content courses were F2F. They noticed that the structure of facilitating a task did not differ significantly in either format. Both had the experience of incorporating multiple representations when working with rich tasks in all of the mathematics content courses. Another critical part of facilitating a rich task is the discourse, which could be challenging in a synchronous online environment. However, through breakout rooms and collaborative slides, the experience was not very different from a F2F setting in which candidates sit around a classroom table. Shruti explained that although she had anticipated feeling disconnected from the other cohort members in an online environment, she found that, due to the synchronous format, the experience was collaborative with a strong focus on discussions. As a result, she never felt that her peers or the professors were not supporting her.

Themes

Through discussions, interviews, and journaling, these four stakeholders discovered three essential themes that were paramount for encouraging mathematical representations in the F2F and synchronous online classes: community, expectations, and mathematical discourse. These themes will be discussed below, first according to the similarities in both F2F and synchronous online settings and then by the characteristics exhibited only in the synchronous online environment. Each stakeholder provides unique perspectives and insights into each theme.

Community

Building a classroom community is critical in all mathematics classrooms, including synchronous online environments (Fisher et al., 2020; Garrison, 2015). Students require interaction and collaboration when exploring various strategies, perspectives, and representations. Theresa and Deborah intentionally planned activities that valued mistakes, persistence, and celebrated risks in solving problems using representations outside of the candidates' comfort zones. From the first day of class, instructors used differentiated "getting to know you" activities for synchronous online students using interactive slides and small breakout groups to ask questions about the technology. In this way, instructors were able to pre-assess the technology, mathematics, and other skills that the candidates would need throughout the class. Candidates had varied levels of expertise; some were technology experts; some were primary grade experts, and some were formulas and abstract notation experts. When instructors created heterogeneous groups based on expertise, they noticed characteristics such as patience, productive struggle, and willingness to make mistakes. These same traits were evident in different groups' abilities to create multiple representations for rich tasks.

Shruti remembers that her age and inexperience with computer programs and websites did not adversely impact her synchronous online learning experience because of the supportive community she was participating in. "At the beginning of each class, the professors would ask them to provide an update about their lives with pictures and a short narrative. It was so wonderful to know when people were getting engaged, receiving promotions, or having babies" (S. Sanghavi, personal communication).

Kate also benefitted from participating in synchronous online communities. She enjoyed the random breakout room feature in the synchronous online class because candidates were able to work with different people and hear multiple in-depth perspectives. In F2F classes, she sat with the same group and did not get to know everyone else in the class. Both formats engendered camaraderie among the students, thus generating another support layer for the cohorts' students.

Building a community is a purposeful act prompted by instructors through activities, observations, and student groupings. As the communities grew, students felt safe taking risks and using digital means of connecting to collaborate and create mathematical representations.

Expectations and Norms

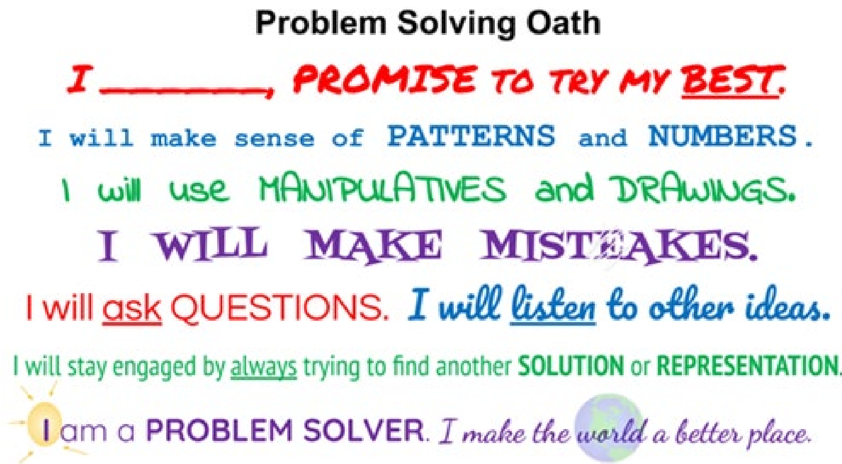
Instructors were explicit in setting expectations and norms to encourage students to create multiple representations. They modeled and practiced these expectations regularly in both F2F and synchronous online classroom settings.

Problem Solving Oath

The problem solving oath (see Figure 3) was an intentional structure implemented in both F2F and synchronous online classes. Students read the problem solving oath aloud in F2F classes and interacted with the oath in synchronous online classes by finding a line in the oath they would focus on during the work time on that particular day. Part of this oath reminded students to consider many different representations and misconceptions. Kate remembers that when everyone said the oath, they committed to using multiple representations.

Deborah was explicitly looking for various types of representations to present during the whole group discussion. When she looked across the representations used by a diverse class of learners, she found many concrete, pictorial, abstract, and even dynamic-pictorial representations. Theresa describes the purposeful planning for encouraging different representations. They found that it was important that they provided a shared space for displaying the representations and reinforced the norm that multiple representations were required. Theresa also anticipated both student voice during class discussions and the multiple modalities necessary for interacting with the representations. For example, a candidate could take a photo of their paper-and-pencil work, share a video of their procedure, create shapes using the tools on the interactive slides, or provide a screen capture of a virtual manipulative. Once the candidates' representations were visible, they could implement the rest of the problem solving oath by asking questions and finding another solution or representation. This structure resulted in all students engaging in the task for the entire work time and provided a plethora of representations for the whole group discussion.

Figure 3
Problem Solving Oath. Reproduced with permission from theresawills.com



Collaboration

Another expectation was that candidates collaborate in small groups as they developed mathematical representations. Instructors in the F2F class observed the collaboration by listening to table discussions, watching candidates point to drawings in a notebook, and seeing pairs of students build a model with manipulatives. Similarly, it was observed in the synchronous online class through listening to small group discussions (each participant used a headset with a microphone within a breakout room), watching candidates use a virtual arrow to point at drawings on a shared slide, and seeing candidates share images of homemade manipulatives. Through collaboration, candidates made connections to different representations even as they were still emerging during an activity, as they developed a deeper conceptual understanding of mathematics.

Shruti remembers that her cohort could interact in real-time, which included seeing each other, communicating, and answering questions together every day of the program. They were continuously able to share their thoughts about a task, question, or assignment. She could see how others answered the problem, asked questions, made side comments, or offered a different solution or strategy. She instantly had a couple of people to bounce ideas off of and knew that they would support her no matter what.

During the whole group discussion, instructors could also see evidence of collaboration. Instructors were deliberate about how they facilitated a discussion by asking questions that required students to make connections with other peers' work. Deborah noticed the candidates collaborated to collect, organize, display, and interpret their data to make decisions about a rich task, scenario, or game. Teams created slides in the class deck to share out their mathematical thinking in a virtual gallery walk. Groups visited each team's slides, giving feedback through comments, symbols such as emojis, emboldening or highlighting, and via the virtual classroom chat box (see Figure 4).

Figure 4
Solution Slide with Peer Feedback Via Emojis

Ownership

A unique characteristic of synchronous online learning was a greater sense of ownership by candidates. In the F2F classes, the slides were static, but since candidates had full editing rights in the synchronous online class, they could add slides, change them, paste screenshots and create unique virtual representations. Deborah recalls that candidates used household items for physical manipulatives and various technology tools to simulate objects' physical movement. Many times, they color-coded their virtual manipulatives to represent their thinking. Others began with drawings and sketches that they uploaded to the slide. If another student wanted to draw on a sketch, they could quickly duplicate it and share a different representation. While many virtual tools will allow students to upload pictures of their mathematical strategies, interactive slides allow for more flexibility as students can upload, modify, duplicate, and collaborate within the same document. The affordance of the interactive slides was critical in obtaining many mathematical representations for a rich task.

Representations within Mathematical Discourse

In both F2F and synchronous online environments, candidates engaged in mathematical discourse around representations developed from rich tasks. The most significant difference between the synchronous online and F2F experiences was the type of representations used for problem solving. Synchronous online students used homemade or virtual manipulatives in place of traditional, hand-held manipulatives. Shruti explored multiple representations through the mathematical tasks in every course. She solved a task using her strategy and posted it to a shared slide as she watched other strategies emerge alongside of hers, and then she tried to connect her work with the work of others through discussion. By communicating with other candidates, she was able to identify the similarities and differences as she developed her conceptual understanding of a mathematical procedure or concept.

Similarly, Kate observed that no matter the location, whether it was at your table in a physical classroom or in a virtual breakout room, representations were used as a springboard for discussions. Both F2F and synchronous online classes began with small group discussions about incomplete representations. However, an advantage to the synchronous online class was duplicating an incomplete representation and modifying it without altering the original work.

Because of this, she experienced shared ownership in synchronous online classes as she modified and shared a different representation.

Facilitating a productive mathematics discussion requires intense multi-tasking by the instructor. Deborah and Theresa watched the representations emerge on the group slides in real-time as groups collaborated in breakout rooms. From the representations viewable on the shared slides, they could see the access point and first representation based on the comfort level and initial problem solving strategies used across different groups. They chose groups to listen to as they discussed their emerging solutions. They observed shifts in thinking as students shared their ideas as well as by how they responded during a small group discussion. They also used feedback to differentiate their responses to advance the thinking of individuals or groups. One group might receive scaffolding to bridge candidates to the next level, while another group might be challenged to think about a related question to extend their thinking beyond the task. The instructors also took copious notes while selecting and sequencing the pieces of student work to present during the whole group discussion. To alleviate instructor overload during the busy class session, Theresa found it critical to anticipate the mathematical strategies (Smith & Stein, 2011), the technical requirements, the applications being used, and also possible candidate misconceptions (Wills, 2021).

Conclusion

Similar to F2F courses, synchronous online mathematics courses must elicit multiple student-created representations of mathematical understanding. Three themes, community, expectations, and mathematics discourse should be explicitly planned before implementation to ensure that students have the required physical, social, and virtual resources to create and share their representations. Students who have a strong sense of community are more likely to participate and share their misconceptions as they explore problem solving. Clear expectations provide the structure for small group time and ensure that students explore multiple representations. Finally, mathematics discourse is the glue that brings the various representations together to form a clear image of the mathematics goal being explored. All of these themes can and should be implemented in synchronous online mathematics courses.

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TEAM TEACHING FOR DISCOURSE: PERSPECTIVES OF INSTRUCTORS AND A STUDENT IN AN ONLINE PROBABILITY AND STATISTICS COURSE FOR PREPARING MATHEMATICS SPECIALISTS

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ABSTRACT

Team teaching is a form of collaborative work where teachers plan lessons and/or teach together. We discuss the strengths of discourse in the planning stage for an intensive, team-taught, three-week probability and statistics course for mathematics specialists as a way to create and sustain a sense of community and show multiple perspectives in an online course. We delve into two cases of lessons—one about stem-and-leaf plots and another on averages—to describe the interactions of and reflections from three online instructors and a preparing mathematics specialist across the phases of planning, enactment, and the resulting student learning. The conversations about our understandings of probability and statistics concepts that arose between the three instructors with differing arenas of expertise—a mathematics educator, a probability instructor, and an expert teacher—often were predictors of conversations that occurred among candidates during class. Through these mirrored conversations, we were able to build off of and expand candidates' conceptions regarding probability and statistics. We argue that when preparing mathematics specialists, having a team with diverse domain expertise but enough overlap to push each other's thinking was crucial to successful planning and enactment in the team teaching setting.

KEYWORDS

team teaching, statistics and probability, online learning, social learning

Mathematics specialists have been and continue to be needed to support teachers (Dossey, 1984; Fennell, 2006). It is crucial, then, that teacher educators provide robust learning opportunities for specialists so that they can, in turn, provide accurate and effective learning experiences for classroom teachers. This reflection on a mathematical content course as described by one candidate (a preparing mathematics specialist) illustrates the importance of experience and community in an online environment:

> Over the duration of the program, and this course, I found myself explaining that I was in an online program, but it really wasn't "online-online." It could be because previously my perceptions of online learning were reading pages and pages, posting to a discussion board, and responding with very little *real* discussion with anyone. Instead, for this program, I had to be "in class." The whole class and small group experiences took my online learning experience to the next level. Knowing my classmates and hearing their thoughts, ideas, and explanations improved my understanding a hundred times over.
>
> Taking time to reflect on my experiences made me realize that what took place during each class was not by chance but rather, the direct result of careful planning and negotiating among teams of instructors. The experiences, learning, and discussions that made our probability and statistics course rise above other courses can be attributed to the diverse group of instructors who not only broadened the view of statistics for their students, but also for themselves. (M. Swoyer, personal communication)

We argue that discourse in the planning phase of team teaching with three instructors who had differing areas of expertise was vital to fostering this sense of community among candidates to bolster their learning.

This paper explores the strengths of discourse within a team teaching approach in an online synchronous probability and statistics course as part of a mathematics specialists' program. Through reflections from the instructional team and a candidate, we examine the impact of an experientially diverse instructional team on the course design process, enactment of lessons, and student learning. We discuss two pivotal scenarios from the course development phase and the online classroom about stem-and-leaf plots and the meaning of the word "average" to illustrate how instructors with differing yet overlapping expertise provide different perspectives that lead to rich class discussions that are beneficial for mathematics specialists.

Literature Review

Team teaching is a form of collaboration among teachers, which can take on various forms: (a) division of responsibilities; (b) cooperative planning but individual instruction; or (c) cooperative planning, instruction, and assessment (Sandholtz, 2000). Here, we use the term "team teaching" to refer to this last version, as it is the most collaborative. Under this view, both students and instructors themselves are exposed to different perspectives (Harris & Harvey, 2000). Effective team teaching requires the honest exchange of ideas between instructors, a clear understanding of individual roles in the team, and adequate time for planning together (Shibley, 2006). Though conversations on content are important, the negotiating of pedagogical decisions that occur during planning is also important for setting the stage for learning.

Just as the curriculum development process in a team teaching environment should provide ample opportunities for instructor interaction, the structure of an online course should also actively engage students. We view learning from a classic social constructivist standpoint, where interactions promote thinking and reasoning through language (Vygotsky, 1978). Online instructors must then plan opportunities for meaningful exchanges between students to foster understanding; social interactions between students and between teachers and students are key for learning in an online classroom (Hill et al., 2009). We use the term *learning community* in this paper to refer to a group of people coming together with shared goals and norms for learning. Even through an online medium, members of a learning community (teachers and students) all share the responsibility to contribute to the overall class learning experience (Harris & Harvey, 2000; Hill et al., 2009).

Context of the Course

The online course, co-taught by three faculty members from Virginia Commonwealth University, was a three-week, online probability and statistics course. The candidates in the course had been together for two years and were comfortable with the online structure, so the candidates knew and were accustomed to active participation. The course covered K–8 statistics and probability concepts. The course was guided by the five practices for orchestrating mathematical conversations: anticipation, monitoring, selecting, sequencing, and connecting (Smith & Stein, 2011). Using these principles, pre-session work completed by candidates prior to in-class meetings and delivered through a course management system (e.g., Blackboard) included case studies, independent activities, and small group discussions to facilitate class sessions. During in-class meetings, through a video conferencing tool (e.g., Blackboard Collaborate), candidates worked independently and in small groups on tasks; their group work was then selected, sequenced, and shared for whole group discussion.

Instructor and Candidate Backgrounds

Kristina is a mathematics educator with over twenty years of experience in the PK–12 and university settings. Her experiences include working with K–8 students and pre-service and in-service teachers. Kristina brought her pedagogical and content knowledge from PK–12 teaching and her prior experience teaching for the online math specialists program to the instructional team.

Mita is a statistics educator with over fifteen years of experience teaching statistics full-time at the undergraduate and graduate levels. She brought expertise with statistics to the instructional team. This was her first time team teaching, as well as teaching an online course on statistics and probability.

Rani is a mathematician and mathematics educator. She brought a focus on student thinking to the instructional team. She has four years of experience teaching pre-service teachers in person. This was her first time teaching in-service teachers, synchronous online courses, and team teaching.

Monica is an elementary school educator with thirteen years of teaching experience, and she is now serving as a K–4 mathematics coach. Prior to joining the mathematics specialist cohort, she participated in a literacy specialist cohort at another university and was a K–4 mathematics interventionist for four years. She was a candidate enrolled in the online probability and statistics course.

Impacts of Team Teaching on Enactment

We first describe the course design process and then two episodes that occurred during class. These episodes show how conversations across the instructional team during course development helped with anticipating and connecting to mathematical and statistical thinking during class online. For each case, Monica provides her reflection as a student. Through reflection, which is vital to improve professional practice (Hart et al., 1992), we illustrate how discourse before and during an online class can impact the learning experience for all.

Course Design

We, the instructors, co-developed the course over a six-week period prior to the first class. We drew on pre-existing materials from the in-person version of the course as had been taught by others and modified them for the online medium. In general, Kristina brought the teacher pedagogy, Rani drew out children's thinking, and Mita provided ways to push candidates' probability and statistics thinking. As a result of our differing lenses, we integrated into the class activities such as reading case studies of children's thinking, watching classroom videos, doing rich mathematical and statistical tasks, and playing probability games.

However, the curriculum design process was more than the sum of its individual instructors' contributions. The group talked about all instructional decisions as we considered what our different perspectives could bring to the class. Individually, we completed all class activities prior to our team instructor meetings in which we expanded each activity by focusing on the big ideas and how to differentiate across the candidates' grade levels. For example, in one activity, Mita and Kristina both looked at the same graphical representation of students and the number of teeth they had lost, and they each viewed the data differently. Mita interpreted the graph as asking, "How many students lost a given number of teeth?" Meanwhile, Kristina, coming from an elementary perspective, thought it was asking, "How many teeth did a given student lose?" This conversation led us to realize these were two different ways to interpret one graphical representation, that interpretation was influenced by grade level, and that it all depended on the question one was asking.

Noticing how our conversations like the one above pushed our thinking, we chose to focus on the activities that pushed each of us in our mathematical thinking to be a driving force for class discussion. Our differing views were rooted in how probability and statistics courses vary; our conceptions were often based on our own learning experiences. Learning to question each other's thinking and reflect on different mathematical and statistical views became a common occurrence during planning.

The *Statistics: Modeling with Data* casebook (Russel, Schifter, & Bastable, 2018), part of the *Developing Mathematical Ideas* series, was our primary source for supporting candidates in working with mathematical concepts and learning to support the development of student understanding. Using the text as the foundation for the course, we planned for a variety of structures: out of class individual and small group pre-session work, in-class direct instruction, individual work, small group work, and whole group discussion. The course structure purposefully led candidates to engage in discourse within different groups, drawing on each person having years of rich and diverse experience to share.

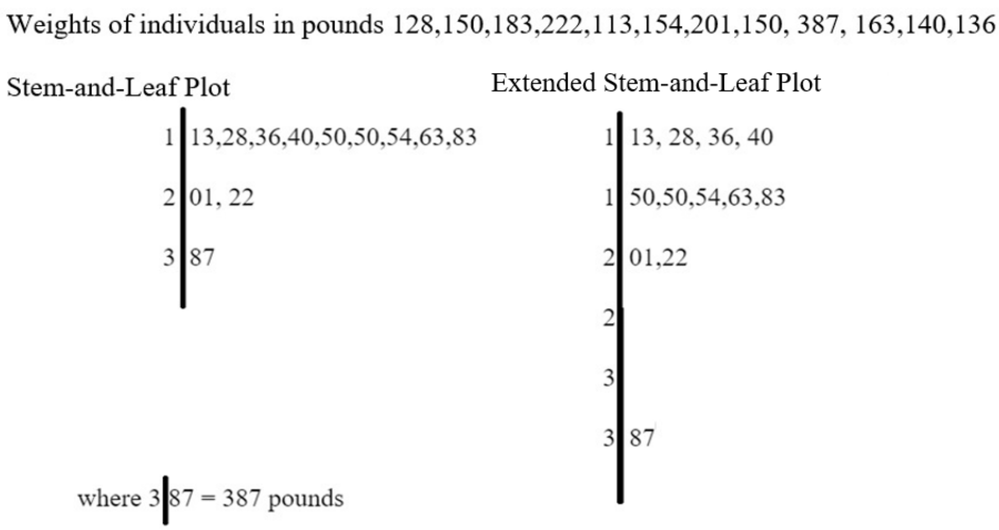
Case A: Stem-and-leaf Plots Discussion

Conversations during Planning

A key idea throughout the course was understanding how to appropriately represent a dataset as a graphical display. One course activity that centered on this concept asked candidates to create a stem-and-leaf plot. Kristina and Rani knew to look at each number, separate the number into a “stem” and a “leaf,” and then organize the stems and then the leaves from least to greatest. Mita shared an extended stem-and-leaf plot for large datasets with a small range, which pushed Kristina’s and Rani’s K–12 understanding. She further shared that stem-and-leaf plots should have anywhere between 6–20 stems. Thus, if it has fewer than 6 stems, it is best to “split the stems” so that there are more stems. An extended stem-and-leaf plot (see Figure 1) better shows the shape and distribution of data, which in turn allows one to better describe and understand the data. Mita’s background expertise was crucial, as this idea was new to Kristina and Rani. But upon further conversation, it made sense when thinking about real-world data and the query: Given a particular “research” question, what would be the best way to display the data?

Figure 1

Standard Stem-and-Leaf and Extended Stem-and-Leaf Plot



Note. The diagram on the left depicts a standard stem-and-leaf plot, drawing from the data at the top. The small number of stems and the multiple leaves for the “1” stem indicate the data may be better illustrated through an extended stem-and-leaf plot (on the right).

Our conversations as an instructional team led us to recognize that K–8 teachers are (like Kristina and Rani) rarely exposed to large datasets, and so students are also rarely exposed to large datasets in the classroom. At first, we questioned the benefit of sharing the extended stem-and-leaf plot: Kristina grappled with it from a K–8 perspective while Rani did from a mathematician’s perspective. Mita, however, showed us its benefits for even moderate sized datasets. We decided that understanding a stem-and-leaf plot involved more than just the construction procedure but also how this graphical display would be used in a research context and thus real-life applications. Sharing this idea would lead the candidates to understand how graphical displays can tell the story of the data: There are different ways to depict a data set, and the shape of the distribution of a data set changes depending on the type of stem-and-leaf plot one constructs.

Kristina and Rani's previous understandings of stem-and-leaf plots allowed the team to predict and relate to where the candidates' beginning understanding may be. This allowed the team to carefully and intentionally design the instruction to move the candidates to this deeper understanding of stem-and-leaf plots.

Enactment

Conversations around stem-and-leaf plots led to a pivotal moment in the first class. We gave candidates a data set and asked them to create a stem-and-leaf plot in small groups. The groups were intentionally a mix of elementary and middle school teachers, as we knew some candidates may not be familiar with this type of graph. We intended for each small group to be a learning community, to share and support each other in their mathematical thinking. Once small groups had completed their stem-and-leaf plots, we shared their representations in whole group discussion. All groups created a standard stem-and-leaf plot with little debate.

Mita shared pictures of a standard and an extended stem-and-leaf plot with split stems for the same data set as seen in Figure 1. She asked the class for their thoughts; many candidates instantly raised their hands and asked questions through the chat feature in the online classroom. This was the first sign that candidates' thinking had been perturbed. Mita, as the statistician, addressed each question, but because of our prior conversations, both Rani, as the mathematician, and Kristina, as the PK–12 teacher, were actively engaged in the conversation. Kristina and Rani shared with the candidates their misgivings and questions about splitting the stem during the planning stage but supported Mita. We explained that the conversations we had as the instructors during the planning stage uncovered our own misconceptions about stem-and-leaf plots, which we now shared to support candidates' questions and misconceptions. This helped the candidates open up even more with the entire class about their current thinking. Then we, as instructors, helped them extend their understanding. By purposefully allowing candidates to question and argue their thinking and by sharing with them our own (lack of) understanding, we supported the candidates in understanding graphical displays from a broader context than a K–8 classroom, further solidifying our online learning community.

Student Perspective

This activity helped solidify my understanding of concepts like stem-and-leaf plots by allowing me to articulate what I understood to others. I had a narrow understanding of the mathematics being explored until I heard perspectives offered by my classmates. There were other times during the discussion when I was the "group expert" and explained the ideas I understood to my classmates. The instructors may not have seen how powerful that type of small group discussion would be for the candidates if they themselves had not grappled with their own understandings of stem-and-leaf plots as they planned and designed our experiences for the session.

Case B: Averages Discussion

Conversations during Planning

Mathematical language played an important role in course development, as we saw in our lesson about averages. Words such as *average* have both a mathematical and everyday meaning.

One activity had candidates identify the average of five numbers in a set, e.g., 6, 7, 7, 7, and 8. Mita and Rani both thought of *average* and *mean* as synonyms, coming from statistical and mathematical perspectives. However, Kristina thought of the mode, arguing that for this set of numbers, the word average might imply to children the number that appeared the most. The mean, median, and mode were all mathematically the same in this problem (seven), so we looked at several variations of five numbers where the mean, median, and mode were the same or different. For example, in the case of 5, 7, 8, 9 and 12, each of us said we would calculate the arithmetic mean for the average based on the relationship of the numbers. We began to notice that depending on the numbers or context of the numbers, our personal choice of whether to use the mean, median, or mode to represent the average changed.

As an instructional team, we had varying interpretations of the meaning of average; our different expertise had come into play. Kristina shared that in the Virginia Department of Education's (2016) curriculum framework for fifth grade mathematics, the mean, median, and mode were all referred to as types of averages. The term *arithmetic average* is used to refer to the mathematical mean. We were forced to justify our thinking to each other, and these conversations helped us recognize the importance of providing a non-judgmental space for the candidates to have the same conversations with each other. Our roles as instructors were to support the candidates in justifying their thinking, so we planned for small group discussions across different grade levels to deepen their use and understanding of mathematical language in K–8 classroom discourse because it would push their thinking.

Enactment

In pre-session work, we prompted participants to think about the word *average* within the assigned case studies, which focused on K–8 students making meaning of the word in conversation and within mathematical content. Next, small groups found the average of various data sets consisting of five numbers. Similar to what had occurred during instructor discourse, when candidates shared their thoughts, the idea of average potentially referring to the *middle* arose. This could come from a person thinking about height, where there are several people shorter or taller than the middle height. Average could also refer to *normal* if thought of as what you see the most in a group. Several participants shared that average meant the *mean* of the data set, thinking about mathematical definitions.

To our surprise, mirroring that of instructor conversations, candidates talked about how K–8 students need opportunities to explore mathematical language in context. The candidates drove the conversation forward on their own without instructor prompting. They sequenced their conceptions of the word average by grade levels. They then moved from mathematical language to representations which supported the class development of mathematical knowledge and addressed misconceptions. Unplanned, our role changed that evening from facilitating to reinforcing and questioning candidates' thoughts. Our prior conversations as an instructional team prepared us for this unexpected turn. Because we had experienced as an instructional team the openness in interpretation of the word average and ensuing confusion, we were better able to support the candidates as they experienced this in real time. Mita, for example, nudged candidates to reflect on the statistical idea that mean and median are the only real measures of center, not mode. This idea challenged the candidates' existing notions of measures of center. But because each of the instructors jumped in organically to add their thoughts, there was a conversational tone to the lesson, with little tension. Ultimately, this back-and-forth in discourse led candidates to a higher level of understanding.

Student Perspective

The discussions around the idea of average and what average meant in different contexts were insightful. I spent a good part of that class building this common understanding and definition of average with the other candidates and instructors. There was definite discomfort in our small groups when some candidates' clarity of the word average was challenged. These conversations made me more willing to look to and learn from my classmates to enhance my learning. I did not realize at the time how powerful my social connection to the other candidates was, nor how intentional the planning for these social connections was as well. I attribute my success and growth in statistics and probability to the intentional design and enactment of the course.

Discussion

Through these two episodes, we have shown how team teaching was beneficial in both the planning and enactment of an online statistics and probability course for preparing mathematics specialists. The conversations within the instructional team were crucial for effective team teaching, as they were often precursors to conversations that occurred in class. This meant we, as instructors, could anticipate candidates' thinking prior to the online meetings, so we could facilitate more productive conversations (Smith & Stein, 2011). Instructors were also able to organically chime in when each other was speaking, to add and build off one another's perspective. This normalized different ways of thinking about concepts and provided a more conversational atmosphere, which invited candidates to join in as well.

We recommend team teaching for all content courses for mathematics specialists in order to draw out rich conversations that specialists will likely witness among teachers and students in the classroom. We especially recommend team teaching for probability and statistics, as this content area brings in ideas from different disciplines and is a struggle for many people. Each of the instructors professed that we would team teach again, with each other and with others.

In terms of recommendations, we believe the facts that we were all new to teaching the course online and that we had set norms for working together were crucial for our success. We were all on equal footing in creating material for a new course together. Second, it is beneficial to co-develop (at least some) lessons together, rather than divide the work, for the sake of the discourse that ensues. Each instructor was aware of all the content, as we had collectively decided what to include and why. This drew out conversation about the content, which led to each individual instructor knowing each other's thoughts, and so we were prepared to build off what one another said in a natural way during class.

This work has implications for the importance and structuring of team teaching in order to develop robust learning experiences for mathematics specialist courses. An instructional team with different background expertise, where each instructor fulfilled a role and was an expert in their domain but with slight overlaps to push each other's thinking, was crucial. Together, we formed a learning community, questioning and sustaining each other, even before the first class meeting. This instructor learning community then supported the creation and strengthening of candidate learning communities. This prior engagement allowed us as instructors to be active in all conversations as we knew what others were thinking, and the candidates' discussions often mirrored ours. It also allowed us to share a common vision for the candidates' online experience.

This work also illustrates the importance of a social environment and interaction for effective online instruction. Monica described how she saw our roles within the community at the end:

> Reflecting on the experience, I can see how I relied on each of the instructors differently during this course. I quickly learned to listen closely when it was Mita's turn to share; she was going to share her vast knowledge of probability and statistics. Rani helped to clarify the big ideas being explored. Lastly, I relied on Kristina's ability to break down the learning into manageable chunks, as I am accustomed to doing in my own elementary teaching experience. (M. Swoyer, personal communication)

By team teaching, our conversations and interactions support our specialists' learning, and through them, we serve communities of teachers across Virginia.

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Equity and Access: Empowering Change Agents

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Abstract

All learners must have access and opportunity to engage meaningfully in the highest levels of mathematics. Mathematics specialists are uniquely situated to contribute to the creation of access and equity for all learners by addressing three target areas with their mathematics teachers and administrators. In order to catalyze change, mathematics specialists need to be prepared to target three obstacles to access and equity: beliefs and expectations, curriculum and instruction, and intervention. This preparation can take place through leadership courses intentionally created to explore the role of change agent and provide practice in negotiating the role.

Keywords

access, equity, catalyzing change, mathematics specialists

The National Council of Teachers of Mathematics' (NCTM) *Catalyzing Change* series (2020) and *Principles to Actions* (2014) call for systematic action and change so that all learners have access and opportunity to engage meaningfully in the highest levels of mathematics. NCTM (2020) makes four key recommendations to catalyze this change: broaden the purposes of learning mathematics, create equitable structures in mathematics, implement equitable mathematics instruction, and develop deep mathematical understanding. These actions require educators who both recognize this call and have the knowledge and skills to be catalysts of change in their schools and school divisions. The mathematics specialist preparation program seeks to intentionally develop a cohort of mathematics teacher leaders who contribute to this purposeful "move from 'pockets of excellence' to 'systemic excellence' by providing mathematics education that supports the learning of all students at the highest possible level" (NCTM, 2014, p. 2). The goal of this paper is to share the work of five mathematics specialists in catalyzing change.

Access and Equity Target Areas

In one university mathematics specialist preparation program, a sequence of three leadership courses ran concurrently with mathematics content courses in order to simultaneously develop the leadership and coaching skills of the candidates as well as their pedagogical content knowledge across the K–8 curriculum. The leadership courses explicitly addressed three target areas identified by NCTM (2014) in order to overcome obstacles to access and equity:

1. Beliefs and expectations of educators,
2. Curriculum and instruction, and
3. Interventions and support personnel (p. 64 – 66).

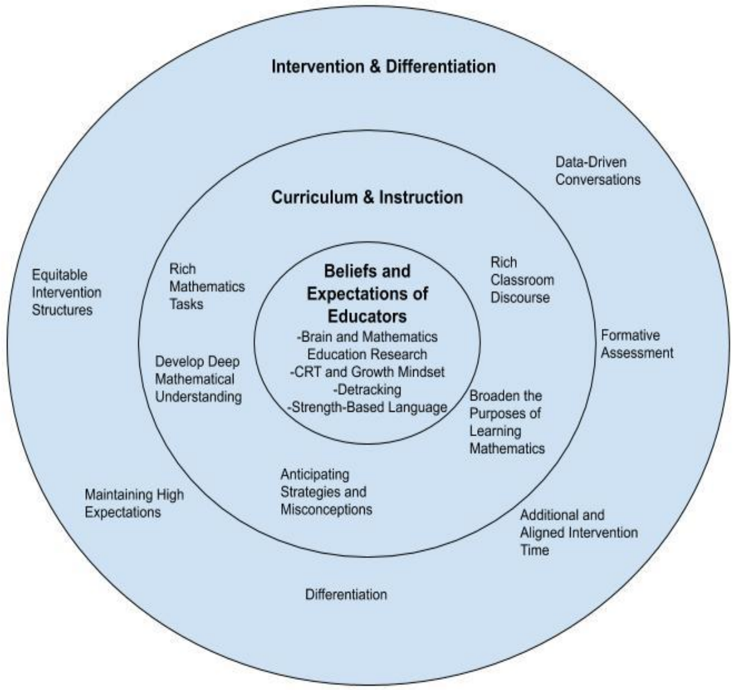
With each leadership course, we cycled back to deepen and broaden our understanding in these areas as well as to facilitate the transfer of knowledge and skills from the roles of mathematics teachers to mathematics coaches to mathematics data coaches.

As mathematics specialists, we are uniquely situated to contribute to the creation of access and equity for all learners by addressing the three target areas with our mathematics teachers and administrators. While systematic change takes time, our initial learning and transfer are important steps along the path to catalyzing change. We will share the ways we learned deeply about these areas and changed our beliefs and expectations, how we used these shifts and insights to learn deeply about equitable mathematics curriculum, instruction, and intervention, and how we began the initial transfer from coursework to our daily practice in order to assume the role of change agents. Figure 1 is a graphic representation of our learning and our transfer work.

Beliefs and Expectations of Educators

In each of the leadership courses, we examined and reexamined our beliefs and expectations as educators in light of both learning theories and current research. We studied the learning theories of equity and access advocates, such as Carol Dweck, Gloria Ladson-Billings, Deborah Ball, and Rico Gutstein. We read and reflected on *Mathematical Mindsets* (Boaler, 2015) and *Culturally Responsive Teaching and the Brain* (Hammond, 2014) in order to use brain and mathematics-education research to explicitly correct myths and misconceptions. We conducted literature reviews in order to understand, appreciate, and teach in ways that engage historically marginalized student populations, including learners with disabilities, English language learners, economically disadvantaged learners, African-American learners, and Latino learners.

Figure 1
Our Learning and Transfer to Practice



We experienced a range of shifts in our beliefs and expectations as educators. We began to see the equitable and inequitable structures in mathematics teaching and learning and have conversations about them. For some, our eyes were opened to inequities, while for others, our lived experiences with inequities were recognized and appreciated.

For example, we learned that students living in poverty are an infinitely diverse group who are repeatedly marginalized in schools through biases, inequitable access, and systemic classism (Gorski, 2013). Economically disadvantaged children may suffer high levels of environmental stress, which threatens brain development. Additionally, complex trauma, which is not confined to a single event, leads to feelings of hopelessness and desensitized emotions and can lead to significant changes in learners' brains that negatively affect their working memory (Hammond, 2014). Many times the historical marginalization of learners who live in poverty leads to challenges when learning and applying mathematics. From an early age, children living in poverty may be placed in groups labeled "low" which begins the track they will inevitably follow. This limits learners' self-concept, having a significant, adverse effect on learning (Haberman, 2010). These learners oftentimes face lower expectations of their thinking and ability, which can lead to many years of learning experiences that are not aligned with evidence-based best practices, such as building sense-making through high cognitive demand tasks and communicating ideas through discourse. Ability grouping continues to be a very common practice that protects and facilitates educators' deficit views and implicit bias related to class.

We reviewed related research to constructively examine inequities, and we developed the tools and learned the language to explicitly disrupt the beliefs and practices that sustain these inequities. This learning empowered us to initiate critical conversations with our teachers and administrators about their beliefs and expectations.

From Deficit to Strength-Based

We began by transferring our growing knowledge of brain research into our work in our schools. To be change agents, it was imperative to help the teachers and administrators in our schools understand neurological research and how it directly relates to learners. For example, Hammond (2014) explained how working memory can be engaged more effectively and efficiently for all learners when mathematical learning is grounded in sense-making, problem-solving, and connections to lived experiences. Boaler (2015) explained the power of brain plasticity to grow all mathematical thinkers through deep mathematical learning. As mathematics specialists who regularly meet with grade-level teams, Professional Learning Communities (PLCs), and Collaborative Learning Teams (CLTs) within our schools, we were able to protect time and provide learning resources to engage teams in unpacking brain research over a series of collaborative meetings. Teams considered questions such as, "How does this new knowledge challenge what we previously knew and did?" and "What will we do differently now?" Then, we facilitated teachers' continual use of brain research in the planning of mathematics teaching and learning for all students.

Our work to revolutionize educator mindsets continued with intentionally planned learning conversations that moved teachers away from deficit models of thinking. For example, one first grade teacher reflected:

> I have started to see just how much students are capable of understanding on their own. When I first started teaching math, I spent a lot of time showing children how to do math and get to the correct solution. Now I allow my kids to explore, experiment, and discuss. Very quickly, I saw that kids didn't need me to tell them what to do or how to do math. They only needed me to give them opportunities to discover and opportunities to share what they know. I now feel willing to push the envelope and take risks in order to help our kids grow by leaps and bounds (personal communication).

Because we learned that systematic marginalization and deficit views often intersect with academic tracking, which, in turn, negatively affects all learners, we targeted the beliefs and expectations of teachers that result from tracking as well as promote and enable tracking. Rather than thinking of learners as those who can and cannot do mathematics, we pushed teams to examine the idea that no one is a "math person" and that everyone can learn mathematics.

Through the lens of brain research, we examined with our teachers how schema and connections are formed via productive struggle and making mistakes. For example, one third grade teacher said, "Giving the students the power to show their own strategies and thinking and teach each other their thoughts was very powerful and I think will help them gain confidence over time." By being members of the team and actively participating in meetings, we were able to disrupt the labeling of learners, such as "low group" or "high group," and instead facilitate in-depth discussions around student work analysis to focus on learners' strengths and the next steps based on learning trajectories.

Gradually, team conversations evolved and became grounded in the belief that every learner deserves the opportunity and has the ability to be a mathematician and to engage in inductive reasoning, mathematical argumentation, and meaningful discourse. Teachers began to recognize themselves as the creators and gatekeepers of these opportunities. Teachers' talk about learners became strength-based. Ultimately, having these sustained conversations around brain research, growth mindset, and systemic marginalization with teachers at our schools led to critical changes in their beliefs and expectations about learners as mathematical doers. As mathematics specialists, we were able to initiate these critical conversations with our teachers about their beliefs and expectations because our coursework prepared us with awareness, language, research, and tools, and because of our unique role within PLCs, CLTs, and team planning meetings in our schools. We were able to catalyze change in educators' beliefs and expectations.

Curriculum and Instruction

Across the three leadership courses, rich mathematical tasks resounded as an essential component of effective, equitable curriculum and instruction. Rich mathematical tasks (or high cognitive demand tasks) are mathematical problems that require learners to make connections among big ideas and do not have a clear, single path to a single solution (Boaler, 2015; Smith & Stein, 2011). We learned the significance of selecting or creating rich mathematical tasks aligned with learning goals, anticipating learner strategies and mistakes, and implementing the tasks in ways that maintain the depth of thinking and problem solving (Smith & Stein, 2011).

Implementing rich mathematical tasks is one way teachers enact their beliefs and expectations that all learners can learn and achieve mathematics at high levels. These tasks have high mathematical ceilings (i.e., can be extended and deepened) and low mathematical floors (i.e., can be accessed through multiple entry points using multiple strategies) so that all learners can engage in high cognitive demand problem solving and discussion. The power of rich mathematical tasks is reflected in the research we studied that shifted our beliefs and expectations (Boaler, 2015; Hammond, 2014) as well as in our deep examination of effective, equitable curriculum and instruction (Fennell et al., 2017; Hattie et al., 2017; Van de Walle et al., 2018). These tasks are inherent in two of NCTM's (2020) key recommendations for catalyzing change: broadening the purposes of learning mathematics and developing deep mathematical understanding. We practiced identifying, anticipating, and implementing rich mathematical tasks so that we could engage teachers in the same process through our roles as mathematics specialists.

Rich Mathematical Tasks and Embedded Professional Learning

Because mathematics specialists lead division-, school-, and PLC-level professional learning, we were able to collaborate with administrators to create sustained, embedded initiatives around rich mathematical tasks. To begin the school year, we engaged the teachers in our schools as learners: they collaborated, communicated, and used multiple representations and strategies to solve rich mathematical tasks. Then we facilitated reflective discussions about the ways rich mathematical tasks make lessons accessible and equitable for all learners. We examined the value of tasks with low mathematical floors and high mathematical ceilings as ways to ensure that historically marginalized learners have access to significant, meaningful, and deep mathematical content. We identified characteristics of rich mathematical tasks that allow for multiple entry points and problem-solving strategies, increase the growth mindset among all learners, and value a broad purpose for using mathematics and a personal connection with mathematics. These initial explorations of rich mathematical tasks provided whole schools and grade-level teams with a common language and criteria for selecting rich mathematical tasks.

The professional engagement continued into grade-level team meetings, PLCs, and individual coaching sessions. As mathematics specialists, we regularly met with teachers throughout the school year in order to deepen and extend their understanding of rich mathematical tasks. Some PLCs worked as a team to plan common, rich mathematical tasks. Then they implemented the tasks in their classrooms and met as a team to analyze student work. Other mathematics specialists coached individual teachers as they planned, implemented, and reflected on their use of rich mathematical tasks.

In each case, we noticed the discussions became meaningfully focused on access and equity when teachers themselves engaged with the mathematical tasks first. This practice, called anticipating student strategies (Smith & Stein, 2015), was a regular part of our leadership coursework that we transferred to our daily work with teachers. When teachers anticipated, they considered tasks from the perspective of the learners, including mistakes that would make sense, common misconceptions, and a variety of strategies and solutions. During anticipation, each teacher solved the problem differently. Their strategies included the use of manipulatives, drawings, equations, and a combination. As the teachers shared their work, the conversation centered around how the task and its deep mathematical ideas were accessible across varying levels of mathematical knowledge.

Teachers began to value the careful selection of rich mathematical tasks that provide all learners with opportunities for productive struggle and rich mathematical discussions with peers. Teachers also valued tasks that enabled learners to reason at multiple levels and to draw upon their personal experiences, contexts, culture, and language. As mathematics specialists, we were able to facilitate these discussions and continue to move teachers' conversation, reflection, and practice forward around rich mathematical tasks. Rich mathematical tasks served as an opportunity to engage all learners, including and especially historically marginalized learners, in deep thinking and meaning making about mathematical concepts and skills. We used rich mathematical tasks as a tangible practice to enact beliefs and expectations that all learners can learn mathematics deeply and, therefore, to catalyze change.

Intervention

With each iteration of our leadership courses, we dove deeper into explicitly developing the tools for catalyzing change by addressing the three target areas that could be obstacles to access and equity in ourselves and in our schools. The third target area, intervention, is founded on the same principle as that of equitable beliefs and expectations and of equitable curriculum and instruction: all learners must have access and opportunity to engage meaningfully in the highest levels of mathematics (NCTM, 2014, 2020; Riccomini & Witzel, 2010; Tapper, 2012). In our coursework, we examined a variety of diagnostic and formative assessments (Fennell et al., 2017; Tapper, 2012) to inform intervention and differentiation strategies. We practiced taking on the roles of data coaches and interventionists by analyzing multiple levels of learner data including state-, division-, and classroom-level assessments, setting goals and adjusting instruction based on this analysis, and creating equitable, data-driven instruction and intervention (DuFour et al., 2016; Love et al., 2008). We increased our pedagogical content knowledge to become change agents through studies of the impact cycle (Knight, 2018) and the content coaching cycle (West & Cameron, 2013) with individuals and teams of teachers. Our course work challenged us to negotiate our agency, or grow our efficacy, to create equity and access both within our cohort of fellow mathematics specialists and in our schools, and to catalyze change.

Data-Driven Conversations

As we engaged teachers and PLCs in data analysis to make instructional decisions, we relied on our growing pedagogical content knowledge to identify the foundational need in order to catalyze change. Many issues arising around intervention were closely linked to other instructional issues: some teams of teachers needed to examine their use of formative assessments, others needed to bolster their differentiation strategies, and others needed to create equitable structures in intervention.

We facilitated PLC meetings where the goal for teachers was to strategically meet each student's needs. In our coursework, we learned instructional time becomes more effective when teachers put a greater emphasis on formative assessments (NCTM, 2014). Utilizing formative assessments often during instruction allows teachers to make learning visible and to proactively adjust instruction in the moment to meet learners' needs (Fennel et al., 2017; Hattie et al., 2017). We put this learning into practice during PLC meetings by presenting different types of formative assessments to measure student progress, including documenting classroom discourse, concrete-representational-abstract (CRA) translations, and learners' recorded work of problem solving strategies and explanations (Berry & Thunder, 2017; Tapper, 2012; Van de Walle et al., 2018). The majority of the teachers were familiar with formative assessments; however, not all teachers were using or analyzing them. By committing to this work as a team and having a mathematics specialist to facilitate the work, teachers realized that intentionally using a variety of formative assessments and regularly analyzing the formative assessment data addressed their needs. They were able to make decisions about lesson pace and effectiveness as well as differentiated next steps, such as reteaching and extending. Formative assessments gave the teachers tools that maximized instructional time and more efficiently enabled them to analyze in-the-moment data.

Most importantly, formative assessments empowered teachers with the efficacy to share their areas of opportunities and best practices within PLCs in order to help their whole team improve access and equity for all learners. As teachers shared and analyzed formative assessments, they recognized areas of strength and opportunity within their own instruction. Together as a team, they supported each other in making intentional changes, using formative assessment to analyze those changes, and differentiating using rich mathematical tasks rather than ability grouping and lowering teacher expectations.

In addition, we identified teachers' need for differentiation strategies that maintained opportunities for deep mathematical understanding for all learners. In our coursework, we learned the importance and effectiveness of flexible grouping rather than tracking and labeling learners (Hattie et al., 2017). We also learned strategies for differentiating rich mathematical tasks, such as tiered problems, parallel tasks, and CRA modeling (Berry & Thunder, 2017; Tapper, 2012; Van de Walle et al., 2018). By coaching individual teachers, we were able to support selecting, practicing, and refining differentiation strategies that met learners' specific needs. These one-on-one conversations with teachers helped illuminate the idea that differentiated instruction comes hand in hand with equity and access by recognizing and appreciating the varying ways that students learn and process information.

Finally, we led regular data meetings where teachers discussed the interventions they put in place and their effectiveness. Based on these conversations, we learned the team's foundational need was to create equitable intervention structures. NCTM (2020) challenges teachers to maintain equitable structures by adding additional intervention time focused on problem solving and conceptual understanding to the grade-level instructional time rather than replacing it. In order for all learners to gain a deep conceptual understanding of mathematics, teachers need to revise and reframe intervention structures to use rich mathematical tasks combined with CRA modeling and focused on significant mathematical concepts and skills (Berry & Thunder, 2017; Riccomini & Witzel, 2010; Tapper, 2012). By analyzing formative assessment data, the team of teachers identified number sense, the foundation of the other content strands, as a pivotal area of need. We facilitated reflection and analysis of their grade-level number sense instruction, and teachers realized they needed to increase both the rigor and time spent developing all learners' number sense. Then, the team systematically planned ways to use intervention as a time for targeted learners to spend additional time growing their number sense with mathematics specialists through aligned instructional strategies, including rich mathematical tasks and CRA modeling. By facilitating data discussions, we were able to support teachers' evaluation and revision of their intervention structures. Together, we put interventions in place so all learners could succeed, and as a result, we catalyzed change.

The Mathematics Specialist: A Role of Advocacy

As mathematics specialists, we are uniquely positioned in our daily work with teachers, grade-level teams, PLCs, administrators, and learners to catalyze change. We can push teachers and administrators outside of their comfort zones in order to engage all learners in meaningful, mathematically rich experiences. At a school-level, we can begin and sustain the work to transform separated classroom instruction into collective mathematics learning (NCTM, 2020).

In order to catalyze change, mathematics specialists need to be prepared to target the three obstacles to access and equity focused on in this paper: beliefs and expectations, curriculum and instruction, and intervention. This preparation can take place through leadership courses intentionally created to teach and practice negotiating the role of change agent. Using tools, language, strategies, and research from coursework, mathematics specialists can then intentionally target teacher beliefs and expectations as well as curriculum, instruction, and intervention practices in their schools and school divisions. As we noted earlier, Figure 1 represents the learning we engaged in through our coursework and our transfer of this learning to practice. We advocate for this structure for mathematics specialists' training in order to systematically grow as change agents.

Mathematics specialists are often perceived as content experts, instructional coaches, interventionists, and data coaches. But at the heart of our work is the role of advocacy. We can help teachers see that "the question is not whether all students can succeed in mathematics but whether the adults organizing mathematics learning opportunities can alter traditional beliefs and practices to promote success for all" (NCTM, 2014, p. 61). Our answer to that question and our ultimate goal is to become the change agents in our schools and school divisions that instill that belief in our teachers and provide the tools for them to help make it a reality. We have begun the work of catalyzing change and will not stop until all learners have the access and opportunity to engage meaningfully in the highest levels of mathematics.

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This paper was developed in part through the project *The Virginia Mathematics Specialist Initiative: An Online Program to Prepare K–8 Mathematics Teacher Leaders for High-Needs School Districts* with support from the National Science Foundation, Noyce Track 3 Award 1660774. The opinions expressed here are those solely of the authors and do not reflect the opinions of the funding agency.

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THE ROLE OF A MATHEMATICS CONTENT- FOCUSED COACHING PROJECT IN PREPARING MATHEMATICS SPECIALIST CANDIDATES TO COACH

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ABSTRACT

The purpose of this paper is to reflect upon the mathematics content-focused coaching (MCFC) process from the perspective of coaches in their work with teachers, specifically, the effective strategies and techniques used by the mathematics coaches as they work with teachers and focus on mathematics and student learning. We discuss the MCFC project, assigned in many mathematics specialist preparation program leadership classes, in detail. Then we discuss the reflection the participants submitted in a course in which this project was assigned at two different universities and the ways in which these submissions can be used as a reflective activity and tool that coaches can employ to support learning. Implications for future research will also be discussed.

KEYWORDS

mathematics coaching, reflective practitioner, video, teacher training, mathematics specialist

Candidates from Virginia universities with mathematics specialist preparation programs complete a Mathematics Content-Focused Coaching (MCFC) video project as part of their leadership coursework. This project can be one of the most powerful experiences participants go through during their teacher leadership preparation, as it provides them with an opportunity to coach a teacher through all phases of a mathematics lesson, one of the critical roles they will undertake as school-based mathematics specialists. We will review the literature, provide an in-depth look at the project, reflect on its strengths, and consider future directions for our research.

Literature Review

Coaching

The term "coach" conjures up many different images: coaches for sporting teams, voice coaches to prepare for choral performances, and even life coaches to help navigate the obstacles of everyday living. When applied to teaching and learning, the term has been defined in many different ways but most broadly as "a person who works collaboratively with a teacher to improve the teacher's practice and content knowledge" (Yopp et al., 2011 p. 50).

Likewise, there are a variety of different models and texts that describe the practice of coaching. Yopp et al. (2017) suggest there are four commonly used approaches: cognitive coaching (Costa & Garmston, 2002), content-focused coaching (West & Staub, 2003), instructional coaching (Knight, 2007), and mathematics coaching (Hull et al., 2009). Yopp et al. (2017) articulate commonalities and differences in the models. All four of these coaching models address the collaboration between the coach and teacher, but the point of emphasis and approaches differ. In cognitive coaching (Costa & Garmston, 2002) the coach helps the teacher negotiate the reflection on and refinement of their practice. The instructional coaching model (Knight, 2007) is that of an on-site professional developer who attends to the skills required of successful coaches. Mathematics coaching (Hull et al., 2009) blends the importance of implementing effective instructional strategies and deep knowledge of mathematical content.

We have chosen to focus on Mathematics Content-Focused Coaching (MCFC) (West & Staub, 2003), as it is the model used by many mathematics specialist preparation programs in the Commonwealth of Virginia. In content-focused coaching, the coach has developed a deep understanding of mathematical pedagogical content knowledge and supports teachers' instructional practices (Gibbons & Cobb, 2016).

In the MCFC model, two types of coaching "moves" are involved in pre- and post-conferences: (a) those that invite teacher contributions, and (b) those that provide the teacher with direct assistance in designing mathematics lessons (West & Staub, 2003). Moves that invite teacher contributions are "statements or questions by the coach that initiate and invite the teacher to verbalize perceptions, thoughts, plans, deliberations, and arguments" (West & Staub, 2003, p. 15). Moves that provide direct guidance are "statements by the coach that provide guidance and explanations for specific designs and ways of implementing a lesson" (West & Staub, 2003, p.15). Indeed, West and Staub (2003) suggest that the goal of a coach, as they gain more experience, is to employ more invitational moves in coaching conversations.

Most MCFC conferences also involve conversations about mathematics content, pedagogical content knowledge, and other topics (e.g., classroom behavior and time management). It has been suggested that the most successful coaching conversations occur when the focus of the conference is maintained on students and their work, not on the teacher (West & Cameron, 2013).

Videos

Videos of teaching have been used prominently in pre-service and in-service learning opportunities (Barlow, 2014; Schoenfield, 2017). Videos capture nuances not found in printed transcripts and reduce ambiguity in trying to denote what is in the mind of the teacher (Hiebert et al., 2002). West and Staub (2003) and West and Cameron (2013) provide videos of the MCFC process featuring expert coaches.

Reflection as a Professional Development Tool

Reflection enables us to make meaning of our experiences. When applied to learning, it is a "reflexive activity which enables the learner to draw upon previous experience to understand and evaluate the present, so as to shape future action and formulate new knowledge" (Abbot & Ryan, 2001, p. 58). In coaching, reflection is a necessary activity for change but must occur before any action is initiated (Askew & Carnell, 2011). It is important for coaches to use reflective discussions with teachers to bring about change, remembering that these conversations must be had in a caring and sensitive way (Askew & Carnell, 2011).

West and Staub (2003) consider reflection an important component of the coaching process that enables teachers to improve students' content-specific learning. West and Staub (2003) discuss how a coach facilitates productive and purposeful conversations centered on supporting students' mathematical learning and teachers' professional expertise. The coach pivots conversations around *what* content the students will learn, *how* the teacher will address these content ideas during the lesson, and *why* the teacher plans to teach content in a particular way (West & Staub, 2003; West & Cameron, 2013). For West and Staub (2003), the *why*, is particularly important. By asking questions about why the teacher plans to structure lessons in a specific manner, the coach encourages teachers to be reflective about their practice.

In his seminal work on reflective practice, Schon writes:

> A coach's legitimacy does not depend on his scholarly attainments or proficiency as a lecturer but on the artistry of his coaching practice. The question is not how much you know, but rather how effectively you can help others to learn. . . . I believe the most effective organizations of the future will be led by coaches committed to helping others learn (1987, as cited in Askew & Carnell, 2011, p. 1).

Description of the Project

Overview

Since the publication of *Content-Focused Coaching: Transforming Mathematics Lessons* (West & Staub, 2003), educators across the United States and Canada have acknowledged that content coaching is a powerful and effective approach to improving teacher practice in service of student learning (Gibbons & Cobb, 2016). MCFC is a very specific process that focuses on the core planning of instruction, teaching, reflecting on, and refining lessons. MCFC uses a three-part cycle: plan, teach, and debrief. Candidates in the Virginia Commonwealth University and

Longwood University mathematics specialist leadership courses video record themselves conducting a pre-conference and post-conference with a teacher in their school. Candidates also view videos that feature experienced coaches working with a variety of teachers (e.g., reluctant teachers, beginning or experienced teachers). Currently, the videos viewed by candidates at these two institutions were created by West and Staub (2003) or West and Cameron (2013).

MCFC Project Assignment Details

Candidates identified a classroom teacher to plan and facilitate one MCFC cycle. The MCFC cycle included: (1) preparing for the pre-conference, (2) facilitating the pre-conference, (3) observing and possibly co-teaching the mathematics lesson, (4) preparing for the post-conference, and (5) facilitating the post-conference. While engaging in the MCFC cycle, candidates maintained notes from the classroom observation of the lesson and from both pre- and post-conferences. After engaging in the MCFC cycle, candidates shared a video segment from the pre- and post-conference and their personal written reflection on the experience with their peers.

The Role of the Coach

Before the Pre-Conference

One of the most important considerations is the selection of the teacher a coach is going to work with, whether the teacher volunteers or is invited by the coach. Equally important are the topics covered in pre- and post- conferences that may vary for a number of reasons (e.g., the experience levels of the coach and teacher, school and district initiatives, personal and professional goals). Before the pre-conference, the coach should approach the teacher for a copy of the lesson plan or lesson topic. The coach explores the mathematics involved in the lesson and investigates possible pedagogical delivery vehicles. Engaging in these activities helps the coach develop a set of written questions to guide the pre-conference and create ways to further develop the teacher's mathematical pedagogical knowledge.

West and Cameron (2013) suggest coaches consider questions like time allocated for the meeting, prioritization of goals for the planning session, teaching and learning issues that present instructional challenges, and the teaching style or experience level of the teacher being coached. A thorough overview of lesson design with potential coaching conversation questions can be found in West and Cameron (2013). Coaches are encouraged to make a concerted effort to include research-based best practices for mathematics teaching, like the five practices for orchestrating productive mathematics discussions (Smith & Stein, 2018) and the seven effective mathematical teaching practices in *Principles to Actions* (NCTM, 2014).

Pre-Conference

The coach will video record and take notes during the pre-conference using the set of questions developed previously to guide the conversation. The coach may guide the conversation to emphasize the mathematics of the lesson, how the mathematics will be developed, and the mathematical learning outcomes for the students.

The Lesson

The coach will observe the lesson while focusing on not only the teacher's actions but also on student interactions, misconceptions, connections, and strategies. Although in practice, coaches often co-teach a lesson, we encourage our candidates to observe for their first coaching experience, looking for how the mathematics content is being taught as well as any other previously agreed-upon topics.

Post-Conference

The coach will video record and take notes during the post-conference. The coach will come with a set of questions that encourage the teacher to think about the teaching and learning that occurred in the observed lesson. The set of questions is not a script but a collection of ideas that encourage the teacher to think deeply about the lesson. Coaches should be mindful of keeping the conversation grounded in evidence of student learning (e.g., observational notes or student work) while attending to common student misconceptions and struggles.

MCFC Project Reflection

Candidates must reflect on all aspects of the MCFC cycle. The reflections center around four components. First, how the candidate and teacher develop mathematical content and pedagogy during the pre-conference. Second, how the candidate, after watching the lesson, suggests possible refinements that could be made to the lesson. Third, how the candidate examines student work and considers the teacher's plan for next instructional steps. Fourth, how the candidate determines next steps for their own professional growth as a coach. The reflection is both a summary of what happened during the MCFC cycle and a blueprint for next steps in the development of the candidate and the teacher they work with.

Reflections from Instructors on the Project

Instructor A

The coaching project is one of the integral projects in my mathematics teacher leadership class. This is the last leadership class in the mathematics specialist preparation program, and it is essential that the candidates have the opportunity to study and practice the art of MCFC. At this point in the program, candidates are ready to put into practice all they have learned about mathematical content and pedagogy. All of the candidates have to share their pre- and post-conference videos and give a presentation about their coaching experience. When the candidates in a recent cohort shared their videos with their colleagues, they began to see that everyone had some areas in which they could improve and where they had performed well. One thing that surprised my students was that they had many shared experiences, which I summarize next.

Candidates realized it was a privilege to work with other teachers, and they wanted to make sure the teacher knew they valued them as the classroom expert. Further, candidates realized that having a productive conference meant they had to be prepared and focused. Candidates recognized the need to become better listeners while focusing on the mathematics content and pedagogy. Lastly, candidates realized they needed to use more invitational coaching moves and less direct guidance moves. For example, an invitational coaching move would be one in which the candidate may ask the teacher, “What task were you planning to use to increase students’ understanding of equality?” as opposed to a direct guidance move in which the candidate provides a copy of an activity for the teacher to use in the lesson.

In my opinion, this assignment shifted the candidates’ view of their own identity to include the role of a mathematics specialist. I believe the candidates greatly benefit from this project. I would have preferred that my students have more than one opportunity to complete the MCFC coaching cycle during the semester, but time constraints make this impossible. A video repository would provide candidates with access to coaching videos made by mathematics specialists. Access to the repository would provide candidates with a reference to help prepare them for future coaching and build confidence in their coaching abilities.

Instructor B

I have used the MCFC assignment for many years in my role as an instructor for cohorts of teachers in the mathematics specialist preparation program. I think this is the most valuable activity coaches participate in during their leadership courses because it gives them the opportunity to practice “coaching.” Most of the candidates who participate in our project have similar opinions. In addition to the activity’s value to them as potential coaches, many participants find that it informs their practice.

One participant from a recent cohort remarked, “I had a lot of uncertainty about what coaching looked, sounded, and felt like. After going through one coaching cycle, I have a clearer picture of what coaching is, and how I see myself in this role” (Monique, personal communication).

Other participants liked it because it aligned with the same best practices they find critical for students:

> The content coaching cycle is the type of personalized professional development that our education system needs. We often talk about differentiating for students, but differentiation for teachers has never been a priority . . . I don’t think I would have known about any of Ms. C’s content needs if I had, for example, just led a workshop on strategies for teaching measurement. When prepared with good questions and resources, the content coaching cycle allows coaches to diagnose and meet individual teacher needs (Jordan, personal communication).

Still others mentioned the fact that the MCFC cycle allows the coach and teacher the opportunity to be reflective about their practice. “The content coaching cycle is important because it provides a time for the teacher to reflect. Whether you reflect on positive aspects, or areas for improvement, reflection allows for growth” (Cho, personal communication).

With that said, I think there are two challenges with the assignment. First, there are no video exemplars, reflections or interviews except for a few that have been done by professional coaches. Most students in the program find these polished examples intimidating as they often feel they do not realistically portray work performed by emerging coaches. Creating an online video repository will make this issue less problematic. The second concern is that after viewing pre- and post-conference videos with prospective coaches, they often ask me, “So, how did I do?” They want targeted feedback about the effectiveness of their conferences. I find my responses to this question very subjective. I can point to what research says should be the goal of conferences, but I know that having an evaluation protocol to help frame the conversation would improve my ability to give productive feedback. I think it would be helpful for me to know how coaches rate the effectiveness of their coaching sessions and what attributes of the sessions contributed to the overall effectiveness of the conference.

Conclusions

After studying the videos and reflections provided by participants while completing the MCFC project, we feel it is the most impactful of all projects completed in mathematics teacher leadership courses. It gives the participants an opportunity to coach and use many of the skills they have learned in their program up to this point. For many it is the first opportunity to do so. Creating their video and reflecting on the experience through the assignment prompts allows them to understand the role they will be expected to play as a mathematics coach. They learn how to offer their teachers personalized support that can have a positive impact on the work they do in the classroom. We also acknowledge that there are ways to improve the tools that support the project. First, we found that candidates and teacher teams who found their coaching conversations to be effective spent the most time employing invitational moves focused on mathematics topics. Indeed, West and Staub (2003) suggest that the goal of a coach, as they gain more experience, is to employ more invitational moves in coaching conversations. Likewise, they suggest the most successful coaching conversations occur when the focus is maintained on students' mathematical thinking and their work. Second, candidates would benefit from a video repository so that they could have examples of coaches at various experience levels as they employ best practices in their work (e.g., working with reluctant teachers, novice teachers, coaching tasks, and coaching small instruction).

Table 1
Video Analysis Template

Coaching Moves	**Topic**		
	Mathematics Content Pedagogy	Pedagogy	Other
Direct Guidance			
Invitation-Guidance (Mixed)			
Invitation			

Note. Numbers recorded in table should represent the percent of time coaches and teachers spent discussing conference topics and coaching moves used by the coach in guiding the discussions. Total of all time should be 100 percent. Once this protocol is formalized and evaluated, we hope it will be useful in providing constructive feedback for participants.

Future Research

One of our next steps is to refine and pilot a video analysis protocol developed to analyze pre- and post-conference videos. This video analysis template was created using information about coaching conversations found in the two MCFC texts (West & Staub, 2003; West & Cameron, 2013). These texts suggest there are two important aspects of a coaching conference, what we call “coaching moves” and “conference focus topics.” We arranged the coaching moves vertically in Table 1. The three categories of topics were placed horizontally in Table 1: (1) mathematics, (2) pedagogy, and (3) other. The Video Analysis Protocol template (Table 1) is the result.

An ancillary benefit of the study we have planned is that we will be creating a collection of MCFC project pre- and post-conference videos. These videos could be the genesis of an online video repository that would feature videos and reflections of pre- and post-conference coaching videos. We have initiated a partnership with the Virginia Council of Mathematics Specialists (VACMS) to host the planned video repository on their website. Videos will be available to mathematics specialists and mathematics teacher leaders in Virginia as a professional development tool.

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ONLINE EDUCATION: TRANSFERRING PERSONAL EXPERIENCES TO PROFESSIONAL DEVELOPMENT

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ABSTRACT
In this paper, we discuss how one candidate's experience as she participated an online mathematics specialist program bolstered her confidence and ability to provide online professional development for her teachers. We include personal accounts by the mathematics specialist program instructors, the mathematics specialist candidate, and an elementary school teacher to illustrate how the experience of completing online graduate courses led to the candidate providing online learning opportunities for teachers. In particular, we highlight the importance of building relationships and using high-quality mathematical tasks in both the online preparation program and the online professional development. This case study provides evidence that exposure to online learning environments as a learner can help lower the barrier of entry for planning and providing online learning experiences as a teacher.

KEYWORDS
online professional development, transfer of learning, relationships, high-quality mathematical tasks

One challenge that mathematics specialists face is finding time to provide professional development during teachers' overscheduled work weeks. In response to this challenge, online and hybrid models of professional development can be accessible and flexible alternatives to in-person professional development. We argue that mathematics specialist candidates who participate in online mathematics specialist courses are uniquely positioned to transfer their online experiences to the task of leading online professional development. Candidates can learn from instructors who are explicit about their pedagogical choices for online learning and who encourage mathematics specialists to incorporate best practices into their own online professional development for teachers.

This paper explores this transfer of practices from an online candidate to an online professional development provider through three perspectives: mathematics specialist instructor, mathematics specialist candidate, and elementary teacher. Erica R. Miller is an instructor at Virginia Commonwealth University (VCU) who has co-taught courses for a fully online mathematics specialist program. In this paper, she discusses the best practices she used in the online format and how they were made explicit to candidates. Tracy Proffitt completed the online VCU mathematics specialist preparation program and now works as a Lead Instructional Coach in an elementary school, focusing on mathematics instruction. She describes her experience as an online candidate, challenges she faces when providing professional development, and online options that she uses to meet those challenges. Elicia Fleshman is a teacher in the school where Tracy coaches. Elicia reflects on her experience as a learner in online professional development experiences facilitated by Tracy. This paper concludes with implications for instructors planning online courses, practicing mathematics specialists, and the next steps for further application of professional development using online tools.

When the terms synchronous and asynchronous are used below to describe modes of instructional delivery, they refer to the definitions provided in the preface of this special issue (Baker et al., 2021). In addition, a hybrid model refers to a combination of in-person and online instruction (Bates et al., 2016). These authors also describe five situations in which online learning is an effective mode of delivery, including two examples that align with the motivation to use online professional development in Tracy and Elicia's school: "teachers' immediate needs prohibit more powerful professional learning experiences" and "particular expertise is not available in a school or district but is available online" (p. 72). When the goal of professional learning is to prepare teachers to facilitate rich tasks for their students, teachers should participate in tasks as a central part of their learning and sensemaking, in both content and pedagogy (Hughes et al., 2015). Tasks are completed collaboratively, and collaboration among teachers has a positive impact on teachers' effectiveness (National Council of Teachers of Mathematics, 2014).

Individual Perspectives

Instructor: Erica R. Miller

Before teaching in the online, synchronous mathematics specialist program at VCU, I had only taught and taken face-to-face classes. However, I have always enjoyed using technology, so I was excited by the invitation to join the instructional team for the online cohort. For both courses that I taught, Jamey Lovin was one of my co-instructors. She had many years of experience teaching and coaching mathematics in the K–12 setting, while I had limited experience teaching undergraduate mathematics courses during graduate school. Jamey's K–12 experience, in addition to her experience teaching other courses in the online cohort, was an invaluable resource. Together we were able to craft our online course to focus on deepening the mathematical knowledge of the candidates in the cohort while also modeling student-centered, constructivist pedagogy through an online platform.

In designing the courses for the preparation program, our main goal was to model best practices that we wanted our online cohort to adopt and model for the teachers in their own schools. We focused on building relationships with the candidates in our cohort by offering virtual office hours and integrating "getting to know you" activities, like sharing recent personal and professional events. We also asked our candidates to work together virtually in small groups outside of class, which provided them with less-formal spaces to form relationships outside of our normal synchronous class meetings. Building upon the *Developing Mathematical Ideas* [DMI] (see Schifter et al., 2016) case-based curriculum, we integrated collaborative explorations of authentic, high-quality mathematical tasks (Smith & Stein, 1998). In order to engage candidates in asynchronous and synchronous mathematical activities online, we utilized digital mathematics platforms (e.g., Desmos Classroom Activities), cloud-based collaborative applications (e.g., GSuite), and our learning management system (Blackboard) to guide candidates through the activities and provide them with a space to share their work.

By asking our candidates to share their work on mathematical tasks, we were able to select and sequence different solution strategies and approaches in order to help them draw connections to the important mathematical concepts during our synchronous class meetings (Smith & Stein, 2018). For many of the mathematics activities, we provided links to online manipulatives that simulated the physical manipulatives that teachers often use in their own classrooms. Candidates were also responsible for posting responses to discussion questions (based on the DMI case studies) on our class discussion board, which provided us with another opportunity to integrate and build upon candidate ideas. During our synchronous meetings, we again used the familiar tools of digital mathematics activities (e.g., Desmos Classroom Activities) and cloud-based collaborative applications (e.g., Google Slides and Google Sheets) in order to monitor small groups as they worked together in breakout rooms. As an instructional team, we then selected and sequenced different groups to share their work during our whole class debriefs.

As candidates progressed through the program, we gave them more and more opportunities to take on leadership roles in different courses. In our final content course, we wanted to provide them with an opportunity to plan and lead part of our class. They had completed a group presentation project in one of their leadership courses, so we used a similar model in the content course. This project provided the candidates with the unique opportunity to lead a professional development session in an online format. Project groups signed up to develop and lead a 45-minute session on one of the chapters from *Connecting Arithmetic to Algebra* (Russell et al., 2011). To support their planning, we provided them with supplemental materials for the book as well as a facilitation plan template. We also met with individual project groups and provided them with feedback on their facilitation plan.

Each project group was encouraged to consider what high-quality mathematical tasks they wanted to include, what focus questions they would use for small and whole group discussion, why these questions were important, how these questions addressed their goals, what additional questions they might want to ask, and how they would adapt the session to meet the needs of the audience. They also were tasked with providing a clear outline of the presentation, all prerequisite work that participants needed to complete before attending the session, concurrent work they would complete during the session, materials and online resources that the participants would need, and a detailed breakdown of the schedule and presenter responsibilities. At the end of the course, candidates in the cohort shared the following reflections about what they learned from the group presentation project:

- The presentation gave me the opportunity to practice planning and implementing a professional learning opportunity for other teachers. I was able to collaborate with two others in creating a presentation that focused [on] making the connections with representations and the laws of arithmetic for students in the middle grades (Candidate 1, personal communication).
- I've learned how to really facilitate good mathematical discussions in the classroom, and I would be able to help other teachers incorporate these ideas in their own classrooms (Candidate 2, personal communication).
- I feel far more comfortable with professional developments and teaching functions (Candidate 3, personal communication).

Candidate: Tracy Proffitt

My experience as a candidate in the VCU mathematics specialist preparation program exposed me to multiple best practices for online instruction. Instructors consistently used rich mathematical tasks and case study reflections as a focus in both in-class and out-of-class work. Candidates practiced video conferencing during impromptu meetings between classes to discuss work and also in small groups during synchronous class sessions. Instructors modeled tools and structures for engaging students online, such as collaborative bulletin boards (e.g., Padlet) and interactive graphing platforms (e.g., Desmos). All candidates were given opportunities to lead online instruction within our cohort and were encouraged to implement best practices modeled by instructors. The learning around these online experiences was amplified by relationships built through frequent and required collaboration with other candidates.

Upon completion of these courses, I began my first year as a mathematics specialist eager to share mathematics content and pedagogical knowledge with teachers. Like other members of the cohort, it was a challenge for me to find time during school hours to meet with teachers due to limited common planning time and a lack of substitutes to cover classes for teachers. After-school hours were often not an option due to after-school programs, second jobs, or family responsibilities. Finally, providing access to experts in mathematics education through attending conferences, bringing in speakers, or purchasing books for a group of teachers can be prohibitively expensive.

After reflecting, I realized that I could leverage online instruction as a tool for meeting some of the challenges I was facing, rather than waiting for face-to-face professional development to become convenient. I tried a variety of formats for delivering mathematics professional development for teachers in my first year as a mathematics specialist. These formats and their advantages are described in Table 1.

Table 1
Professional Development Formats and Advantages

Format	Example	Advantages of the Format
Synchronous video course	Presented a "Pajama PD" workshop on the basics of number talks offered through video conference at a late evening hour.	Synchronous formats work well for professional development in which participant interaction is essential.
Asynchronous online video module	Recorded a video about rounding on a number line, prepared accompanying questions, and shared the module with teachers to complete at a time convenient for them.	Asynchronous modules can be created ahead of time and accessed by the teacher when they need to know the new content or skill.
Hybrid course	Facilitated an online course, Empowered Problem Solving (Kaplinksy, 2020), for six teachers. Some of the course work was completed together and some at home on their own time.	An advantage of the hybrid model is that it provides the opportunity for teachers to learn from each other while completing rich mathematical tasks, followed by reflection and further teaching from an expert online.
Video recording of an in-person training or lesson	Recorded trainings, book study meetings, or modeled lessons so that teachers who were unable to attend could watch them at a later time.	This is helpful for professional development that needs to be repeated each year for new teachers.
One-on-one coaching via video conferencing	Met with teachers virtually to discuss upcoming mathematics content or lessons observed.	This format works when traditional face-to-face coaching is needed but not feasible due to scheduling conflicts.
Podcast	Recommended specific episodes of a mathematics education podcast to teachers who wanted to learn more about a certain topic.	This format works well for teachers who have specific questions because their needs can be matched to episodes that they can listen to while driving, exercising, etc.

The online mathematics specialist courses I completed as a candidate impacted my facilitation of online professional development in several ways. First, participating in these courses and being required to lead classes through video conferencing helped to reduce my fear of using various online formats. Throughout my program, I was exposed to multiple tools and formats for collaborative, engaging, online instruction, and this familiarity as a candidate transferred into a willingness to try the same when planning and leading professional development. Second, as I planned various professional learning experiences, I tried to model my instruction after best practices I observed in my mathematics specialist courses. For example, I avoided the "sit and get" structure by incorporating frequent discussion opportunities and formative checks for understanding. When applicable, professional development included high-quality mathematical tasks, especially in the hybrid Empowered Problem Solving course discussed below by Elicia. Finally, I constantly considered the importance of building and sustaining positive, supportive relationships throughout professional development offerings. Much like I had learned to do as a candidate, I encouraged participants to ask questions, maintained a curious, facilitator disposition while leading activities, and focused on teacher growth in all situations. I also provided teachers with time to have one-on-one follow up meetings with me for reflection and individual goal setting.

Teacher: Elicia Fleshman

I have been an elementary educator for 20 years, and I am of the belief that as an educator it is incumbent upon me to remain a continuous learner. As a continuous learner, I have always had an interest in effective mathematics pedagogy and the potential impacts it has on student achievement. An opportunity arose to work with my school's mathematics specialist and other colleagues through a National Science Foundation Noyce-funded workshop. I welcomed the challenge to enrich my learning of teaching mathematics and to collaborate with other educators.

The Empowered Problem Solving Workshop (EPS) is typically offered as an asynchronous course. The EPS Workshop's mission was to bring to light the ineffective teaching strategies educators have been instituting for years, such as a heavy reliance on algorithmic instruction and rote memorization techniques. These antiquated techniques bypassed essential, rigorous, critical thinking instruction that promotes learner agency and engagement.
My colleagues and I met weekly to engage with the online course modules. During our meetings, we openly processed new learning goals through collaboration. We considered this format to be that of a hybrid model because we began the week's learning objectives together, processed new information, worked through high-quality mathematical tasks, discussed strategies, and then continued individual learning outside of contract hours to ensure we were adhering to program fidelity. Though the course was designed for individual teachers to complete asynchronously, our group's redevelopment of the course to a more hybrid approach met the diverse needs of each participant.

One of the key elements that made this experience successful was that Tracy, our mathematics specialist, was skilled at developing meaningful, trustworthy relationships. Also, she created an environment of shared responsibility, creativity, and varied skill sets in which each member of the team was able to share openly and honestly. Another key element that she brought to the table was her level of expertise. She had a firm grasp of the mathematics and technology fields, and that in and of itself was invaluable. Having a knowledgeable, capable, and approachable facilitator to guide us through the hybrid course helped to provide an element of ease in which each team member could apply the new knowledge to their own classroom.
Moving forward with my experiences regarding the aforementioned learning opportunities and the new norm of teaching and learning during the COVID-19 pandemic, I feel more prepared and confident within the realm of remote learning. Working with and learning from Tracy has

enabled me to pick up new and exciting ideas about online teaching and learning. The technological platforms themselves and how she modeled their use has led me to believe that these strategies are universal. They can be just as effective in online learning environments as in face-to-face environments. I feel more confident in taking risks with my instructional pedagogy than before we embarked on this journey.

Conclusion

Mathematics specialist candidates who complete online course work are exposed to a variety of best practices and tools for online learning. When instructors are explicit about the choices they make for online instruction, conversations may arise between instructors and candidates about the pedagogical and technological choices. This may lead to candidates transferring online experience into practice in effective professional development experiences for the teachers they serve. The anecdotal account described above prompted one mathematics specialist to explore synchronous, asynchronous, and hybrid options to meet the professional development needs of the teachers in her school building. Instructors and mathematics specialists are encouraged to consider ways in which online learning can be paired with collaborative tasks in order to increase learning and engagement (Bates et al., 2016).

The idea for this article originated before the COVID-19 pandemic forced closures of schools across the United States in March of 2020. As school systems faced the challenge of preparing teachers for online instruction, the conversation surrounding online professional development became even more widespread and necessary. Mathematics specialists who already have experience with the online learning format can pave the way in preparing teachers for concept-based, student-focused online and hybrid learning. In addition to group professional development, one-on-one virtual coaching (Matsumura et al., 2016) can also be explored and expanded. Another area that can be explored is the further transfer of best practices from online professional development into online mathematics teaching. Mathematics specialists who make their online professional development choices explicit to teachers may better prepare teachers as they navigate various online teaching platforms.

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VIRTUAL MENTORSHIP OF TEACHER LEADERS: THE RIPPLE EFFECT

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ABSTRACT

In this paper, the authors reflect on the first completely online mathematics specialist preparation and mentoring program. Candidates across Virginia successfully completed this program and are now serving as teacher leaders, interventionists, department leaders, instructional coaches, mentors, and program specialists. They are impacting mathematics instruction across the state at all levels. As two mathematics specialists serving as mentors and two candidates, we share our thoughts and ideas as we continue to learn from our mentorship process. The goal is to provide continuous professional development as candidates share problems, successes, research, and best practices to improve mathematics teaching and learning. In our situation, virtual mentoring is a vital support to long-term development, growth, and success of mathematics teacher leaders. Keeping in touch with fellow leaders has benefitted us personally and professionally. We will discuss the benefits and constraints of online mentoring and how it can be a model for other virtual mentorship programs.

KEYWORDS

online mentoring, teacher education, virtual learning, mathematics education, mathematics specialists, professional learning community

In August 2019, twenty mathematics specialist candidates completed Virginia Commonwealth University's (VCU's) first fully online mathematics specialist preparation program. The program was developed and offered as part of a National Science Foundation grant project entitled *Virginia Mathematics Specialist Initiative to Prepare K–8 Mathematics Teacher Leaders for High-Needs School Districts*. As outlined in the grant proposal, this project was

> designed to increase the number and retention of highly qualified, diverse mathematics teacher leaders in Virginia's high-needs, K–8 schools and provide an online professional development and certification program to prepare teachers for these roles. These leaders … support teachers in their schools and, in turn, help increase student achievement in mathematics. (National Science Foundation, 2017, p. 1)

This paper describes a mentoring program the candidates participated in following the completion of the mathematics specialist preparation program and shares the facilitators' and candidates' feedback about the successes and challenges of the mentoring program.

The candidates serve in a variety of roles including teacher, interventionist, department leader, coach/mentor, program specialist, or a combination of these roles. As shown in Figure 1, the school divisions in which the candidates work are diverse. The divisions represented are large and small. They are urban, rural, or suburban. They represent diverse populations of teachers and students across the Commonwealth of Virginia. The question for the mentorship program developers was, "How do we support and give professional development to new candidates in order to impact mathematics teaching and student learning across five geographical regions and 42,775 square miles?" The answer to this question was to create a virtual platform for monthly mentoring sessions. Figure 1 below shows the locations of each candidate within the state. The color of the pin represents the definition of candidates' territorial locations as described below.

According to William Haver, Ph.D., early developer and advocate for mathematics specialist training,

> …grant-supported cohorts tended to have mathematics coaches in the same school system. We did a lot of work with division leaders in preparing them to provide ongoing support and mentoring to their people once they were on the job as mathematics specialists. (personal communication, June 2020)

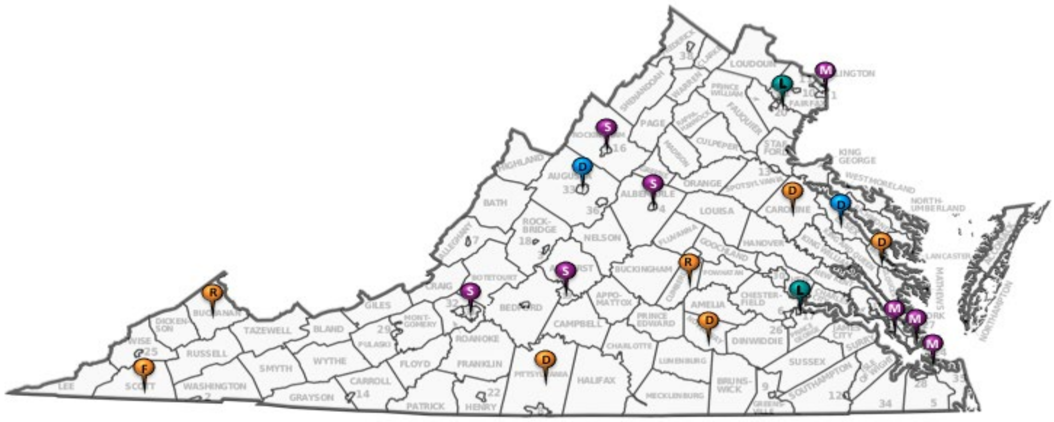
This was not an option for the online cohort. Aimee Ellington, Ph.D. devised a mentoring program to support the needs of candidates in their first years of work as mathematics specialists. The idea was to have a supportive peer group for coaches to rely on for problem-solving, success sharing, and collaboration (W. Haver, personal communication, June 2020).

Ellington, together with a group of mathematics professionals including instructors and previous candidates from Virginia Commonwealth University and George Mason University, brainstormed and planned for this different approach to mentorship. The mentors who facilitate the monthly virtual sessions are both National Board Certified, K–8 Mathematics Specialists. One works in a rural, middle school setting and one in an urban, district-level setting. Both have been instructors for previous cohorts in the mathematics specialist preparation program.

The goal of the mentoring program is to provide continuing professional development in a synchronous setting in the areas of mathematics and pedagogy to increase student achievement. Members share problems, successes, current research, and best practices to improve mathematics teaching and learning. During the first year, the focus was on building relationships with teachers and administrators to advocate for high-quality instruction. The discussions and activities that took place during the mentoring sessions were developed in part to reinforce and enhance the

Figure 1

Geographical Locations of Math Specialist Candidates in Cohort



Locale	Definition	Locale	Definition
City		**Town**	
Large	Territory inside an urbanized area and inside a principal city with population of 250,000 or more	Fringe	Territory inside an urban cluster that is less than or equal to 10 miles from an urbanized area
Midsize	Territory inside an urbanized area and inside a principal city with population less than 250,000 and greater than or equal to 100,000	Distant	Territory inside an urban cluster that is more than 10 miles and less than or equal to 35 miles from an urbanized area
Small	Territory inside an urbanized area and inside a principal city with population less than 100,000	Remote	Territory inside an urban cluster that is more than 35 miles from an urbanized area
Suburb		**Rural**	
Large	Territory outside a principal city and inside an urbanized area with population of 250,000 or more	Fringe	Census-defined rural territory that is less than or equal to 5 miles from an urbanized area, as well as rural territory that is less than or equal to 2.5 miles from an urban cluster
Midsize	Territory outside a principal city and inside an urbanized area with population less than 250,000 and greater than or equal to 100,000	Distant	Census-defined rural territory that is more than 5 miles but less than or equal to 25 miles from an urbanized area, as well as rural territory that is more than 2.5 miles but less than or equal to 10 miles from an urban cluster
Small	Territory outside a principal city and inside an urbanized area with population less than 100,000	Remote	Census-defined rural territory that is more than 25 miles from an urbanized area and is also more than 10 miles from an urban cluster

Note. Location designations and sizes by color on pins (Wikimedia Commons, n.d.). Letter on pins corresponds to size of locale. Legend (National Center of Education Statistics, n.d.).

connections that had formed among candidates while they completed their coursework. This communication was also designed to alleviate the feeling of isolation that candidates might experience when serving in the unique role of a mathematics coach in a school situation. Throughout the mentoring program, the mentors took a continual needs assessment to make the adjustments necessary to provide just-in-time professional development to benefit the evolving needs of the candidates.

Literature Review

Virtual mentoring is vital for the successful development, implementation, and long-term impact of candidate mathematics specialists in K–12 education. Paulus and Scherff (2008) state that “isolation and a lack of support” are two major challenges facing beginning educators that hinder success and lead candidates to abandon the profession (p. 113). Virtual mentors contribute greatly to overcoming these challenges by supporting mathematics specialists in navigating professional relationships with administration, peers, teachers, and students. Sherman and Camilli (2014) found this to be especially powerful when the mentor is an expert in mathematics or science (p. 114).

Isolation is overcome by actively participating in a mathematics learning community. This community connects educators who are geographically spread across diverse settings through a virtual, real-time network of mathematics specialists and mentors. Members communicate using a variety of platforms such as Facebook, text message, email, Twitter, Messenger, Zoom, and Google Suite applications. Li et al. (2010) found that “technological advances…provide a useful tool to facilitate mentoring” (p. 730). And Reese (2016) states, “the rapid boom in technology-based professional development and the myriad options for mentorship opportunities for educational mentoring and professional development experiences—via the Internet and various virtual technologies—are increasing exponentially” (p. 49).

The COVID-19 pandemic of 2020 has highlighted and reinforced the importance of virtual mentoring and learning opportunities. During this type of crisis, colleges, universities, and K–12 schools must limit or eliminate face-to-face instruction to protect students and staff. However, educators in this situation must continue to provide educational opportunities and support for all students (Hodges et al., 2020). Virtual mentoring provides the critical support K–12 candidate mathematics specialists need.

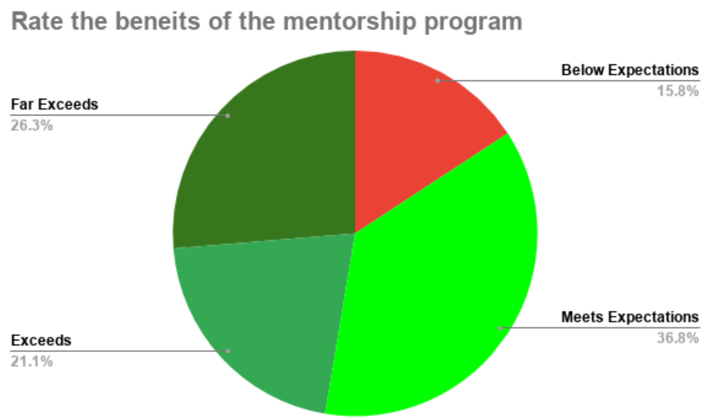
Methodology

Since completing the VCU online mathematics specialist preparation program in August 2019, candidates were provided continued support via a monthly online meeting with mentors. Experienced mathematics specialists served as mentors, and the candidates were their mentees. After the first year of the two-year mentorship program, participants completed a survey to provide feedback on the effectiveness of the program and to inform the plans for future sessions. The participants were asked to elaborate on the benefits and constraints of online mentoring and offer suggestions for improvement. Candidates were asked to share a specific instance in which they personally benefited from or were supported in their role as a mathematics leader. Additional questions were asked about communication methods, frequency, and using mentors as a resource between sessions. Follow-up interviews were conducted to clarify responses and gather anecdotal details.

Data Collection

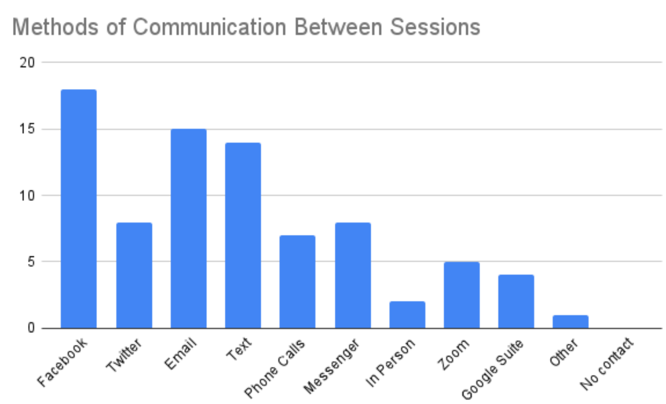
Candidates rated the mentoring program on scale of one to five, with 1 being below expectations, 3 meeting expectations, and 5 exceeding expectations. Figure 2 presents a circle graph of the survey data on how candidates rated the benefits of the mentorship program. Based on a rating of 3 or higher, 84% of participants felt the mentoring program was helpful to them with respect to the different types of professional relationships in which they engage. Of those candidates, 26% said the program far exceeded expectations.

Figure 2
Candidates Ratings of the Benefits of Mentorship Program



There are many platforms that were utilized for continued communication between mentors and participants. According to data from interviews, approximately 95% of respondents stated that Facebook was their preferred method of communication. Most participants belong to a private Facebook group for this purpose. The graph in Figure 3 shows that candidates utilized a variety of communications methods. The preferred methods for connecting were email and text messaging. The data point that stood out to the authors was that 100% of respondents were communicating with each other outside of the monthly mentoring session.

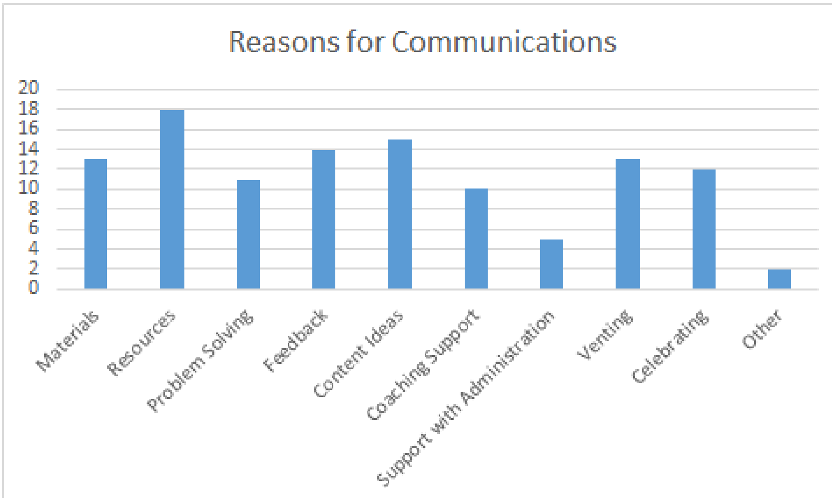
Figure 3
Methods of Communication Between Candidates



Note. All candidates are communicating between sessions in some form.

Figure 4 represents the reasons candidates were communicating and reaching out for support. Sharing resources ranked highest, with ideas for teaching content being the second most frequent reason. Candidates solicited feedback from their peers on ideas they presented and also felt comfortable venting frustrations with each other.

Figure 4
Reasons for Communication Between Candidates



Note. Candidates are communicating for many reasons.

Two questions were asked during candid interviews to gather information about communication among candidates and their peers: (1) How has the online mentoring program been specifically helpful for you? and (2) Can you share a specific time or situation where you reached out to your peers? In the following paragraphs, we will discuss the responses to these questions.

A sixth-grade mathematics teacher in a rural district explained that when her school announced in March they were transitioning to remote learning due to the COVID-19 pandemic, she reached out to the cohort through the private Facebook group for online resources and promptly received ideas from another sixth-grade teacher. A mathematics interventionist for two schools in an urban Virginia district on the opposite side of the state shared his district's website in response to her request for resources. He also directed her to a great website with quality activities**.** She was immediately grateful for the support. Based on this interaction, she knew the resources had been vetted by her peers. We wonder if this collegiality and support would have transpired prior to the online preparation program and monthly mentoring meetings. There was a significant geographic distance between these candidates. Candidates may have had the opportunity to meet at a state or national conference, but the ongoing building of relationships and efficacy would not have been as likely to occur. Another candidate explained how the small-group breakout times during online monthly sessions allowed participants to share ideas and strategies and provide feedback to each other, which she found very helpful: "Iron sharpens iron. We're all good at math. There's always something to be learned [from peers]" (personal communication, June 2020).

Two other candidates were interviewed and asked the aforementioned questions. One candidate was from an urban district in the eastern part of Virginia and the other candidate was hundreds of miles away in the southwestern part of the state. They described the great partnership that formed between them.

We found it interesting that candidates spoke of the comfort level they experienced using the online platform. A candidate shared, “It is easier for me to reach out online. I tend to be reluctant when it comes to face-to-face interactions. I’ve often felt self-conscious about my ‘twang.’ I’m much more comfortable talking to people online” (personal communication, 2020). This has implications for any teacher or student working through an online platform. One may become more engaged and open to sharing when in an online setting. Another candidate confirmed the overall sense of this powerful professional learning community that the authors had similarly deduced from interviewing candidates. “We have such a great network and wealth of knowledge. There’s always something to take away from others’ ideas” (personal communication, 2020). Each person has a part to play in this collaborative mentorship. The mentors learn from the mentees and mentees learn from each other. We agree that “well designed and managed mentoring programs can have a dramatic impact on workplace culture and people engagement. A strategic mentoring program transcends hierarchy, creating relationships and interactions to build individual and hence organisational value” (Art of Mentoring, 2019). In other words, mentoring programs can have a ripple effect on mathematics education.

Benefits and Constraints

This synchronous meeting format has benefits and constraints. According to our interviews, eighteen out of eighteen candidates stated they felt supported with the synchronous meeting format, and the meetings satisfied their expectations to date. Though the mentors and candidates are geographically distanced, this does not appear to be an obstacle because 100% of candidates stated mentors are always accessible in the online format. One of the positive comments shared several times by mentees was their appreciation for opportunities to share ideas and resources and to hear what is happening in other districts. One said, “Sharing successes and struggles with others in the same teacher leader positions confirms and validates what we are doing.” The meetings provide opportunities “to stay connected with cohort colleagues” and “encourage collaboration with people across Virginia.” A coach from a small city stated, “It opens doors to receive expert advice anytime and anywhere.” Being able to give and receive feedback, to share, and to grow together are significant benefits to meeting in an online format.

Even so, there are some constraints with an online synchronous meeting format that can provide some challenges. As one candidate put it, “Technology is wonderful when it works.” Trying to connect to a web-based platform from across the state can be difficult. There are technological challenges and certain candidates have to connect by phone and listen, not having access to the visuals being presented due to rural internet issues. To overcome this obstacle, meetings are recorded and are accessible to candidates at a later time. Data from our interviews revealed another issue: candidates are not all serving in the same positions at their schools. Some are mathematics coaches who work with teachers, others are mathematics leaders in their buildings who work with remediation, and others are classroom teachers. This provides a challenge with certain topic discussions, where individuals are coming from different perspectives and the information may not be relevant to their position. To address this, mentors set up breakout rooms based on teaching positions. In whole group discussions candidates shared topics discussed in breakout rooms. Anything shared can be adapted to different grade levels and situations. Mentors must be cognizant of the various roles of each member. Time can also be a challenge for these scheduled meetings. An elementary coach from a large city stated, “One of the constraints is time. Some of the best discussions we have are when we are in breakout rooms, but there isn't enough time to discuss in detail what each individual would like to share” (personal communication, June 2020).

The cohort stays in contact through many other online formats and often continues discussions after the mentoring session. The online synchronous meetings provide many positive opportunities and some challenges for this mentoring program.

Next Steps

The focus for year two of the mentoring program will be to have mentees take on leadership roles in the planning and delivery of the mentoring session activities. Because of our current COVID-19 social distance reality, virtual teaching and sharing will be a subject for exploration. Candidates will be asked to share monthly on a variety of topics including mathematics pedagogy and content, blended learning, building relationships, observations, collecting evidence of learning, and the impact of virtual teaching and coaching on student learning. The topics will be based on current needs, new research, and best practices. This will allow candidates to learn from each other and grow professionally.

Things to Consider

This model of coaching and mentoring can be adapted for other virtual mentorship programs. It can be scaled up or down to district or individual school levels and could pivot to different subject areas. Professional development on a virtual platform allows people across geographical distances to meet without traveling, saves time, and offers real-time mutual support.

When preparing mentoring sessions, facilitators may want to over plan, but remain flexible with the agenda. One never knows how long or short a discussion will be. Some topics demand more time from the group. Occasionally, a candidate will have a problem that needs the immediate attention of the mentors and candidates.

Use the collective knowledge of the members. Everyone has experience to share that will benefit the group. Become very familiar with the virtual platform being used. Technology issues will arise; attendees will need to be patient in order to overcome glitches.

Facilitators should be prepared to pivot completely. The ever present nature of the COVID-19 pandemic caused our sessions to move away from the topic of exploring coaching cycles to focus on virtual teaching and transitioning students to learn at home. It is important to remain consistent with mentor meetings. The group can continue to support each other through challenges.

Conclusion

In conclusion, this online mentoring program has been beneficial to the candidates. It is evolving and growing to build leadership capacity. While there are some constraints, they can be overcome. The basic purpose of this mentoring program should remain to support educators in content and pedagogy in order to increase students’ understanding and enjoyment of mathematics. Each candidate in this mathematics community is like a drop of water in a pond sending ripples across Virginia, impacting mathematics teaching and learning.

Acknowledgment

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PROVIDING JOB-EMBEDDED PROFESSIONAL LEARNING FOR MATHEMATICS SPECIALISTS

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ABSTRACT
We know that if professional learning opportunities are to be meaningful and create long-lasting and systemic change, they must be ongoing and job-embedded. One of the most beneficial aspects of having mathematics specialists in schools is that they can provide job-embedded professional learning directly to teachers. Perhaps due to the strong impact mathematics specialists have on teaching and learning, we may overlook the need to provide professional learning to support the growth of mathematics specialists themselves. Just as we provide coaching to teachers to affect their professional growth, we must identify similar opportunities to affect the growth of mathematics specialists. This paper will identify the purposes of these opportunities to include supporting growth in content knowledge, pedagogical expertise, coaching skills, and professionalism and leadership. We recognize that sustained efforts must be undertaken to see significant growth in these areas. Through interviews with individuals from Virginia school divisions and professional organizations, we identify models that can be replicated to provide the recommended professional learning for mathematics specialists.

KEYWORDS
professional learning, mathematics specialists, professional organizations, mathematics supervisors, job-embedded

Mathematics specialist preparation programs prepare teacher leaders to serve in this role in a similar manner to how teacher preparation programs prepare individuals to serve as classroom teachers. However, once teachers begin their work, they are, or should be, provided ongoing professional learning to support their continued growth (Campbell & Malkus, 2013). Too often, we find that mathematics specialists do not have the same type of ongoing support. One reason for this oversight may be that schools and school divisions traditionally designate days in the school calendar when professional development workshops are offered. Mathematics specialists are often called on to lead these workshops for teachers, thus preventing them from receiving assistance toward their own professional growth during this time. Yet, just as teachers need continued professional learning for their growth, school divisions must provide opportunities for mathematics specialists to continue their growth.

Guskey (2002) found the most beneficial form of professional learning for teachers is ongoing and job-embedded. Building on this finding, we posit that professional learning for specialists should strive to provide similar experiences. Professional learning experiences for mathematics specialists should target at least one of four broad areas: content knowledge, pedagogical expertise, coaching skills, and professionalism and leadership (Association of Mathematics Teacher Educators [AMTE], 2013; Campbell & Ellington, 2013). The modes and methods used to present this information may differ depending on the size of the school division and the number of mathematics specialists serving that division.

In this paper, we provide further information on the areas of professional learning needed for mathematics specialists and outline multiple delivery methods. Informed by interviews and surveys of school division mathematics specialists and their supervisors, this paper describes professional learning experiences that have been offered by school divisions and professional organizations across Virginia. The authors of this paper advocate for increased professional learning opportunities for all mathematics specialists and offer recommendations for how these opportunities can be provided in areas where there is not a large group of mathematics specialists.

Professional Learning for Mathematics Specialists

As noted earlier in this paper, the AMTE (2013) and Campbell and Ellington (2013) identified four broad areas of continued learning that will strengthen mathematics specialists' knowledge and skills to carry out their responsibilities. The four areas of learning and examples of how some school divisions in Virginia offer opportunities for professional growth are discussed below.

Professional Learning to Support Content Knowledge

Mathematics specialists must have a deep understanding of the mathematics content when working with teachers in their school building. An informative conclusion based on a study about teaching and learning in grades PK–8 and published in *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick et al., 2001) is that teachers need to deeply understand the mathematics they teach, how students learn mathematical ideas, and how to implement instruction that supports student learning. It reasonably follows that mathematics specialists who are supporting teachers in schools must also have this same deep understanding that teachers need, but for multiple grade levels.

We acknowledge that it is unlikely that a mathematics specialist would have direct experience teaching every mathematics course offered in a school. Because of this, it would be tempting to provide targeted support in understanding the curriculum for each mathematics course. While this approach has its uses, we believe professional learning should focus on what Ball et al. (2008) define as specialized content knowledge and horizon knowledge. Specialized content knowledge is the knowledge that supports structuring and representing mathematics concepts, identifying the mathematics that underpins an instructional task, and anticipating different ways students might think about concepts, including their misconceptions. Horizon knowledge is an understanding of the vertical progression across grade levels which is necessary for teaching a topic at a particular time. This knowledge also requires making connections to what content is to come.

To provide professional learning in specialized content knowledge and horizon knowledge, school divisions have often used rich mathematical tasks. One suburban school division frequently uses these tasks as opening activities during bi-weekly mathematics specialist meetings. This school division employs multiple building-level mathematics specialists who support teachers at one or two schools. The specialists represent elementary, middle, and high schools. The division reports that working on a task is especially beneficial with this diverse group of specialists due to the wide range of mathematics background that the specialists use to approach the task. The specialists begin by individually completing the task and anticipating student strategies and misconceptions. The specialists then engage in discussions about connections among the solution methods and the different mathematical ideas that have come out of the task. The specialists end with a discussion about how they could coach a classroom teacher to effectively implement the task in the classroom.

One Virginia superintendent's region (Virginia Department of Education, n.d.) uses region level meetings for multiple school divisions to provide professional learning for mathematics content to their specialists. The region also holds regular meetings where mathematics specialists work together to curate resources. While the meetings may have the appearance of working meetings, specialists all participate in deep discussions around the learning targets, essential questions, and the essential understandings of state and national standards.

Professional Learning to Support Pedagogical Expertise

Mathematics specialists must have knowledge of a variety of instructional strategies to support mathematics understanding. As the educational landscape moves toward increases in the use of digital curricula and virtual learning, mathematics specialists must have knowledge of the advantages and limitations these types of tools have to offer. The specialist must be able to guide teachers to use tools and strategies that enhance students' understanding of essential mathematics concepts.

Professional learning to support pedagogical expertise includes work with instructional strategies, planning lessons, and understanding student motivation. School divisions often approach professional learning in this domain through book studies. The books chosen for mathematics specialists' book studies might preview instructional methods and models to be rolled out to teachers throughout the division. For example, we have seen school divisions use this approach before implementing a new instructional model with teachers.

Another area for providing professional learning in pedagogical expertise can occur within professional organizations. From the authors' experiences, organizations such as the Virginia Council of Mathematics Specialists (VACMS) and the Virginia Council of Teachers of Mathematics (VCTM) have an easier time bringing in nationally recognized speakers to provide training to a statewide audience as part of a conference or meeting. As with any conference, it is imperative that mathematics specialists participate in follow-up events or conversations to ensure that the experience is ongoing rather than a one-and-done experience.

An urban school division in Virginia uses lesson study (Wang-Iverson & Yoshida, 2005) to support both content knowledge and pedagogical expertise. The division uses monthly meetings of mathematics specialists and mathematics resource teachers to carry out this process. The teams use student-level data to determine the concept and grade level to be studied. After identification of the underlying content, the teams plan a lesson, conduct the lesson, and then reflect on the effectiveness of the lesson.

Professional Learning to Support Coaching Skills

For mathematics specialists to effectively enact change in instruction, they must have the ability to work with teachers as adult learners. When working with an adult learner, it is especially important to devote time to building relationships and developing trust. These skills are part of a broader spectrum of coaching skills that must be supported through sustained professional learning.

As coaching skills may be universal to all instructional coaches and not specific to mathematics, some school divisions provide professional learning to support coaching skills to all individuals who serve as instructional coaches. Professional learning for these cross-content groups has been provided through conferences, book studies, and in-house opportunities.

A large suburban division provides professional learning to support coaching skills through peer observation. Groups of mathematics specialists conduct instructional rounds where they jointly observe several teachers in a school. Afterwards, the specialists debrief and reflect on the experience, focusing on follow-up coaching opportunities that may be necessary. As a variation on peer observation, coaches may also observe each other interacting with a teacher during a coaching cycle. Follow-up discussions focus on the coaching moves utilized and recommendations for ongoing coaching.

Professional Learning to Support Professionalism and Leadership

A mathematics specialist often functions in between two implicitly defined roles in a school—a classroom teacher and an administrator. Mathematics specialists must move in and out of both of those roles while doing their work. This transition between two seemingly different worlds is difficult and requires professionalism and leadership skills. Ongoing professional learning is necessary in this area, particularly as the mathematics specialist works with changing teaching staffs and administration.

Professional learning in this area can occur during mentoring opportunities between a new mathematics specialist and an experienced specialist. In a suburban division, first year specialists are assigned a veteran specialist as a mentor. During structured mentorship meetings, the new specialist shares specific cases and concerns, while the experienced specialist utilizes coaching language to help the new specialist navigate the concerns. Of note to mathematics specialists who may be the only specialist in their division or region, successful mentorship meetings have occurred virtually. In some cases, instructors of mathematics specialist programs have served as mentors of program participants after they begin working as a mathematics specialist.

Modes and Methods of Providing Professional Learning

Once a decision has been made about the specific content or focus area(s) for professional learning, we turn to the question of how the specialist will access these experiences. In some cases, specialists will benefit from participating along with teachers. Darling-Hammond et al. (2017) suggest that "teacher learning experiences should: (a) be intensive, ongoing, and connected to practice; (b) focus on student learning and address the teaching of specific curriculum content; (c) align with school improvement priorities and goals; and (d) build strong working relationships among teachers" (pp. 9 – 11). But, in addition, mathematics specialists need opportunities to reflect on their coaching practice and to engage in a professional learning community with their peers. The specialists need professional learning to address their specific role and responsibilities.

School divisions across Virginia have used various modes and methods to provide the professional learning described above. We recognize the type of professional learning experiences available to a mathematics specialist may be dictated by the size of the division and the number of mathematics specialists employed by the division. Divisions that have a larger number of mathematics specialists have the flexibility to provide ongoing professional learning experiences through regular meetings. However, Virginia also has school divisions with only one mathematics specialist. In this case, large-scale professional learning is not possible, and it may be up to the individual to assume responsibility for his or her own professional growth. We provide a summary of various approaches throughout Virginia that have been shared with the authors.

Group-Based Professional Learning through Regular Meetings

School divisions with multiple mathematics specialists can pull these specialists together on a regular basis. If these meetings are to occur, and we concur with McGatha and Rigelman (2017, p. 15) who strongly recommend they do, they must be purposefully planned to justify pulling mathematics specialists away from their other roles. Specifically, division-level leaders must use these meetings as an opportunity to provide ongoing professional learning in the areas we outlined in the first section.

These ongoing meetings provide an excellent opportunity for professional learning that serves each of the four purposes described above. Meeting time can be devoted to book studies, work with an outside consultant, peer observation and discussion, building content knowledge, and mentoring conversations. One division reports that professional learning can be obtained while creating and implementing division-level resources. This division had a year-long project where mathematics specialists developed cognitively demanding tasks and associated scoring rubrics. These tasks were then piloted in the specialists' schools. Throughout the process, teachers grew in their understanding of tasks and authentic assessment. At the same time, the project yielded student work that became the source of mathematical conversations among the specialists. As a result, mathematics specialists were able to grow in their practice by sharing ideas about how to work with teachers to determine their next steps in the classroom.

Virtual Opportunities

Some mathematics specialists may be in a position where they are the only specialist for their division. In this case, we advocate for a network of mathematics specialists in similar positions. Technology has improved to the point where this network can be virtual, connecting mathematics specialists from across Virginia. These virtual opportunities are particularly helpful when addressing professional learning to support professionalism and leadership skills. Care must be taken to ensure that these virtual check-ins are structured and purposeful. Virginia specialists who participated in the same university preparation program as a cohort have reported that they often meet virtually with each other after the cohort's official work has ended. It is helpful if the rationale for this way of networking is brought up during their course work. Some specialists who have attended the state professional conferences have shared that they continue to meet and consult virtually with specialists from other divisions. One specialist appreciates that she can meet with and gain ideas from specialists in schools and school divisions that are like hers as well as with those that are different from hers.

Professional Organizations

Professional organizations provide another avenue for mathematics specialists to network with each other. Virginia's mathematics specialist community has several state-wide professional organizations to turn to for guidance. These include the VACMS, VCTM, and the Virginia Council for Mathematics Supervision. Each of these organizations provides an annual meeting with opportunities for specialists to learn from established specialists and presenters and network with each other. In addition, these and similar national organizations (e.g., National Council of Teachers of Mathematics and NCSM) provide research and literature dedicated to mathematics coaching. These professional organizations also provide an opportunity for mentorship as new and experienced specialists look out for each other. Some division central office mathematics leaders attend along with the specialists and then use shared experiences during the conference to support follow-up in the school division. Specialists have also brought their administrators to conferences and used this time together to build a partnership and identify strategies that will move the school's mathematics program forward.

Individual Professional Learning

Mathematics specialists, whether they are one of many in a school division or work in more isolated circumstances, must also take the initiative to reflect on areas where they want to grow themselves and seek out opportunities to strengthen those areas. This may include reading journals, participating in conferences, accessing reputable web-based sources for professional learning, participating in grant-supported opportunities with local institutes of higher education, or reaching out to others in similar situations. Mathematics specialists, whether they are part of a larger group of specialists or not, are encouraged to seek out new research about what makes a mathematics specialist effective in their roles.

Conclusion

Throughout this paper, we have defined four different purposes for professional learning for mathematics specialists and explored different models for offering that professional learning. In no way do we insinuate that a professional learning experience should focus on only one of the four purposes. Instead, we recognize that the most effective professional learning may address all four. For example, the application of lesson study addresses content knowledge and pedagogical expertise through researching the mathematics standard and planning the lesson. Professional learning in coaching skills and professionalism and leadership is provided during the conducting of the lesson and debriefing on its effectiveness.

Similarly, we do not intend to state that one delivery model of professional learning is more effective than any other. We do stress that whichever model is used, there must be opportunities for ongoing professional learning that extends beyond traditional professional development workshops. Additionally, the opportunities should be job-embedded so that the specialist can have real-time feedback and learning on the support they are offering for teachers.

School divisions know that professional learning is crucial for teacher growth and thus require teachers to participate in a minimum number of professional improvement activities per year. School divisions need to recognize that professional learning is equally important for the growth of mathematics specialists and should have a similar requirement for completing professional improvement activities. It is the role of the school or school division to determine the quantity of professional learning activities required. It is also the role of the school or school division to provide or support activities that are designed for mathematics specialists to experience growth in one or more of the areas we have outlined. This action will produce mathematics specialists who are continually improving in how they support teaching and learning, resulting in teachers who are better equipped to provide instruction that allows for students to have deep and rich mathematics understanding.

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A RELATIONSHIP BUILT TO IMPACT INSTRUCTION: DEVELOPING AND SUSTAINING PRODUCTIVE PARTNERSHIPS BETWEEN MATHEMATICS SPECIALISTS AND PRINCIPALS

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ABSTRACT
How does the mathematics specialist provide a profound and lasting impact on instruction? We believe that a productive partnership between the principal and specialist, which we will call the principal-specialist relationship, is at the crux of the matter. When the principal-specialist relationship is built upon a foundation of a shared vision, clear roles, communication, and trust, both the teachers and students in the school benefit. We will explore the impact of the principal-specialist relationship on teacher success during the era of distance learning as necessitated by the COVID-19 pandemic. In order to explore how these ideas come alive in the field, we gathered survey responses and conducted personal interviews with mathematics specialists in a variety of roles. This article examines ways in which the principal-specialist relationship supports successful mathematics instruction beginning with a review of contemporary literature. In the form of short vignettes throughout the paper, we illustrate the roles of the mathematics specialist and how those roles were adapted for online learning environments. Our findings revealed that a unifying vision for mathematics instruction is essential for attaining maximum impact on student achievement.

KEYWORDS
principal-specialist relationship, productive partnerships, vision, trust, collaboration, communication

Alone we can do so little; together we can do so much.
— Helen Keller, as cited in Lash, p. 489

While Helen Keller is certainly not referring to school social dynamics, she adequately describes the impact of a mathematics specialist purposefully developing and maintaining a trusting partnership with the principal.

Productive partnerships are vital for mathematics specialists to be successful in their roles and to optimize their impact in schools. Moreover, the principal-specialist relationship may help to establish the specialist as a positive influence on instruction or result in a disruption in the specialist's development of trust and rapport with other educators in the building. A shared vision, clarity in communication and roles, mutual support, and established trust between the specialist and the principal yields an effective partnership. When all of these standards are achieved, the principal can expect the specialist to serve as a lynchpin, connecting administrative goals and initiatives to instructional decisions and pedagogical action.

The National Council of Supervisors of Mathematics [NCSM] (2019) points out that "relationships are the vehicle from which coaching is delivered" (figure 2.3g, p. 1). It is through this lens that we will examine the relationship between the mathematics specialist and building principal. Research and anecdotal evidence will illuminate the benefits of building positive and productive relationships with principals. We will also highlight how time spent getting to know each other can help in navigating unforeseeable obstacles such as those that resulted from the COVID-19 pandemic.

In this paper we will share results from research studies, illustrative stories, and interviews with teachers, principals, and mathematics specialists that highlight the importance of productive relationships. Qualitative data was collected data through an online survey sent to more than 30 mathematics specialists, coaches, and administrators. The combination of research and firsthand accounts allows us to describe the current context and emphasize the impact of the principal-specialist relationship on learning in schools.

Literature Review

Whether acting as an intermediary or instructional support, it is vital that the mathematics specialist establishes relationships and builds strong partnerships with administrators and teachers (Bengo, 2016). Davis et al. (n.d.) suggest that partnerships between the mathematics specialist and the principal can be developed by meeting regularly to share teacher success stories and relevant research while also discussing the goals for the mathematics program, mathematics content, and achievement data. Beyond communicating with the principal, quality partnerships between the mathematics specialist and teachers will depend on inclusive collaboration, personalized planning, and differentiated coaching (Campbell & Ellington, 2013; Inge et al., 2013). Regardless of the methods utilized to build the partnerships, these relationships are a lifeline for a successful mathematics specialist.

Trust is the glue that holds partnerships together. Sticking to the data and facts strengthens the trust in the mathematics specialist with everyone involved. The principal's confidence and trust is bolstered when the mathematics specialist offers relevant data from instruction, assessments, policies, research, and initiatives aimed at moving the school's mathematics program forward (Campbell & Ellington, 2013). Teachers embrace that same level of confidence and trust when the mathematics specialist utilizes data to maintain a focus on mathematics instruction and exercises discretion to avoid the pitfalls like gossip and breached confidentiality (Inge et al., 2013). This demonstration of knowledge and professionalism enhances collaborative work efforts and sustains credibility and trust within the partnerships.

Traditionally, mathematics specialists have provided face-to-face assistance, support, collaboration, and coaching (Rock et al., 2011). Statewide governances resulting from the COVID-19 pandemic called for an immediate conversion from traditional learning, instruction, and coaching to remote learning with virtual instruction and coaching for all mathematics specialists, teachers, and administrators (Natanson, 2020). Mathematics specialists were forced to change instructional support vehicles and proceed through uncharted territory while maintaining their partnerships with administrators and teachers. In places where shared understandings had already been developed regarding visions and roles within the program, mathematics specialists were more likely to navigate this change as well as other changes without a problem (Inge et al., 2013). Additionally, due to the anticipation of returning to school buildings for fall 2021, these established partnerships can be significant keys to sustaining professional development, support, and collaboration, whether in a face-to-face or virtual environment (National Council of Teachers of Mathematics [NCTM] & NCSM, 2020).

Establishing Yourself as a Specialist

Affecting sustainable positive change in a school can be achieved through productive relationships. While collegial relationships benefit classroom teachers in many ways, they are absolutely critical for the success of a mathematics specialist. Part of the mathematics specialist's role in the first few years in a school or district involves laying the groundwork for how relationships between the specialist, educators and administrators will be established and maintained in a way that leads to a dynamic education environment.

While there exists no specific formula for how to establish oneself as a leader in a school building, there are recommendations that have proven over time to be effective. Each school's environment and culture is different, so approaching the principal to outline the expectations of the specialist's role is an important first step. As indicated by a retired mathematics supervisor and mathematics specialist program coordinator, one of the biggest challenges arose "when the principal and the specialist held very different beliefs about what it means to know mathematics and the power of student-centered learning" (I. Vance, personal communication, June 1, 2020). If this happens to be the case, it is important to keep the focus on students' success to determine effective strategies for instruction and assessment. Once a shared vision is clearly defined, the specialist can then work alongside teachers to develop goals that are concurrent with the principal's vision. Without establishing a plan, the specialist could fall into reacting to issues that arise rather than acting as a proactive agent of change in the school building (I. Vance, personal communication, June 1, 2020).

As experience and research show us, being an effective specialist is reliant on taking the time to better understand the teachers and others in the school community that the specialist is servicing. Heather Nunnally, currently a teaching assistant professor at Virginia Commonwealth University, shared that during her first years as a mathematics specialist in a school, "being willing to help in any way that I could was one way I was able to maintain the relationships with teachers" (H. Nunnally, personal communication, June, 2, 2020). When teachers and administration see the specialist's resolve to achieve student success, it will serve to strengthen the developing partnerships.

After taking time to understand the established relationships within the school, the specialist should become a link between the principal and teachers. The principal's goals, while providing a wide-angle perspective of the school's vision, may be challenging to translate into practical application in classrooms. The mathematics specialist can digest the broader goals and break them down into actionable steps for classroom teachers. Conversely, while teachers may have difficulty voicing their day-to-day struggles with the administration due to the evaluative nature of their relationship, the mathematics specialist can synthesize the needs and concerns of the classroom teachers and approach the principal as a mediator after carefully considered everyone's ideas and reasonable requests.

The following is an excerpt from an interview with Ms. Keo, a district-wide instructional coach from a rural district in her second year, who shares her perspective on establishing oneself as an essential link between administration and instruction.

Ms. Keo Depicts Dynamics of Collaborative Relationships

> Teachers see me as an ally for their students and themselves. Administrators see me as supporting their efforts to provide a rich instructional environment with a focus on teacher and student issues and needs. Bridging the gap that can exist between administrators and teachers provides for a better instructional environment and builds trust for my own role, as well as between others (A. Keo, personal communication, June 16, 2020).

The Impact of Vision on Progress

A principal with a clear, achievable vision for the school's growth in mathematics unlocks the possibilities for what a specialist can accomplish in a school. In the following vignette, Ms. Keo describes her work with an administrator with whom she had positive rapport, who had recently become the school's new principal. She discovered that vision can be an anchoring feature in the principal-specialist relationship.

Ms. Keo Emphasizes the Importance of a Shared Vision

> This principal really had a strong vision for strong instruction…taking these traditional teachers and really pushing them to move more into current and progressive teaching,… small groups, really trying to do more inquiry, really trying to do more performance tasks, really trying to move away from [the traditional method of] stand and deliver and practice. She has such a great way of challenging her teachers in a positive way, and so her attitude has helped [mathematics specialists] take the lead and facilitate some of this change in some of her departments… Her leadership and her vision and, I think, her [prior] experience in elementary school has really helped lead the charge. And she has created such a positive buzz about us and how we can be a resource and support for teachers that [her efforts have] just kind of kicked off [an effort to generate] the amount of support that we can have. She knows her teachers are good and she wants them to feel supported as she challenges and pushes them further. I think that strong vision…I think it makes or breaks a school (A. Keo, personal communication, June 16, 2020).

When a principal's vision is clearly communicated, whether as expectations or aspirations, the mathematics specialist can take action knowing that administrative support is present and strong. Ms. Leath, a veteran mathematics specialist from a large suburban district, illustrates the importance of clearly communicated expectations in the following interview response.

Ms. Leath Illuminates the Impact of Clear Expectations on Progress

> I've had an experience where the principal, at one point, you know, I didn't think she cared for me…Her teachers weren't listening to her so she wasn't too sure how much they would actually listen to *me*. And then when I told her, "give me a chance, you know, I'll back you, I'll support you, what do you want to see when it comes to math?" And she was like, "I wanna see this, I wanna see this, I wanna see that," and I'm like, "let's do it!" And she's like, "you don't know these teachers." And I'm like, "what do *you* want to see?" And she's like, "I wanna see those things," and I'm like "let's do it."…Being a mathematics specialist is not for the faint of heart. You have to know how to be able to smile, but you're also not a "yes" person all the time. People have to understand that you're there to accomplish something. You're not there to become people's friends and people's buddies, so I'm not going to say yes to everything. …I tell them this is what your principal wants, and I'm here to make sure it gets done. Because why? This is what's best for our children… I tell you within a year, they changed, and the principal just rode the wave. But all I needed was the backing of administration. And the fact that she wanted something so desperately to [take place] in her school. We were a perfect duo. …It was those four words: What do you want? You tell me what you want and I will let you know if I can't do it. And if I can't do it, I know a slew of people who can help me get it done… And honestly what I shared with you is no different. If you don't have clear expectations as a teacher in your classroom for your students, what is going to happen? But you go across the hall, and you look at somebody who has clear expectations and makes it known, it's a different story. It's not that these children are "better" or whatever. It's just that the expectations are *clear*. They are specific and they speak to whatever the goal and the objectives are. That's it! (J. Leath, personal communication, July 9, 2020).

The previous vignettes exemplify the standards needed for effective partnerships. The following will highlight the importance of purposefully maintaining this newly developed and productive partnership.

Maintaining Trust and Relationships with Teachers and Principals

When working as a mathematics specialist, there is a challenging duality between maintaining the trust that has been developed, while also serving as a bridge between the different needs and different agendas of the principal and teachers. While infrequent, there are times when, after coaching a teacher and not seeing improvement, the specialist may have to consult with the principal about the next steps regarding inappropriate teaching practices. However, the specialist must be careful not to view their teachers with a deficit mindset.

> At the end of it, we all have our strengths and we all have our weaknesses. As a specialist I have to understand that nobody has it all. And the only time when you have to break that "trust" is if there is danger. And when I say danger I mean…I don't want to call

> teaching 2009 [National Council of Teachers of Mathematics] Standards when you are supposed to be teaching 2016 [NCTM] Standards dangerous, but if I'm telling you not to do that and giving you something to do instead, and you're still doing your own thing, you do need to be called out on the carpet. (J. Leath, personal communication, July 9, 2020).

Ms. Keo provides another perspective on the importance of balancing trust while maintaining high standards.

> Teachers and administrators are looking at different pieces to the puzzle. We're all working on the same puzzle, we're just coming at it from different angles and so somebody's got to hold up the box to say, "Look! This is what it looks like in the end! You're all working on the same thing!" (A. Keo, personal communication, 2020)

Maintaining The Principal-Specialist Vision at a Distance

How can the mathematics specialist continue to partner with the principal in order to help realize their vision for the school at a distance? As COVID-19 spread through the United States in 2020, Virginia school districts sought to keep children safe from this potentially deadly disease. Schools were deemed unsafe, and for the first time in history, teachers were responsible for encouraging students' academic development in a virtual learning environment. As a result, mathematics specialists became even more essential to crafting successful lessons. However, working remotely meant that the trust that had been previously developed between the principal and specialist was tested. Remembering the principal's key values and the metrics for success enabled the specialist to make decisions about instruction, assessment, pedagogy, and technology without needing to knock on the principal's door for input.

In the vignette that follows, Mr. Potter, a school-based mathematics specialist with six years of experience in a rural school district, highlights the importance of having clarity of vision, trust, and mutual support with his principal to realize the potential for positive effects on students during a global pandemic.

Mr. Potter Maintains Relationships During COVID-19

My elementary school, like many, abruptly closed in March of 2020 for what we were told would be two weeks and, at present, remains locked. Because the principal and I have a shared understanding of each other's priorities, our entire school was better able to navigate the unexpected closure (Inge et al., 2013). I know that my principal loves his teachers and their well-being is of utmost importance to him. He believes that, when teachers feel loved and supported, students will feel likewise. To ensure the love and support continued, each week our staff was invited to come together virtually through Google Meet just to talk. We had a weekly highlight in which, for example, the music teacher led a sing-along or the P.E. teacher facilitated a warm up activity. These activities were designed to bring us closer while our physical distance remained. Each week we listened and shared stories about our newly adopted pets, our children and grandchildren, and how we were all spending our time. There were tears and laughter. Some stood outside the school for the Wi-Fi connection, and others drove to other places to achieve a better Wi-Fi signal. We all needed to connect to our faculty-community. And yes, our principal attended regularly with his grandson in his lap.

During these weekly staff connection meetings, many teachers asked specific questions about delivering mathematics instruction at a distance. One teacher connected me to her grandson after the weekly meeting was over so that he could receive tutoring on long division. Another teacher asked if I would be a guest teacher for her gifted mathematics class. We co-planned and co-taught multiple classes through Google Meet as a result of that request. But the instructional decisions that were made during that time with long lasting impact are yet to be seen. Every teacher from preschool to the gifted sixth grade class in my district was asked to develop a comprehensive learning plan in the event that students are not able to return to class in the fall of 2020. At every level, I was asked for resources and advice on how best to help our students learn. Lead teachers across the district invited me to be a part of the development meetings. Each of these instructional opportunities stemmed from our weekly social connections at a distance. It was because of our mutually strong principal-specialist relationship that I knew my principal would want his staff to meet together in any way they could and he knew that I would coordinate and facilitate positive conversations. Because of our shared trust, vision, and mutual support, we both could be confident in each other's actions without needing approval from one another.

As Ms. Keo explained, "A principal's role is very important in building and maintaining a thriving network of instructional support within a school" (A. Keo, personal communication, June 16, 2020). This network is strengthened as the mathematics specialist meets the needs of the administrators.

The common denominator for success while engaging in virtual learning has been communication. This collaborative element has been the key to unlocking the true potential for both educators and students in a face-to-face or virtual learning environment. While for some, the relationships that were already established before the COVID-19 pandemic necessitated closing schools were the only ones that grew during our forced distance-learning experiences, Mr. Potter was able to facilitate a virtual space that fostered new relationships leading to more coaching opportunities.

Conclusion

A shared vision, clarity in communication and roles, mutual support, and building trust between the specialist and the principal yields an effective partnership. This principal-specialist partnership paves the path for supporting instruction and, ultimately, student success. The mathematics specialist holds a unique position as an instructional leader who also serves in a supporting role. It is this paradox that can yield tremendous outcomes when carried out with purpose in a team environment.

This analysis was conducted in the spring of 2020 during the start of the COVID-19 pandemic. Teachers were faced with the hurdle of delivering quality instruction during abrupt, seemingly short-term school closures across the country. Our findings were impacted by those circumstances. This leaves an opening for future research regarding ways in which the mathematics specialist can build and sustain quality relationships with school personnel during long-term or perhaps permanent distance-learning arrangements.

For the mathematics specialist, the time and effort invested in building a positive and productive relationship with the principal will be worthwhile. We urge all new and existing mathematics specialists to reevaluate their relationship with the principal to ensure that you realize its full potential. Find common ground, even if it requires some searching. Clarify values and the expectations for each person's role. Ask what the principal needs and deliver creative solutions that lead to the realization of a shared vision, even in the midst of a global pandemic.

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A CASE OF MISALIGNMENT OF REASONING, AFFECT, AND PERFORMANCE IN THE TRANSITION-TO-PROOF

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ABSTRACT
Learning how to prove is known to be difficult for undergraduate students. Understanding students' growth in the multiple arenas that make up proving is crucial for supporting them. Across four interviews over a semester, I examine one student who showed growth in his reasoning but whose proofs were still incorrect, yet he showed high levels of positive affect including confidence throughout. Investigating this single-subject case serves as an example of the interplay between development and performance. The question of whether we can say this student is a better prover than before—fundamentally, how to weigh reasoning versus affect versus performance—motivates the need for robust frameworks to characterize a student's progress in proving.

KEYWORDS
Transition-to-Proof, Problem Solving, Affect, Undergraduate Students

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Learning how to prove is well known to be difficult for undergraduate students (Moore, 1994), as there are multiple components that comprise the activity we call "proving" (Mejia-Ramos & Inglis, 2009). One such component is reasoning about logical statements in order to justify and write arguments, which is a shift from computation and exercises in students' mathematical experience (Smith et al., 2017). There is also a strong problem-solving component to proving (Stylianides et al., 2017; Savić, 2012), where the solution path is not apparent from the start and not all mathematical work and reasoning is included in the final written product.

Additionally, student affect (beliefs, attitudes, emotions, etc.) is also a component of learning how to prove successfully, as maintaining feelings of enjoyment (and a sense of self-efficacy) with mathematics during this process can be difficult (Smith et al., 2017). This is yet another transition for students, as they often come to transition to proof courses viewing themselves as "good at mathematics." However, as United States students have little prior experience with proving outside of high school geometry (Anderson, 1994), they often feel frustrated with this new mathematical work, as well as showing other forms of negative affect (Smith et al., 2017).

While much is known about students' errors (e.g., Selden & Selden, 1987), less is known about students' growth—how the learning process of proving unfolds over time. Understanding students' growth is crucial for helping undergraduate students through this difficult transition point in their upper-level mathematical career. Through more research on the various stages students step through while learning how to prove, instructors can better design transition-to-proof courses to support undergraduates along these expected pathways, as they grapple with these difficult mathematical ideas. There is also a need for frameworks to assess students' proving skills and processes (Savić, 2012; Selden & Selden, 2007): "We need a richer framework for keeping track of students' progress than the everyday one" (Selden & Selden, 2007, p. 1).

I present a short-term, longitudinal case of a student, Leonhard, whose reasoning, performance, and affect while learning how to prove are out of alignment in an unexpected way: the growth he shows in proving is not captured by his performance, yet he shows high positive affect throughout. I analyze his decision-making (reasoning), the correctness of his proofs (performance), and his emotions (affect) to illustrate how a student can have sophisticated decision-making and an overall high confidence yet not produce correct proofs. In doing so, the aim is not to fault Leonhard but to consider that written work, especially for proving, does not necessarily capture students' growth in crucial thinking processes—and that a robust framework to assess all the facets of students' proving skills and processes is needed.

Background & Conceptual Framework

There are multiple perspectives from which to approach research in proof (Stylianides, Stylianides, & Weber, 2017). One common perspective is to consider proving as a form of problem solving (e.g., Savić, 2012). However, the relationship between proving and problem solving is not purely that of one being a subset of the other.

Selden and Selden (2007) discussed two major sources of difficulty for students when writing proofs. The *formal-rhetorical* aspect of proving involves the logical structure of the proof, e.g., determining the first and last lines of a proof. Meanwhile, the *problem-centered* aspect of proving involves the decisions and key insights made to solve the embedded problem at the core of a proof (Raman, 2003), oftentimes with no set procedure. Both aspects are necessary for students to interpret mathematical statements and prove them, although students may favor one approach to proving over the other.

The formal-rhetorical versus problem-centered dichotomy parallels the notion of syntactic versus semantic proof production (Weber & Alcock, 2004). Under *syntactic proof production*, a person generates a proof by attending to the logical structure of a statement, oftentimes through manipulating symbols. In contrast, *semantic proof production* is where a person attends to the meaning of the mathematical objects and concepts in the statement to formulate the steps of a proof. While the specifics of a mathematical statement may lend themselves to one approach over another, it is important that students can work both syntactically and semantically in learning how to prove a variety of statements.

In terms of statements, students learn to determine the meaning of not only formal but also informal mathematical statements in the transition-to-proof. Formal statements use quantifiers, an *if-then* structure, logical operators such as *and*, *or*, and *not*, etc. Students must also learn how to unpack the meaning of *informal statements* (Selden, 2010; Selden & Selden, 1995), which are not written in their purely logical structure and may use words where mathematical meaning is inferred. For example, "All multiples of 6 are divisible by 3" is an informal statement in that to formally prove this, a person must infer the logical meaning of "all" and "are." There can be degrees of informality, in that one statement can be more informally worded than another. Students will see informal statements in their mathematical future: "Such statements are commonplace in everyday mathematical conversations, lectures, and books. They are not generally considered ambiguous or ill-formed, apparently because widely understood, but rarely articulated, conventions permit their precise interpretation by mathematicians and, less reliably, by students" (Selden & Selden, 1995, p. 127).

Students working with informal statements to identify their meaning and the analogue formal statement to then use in a proof is a crucial part of learning how to prove. Given a formal statement, there are a myriad of differently phrased equivalent informal statements. Selden (2010) reported students' struggles with informal statements: "When asked to unpack the logical structure of informally worded statements, but not to prove them, U.S. undergraduate mathematics students, many in their third or fourth year, did so correctly just 8.5% of the time" (p. 7). Yet, informal statements may help students build an intuitive understanding of the meaning of concepts and how they relate to each other (Selden & Selden, 1995).

Conceptual Framework: Reasoning, Performance, and Affect

I adopt the perspective of proving as a form of problem solving and draw from its literature base. Research on problem solving is vast and was a common theme of mathematics education research in the 1980s and early 1990s (Schoenfeld, 1992; Silver, 1985). Since Polya's (1945) work on problem solving, a number of theoretical frameworks for investigating problem solving have been created that build off that lineage (e.g., Carlson & Bloom, 2005; Garofalo & Lester, 1985). Schoenfeld (1992) identified five components of problem solving: a knowledge base, problem solving strategies and heuristics, monitoring and control, practices, and beliefs and affect.

Based on Schoenfeld's problem solving work, I take a three-pronged approach to analyzing student growth in proving by looking at aspects of reasoning, performance, and affect. Within reasoning, I focus on students' decision-making for how they choose which proof technique to pursue when constructing a proof. Proof techniques include direct proof, cases, proof by contradiction, and proof by contrapositive. Students' rationales for their approaches do not of course encompass all of what it means to reason when attempting to prove a statement, but the act of decision-making is a clearly defined moment of reasoning. The choice to study this aspect of reasoning comes from more extensive findings about student proof development from Satyam (2018). Performance refers to whether the mathematical proof the student produced was correct or if there were invalid mathematical steps.

Lastly, within affect, I focus on emotions. Affect is generally thought of as the domain involving feeling (Middleton et al., 2017), including beliefs, attitudes, emotions, motivation, engagement, confidence, etc. Beliefs, attitudes, and emotions as a trio have commanded attention, but among these three, emotions remain a relatively understudied subfield (McLeod, 1992). Emotions may be described as "rapidly-changing states of feeling experienced consciously or occurring preconsciously or unconsciously" (DeBellis & Goldin, 2006, p. 135). Emotions can be seen as responses to events; they tend to be short in duration but can reach high intensity. This leads to methodological difficulties in collecting data on and studying them. However, understanding emotions is crucial for understanding other affective structures with strong ties to learning: repeated emotional responses of a kind (positive or negative) may influence deeper-seated affect, like attitudes and beliefs (Grootenboer & Marshman, 2016; McLeod, 1992). Emotion may therefore be a vehicle through which to enact affective change.

I examine aspects of one student's reasoning, performance, and affect over the course of a transition-to-proof class. The purpose of this case is to illustrate how growth in reasoning does not necessarily lead to correct work, even in a proof course, and is moreover not captured by written work, and to examine implications of this situation when coupled with high confidence.

Methods

This work is part of a larger study focusing on the cognitive and emotional aspects involved in the transition-to-proof (Satyam, 2018). The full set of participants were N = 11 undergraduate students taking a transition-to-proof course at a large, public Midwestern university. The transition-to-proof course was designed to ease the change from computation-based courses to upper-level mathematics courses that involve writing proofs. The content taught in the course included logic, quantifiers, proof techniques (direct proof, proof by cases, proof by contrapositive, proof by contradiction, mathematical induction), and it provided a sampling of topics from analysis, linear algebra, and number theory. The population was students majoring or minoring in mathematics.

A series of four, semi-structured, task-based interviews was conducted with each of the participants across one semester. Interviews were spaced two to three weeks apart. Students had seen all proof methods by the time of the first interview. Within each interview, participants were given two proof construction tasks, where they were given a statement and asked to write a proof for it.

Design of Proof Construction Tasks

All proof construction tasks were on basic number theory: properties of integers and real numbers, even and odd integers, divisibility, etc. Tasks were designed so that the content area would be the same and to minimize any special domain knowledge as much as possible; care should be taken, however, in making content-free claims about proving (Dawkins &

Karunakaran, 2016). One proof construction task in each interview used a definition to test students' skills at making sense of definitions; however, students had often been exposed to these definitions earlier through homework.

Tasks were also worded to incorporate some degree of informality, given the importance of informal statements in proving (Selden, 2010; Selden & Selden, 1995). An example of this can be seen in Task 3: *Suppose x, y, and z are positive integers. If x, y, and z are a Pythagorean triple, then one number is even or all three numbers are even*. The conclusion, *one number is even or three numbers are even*, is an informal statement, as it formally means exactly one of x, y, and z is an even integer (and exactly two of x, y, and z are odd integers) or x, y, z are all even integers. A more informal conclusion to the statement could be *one is even or all three are even*.

Data Collection

Participants were given fifteen minutes to construct a proof. Each proof construction task was administered as a think-aloud (Ericsson & Simon, 1980). Students were asked to verbalize their thinking, but the researcher did not ask questions while they were working in order to not interrupt their problem-solving process (Schoenfeld, 1985). Instead, a debrief was conducted with the participant immediately after each task, during which they were asked questions about their thought process, places where they perceived they were stuck, and other points of interest. Participants were not told whether their work was correct or not unless they asked after the interview was over.

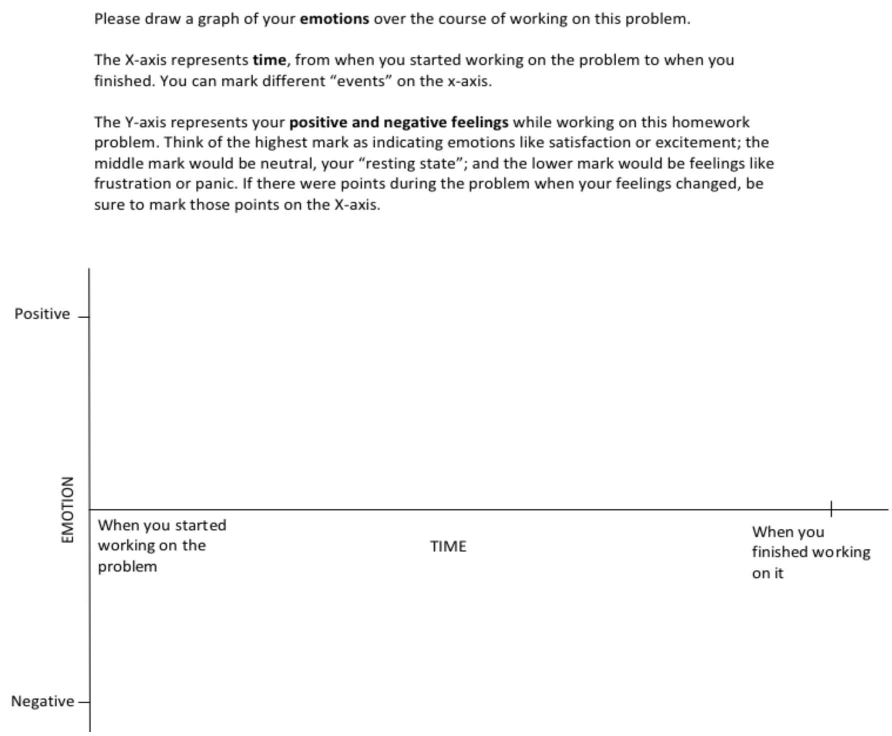
Participants were also asked after the task to talk about the emotions they experienced while constructing the proof, through an *emotion graph* task (adapted from McLeod et al., 1990 and Smith et al., 2017). Students drew by hand a graph of their emotions over the course of the task, where the x-axis represented time and the y-axis represented the intensity of emotion felt (see Figure 1). Students also textually annotated their graphs to describe what was happening at a certain point, the reason(s) for a shift in emotion, or specific emotions.

Data collected and analyzed here include the audio- and video-recorded think-aloud and debrief portions of the interviews, student written work, and their emotion graphs. From the audio-recordings, the interviews were then transcribed. Students' verbal responses were analyzed for their reasoning, and emotion graphs were analyzed qualitatively for dips and rises. A coding rubric was developed for assessing performance (correct, partially correct, or incorrect) on the proof construction tasks but is not used here due to the single case structure of this study.

Case Study

In this work, I examine a single participant, Leonhard, as a case study. In keeping with case study methodology, this work does not generalize nor is it representative of the data set. Leonhard serves as a *unique case* (Yin, 2009) of a phenomenon and is why I discuss a singular case (rather than compare and contrast multiple cases). Across the set of participants, some participants showed strong reasoning, performance, and affect from the start, some struggled with these throughout, and some showed gradual growth across these three metrics. I have chosen Leonhard's case in particular due to his atypicality from the expected development: he grows in reasoning but not in performance, yet shows high affect throughout. His case serves as an example where reasoning and performance are in misalignment, showing how assessing a student's growth in proving can be difficult.

Figure 1
Blank Emotion Graph as an Instrument



Leonhard was a white male freshman majoring in mathematics. He wanted to be either a high school teacher or a mathematician in aerospace engineering. Leonhard had many thoughts about mathematics, which he effusively shared. He chose his own pseudonym, Leonhard, after Leonhard Euler, which shows the extent to which he enjoyed and identified with mathematics.

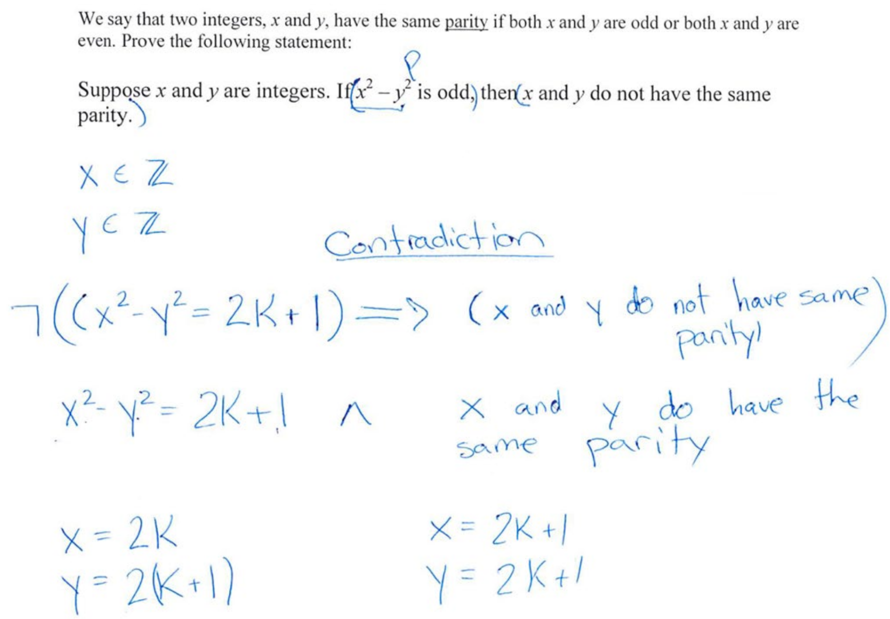
Results

I trace through a task from each of Leonhard's four interviews to illustrate his affect and the growth in his decision-making as reasoning in response to each task. As there were two tasks to choose from for each interview, I selected the tasks in the following way. The first three tasks concern proof by contradiction, so we may see how Leonhard's reasoning particular to that technique changed. The last task concerns proof by contrapositive, to show that his decision-making extended to other proof techniques as well. Leonhard had seen all proof techniques in class by the first interview.

Interview 1: Little Rationale for Choice of Proof Technique

In the beginning, Leonhard's baseline practice was to choose proof techniques based on what he knew and was familiar with. The first task of the first interview was to prove the statement: *Suppose x and y are integers. If* $x^2 - y^2$ *is odd, then x and y do not have the same parity.* The definition of two numbers having the same parity—both being even or odd—was given in the task. Leonhard was stuck on how to start the proof. Having seen all standard proof techniques in class at this point (direct proof, cases, etc.), he decided to use proof by contradiction, despite not being sure how to negate the conclusion. He carefully wrote down the parts of the statement to find its negation (see Figure 2). His rationale for his choice of proof technique was, “A lot of time in class whenever we’re proving an implication, we use contradiction, I guess, so that’s why it’s my first thought.” He used contradiction because he noticed the instructor often used it in class, and he was used to it.

Figure 2
Student Work in Interview 1



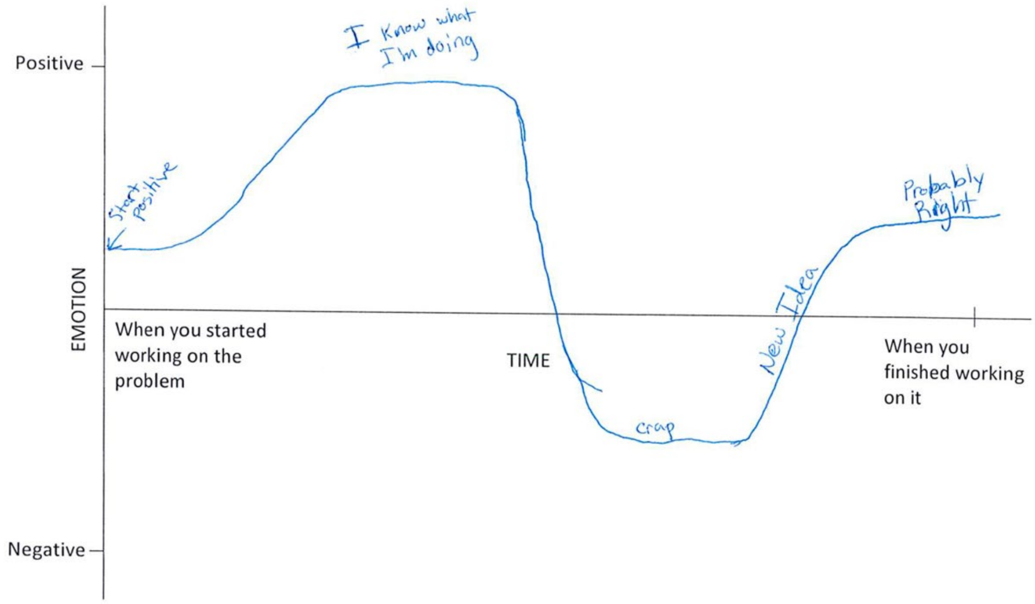
Note. The start of Leonhard’s work on this task is shown (not his complete work), as he tried to find the negation of the statement and mistakenly used the same variable for both x and y.

Leonhard set up the proof well, but ultimately, it was not a fully correct proof: while x and y are both even (or both odd) in his approach, he made an error in using the same variable for both x and y, which implies they are the same number. He changed the variables in his proof later down (not shown) but then went back and changed $y = 2k$ to $2(k + 1)$ so “he’d have something left over” to reach a contradiction that an even integer would equal an odd integer. For these reasons, his errors led to his proof being incorrect.

Leonhard’s emotion graph for this first task revealed big shifts in emotions throughout this attempt (see Figure 3). His emotions grew to a peak early on, remaining positive for a period of a time (“I know what I’m doing”). He then realized something in his work was wrong as indicated by the dip below the x-axis, but then the graph ended slightly positive (“probably right”).

In summary, Leonhard used proof by contradiction because it was what was done in class, even when he found it difficult to take the negation. His proof contained errors, so it was not correct. His emotions dropped negatively when he was stuck, but he felt positively about his work in the end.

Figure 3
Emotion Graph in Interview 1



Interview 2: Choosing a Proof Technique Based on Fluency

In the first task of the second interview, the statement to prove was: *If x and y are consecutive numbers, then xy is even.* As students had already been exposed to the definition of consecutive integers in class, a more informal definition for "consecutive number" using everyday language was intentionally given. Moreover, the definition of consecutive integers x and y as $y = x + 1$ leads to $xy = x(x + 1) = x^2 + x$, which does not contain enough information without further work to be shown as an even integer. The task was intentionally chosen for this disconnect between the definition of consecutive integer and the solution path. As seen in Figure 4, Leonhard wanted to use direct proof but became stuck, as he was unsure if direct proof would work.

Leonhard immediately knew to not use the direct definition of consecutive integers but instead set $x = 2k$ and $y = 2k + 1$, albeit leaving off that k must be an integer as well. When asked why he used $2k$ and $2k + 1$, he explained that his thought process was that an odd integer comes after an even integer and an even integer comes after an odd integer. Leonhard also took liberties in assuming that x was the even integer; for a fully correct proof for students at this level, he should have done another case where x was an odd integer and y the subsequent even integer or potentially use a "without loss of generality" argument.

He was then stuck again over what technique to use: direct proof versus proof by contradiction. He chose to use proof by contradiction, saying, "I decided to do contradiction because I know how to do it." Leonhard decided what method to use based on what he felt he could do at that point in time, i.e., his sense of fluency with proof techniques. Interestingly, the direct proof is embedded in here; his finding that xy is even is the conclusion to the direct proof. Given that direct proof was the more efficient proof, Leonhard's work suggests he felt more comfortable with proof by contradiction.

Leonhard's proof was overall correct, albeit missing details we want to see in students at this level, and his affect matches this. His emotion graph (see Figure 5) shows that this was a positive experience overall, with little variation in emotion. He was slightly confused at the beginning in deciding between direct proof or proof by contradiction, but he felt that he knew what he was doing after that. His annotation of "Yeah! (I got this)" reveals his sense of pride as he completed his proof.

Figure 4
Student Work in Interview 2

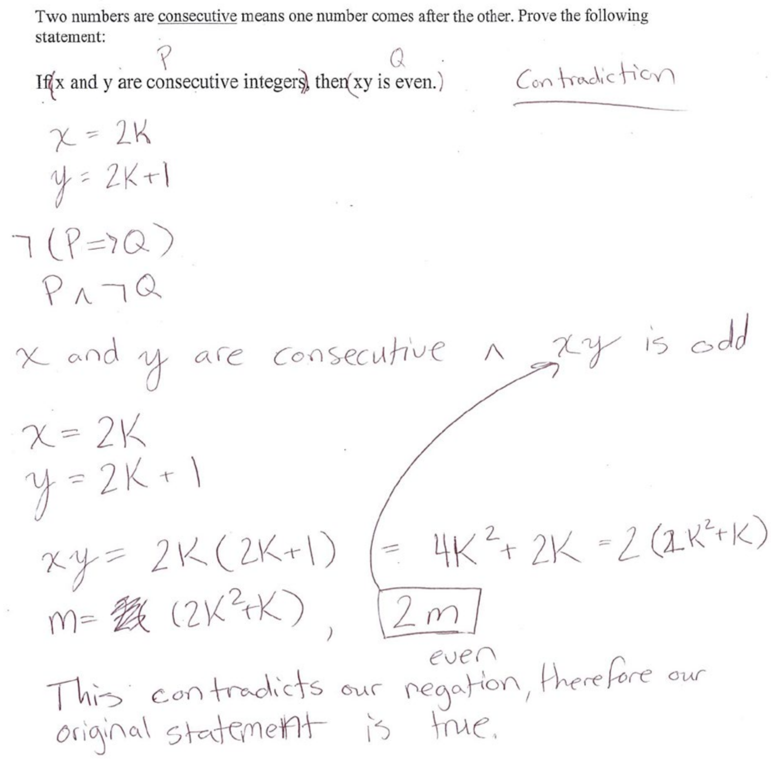
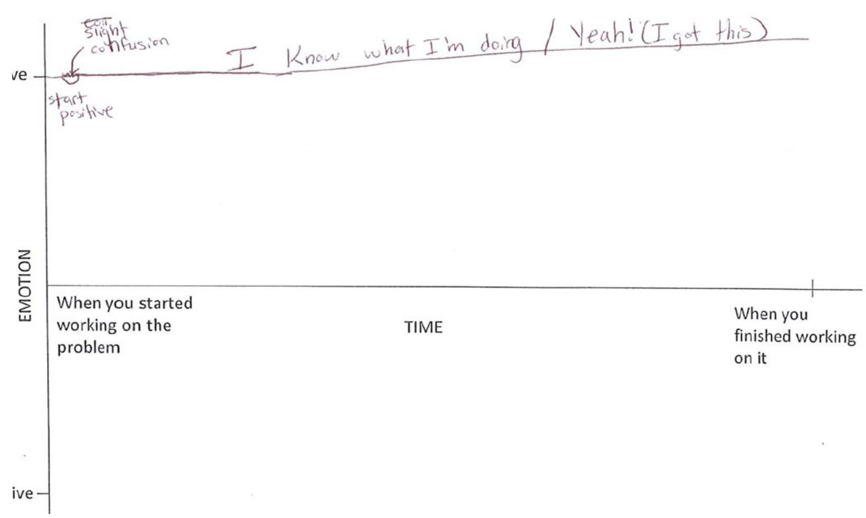


Figure 5
Emotion Graph in Interview 2

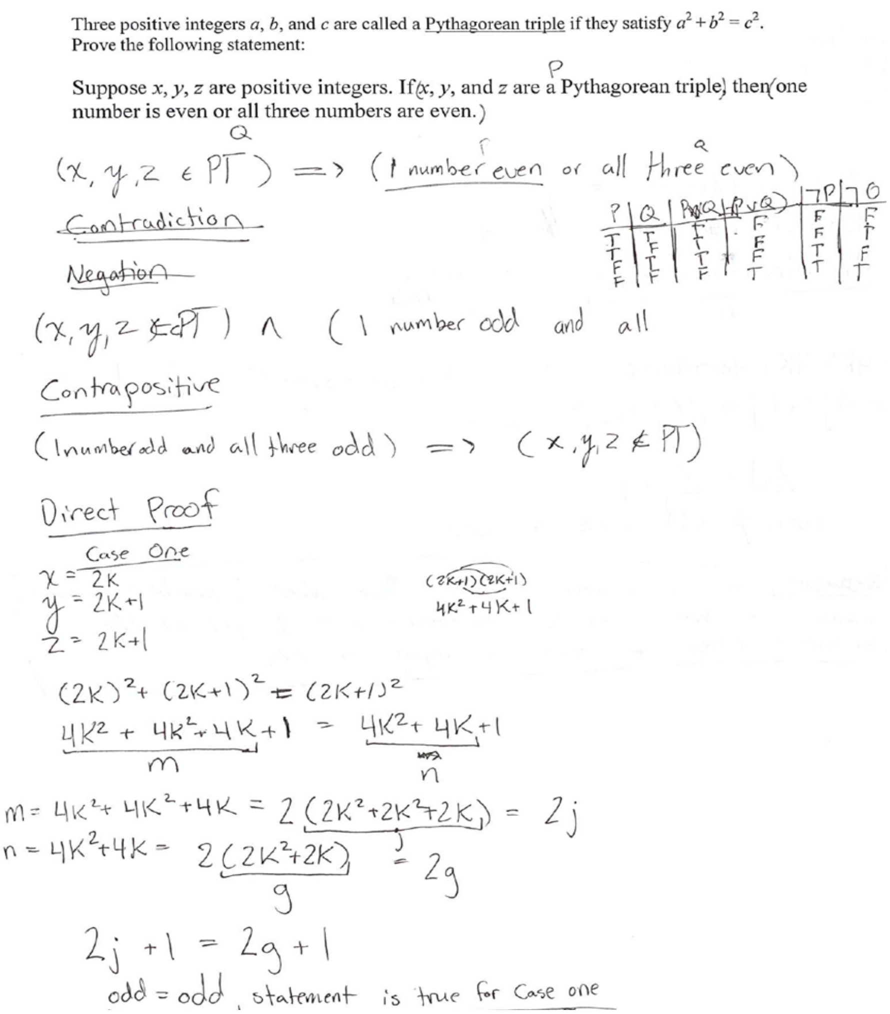


In summary, Leonhard used a technique that he felt he knew how to do well (proof by contradiction), even though it was not the simplest one and the direct proof was embedded in his work. His work was generally correct, and his affect was positive with no dips, except for slight confusion at the start, which abated when he decided on a technique and went with it.

Interview 3: Proof by Contradiction as a Default Choice

As time progressed, there was clear growth in Leonhard's reasoning related to the proof techniques he pursued in a problem. This task from the third interview provides an example of where Leonhard cycled through a few options for proof techniques, as seen in his written work (see Figure 6). The statement to prove was: *Suppose x, y, and z are positive integers. If x, y, and z are a Pythagorean triple, then one number is even or all three numbers are even.*[1] He used proof

Figure 6
Student Work in Interview 3



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[1] See Design of Proof Construction Tasks in the Methods section above for an explanation for the phrasing of this task.

by contradiction but then became stuck when writing the negation, because his negation of the conclusion did not make sense to him: "One number is odd and all three numbers are odd" did not seem possible, and he stopped writing the negation midway through his work. He had negated the "or" when it was in fact not a logical operator; the task was intentionally structured to check if students thought about the meaning or took the negation mechanically. The correct formal negation was "none or exactly two of *x, y,* and *z* are even integers." He switched to proof by contrapositive but realized he had the same issue with how to negate the conclusion as before. He then switched to direct proof. While he again used the same variable k in setting *x,y,* and *z* equal to even or odd integers, he realized his mistake near the end but did not change his answer as it would not change his overall result.

His rationale for using proof by contradiction in the beginning was: "I'm biased towards contradiction so I usually like to do that…my mind goes straight there. I like it the most because…at some point you usually run into something that just comes out sounding weird." Leonhard admitted that proof by contradiction was his go-to technique; it was his favorite and so he tended to use it. He liked proof by contradiction for its unique nature in producing something nonsensical. He later remarked on his proof by contrapositive attempt, "I don't know what possessed me to write this [contrapositive]," because he ran into the same issue, needing to negate the conclusion. Leonhard knew he liked certain techniques over others and had some rationale grounded in the techniques themselves, namely that a proof by contradiction results in a nonsensical claim and that proof by contrapositive has no advantage over proof by contradiction here. His rationale was still relatively general, however, in that proof by contradiction was a technique he liked and his fondness for it drove his usage of it.

His use of direct proof as his third attempt suggests he came to it through a process of elimination. He posited that his underlying idea may have been to check which proof techniques did not work well here and see what was left over: "I guess this was a good way of crossing out the things that you can't do so you can find the things that you can do."

Unfortunately, Leonard's proof was not correct. He started with one of the cases in the conclusion, reached a point where an odd integer was equal to an odd integer, and thought this meant he had shown the statement. Leonhard had used backwards reasoning on one case and shown there was logical consistency, but this was not a proof.

Leonhard's emotion graph shows this was a positive experience for him (see Figure 7). While there was a dip in emotion when he was confused ("eh, what"), his emotions grew steadily more positive as he continued on. His experience was so positive that he labeled a period of time as "The Zone," annotating his self-talk on the graph, "I'm doing it! I'm doing it! Almost there." His annotations also show his confidence, with humor ("Dope, I'm smart.") The "oops" near the end referred to his realization that he had used the same variable k in all three of *x, y,* and *z,* but he felt it did not fundamentally affect the correctness of his work. His emotion graph suggests that Leonhard was confident about his work and that he thought it was correct.

Interview 4: A Rationale Based on the Statement

By the fourth interview, Leonhard's rationales for his choice of proof technique displayed more precision. In the second task of the last interview, the statement was: *If x, y are positive real numbers and* $x \neq y$, *then* $\frac{x}{y} + \frac{y}{x} > 2$. He was stuck over how to start; he then identified the assumption and conclusion, tested a couple examples for x and y, and then tried proof by contrapositive (see Figure 8).

Figure 7
Emotion Graph in Interview 3

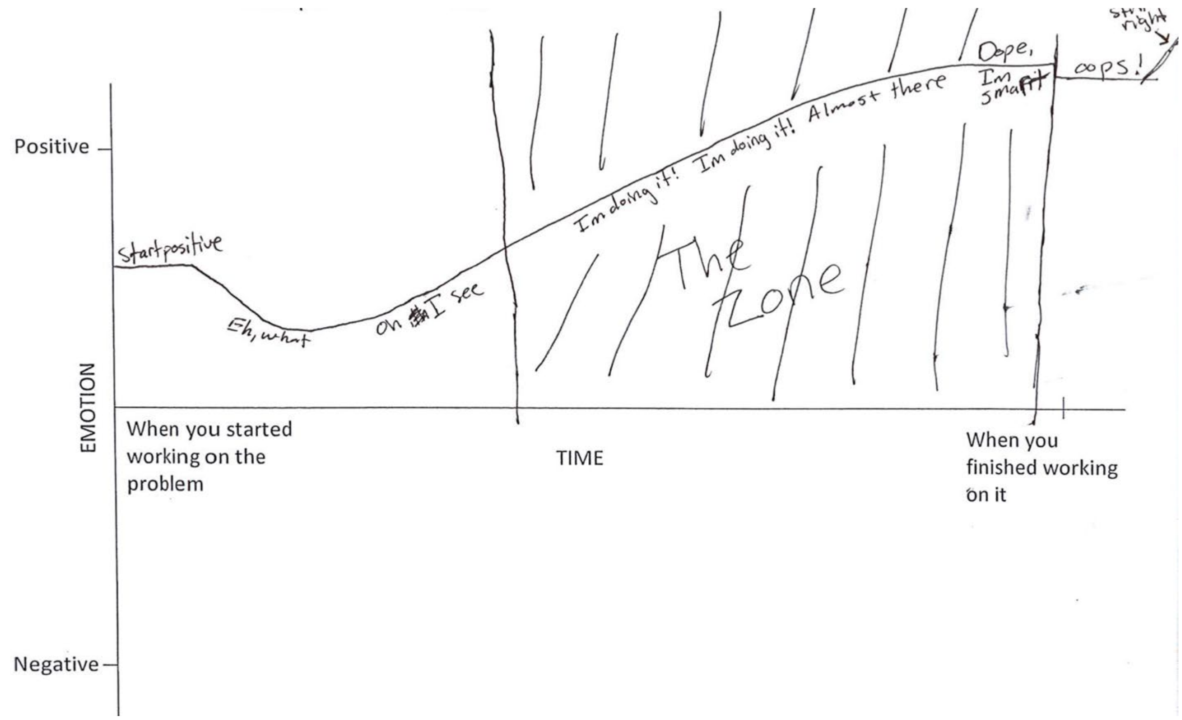
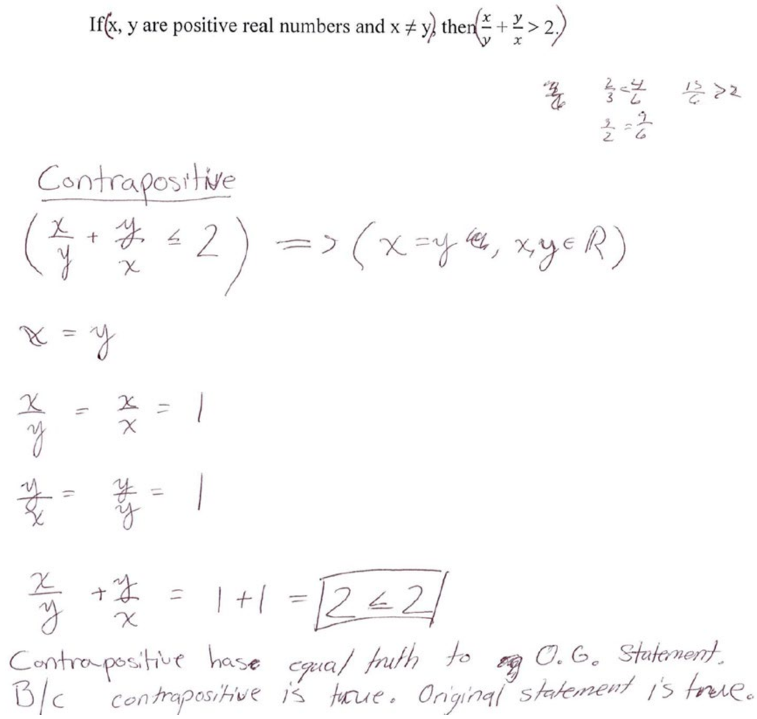


Figure 8
Student Work in Interview 4

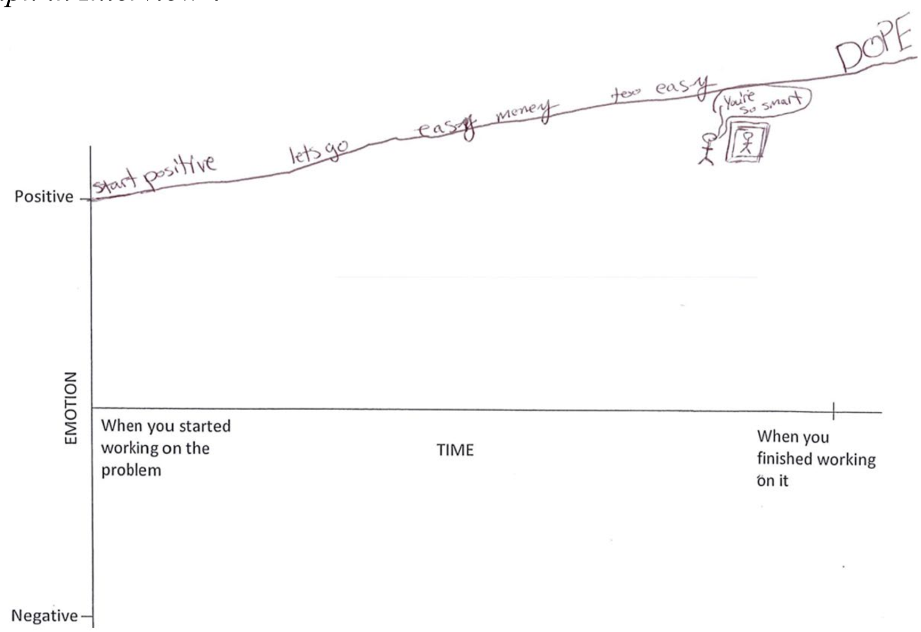


His rationale for proof by contrapositive was, "You can't really do much with x not equal to y. But you can do a whole lot with x is equal to y." He also explained why proof by contradiction would not be helpful: "The contradiction wouldn't give me anything to work with." He wanted to start with $x = y$ because he saw how an equality was more useful than having objects not equal to each other when proving. Neither direct proof nor proof by contradiction would provide an equality here. We see that Leonhard decided which proof technique to use based on specifics of the statement to be proven. His rationale also specifically explained why another proof technique (proof by contradiction) would be less useful here. In summary, he had a rationale for why his chosen proof technique was a helpful approach and why other techniques would be less helpful.

Although his rationale for why to use contrapositive was coherent and his affect overwhelmingly positive, his proof was incorrect. He used backwards reasoning to work off the conclusion (of his contrapositive) and then reached a true statement ($2 \leq 2$); he had still not realized that this was not the same as showing the original statement is true.

Leonhard's emotion graph in Figure 9, however, depicts a student who feels comfortable and confident with proving. His graph was entirely positive; he started the graph at the positive tick-mark, and the graph rose even more. Although he was not sure how to start, it did not appear to impact his emotions based on the graph drawn afterwards. His annotations, "easy money" and "too easy," suggests not only that he wrote this proof with ease, but that he enjoyed it.

Figure 9
Emotion Graph in Interview 4



Looking Across Leonhard's Reasoning, Performance & Affect

Over the course of these four interviews, the rationales Leonhard gave for why he chose the proof techniques that he did became more nuanced. He moved from choosing certain techniques because it was done in class (no rationale), to what he was comfortable with, to deciding based on the particulars of the statement itself. By the end of the series of interviews, Leonhard also articulated why other proof techniques would not be helpful (so as to not go down that path). Leonhard showed clear growth in his reasoning for how he decided which proof technique to pursue.

However, if we look at his performance, Leonhard's work was oftentimes incorrect. Across the four tasks shown here, he only proved one statement correctly (Interview 2); he was partially correct in Interview 1, and his work for both Interview 3 and Interview 4 was incorrect. In fact, across the entire set of eight tasks (two per interview), the one task from Interview 2 was the only statement he proved entirely correctly. Moreover, his work on the last two interviews (all four tasks) was all incorrect due to substantial errors or missing crucial pieces of the proof. Leonhard would repeatedly work from the conclusion until he found a statement that was logically consistent, e.g., an even integer is equal to an even integer, and took that to mean he had proved the statement. Given that the interviews were weeks apart and Leonhard continued to use this logic, this is evidence his misconception had not been dislodged.

Interestingly, Leonhard's perception was that his work was correct. Looking across the set of emotion graphs, Leonhard's affect was overwhelmingly positive. They paint a portrait of a person who is confident with and feels at ease proving. He recovered from dips in emotion, felt good about writing proofs ("I'm doing this"), referenced being "in the zone," and believed in his abilities. Leonhard genuinely enjoyed doing this work; he displayed the positive affect we hope to see in students regarding proving. That his work was oftentimes incorrect and he did not realize it is troublesome.

Discussion

Through this case of Leonhard, we explored one transition-to-proof student's reasoning, performance, and affect over a series of four tasks and interviews. Over time, Leonhard's rationales in deciding which proof techniques to pursue became more sophisticated while his performance declined, yet his affect was quite positive. He went from using one proof technique (proof by contradiction) for everything, at first because it was done in class to later because he felt the most comfortable with it, to analyzing the structure of the statement itself for what technique would make sense. He also articulated why other techniques would not work well. Leonhard showed relatively favorable affect through many of the tasks, in that he had a positive orientation to his work: he felt at ease, enjoyed proving, and displayed confidence about his proofs and his competencies. However, Leonhard's work was often incorrect, with major logical flaws regarding backwards reasoning and about what it meant to prove a statement. While he had a positive orientation towards his work, he did not notice major logical flaws in his work.

Leonhard is an example of a student who has strong positive affect towards proving and their reasoning—specifically their rationale for their decisions, is strong—but these do not necessarily lead to correct work. There is a difference between reasoning and execution: can we say Leonhard knows how to prove or that he is better at proving than when he started? How do we weight reasoning versus performance versus affect here?

This work—the misalignment of reasoning, performance, and affect—highlights multiple implications for the transition-to-proof. First, thinking that reaching a true statement (often of the form $1 = 1$ or $2k = 2j$) is equivalent to proving a statement is true is a stubborn and pervasive error. In noticing that two sides match, students have verified that the mathematical situation is valid, that there are no inconsistencies—but writing a formal proof to in fact prove the statement is different. Further research is needed on this particular error, on how to help students notice when they make this error in their work, see why it is incorrect, and how to fix their proof. One recommendation is for transition-to-proof classes to more regularly task students to read and critique sample proofs with errors such as this one and discuss them. Misconceptions like these may in fact be developmental stepping-stones in learning how to prove, and rather than attempt to dislodge and replace such errors, we can help students refine and reorganize their knowledge (Smith et al., 1994). Continued work with students could include differentiating between the mathematical process of proving and the final written product (Karunakaran, 2018) and could reinforce the importance of keeping track of the conclusion one wishes to show.

Second, what do we do with students who are in fact overly confident about their work, not realizing they are making errors? On one hand, overconfidence with one's work can lead to not noticing errors, as happened here. More caution would have helped to catch errors. On the other hand, students who are overconfident tend to at least put down a written solution; because their thinking is now visible, their errors can be addressed. Meanwhile, students who are underconfident may doubt their thinking and not write down much or any of their thoughts. It is difficult for instructors to know that this is the case and determine how to help without talking to the students. This also brings up questions about the role of confidence in mathematics, whether overconfidence is beneficial for learning how to prove in that the positive affect helps students move forward through what may otherwise feel paralyzing. This has implications for students who come from backgrounds that have been historically marginalized in mathematics in the United States (African Americans, Native Americans, underrepresented Asians, Latinos, women, etc.), on whom mathematical confidence has not culturally been bestowed by society. Lundeberg et al. (1994) found that undergraduate men were more overconfident over incorrect answers than women. One recommendation is for instructors to address what makes for a healthy sense of confidence in proving—and provide strategies for all students in dealing with under- and over-confidence, but with special attention to gender and racial dynamics.

Third, the misalignment in reasoning, performance, and affect indicates the continued need for a framework for assessing students' proving (Savić, 2012; Selden & Selden, 2007) that encompasses these multiple components. While not typically thought of as part of the work of proving, affect can be a supplementary or even central component, much like how beliefs and affect are components of Schoenfeld's (1992) problem solving framework. Skills assessed should include common ones such as applying definitions and taking negations but also skills seen in this case, such as interpreting informal statements, negating informal statements, and differentiating valid statements from one's conclusion. Processes assessed should include how students choose a proof technique; a framework for students' development in this domain is provided in Satyam (2020). Such a framework would support the characterization of and assessment of students' proving as a process over short and potentially longitudinal timescales.

Lastly, this case serves as a reminder that progress in learning how to prove does not always manifest itself in performance as measured by objective correctness. Through interviews, Leonhard's more nuanced decision-making and positive affect shone through. Assessing a student solely through their written work does not capture the thinking and reasoning behind their choices that may have been valid, which, when taken alone, is valuable growth in proving.

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AN EXAMINATION OF MIDDLE SCHOOL STUDENTS' ATTITUDES TOWARD SCIENCE

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ABSTRACT
For more than 40 years, researchers have been studying the persistent underrepresentation of women in science. Today, the gender gap has narrowed in some, but not all, disciplines of science. To better understand the impetus of this continuing problem, the attitudes of middle school students toward science were examined using a causal-comparative design based on biological sex across four attitude constructs: attitudes toward school science, desire to become a scientist, value of science to society, and perceptions of scientists. A sample of 450 sixth-, seventh-, and eighth-grade science students located in suburban, central New Jersey responded to Likert-type items on the My Attitudes Toward Science (MATS) survey during their regularly scheduled science class periods. Data analysis was performed through a multivariate analysis of variance. The findings indicated no statistically significant differences in middle school students' attitudes toward school science, desire to become a scientist, value of science to society, and perceptions of scientists based on biological sex of the students. Implications for the findings are discussed.

KEYWORDS
science attitudes, middle school science, gender gap, biological sex

It is widely recognized that women historically have been underrepresented in science (National Science Foundation [NSF], 2019). Today, the problem persists within some disciplines of science, though female representation in other science careers, such as those in the health professions, veterinary medicine, and biology, has become more equitable (Jones et al., 2000; Wang & Degol, 2017). The unbalanced distribution of women in science, and blatant underrepresentation in some fields, is a two-fold problem: it has the potential to greatly impact the diversity, creativity, and productivity of the larger society (National Academies of Sciences, Engineering, and Medicine [NASEM], 2019; NSF, 2019), and it places women at a disadvantage by diminishing their earning potential in comparison to men (Beyer, 2014; Oh & Lewis, 2011; Xu, 2015).

To better understand and combat the problem of female inequities in science, educational researchers have been studying how male and female students participate in science for over 40 years (Buck et al., 2014; Bybee & McCrae, 2011; Naizer et al., 2014; Osborne et al., 2003). These studies have considered the problem through different lenses, such as disparities in science achievement based on biological sex or students' attitude toward or appreciation of science overall. Even with extensive study in this area, differences in engagement in science based on biological sex—referred to in the literature as the gender gap—continue to pervade the realm of science. Current research calls for continued investigation (Reilly et al., 2019; Wieselmann et al., 2020).

The research presented here investigated differences in science attitude constructs in middle school students based on biological sex. The aim of the research was to complement prior research that mainly focused on overall science attitudes or achievement differences based on sex or gender (e.g., Gokhale et al., 2015; Guzey et al., 2016; Quinn & Cooc, 2015) by examining the multidimensional nature of students' science attitudes. The study also used the My Attitudes Toward Science (MATS) instrument (Hillman et al., 2016), which has shown promise in examining the nuances of science attitudes.

Attitude Toward Science

Hillman et al. (2016) report that a child's attitude toward science can be broken into four main domains or constructs: attitude toward school science, desire to become a scientist, value of science to society, and perceptions of scientists. It is not enough then to measure only how positively or negatively a student views science as a whole. Instead, researchers need to determine on which specific attitude constructs male and female students differ to ultimately determine why fewer women historically participate in science across different fields.

Hillman et al. (2016) describe the attitudes toward school science construct as a student's feelings toward the behavior of participating in school science classes. A student's attitude can affect the way he or she engages with science coursework (Teodorescu et al., 2014). Male students traditionally have "a consistently more positive attitude [toward] school science than girls" (Chen & Howard, 2010, p. 138). Studies suggest that, overall, male students tend to appreciate the use of technology in school, show interest in learning by discovery, and are willing to take risks (Chen & Howard, 2010; Eagly & Wood, 2013; Incantalupo et al., 2014). In contrast, female students are often less confident in school science, which may cause them to dislike the subject in school, particularly in the middle grades (Smith et al., 2014).

A student's desire to become a scientist is broadly defined as his or her interest in pursuing any career in a scientific, medical, or technological field (Hillman et al., 2016). Ajzen and Fishbein (1977) explain that behavior is directed by attitude. Attitudes toward science can therefore influence students' choices to engage in science-related courses, potentially translating into improved academic achievement (Barnes et al., 2005; Leibham et al., 2013). It is then surmised that if one possesses a positive attitude toward science, then the resulting behavior would likely be engagement in or increased achievement in science (Singh et al., 2002). Male students in middle school have been found to express a desire for careers relating to technology, engineering, or mathematics at a significantly higher rate than middle school girls (Desy et al., 2011; Jenkins & Nelson, 2005; Stoet & Geary, 2018). Of the female students who do indicate a desire to become a scientist, the most popular career choices are veterinary medicine and healthcare professions (Desy et al., 2011; Jones et al., 2000)—so called "helping professions." Further, reports indicate that females are less likely to persist in science as a career broadly across disciplines (NSF, 2019). Thus, numerous efforts within the United States continue to focus on broadening participation of women in science, especially in "non-helping" fields (NSF, 2019).

Value of science, as defined by Hillman et al. (2016), is a student's awareness of how discoveries and technological advances aid society through STEM. Students become more interested in science when they see the practical significance of science as a contributor toward society (George, 2006). Differences in the perceived value of science to society based on biological sex are contradictory within the literature. Some studies have shown that female students recognize the value of science more readily than male students, though this perceived value does not necessarily correlate to an increased pursuit of STEM careers (Blanchard Kyte & Riegle-Crumb, 2017; Else-Quest et al., 2013), while Blanchard Kyte and Riegle-Crumb (2017) report that male students' choices for careers in science appear to be unaffected by their perception of science's societal value.

The stereotypical belief that scientists are male and that science is a masculine domain is referred to in the literature as the gender-science stereotype (Cai et al., 2016; Miller et al., 2015). It is a well-known phenomenon and has been shown to have a negative impact on female students interested in science (Hong & Lin, 2011; Quinn & Cooc, 2015; Reilly et al., 2019). Men working in STEM professions more readily endorse this stereotype, as their own actions serve as reinforcement of their perception that science is predominantly a male domain (Smyth & Nosek, 2015). Similarly, women who work outside of STEM professions continue to uphold this stereotype, whereas women working in STEM endorse the stereotype far less (Smyth & Nosek, 2015). The endorsement of the gender-science stereotype has led to fewer women participating in some fields of science such as computer sciences, engineering, and physics (NSF, 2019). The NSF (2017) has shown that female students are less likely than male students to pursue advanced science courses in high school and college, precluding women from entering science professions in an equitable manner when compared to men.

While the gender gap in science has been researched extensively, the gap still persists and many questions remain about the cause, continued perpetuation, and methods for closing the gap. If the goal of researchers is to identify disparities in science based on biological sex with the intention of drawing more women into all disciplines of the science-related workforce, it is important to understand how student attitudes differ based on sex at the critical middle school level. Attitudes, in fact, have been cited as one of the most important factors in determining females' participation in science (Else-Quest et al., 2013; Reilly et al., 2019; Smeding, 2012), yet they remain under-researched. As the pursuit of science-related careers is directly related to attitude, it becomes increasingly imperative to examine student attitudes across specific constructs rather than as a whole. These nuances may provide greater insight into student choices related to science than examining students' attitudes as a single domain, allowing educators and researchers to better provide interventions to keep female students interested in STEM.

Theories Related to Gender Stereotypes in Science

There are several theories that may provide insight into the observed STEM gender gap in terms of male and female students' interest in, attitudes towards, participation in, and persistence in science. Eagly's social role theory (1987), for instance, postulates that gender stereotypes may impact children's attitudes toward science. Social role theory suggests that children learn what social roles are acceptable and expected of them based on their observations of adults in their society (Eagly & Karau, 2002; Miller et al., 2015). Historically, men and women have performed different jobs within and outside of the household. These traditional gender roles are often observed by children and then perpetuated through subsequent generations (Eagly & Karau, 2002). Importantly, despite some shift in societal attitudes, these traditional gender roles persist today (Rennison & Bonomi, 2020). Research examining science identity aligns closely with social role theory in that individuals who do not have opportunities to see others that look like them participating in their selected field of study may not believe that they belong in the profession (Carlone & Johnson, 2007; Hill et al., 2010; Rockinson-Szapkiw et al., 2021). When this happens, the female students endorse and perpetuate stereotypes, allowing these beliefs to continue through to yet another generation. Though the perception increases in magnitude with age for both male and female students, Liu et al. (2010) found that it is stronger for female students than it is for male students in middle school.

Social role theory serves as a foundation to explain the historical disparities observed in STEM fields based on biological sex. As female representation in some science disciplines has improved, however, it is also important to consider how gender theories have evolved over time in response to social change and how changes to these theories may help to explain the inequities still observed in the other science disciplines. Gender identity theory, for example, reconsiders gender differences and isolates the term gender from biological sex (Vantieghem et al., 2014). Egan and Perry (2001) posit that individuals engage in gender as a multidimensional process rather than as a singular identity attribute, identifying these dimensions to include gender typicality, gender contentedness, pressure for gender conformity, and gender superiority. Individuals may express gender in typical or atypical ways for their biological sex or feel pressure to conform to expected gender roles (Egan & Perry, 2001; Lagaert et al., 2017; Vantieghem et al., 2014). Gender identity theory then realigns behaviors and attributes to the domains of masculinity and femininity rather than to each biological sex (Vantieghem et al., 2014). The dimensions of gender identity have been shown to serve as powerful mechanisms reinforcing the gender gap in non-STEM disciplines (Lagaert et al., 2017) and STEM disciplines alike (Sibley & Crane-Seeber, 2020). Given that gender governs interactions and self-perceptions in academic, occupational, recreational, and interpersonal aspects of an individual's life (Egan & Perry, 2001), it is likely that gender identity serves as a similar mechanism, reinforcing gender gaps in other disciplines, including STEM.

Negative Effect of Stereotypes in Science

The disproportionate abundance of men compared to women in STEM professions has led to a long-standing stereotype that science is mainly for men (Farland-Smith, 2009; Quinn & Cooc, 2015). Studies have shown that, when young students are asked to provide a depiction of what they believe a scientist looks like, scientists are typically believed to be White men (Farland-Smith, 2009; Farland-Smith et al., 2014; Miller et al., 2018). Recent analysis has indicated the frequency by which young female students draw depictions of scientists as male has decreased as compared to past decades (Miller et al., 2018). However, scientists are still overwhelmingly perceived as male by both young girls and young boys. Further, despite efforts to engage young girls in STEM, female elementary student participants still overwhelmingly believe that STEM is better suited for males (Wieselmann et al., 2020) and, importantly, view "mathematics as a gatekeeper for STEM participation" (p. 304).

In some cases, parents, teachers, and other role models may intentionally or unintentionally model gender stereotypes while encouraging male students to engage in science-related activities and encouraging female students to engage in more feminine activities (Farland-Smith, 2009; Venkataraman et al., 2019). The perceptions of such role models have been shown to influence students' views of whether or not they belong in science fields (Gokhale et al., 2015; McGuire et al., 2020; Ochsenfeld, 2016). Further, given the disparity in representation of females in science fields (NSF, 2019), female students may have fewer known female role models to alter the perspective that science is a masculine endeavor (McGuire et al., 2020; Stearns et al., 2016). Thus, there are fewer like others to view (Venkataraman et al., 2019; Wendt et al., 2019), which may influence students' identities and their ability to see themselves as belonging in science (Archer et al., 2013).

Some fields of science, such as computer science, experience larger gender gaps than other science fields (Venkataraman et al., 2019) and may elicit additional stereotypes. Computer scientists, for example, are often stereotyped as "nerds, geeks, or hackers" who lack interpersonal skills (Beyer, 2014, p. 155). This stereotype is carried over to other scientific professions as many people perceive scientists to be individuals who work alone in laboratories filled with test tubes and scientific equipment (Farland-Smith et al., 2014). Women and girls may avoid these fields because they believe them to be isolating (Beyer, 2014; Venkataraman et al., 2019). Though the stereotype of science being only for men is untrue, the perception and feelings of not belonging may prevent women from choosing to pursue science as a career or remaining in a science career (Archer et al., 2013). Previous research reports that women also choose to leave science fields and careers due to external pressures, such as family responsibilities, a "chilly climate," and incongruence between personal values and job expectations (see Brue, 2019; Dawson et al., 2015; Fouad et al., 2016; Jensen & Deemer, 2019; Rockinson-Szapkiw et al., 2021).

Women who are impacted by gender stereotypes in science often find themselves at a disadvantage (McGuire et al., 2020). By not pursuing STEM careers, their earning potential is lowered in comparison to men (Beyer, 2014; Oh & Lewis, 2011; Xu, 2015). The potential loss of talent and the need for increasing the diversity of the STEM workforce are undeniable (NASEM, 2019).

Methods

This study used a causal-comparative research design with the students' self-reported biological sex as the independent variable and the students' attitudes toward school science, desire to become scientists, value of science to society, and perceptions of scientists as the dependent variables.

> **RQ**: To what extent do attitudes toward school science, desire to become a scientist, value of science to society, and perceptions of scientists of male and female middle school students differ as measured by the MATS instrument?

Sample

A convenience sample of middle school students was selected from a suburban school district in central New Jersey in the United States during the 2017–2018 school year. Eighteen classes each from two middle schools were included in the sample for a total of 36 classes. Participants for this study were selected from general education science classes in the sixth, seventh, and eighth grades in each of the participating schools. The sample did not include advanced placement, honors, or resource level classes, but instead focused solely on general education track students. The resulting sample consisted of 198 male students and 252 female students for a total sample size of 450 participants. The ethnic breakdown of participant groups is shown in Table 1. All classes recruited used a spiral curriculum model, which shares instructional time among the major science disciplines—Earth, life, and physical—throughout the year at each grade level.

Table 1
Demographic Data of Middle School Students

Category	Gender	
Grade Level	Male	Female
6	22.7%	28.6%
7	42.9%	44.1%
8	34.3%	27.4%
Average Age	12.4 years	12.2 years
Self-Reported Ethnicity		
Caucasian	44.9%	45.6%
Asian	12.6%	20.2%
African American	8.7%	4.4%
Latino/Hispanic	8.3%	7.5%
Biracial	6.7%	11.1%
Other race(s)	9.5%	10.3%

Note: N = 450

Instrumentation

The MATS instrument, designed by Hillman et al. (2016), was developed to measure the multidimensional nature of a child's attitude towards science. This instrument measures a student's science attitude across the four specific attitude constructs: attitudes toward school science, desire to become a scientist, value of science to society, and perceptions of scientists.

Prior to the current study, the instrument was subjected to several rigorous field tests to demonstrate its reliability and validity. Expert review was conducted by teachers, researchers, and graduate students (Hillman et al., 2016). Cronbach's alpha coefficients showed internal consistency for each of the subscales across elementary, middle, and high school grade levels. The Cronbach's alpha coefficients for the attitude toward school science, desire to become a scientist, and value of science to society subscales were 0.866, 0.700, and 0.794 respectively for all grade levels (Hillman et al., 2016). The same subscales revealed coefficients of 0.841, 0.658, and 0.780 at the middle school (grades six–eight) level. In a previous study, the perception of scientists subscale showed a lower coefficient (0.539 total and 0.495 at the middle school level), indicating students' perceptions were not homogenous (Hillman et al., 2016). Cronbach's alpha for the current study is reported in the Results section below.

The MATS instrument consists of 40 items representing the four subscales of students' attitudes using 5-point, Likert-type responses. The attitude toward school science subscale contains 14 items, allowing each student's score to total between 14 points, indicating the most negative attitude toward school science, and 70 points, indicating the most positive attitude toward school science. The desire to become a scientist subscale only contains two items so that each student's score could fall between 2 and 10 points. The value of science to society subscale has 12 items allowing potential scores to fall between 12 and 60 points. These three subscales are comprised of an equal number of positively phrased and negatively phrased statements. For the perceptions of scientists subscale, a higher score represents a more stereotypical ideation of scientists, where 60 is the highest possible score and 12 is the lowest possible score (Hillman et al., 2016). No composite score was calculated, as the instrument is designed and used to interpret multiple components of a student's attitude rather than an overall positive or negative attitude. The subscales of the instrument allow its findings to be interpreted to the extent that researchers can identify the specific attitude constructs on which students differ based on biological sex.

Procedures

After receiving ethics approval and obtaining consent and assent forms, students electing to participate in the study were asked to complete the MATS instrument during their normal science class periods. After obtaining the completed instruments, the researchers combined data from all classes, entered them into an Excel spreadsheet, and analyzed the data with the use of IBM SPSS software. Because the first three subscales included positively and negatively worded statements, reverse coding was necessary for the negative statements before data analysis could take place.

A one-way MANOVA at the 95% confidence level was conducted to determine if there was a difference in attitudes towards school science, desire to become a scientist, value of science to society, and perceptions of scientists of male and female middle school students. Prior to conducting the MANOVA, data screening was performed. Several outliers were identified and removed from the study. A Kolmogorov-Smirnov test and generation of histograms indicated that the assumption of normality was violated, a problem inherent in the use of Likert-type surveys. Thus, QQ plots were created and subsequently showed normal distribution patterns. Additionally, "even when the data are not multivariate normal, the multivariate normal may serve as useful approximation" (Rencher, 1995, p. 94). The central limit theorem permits normality violation with large enough sample sizes, as those seen in the present study, to the extent that analysis could be continued (Rencher, 1995). Therefore, with a sample size of 450 students, the assumption of normality was deemed tenable. The Box's *M* test was used to test the equality of covariance matrices. The assumption of covariance matrices was met ($p = 0.467$).

Results

The results indicated that male students' attitudes toward school science ($M = 56.07$, $SD = 9.70$) were not statistically significantly different from female students' attitudes toward school science ($M = 54.04$, $SD = 10.84$). Male students' desire to become scientists ($M = 5.49$, $SD = 2.28$) was also found to be no different, statistically, than that of female students ($M = 5.17$, $SD = 2.34$). Similarly, no statistically significant differences were found in male students' ($M = 48.15$, $SD = 7.28$) and female students' ($M = 48.52$, $SD = 6.65$) perceived values of science to society. When examining the descriptive statistics, male ($M = 27.56$, $SD = 5.16$) and female ($M = 27.47$, $SD = 4.86$) students' perceptions of scientists were nearly the same.

A Wilks' Lambda statistic was used to measure the proportion of variance in the functions of student attitudes that is not associated with group membership (Warner, 2013). The result of the MANOVA was not statistically significant at an alpha level of 0.05, where $F(4, 445) = 1.96$, $p = 0.10$, partial $\eta^2 = 0.02$, which suggests there were no statistically significant differences in male and female middle school students' attitudes toward school science, desire to become a scientist, value of science to society, and perceptions of scientists. The effect size, as measured by partial eta squared, was small (Warner, 2013).

In order to ensure internal consistency and report on the instrument used in the study, Cronbach's alpha coefficients were calculated for each subscale: attitude toward school science ($\alpha = 0.893$), desire to become a scientist ($\alpha = 0.774$), value of science to society ($\alpha = 0.781$), and perception of scientists ($\alpha = 0.534$). The attitude toward school science, desire to become a scientist, and value of science to society subscales demonstrated high reliability (Rovai et al., 2013). The perception of scientists subscale, however, demonstrated only moderate reliability (Rovai et al., 2013), aligning with previous findings of the instrument developers (Hillman et al., 2016).

Discussion

This study aimed to examine the differences, if any, that exist among middle school students' attitudes toward science from a multidimensional perspective based on students' biological sex. The results of this study indicated that male and female students' attitudes toward science are not statistically different at the middle school level among the sample population. A comparison across each attitude construct measured by the MATS instrument based on descriptive statistics revealed similar scores for male and female students. When considering the subscales, the attitudes toward school science of male and female middle school students was positive. The students' desire to become scientists was almost neutral for both males and females. Students of both biological sexes also shared positive views of the value of science to society and indicated a low ideation of scientist stereotypes. This finding aligns with previous research, albeit limited, that indicates a shift in attitudes toward science around middle school, with girls demonstrating more equitable attitudes than boys (Desy et al., 2011; McGuire et al., 2020). However, causation for this shift in attitudes still remains undetermined (McGuire et al.,

2020) and is an important component of understanding how efforts to broaden female participation across all science disciplines may be made effective.

Though attitudes toward school science, value of science to society, and perceptions of scientists remained positive for students of both biological sexes, male ($M = 5.49$, $SD = 2.28$) and female ($M = 5.17$, $SD = 2.34$) students only indicated a neutral desire for careers in science. In this case, the students indicate that they enjoy science in school, believe it has value in society, and no longer endorse science stereotypes, yet neither male nor female students showed a great desire to become scientists themselves. It appears that the belief that science is a male domain could be waning, but the draw of new students into STEM professions is not keeping pace with current and projected economic needs (Huderson & Huderson, 2019). While recent NSF (2019) reports indicate that women hold the majority of degrees in psychology, biology, and social sciences, they continue to be underrepresented in computer science, engineering, mathematics, and physical sciences. Thus, efforts should be focused on determining how attitudes may impact women's choices to pursue specific science fields over others.

Because there are persistent gender gaps within STEM fields, it would be logical to expect statistically different results in the science attitudes male and female students express. No significant differences, however, were shown in the data from this study. Social role theory, and even gender schema, may not be enough to explain the differences observed in men and women in the STEM workforce. Gender identity may be a greater factor in the results observed in this study. Prior research has shown that more women are drawn to biological, psychological, and health professions than to physical science, technology, or engineering (Jones et al., 2000; Wang & Degol, 2017). These professions are dubbed "helping professions" because the work associated with these professions often translates to caring for or helping others. Differences in attitudes may not be perceived as based on biological sex alone. Instead, it will be important for future studies to examine any differences that may exist based on students' gender identities.

It should also be noted that the MATS instrument is relatively new and has not yet been used extensively. Thus, more extensive use of the instrument may lead to its further refinement based on current and subsequent findings. For example, the desire to become a scientist subscale only has two items stating "I would like a job as a scientist" and "I don't want a job as a scientist, because I have no interest in it." As previously discussed, many of the differences based on biological sex currently found in STEM professions are related to the specific fields of science. Additionally, some careers requiring STEM skills, such as nursing, may not be considered STEM professions by students (Stoet & Geary, 2018). Because this subscale does not enumerate the various professions students may choose within the sciences, students in the present study considering careers that they would not label with the term "scientist" may not have answered these survey items to reflect their true career plans. The neutral findings on this particular subscale may be due to a lack of agreement on what it means to have a career as a scientist. Students surveyed did not show strong preferences for or against scientific careers, and no statistically significant difference was found based on biological sex. If the instrument had listed specific careers within science such as veterinarian, computer scientist, astronomer, or botanist rather than simply using the term "scientist," students may have been better able to envision themselves within the larger STEM professional community. This type of change to the instrument could allow for a better overall comparison of students' desire to enter STEM professions based on biological sex as well as demonstrating how biological sex affects students' choice of career fields within STEM.

Similarly, the perception of scientists subscale demonstrated only moderate reliability in a previous study (Hillman et al., 2016), as well as in the current study. This finding indicates that additional refinement of the perception of scientists subscale may be needed. The low Cronbach's alpha coefficient calculated for this subscale in both studies may indicate a shift in students' views of the stereotypical traits that scientists possess; however, it is also likely that the subscale lacks internal consistency as it attempts to measure many stereotypical perceptions within a single subscale. Some of the statements, for example, apply to the masculine domain of science while other statements are made regarding scientists' presumed lack of social skills or the stereotype that all scientists work in laboratories (Hillman et al., 2016). Students may endorse some, but not all, of these stereotypes, leading to the low internal consistency score. Separating the specific stereotypes out into their own subscales, or onto a separate instrument completely, could improve the reliability and validity of this instrument.

Limitations and Recommendations for Future Research

The sample used in this study was drawn from middle schools residing within the same suburban school district, which could limit the generalizability of the results. The study could be replicated in other geographical locations to ascertain the climate of students' attitudes at a national or international scale. Additionally, the numbers of male and female students used in the study were not equivalent, nor were the numbers of students in each of the three grade levels. Using a sample that is more equivalent in representation of biological sex, as well as a larger sample, could yield different results.

The MATS instrument itself also represents a limitation. It is a relatively new instrument, and it yielded a low Cronbach's alpha for the perceptions of scientists subscale during its field testing. A similarly low Cronbach's alpha was calculated for the perceptions of scientists subscale during the present study ($\alpha = 0.53$). Thus, the development of a more robust measurement of perceptions of scientists could be beneficial in future studies. Future measurement should also account for the multitude of careers that relate to science.

Studying students' attitudes at one point in time may not provide the same depth of knowledge as studying how students' attitudes change over time. Therefore, a longitudinal study allowing researchers to compare students' attitudes toward school science, desire to become a scientist, value of science to society, and perceptions of scientists could yield different results measuring how these attitude constructs change over time. Increasing the diversity of the STEM workforce should not end with attracting more women to the different fields of science. Future studies should also be performed measuring students' attitudes across the four constructs based on race and ethnicity to further inform curricular reform that may diversify science professions.

Further, the authors recognize that sex and gender are complex characteristics. While the current study has limited the examination of attitude constructs to comparisons among biological sex, future study should examine variations of sex and gender, including gender identities, to further add to the research literature. Research that focuses on those with diverse gender identities remains sparse (Sibley & Crane-Seeber, 2020).

Conclusion

While this study demonstrated that no statistically significant differences among middle school students' attitudes toward science existed among the sample population studied, the findings contribute to the body of knowledge by supporting and upholding previous studies (Desy et al., 2011; McGuire et al., 2020). The findings indicate that a shift in attitudes may have occurred in recent years, resulting in more equitable attitudes toward science among male and female students. However, future research should consider what factors impact students' attitudes toward science, whether attitudes remain consistent as students matriculate into high school and beyond, and whether findings are generalizable among populations who have diverse gender identities. The findings, regardless, indicate an encouraging trend within the field of education in supporting the construction of attitudes that embrace science, breaking from traditional gender roles, identities, and expectations.

Disclosure Statement

The authors reported no potential conflict of interest.

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