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Allocating Homeland Security Screening Resources Using Knapsack Problem Models

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science at
Virginia Commonwealth University.

By

Rebecca Ann Dreiding

Master of Science in Mathematical Sciences with a concentration in Operations Research
December, 2010

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Richmond, Virginia
December, 2010

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Dedications

To my wonderful, loving, and supportive family, who have been through this entire process with me. In particular, my parents, Patrick and Nancy Dreiding, who have also believed and encouraged me, as well as listened to the many challenges along the way.

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Table of Contents

1	Introduction	1
2	Maritime Security and Multi-layered Prescreening Classifications	5
2.1	Introduction	5
2.2	Literature Review	8
2.3	Screening Framework and Model	9
2.4	Structural Properties	14
2.5	Computational Example and Results	18
2.6	Conclusions	23
3	Maritime Security and Bi-Level Screening Procedures	30
3.1	Introduction	30
3.2	NSP Model	32
3.3	Structural Framework	35
3.4	Computational Example and Results	38
3.5	Conclusions	52
4	Aviation Security and Direct-to-Target Nuclear Attacks	61
4.1	Introduction	61
4.2	Literature Review	63
4.3	Performance Measures	65
4.4	The Single Objective Discrete Optimization Models	67
4.5	Goal Programming Models	73
4.6	Greedy Heuristics	75
4.7	Computational Results	77
4.7.1	Optimization Results	79
4.7.2	Greedy Heuristics Results	89
4.8	Conclusions	96
5	Conclusions	98
	References	101
	Vita	106

List of Tables

2.1	Base case parameter values	20
3.1	Base case parameter values	40
3.2	Types of threats	41
3.3	Legend for primary screening levels and outcomes	41
3.4	Primary and secondary screening for $\beta = 1$ and $P_{I 2} = 0$	45
3.5	Primary and secondary screening for $\beta = 10$ and $P_{I 2} = 0$	46
3.6	Primary and secondary screening for $\beta = 100$ and $P_{I 2} = 0$	47
3.7	Primary and secondary screening for $\beta = 1$ and $P_{I 2} = 0$ for next-generation technology	58
3.8	Primary and secondary screening for $\beta = 10$ and $P_{I 2} = 0$ for next-generation technology	59
3.9	Primary and secondary screening for $\beta = 100$ and $P_{I 2} = 0$ for next-generation technology	60
4.1	Illustrative example parameters	67
4.2	Airport codes of origins and targets	82
4.3	Total number of flights, passengers, and bags from origins to targets	83
4.4	FCP, PCP, BCP, and TCP heuristic results for FC measure	92
4.5	FCP, PCP, BCP, and TCP heuristic results for PC measure	92
4.6	FCP, PCP, BCP, and TCP heuristic results for BC measure	93
4.7	FCP, PCP, BCP, and TCP heuristic results for TC measure	93
4.8	FC measure for TCFCP, TCPCP, and TCBCP heuristics for $B = \$275K$	94
4.9	PC measure for TCFCP, TCPCP, and TCBCP heuristics for $B = \$275K$	95
4.10	BC measure for TCFCP, TCPCP, and TCBCP heuristics for $B = \$275K$	96

List of Figures

2.1	Detection probability Z_n as a function of β	24
2.2	Marginal increase in the detection probability ΔZ_n as a function of β	25
2.3	Marginal increase in the detection probability ΔZ_n as a function of n	26
2.4	Detection probability as a function of $P_{A NM\cap T}$ for $n = 1, 3, 5$	27
2.5	System alarm as a function of $P_{A NM\cap T}$ for $\beta=10$ and $n = 5$	28
2.6	System alarm as a function of $P_{A NM\cap T}$ for $\beta=100$ and $n = 5$	29
3.1	Decision Tree	42
3.2	Overall interdiction probability as a function of the exogenous interdiction probability	48
3.3	Overall interdiction probability as a function of the budget	49
3.4	Overall interdiction probability as a function of the budget for different exogenous interdiction probabilities	50
3.5	Primary screening decisions for high-risk containers	51
3.6	Primary screening decisions for low-risk containers	52
3.7	Similarities and differences between the primary screening decisions for high-risk and low-risk containers	53
3.8	Overall interdiction probability for $\beta = 1, 10, 100$ as a function of secondary screening costs	54
3.9	Overall interdiction probability for $\beta = 1, 10, 100$ as a function of budget for current and next-generation technology and $P_{I 2} = 0$	55
3.10	Sensitivity analysis for primary screening decisions for high-risk containers	56
3.11	Sensitivity analysis for primary screening decisions for low-risk containers	57
4.1	Optimal solution values for FCP, PCP, BCP, and TCP as a function of budget	84
4.2	Optimal solution values for FCP, PCP, BCP, and TCP as a function of the EN	85
4.3	Corresponding FC, PC, BC, and TC measures for FCP, PCP, BCP, TCP and EN as a function of budget for $C = 2500$	86
4.4	TCFCP, TCPCP, and TCBCP for $C = 2500$ and $B = \$275K$ USD	87
4.5	TCFCP, TCPCP, and TCBCP optimal solution values as a function of budget for $C = 1250, 2500$ and $T = 0, 7, 10$	88

Abstract

ALLOCATING HOMELAND SECURITY SCREENING RESOURCES USING KNAPSACK PROBLEM MODELS

By

Rebecca Ann Dreiding

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A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science at
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Director: Dr. Laura A. McLay

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Since the events of September 11, 2001, the federal government is focused on homeland security and the fight against terrorism. This thesis addresses the idea of terrorist groups smuggling nuclear weapons through the borders of the United States. Security screening decisions are analyzed within maritime and aviation domains using discrete optimization models, specifically knapsack problems. The focus of the maritime chapters involves a risk-based approach for prescreening intelligence classifications for primary and secondary screening decisions given limited budget and resources. Results reveal that screening decisions are dependent on prescreening classification and the efficacy of the screening technologies. The screening decisions in the aviation security chapter highlight different performance measures to quantify the effectiveness of covering flights with the intent of covering targets. Results reveal that given scarce resources, such as screening devices capacities and budget, flights and targets can be covered with minimal expense to the system.

Chapter 1

Introduction

The horrific events of September 11, 2001, have led national security and other governmental agencies to assess threats, especially terrorist threats, towards the United States. As many lives were lost that day, and families broken, the goals have thus been to prevent another attack from occurring. Since it is known that terrorist groups are looking into creating an even bigger attack and statement against western countries, the United States has been preparing and attempting to prevent that occurrence. The bigger attack that terrorists are considering involve the use of weapons of mass destruction (WMD). A WMD is also considered a nuclear-type attack and as the United States Homeland Security Council (2009) has reported that a detonated nuclear weapon would not only be catastrophic, but would lead to numerous lives lost and negative impacts to the economy. The current nuclear weapons around the world are accounted for, yet prior to doing proper inventory years ago, has led countries, such as Russia, to have inaccurately acknowledged the number of nuclear weapons that the country has had. Therefore, it is very possible that these unaccounted for nuclear weapons are potentially located in the black market. The International Atomic Energy Agency (IAEA) has reported that not only that there have been confirmed cases of weapons-grade nuclear material being trafficked, but that terrorist groups have attempted to buy nuclear and radioactive materials (IAEA 2007). Since prevention, the first step in a defensive scenario, is potentially unattainable since nuclear weapons are missing, the next step is creating a preparation and contingency plan, 'how does the United States defend our country if a nuclear weapon is in the hands of our enemies and how, why and where would an attack with a nuclear weapon occur'? This question has opened the minds of the country's defensive agencies to view

scenarios and seek out vulnerabilities within the nation's security. If a nuclear weapon is in the hands of a terrorist group and their desire is to detonate it within the U.S. borders, transportation of the weapon is required, and sea and air are the main options for transportation.

Since sea and air are the main transportation methods of a nuclear weapon that must cross the oceans, it is important for strong security within these sectors, as well as good foreign relationships. Depending on the target of a terrorist's mission, a container ship, truck, or aircraft could be used as a means of transportation of the nuclear weapon or help aid the physical detonation of the weapon. There are many vulnerabilities within the maritime and aviation industries. There are approximately 11.4M containers incoming to the United States ports every year for trade purposes, and currently all containers are prescreened by the Automated Targeting System (ATS) which classifies the containers as either high- or low-risk (Strohm 2006). Given that illegal substances, including drugs and conventional weapons, have been smuggled in containers through the ports has created the idea that a terrorist group may decide to smuggle a nuclear weapon in through the ports as well. In this case, the nuclear weapon may be placed in a container in a foreign port in hopes that certain screening procedures will not detect the weapon. This is just one vulnerability of maritime security, and a lot of emphasis is placed on the foreign port. If the foreign port has effective technology, a terrorist group may be deterred from trying this method. Another vulnerability within maritime security is when the successful smuggled nuclear weapon arrives at the United States port. The screening technology at ports is not as effective as hoped for, and a nuclear weapon could pass through the screening procedures undetected, and continue on its planned path. Another option that terrorists could consider is detonating a nuclear weapon prior to a container being screened at a port. Therefore, that leads to the importance of foreign relationships in hopes that foreign countries would detect and interdict the nuclear weapon. These vulnerabilities can also be applied to other sectors of transportation, such as the aviation industry. Currently, airports do not screen for nuclear weapons (and there is no prescreening classification for baggage or passengers), the focus is on conventional weapons, for the commercial aviation industry. One such scenario could include foreign airports destined for a United States city, and an aircraft hijacking in which a terrorist group contains a nuclear weapon. If the events of September 11, 2001, occurred with the addition of a nuclear weapon that was detonated prior to the aircraft crashing, the events would be even more catastrophic than what happened in reality. Contrary to this idea, an aircraft could

easily just be used as a means of transporting a nuclear weapon for a different destination. Since there are vulnerabilities within these sectors, this contributes to the critical need of ensuring that technology is available and would be able to detect potential threats, especially WMD.

The maritime industry currently employs devices that can potentially detect nuclear weapons. The devices used consist of Radiation Isotope Identification Devices (RIIDs), Radiation Portal Monitors (RPMs) and X-Ray imaging devices. However this technology cannot detect 100 percent of the threats, due to their current capabilities. RPMs can detect radiation from a container, in which nuclear material emits some radiation, yet many other items also emit radiation, such as bananas, kitty litter, and medical equipment. This often leads to false alarms, which can end up being costly to the system, as well as time-consuming if further screening, such as physical inspection is required (Huizenga 2005, The Royal Society 2008). X-Ray imaging leads to pictures of the contents of the container, without having to open the container. However, similar to drugs and conventional weapon trafficking, a nuclear weapon may be hidden among normal items, and since an X-Ray is subject to human interpretation, therefore it may not detect a nuclear weapon. In the aviation industry, currently X-Rays, Explosives Detection Systems (EDSs) and Explosives Trace Detection (ETDs) are employed to detect threats, only pertaining to conventional weapons. There are current aviation test markets employing nuclear material detection devices, such as RIIDs for checked-in baggage within the United States, however this appears to be very costly and therefore it will be a matter of time before these procedures and devices are installed at all airports (Sammon 2009). Furthermore, screening of containers and baggage within maritime and aviation industries is a critical component in the ability of interdicting a nuclear weapon.

Within the maritime industry, the prescreening classification of the containers appear to advantageous to the system, if the classification is accurate. The types and detection capabilities of the different technologies also benefit the system if used properly. Multi-layered and bi-level screening also improve the probability of interdicting a nuclear weapon. The contingency plan of detecting a nuclear weapon before an attack occurs is very possible, as long as procedures continue to be followed and improved. The commercial aviation industry, slightly behind in the progress of detecting nuclear weapons, has to ability to be strengthened to the level of maritime security, and can therefore play an equal role in securing the safety of the nation.

The chapters are organized as follows. Chapter 2 discusses risk-based policies for screen-

ing procedures given two prescreening classifications of cargo containers in a maritime security screening system. A computational example is analyzed and primary screening alarms are defined. Results illustrate the tradeoffs between the prescreening classification accuracy and the efficacy of the radiation detectors. Chapter 3 extends Chapter 2 by limiting the prescreening classification to one level, while including a second layer of screening. Primary and secondary screening decisions are given for a computational example given particular nuclear screening technology, as well as includes sensitivity analysis for next-generation technology and changes in the secondary screening costs. Results also conclude that prescreening intelligence is critical in finding a nuclear threat within the system. Chapter 4 explores checked-in baggage screening decisions within aviation security by using performance measures to quantify the effectiveness of the screening system. Screening resources are allocated in order to optimize specific performance measures. Tradeoffs between the performance measures are analyzed and reveal that targets and flights can be covered at minimal expense. Also, greedy heuristics are applied to a computational example in order to improve processing time and find near-optimal solutions. Some of the greedy heuristic result in optimal solutions, while the others do not fall far from being optimal solutions. Chapter 5 includes conclusions and future work ideas for making screening decisions for Homeland Security.

Chapter 2

Maritime Security and Multi-layered Prescreening Classifications

2.1 Introduction

There are many challenges associated with screening cargo containers. One challenge is that RPMs set off a large number of alarms. However, the vast majority of these alarms are due to naturally occurring radioactive material (NORM) alarms, not nuclear material (Huizenga 2005, The Royal Society 2008). Many experts have identified that port security operations largely depend on the characteristics of the cargo contents (such as NORM alarms), yet no systematic, prescriptive guidelines exist for handling NORM alarms from a security point of view (Rooney 2005, Lava 2008). As a result, a small proportion of cargo containers entering United States ports are inspected for nuclear and radiological material using highly effective techniques and technologies, since it is expensive to inspect cargo by physically unpacking the containers or to use non-intrusive inspection technologies. This paper provides a prescriptive framework for investigating security system design given the influence of NORM alarms and classification errors associated with identifying NORM and threat containers.

The Automated Targeting System (ATS) is used to prescreen each cargo container and classify it as high-risk or low-risk (Strohm 2006). Prescreening can also be used to identify which containers have high levels of naturally occurring radiation, resulting in a prescreening paradigm in which each cargo container can be classified according to its likelihood of containing NORM as

classified according to its level of risk. These two levels of classification could be used to create a risk-based screening framework that depends on prescreening intelligence and the physical contents and characteristics of the containers. Although a risk-based approach to cargo screening is part of the U.S. Customs and Border Patrol (CBP) plan for security, few guidelines are given to implement and assess such a strategy.

Note that a prescreening system, such as ATS, could also be used to identify or verify the contents of cargo containers by examining shipping manifests. This is useful, since their physical contents often influence the alarm probabilities for several security devices (e.g. radiography that utilizes X-Rays depends on z-values). In this paper, screening refers to the entire inspection process in a layered port security system, which may involve scanning with a RPM, non-intrusive inspection using imaging technologies, document checking, and physically unpacking containers. Screening may take place at several locations, such as foreign ports, U.S. seaports, land border crossings, as well as other locations. Note that more advanced and expensive screening procedures, such as non-intrusive inspection and unpacking containers, are used more sparingly and are targeted at high-risk containers (US CBP 2007). Such inspections are assumed to be performed at a predetermined security station. Identifying optimal ways to use these limited screening resources is an important part of the homeland security system.

This chapter introduces a linear programming model for using existing screening technologies (e.g., RPMs) to screen cargo containers at a security station using knapsack problem models. The approach determines how to define a *primary screening alarm* and hence, designs and analyzes security system architectures. Containers are sent to secondary screening (or cleared) at a specific location (e.g., the exit lanes at a single port). Containers that yield a primary screening alarm undergo secondary screening, where inspection methods are more effective for nuclear and radiological material examinations. It is assumed that there is a budget that limits the number of containers that can be sent to secondary screening. Note that this approach of exploring how to define a primary screening alarm complements the approach of determining individual sensor operating characteristics (by changing the threshold for defining a primary screening alarm based on receiver-operating characteristic curves) while assuming that an escalating security policy is in place. This latter approach is explored in several papers (e.g. Wein et al. 2007, Boros et al. 2009). Defining a primary screening alarm and defining individual sensor operating characteristics

are interrelated problems that should receive care when modeling. However, the objective of this chapter focuses on the definition of a primary screening alarm to isolate the effect of this type of decision and to highlight its importance in a risk-based security context given the presence of NORM alarms, particularly since any layer in the security system could cumulatively use information from previous security checks to more effectively detect nuclear material.

This chapter extends the analysis by McLay et al. (2010), who explore the relationships and tradeoffs between prescreening intelligence, secondary screening costs, and the efficacy of radiation detectors when there is a single layer of prescreening. This chapter expands this analysis to consider two layers of prescreening to examine the impact of cargo content characteristics (e.g. NORM) on system performance. In particular, each cargo container is classified in two ways: (1) high-risk or low-risk to assess whether the container is a threat (2) high-background or low-background to assess whether the container is a NORM container. The key contribution of this analysis is that it provides a two-level, risk-based framework for determining how to define a system alarm when screening cargo containers given limited secondary screening resources. It illustrates that resources are fundamentally used differently when cargo container contents are part of the decision context. Note that this approach can be trivially generalized to consider three or more layers of prescreening, where the type of prescreening performed in each layer can be defined arbitrarily. The analysis indicates that a threshold-based definition for the system alarm may not be optimal under reasonable assumptions. An illustrative example suggests that the risk prescreening layer is an important factor for effective screening, particularly when sensors are not effective at differentiating threat containers from containers with naturally occurring radioactive material. It also suggests that the NORM prescreening layer is not important for effectively using scarce screening resources.

This chapter is organized as follows. Section 2.2 provides a literature review for security screening problems and research models for detecting nuclear material. Section 2.3 introduces parameters and notation used in the models, and then it introduces the proposed model. Its structural properties are analyzed in Section 2.4. A computational example is analyzed in Section 2.5. Concluding remarks and directions for future research are given in Section 2.6.

2.2 Literature Review

There are few papers that address the detection of nuclear material using operations research methodologies, particularly when considering the effect of screening and NORM alarms. Wein et al. (2006) analyze an 11-layer screening system for containers entering the United States by considering a fixed budget and port congestion. They consider the effects of prescreening from ATS and whether a terrorist enrolls in the Customs-Trade Partnership Against Terrorism (C-TPAT) program.

Bakir (2008) presents a decision tree model to analyze the screening of cargo containers at commercial truck crossings on the United States border with Mexico. They do not recommend routine screening with next-generation technologies at such commercial truck crossings. Bakir's results largely depend on the probability of an attack, and the analysis motivates the need for improved next-generation RPMs. Merrick and McLay (2009) extend Bakir's model to include the effect of NORM alarms, deterrence, and performance measures other than cost. They draw different conclusions than Bakir (2008), namely, that each container should receive at least some minimum level of screening under reasonable assumptions. This suggests that taking NORM alarms into account is important for port security models. Gaukler et al. (2009) also considers the impact of prescreening and cargo contents on cargo containers screening systems. Their analysis suggest that taking cargo container contents into account can be used to effectively augment prescreening.

Several research papers examine inspection strategies for cargo containers that use several types of screening tests. Wein et al. (2007) apply queuing theory and optimization to analyze cargo containers on truck trailers passing by a series of RPMs. They determine the optimal spatial positioning and scanning time for RPMs such that a desired detection probability is achieved. Ramirez-Marquez (2008) use decision trees to find cargo container inspection strategies that minimize inspection costs. Each strategy selects sensors that have varying reliability and costs. The strategy presented maintains a required detection rate in which follows a minimum cost, order-dependent inspection. Boros et al. (2009) determine how to optimally inspect cargo containers by using a large scale linear programming model. Goldberg et al. (2008) extend this approach using decision trees and knapsack problem models by using dynamic programming to identify optimal inspection policies. Kantor and Boros (2010) use principles of game theory to determine

when to unpack and inspect cargo containers when considering mixed inspection strategies. Bakir (2010) uses game theory to explore the strategies of resource allocation within port security. This defender-attacker-defender model incorporates the idea of an adaptive adversary. Note that none of these efforts explicitly consider the effects of prescreening to identify high-risk cargo containers. Morton et al. (2007) propose two different stochastic network interdiction models to minimize the success of a potential terrorist. The first model is deterministic, which assumes that the path and location of the radiation detectors are known by all. In the second model, only a subset of radiation detectors is known by the adversary and the views of the interceptor and smuggler differ. These models help to select sensor locations to minimize a terrorist's probability of being successful at smuggling nuclear material across the borders.

In contrast to previous work in this area that seeks to determine sensor operating characteristics while assuming that an escalating primary security alarm is in place (i.e., at least one alarm yields a primary screening alarm), this chapter explores the complementary issue of how to define a primary screening alarm in a risk-based security system while assuming that the sensor operating characteristics remain constant to shed light on optimal, risk-based security system design and operation. While examining the multi-layered port security system, it is clear that the impact of the cargo containers contents could be very important in the development and design of next-generation port security systems.

2.3 Screening Framework and Model

In this section, terminology and parameters are introduced for the proposed model, and the model is formally stated. The model examines the particular case when containers receive two layers of prescreening, based on ATS and other forms of prescreening, and it investigates how to define a primary screening alarm given these prescreening classifications and the number of sensor alarms. First, a prescreening system is used to classify each container as (1) high-risk or low risk based on whether the container is perceived as a threat and (2) high-background or low-background based on whether the container has high levels of background radiation due to NORM. This results in four prescreening classifications based on combinations of these two prescreening layers.

Cargo containers enter a security station (e.g., exit lanes at a port) to undergo *primary screening*, where n sensors screen each container. These sensors could be radiation detectors such as

RPMs, which screen each cargo container for radiation that is emitted by nuclear material such as plutonium and highly enriched uranium (HEU). Each sensor yields an alarm or clear response, based on how the sensor operates and the characteristics of the cargo container, and hence, the total number of sensor alarms is between zero and n . The sensor alarms depend on the true underlying container contents (whether a threat or NORM is in the container). NORM containers that signal alarms often result in false positives. Reducing these alarms is critical, since it takes resources to ensure that a threat is not present. Ideally, the system yields a clear response for all of the non-threat containers irrespective of whether NORM is present and yields an alarm response for all of the threat containers.

Based on the total number of sensor alarms, a primary screening system response is given. This allows the system response (either alarm or clear) to be defined in one of several ways (Kobza and Jacobson 1996, 1997). The primary screening system response has one of two outcomes, either an alarm is given or the container is cleared. If the cargo container is cleared, it exits the security station and continues along its path to its destination. The cargo containers that yield a primary screening alarm undergo *secondary screening*. The objective is to determine which containers yield a primary screening alarm in order to maximize the expected number of threat containers that are selected for secondary screening. Note that this framework is defined generally for any type of radiological and nuclear sensor, and it makes no assumptions about how the sensors work together. Moreover, prescreening classifies containers as either high-risk/low-risk and high background/low background resulting in four classes of containers. However, this approach can be used for any type of prescreening that classifies containers into one of a set of possible risk classes. Therefore, this framework captures a broad range of security screening operations.

The parameters are:

- N = number of cargo containers screened at the security station,
- B = total secondary screening budget (in terms of the total number of the N containers that can be selected for secondary screening)
- P_{HR} = the probability that a cargo container is classified as high-risk,
- $P_{LR} = 1 - P_{HR}$ = the probability that a cargo container is classified as low-risk,
- P_T (P_{NT}) = the probability a cargo container (is a threat and contains nuclear material (not a threat),

- P_{HB} = the probability that a cargo container is classified as high-background,
- $P_{LB} = 1 - P_{HB}$ = the probability that a cargo container is classified as low-background,
- P_M (P_{NM}) = the probability that a cargo container contains (does not contain) NORM,
- $P_{M|T}$ = the probability that a container contains NORM given that it is a threat,
- P_{kA} = the probability that a cargo container yields k alarms (of the n sensors), $k = 0, 1, \dots, n$,
- $P_{A|T \cap M}^i$ = the probability that a threat and NORM container yields a true alarm (false clear) at sensor i , $i = 1, 2, \dots, n$,
- $P_{A|T \cap NM}^i$ = the probability that a threat and non-NORM container yields a true alarm (false clear) at sensor i , $i = 1, 2, \dots, n$,
- $P_{A|NT \cap M}^i$ = the probability that a non-threat and NORM container yields a false alarm (true clear) at sensor i , $i = 1, 2, \dots, n$,
- $P_{A|NT \cap NM}^i$ = the probability that a non-threat and non-NORM container yields a false alarm (true clear) at sensor i , $i = 1, 2, \dots, n$,
- $\beta = P_{T|HR}/P_{T|LR}$ = ratio of high-risk containers that are threats to low-risk containers that are threats,
- $\gamma = P_{M|HB}/P_{M|LB}$ = ratio of high-background containers that are NORMS to low-background containers that are NORMS.

The total number of containers N is a deterministic value that represents the number of cargo containers that pass through a given station in a year, or another period of time. The budget for secondary screening B is a deterministic value based on available resources, and the cost to perform secondary screening is a deterministic value based on information collected and analyzed by the Department of Homeland Security (DHS) and CBP. It is in part based on salaries paid to the employees hired to perform secondary screening. Note that the budget can be selected to take delay costs into account, and hence delays are implicitly handled by this model, which assumes that the cost to resolve an alarm with secondary screening is the same for all types of containers.

The probability that a cargo container is a threat P_T is a deterministic value that is assessed by personnel within DHS based on the perceived threat level. This value is considered highly sensitive and may change based on changes in national or international situations, intelligence information, or the risk level of the Homeland Security Advisory System. The probability that a cargo container is a NORM container P_M is a deterministic value that represents the proportion of NORM containers. The conditional probability that a threat container is a NORM container $P_{M|T}$

is assumed to be a deterministic parameter based on intelligence. Note that this value reflects the decision of an intelligent adversary, which is not likely to be random.

The values of β and γ are deterministic values that quantify the quality of prescreening provided by ATS. The probability that a container is classified as high-risk P_{HR} (or high-background P_{HB}) is based on the proportion of containers passing through a security station that are classified as high-risk (or high-background), once a large number of cargo containers has been evaluated. The probability that a cargo container yields k (of n) alarms depends on how the sensors operate, and it is assumed to only depend on whether a container is a threat and whether NORM is present.

The proposed model determines which containers yield a primary screening alarm and undergo secondary screening. Although $P_{M|T}$ is deterministic from the point of view of terrorists, it is treated as a probability to reflect uncertainty from the point of view of the defender. It is assumed that each container is screened independently of the other containers. After each container is screened by the sensors, the number of alarms is known, and the decision is made about whether to inspect the container using secondary screening.

The variables implicitly assume that a container is selected for secondary screening (SS) based on the number of primary screening alarms and its classification (LR/HR and LB/HB) rather than its true state (T/NT and M/NM).

- $x_{HR \cap HB}^k = P_{SS|kA \cap HB \cap HR}$ = proportion of high-risk, high-background containers with k -of- n alarms that yield a primary screening alarm.

Note that $x_{HR \cap LB}^k$, $x_{LR \cap HB}^k$, and $x_{LR \cap LB}^k$ are defined analogously.

First, the proposed model is stated as an linear programming model in terms of the number of sensors n . Then, its objective function and constraint coefficients are derived using the parameters introduced earlier in this section.

$$\begin{aligned}
Z_n = \max \quad & N \sum_{k=0}^n \left(P_{kA \cap HB \cap HR \cap NT} x_{HR \cap HB}^k + P_{kA \cap LB \cap HR \cap NT} x_{HR \cap LB}^k \right. \\
& \left. + P_{kA \cap HB \cap LR \cap NT} x_{LR \cap HB}^k + P_{kA \cap LB \cap LR \cap NT} x_{LR \cap LB}^k \right) \quad (2.1) \\
\text{s.t.} \quad & N \sum_{k=0}^n \left((P_{kA \cap HB \cap HR \cap NT} + P_{kA \cap HB \cap HR \cap NT}) x_{HR \cap HB}^k + (P_{kA \cap LB \cap HR \cap NT} + \right. \\
& \left. P_{kA \cap LB \cap HR \cap NT}) x_{HR \cap LB}^k + (P_{kA \cap HB \cap LR \cap NT} + P_{kA \cap HB \cap LR \cap NT}) x_{LR \cap HB}^k + \right.
\end{aligned}$$

$$(P_{kA \cap LB \cap LR \cap T} + P_{kA \cap LB \cap LR \cap NT})x_{LR \cap LB}^k \leq B \quad (2.2)$$

$$0 \leq x_{HR \cap HB}^k \leq 1, k = 0, 1, \dots, n \quad (2.3)$$

$$0 \leq x_{HR \cap LB}^k \leq 1, k = 0, 1, \dots, n \quad (2.4)$$

$$0 \leq x_{LR \cap HB}^k \leq 1, k = 0, 1, \dots, n \quad (2.5)$$

$$0 \leq x_{LR \cap LB}^k \leq 1, k = 0, 1, \dots, n \quad (2.6)$$

The objective function value (2.1) captures the the expected number of threats that yield a primary screening alarm. It can be computed by conditioning on whether the container is a NORM container, for example,

$$\begin{aligned} P_{kA \cap HB \cap HR \cap T} &= P_{kA|HB \cap M \cap HR \cap T} P_{HB|M \cap HR \cap T} P_{M|HR \cap T} P_{HR|T} P_T \\ &\quad + P_{kA|HB \cap NM \cap HR \cap T} P_{HB|NM \cap HR \cap T} P_{NM|HR \cap T} P_{HR|T} P_T. \end{aligned} \quad (2.7)$$

There is a single knapsack (i.e., budget) constraint (2.2) that ensures that the expected number of containers that undergo secondary screening is less than B . This constraint can be computed by conditioning on the two prescreening in an analogous way as in (2.7). Constraints (2.3) – (2.6) set the variable lower and upper bounds.

In order to obtain insight into screening operations, consider two simplifying assumptions.

ASSUMPTION 1: The event that a container is a NORM container is independent of whether it is classified as high-risk or low-risk. (i.e., $P_{M|T} = P_{M|HR \cap T} = P_{M|LR \cap T}$ and $P_{M|NT} = P_{M|HR \cap NT} = P_{M|LR \cap NT}$).

ASSUMPTION 2: The event that a container is classified as high-background or low-background is independent of whether the container is a threat and of whether the container is classified as high-risk or low-risk. (i.e., $P_{HB|M} = P_{HB|M \cap HR \cap T} = P_{HB|M \cap HR \cap NT} = P_{HB|M \cap LR \cap T} = P_{HB|M \cap LR \cap NT}$ and $P_{HB|NM} = P_{HB|NM \cap HR \cap T} = P_{HB|NM \cap HR \cap NT} = P_{HB|NM \cap LR \cap T} = P_{HB|NM \cap LR \cap NT}$).

Given these two assumptions, then the objective function and budget constraint coefficients in (2.1) and (2.2) can be obtained as follows. First, consider the objective function coefficients for high-risk, high-background containers, which simplifies (2.7).

$$\begin{aligned} P_{kA \cap HB \cap HR \cap T} &= P_{kA|M \cap T} P_{HB|M} P_{M|T} P_{HR|T} P_T \\ &\quad + P_{kA|NM \cap T} P_{HB|NM} P_{NM|T} P_{HR|T} P_T, k = 0, 1, \dots, n. \end{aligned} \quad (2.8)$$

Ideally, all threat containers are classified as high-risk. The probability that a threat container is classified as high-risk is given by $P_{HR|T}$. Given that $\beta = P_{T|HR}/P_{T|LR}$ and $P_{HR|T} + P_{LR|T} = 1$ and using Bayes Rule,

$$P_{HR|T} = \frac{\beta P_{HR}}{1 - P_{HR} + \beta P_{HR}}, \quad (2.9)$$

and

$$P_{HR|NT} = \frac{P_{HR} - P_{HR|T}P_T}{1 - P_T}. \quad (2.10)$$

Likewise,

$$P_{HB|M} = \frac{\beta P_{HB}}{1 - P_{HB} + \beta P_{HB}}, \quad (2.11)$$

and

$$P_{HB|NM} = \frac{P_{HB} - P_{HB|M}P_M}{1 - P_M}. \quad (2.12)$$

The method used by McLay et al. (2010) can be used to compute the conditional probabilities that k -of- n alarms are signaled given that a container is a threat/non-threat and whether it is a NORM/non-NORM container. These k -of- n alarm probabilities ($P_{kA|M \cap T}$, $P_{kA|NM \cap T}$, $P_{kA|M \cap NT}$, and $P_{kA|NM \cap NT}$) can be computed using a reliability model (Koucky 2003). In the case when each sensor operates independently and identically with the probability of a single sensor alarm P_A , then the number of alarms can be modeled as Binomial random variables with parameters n and P_A .

2.4 Structural Properties

This section summarizes the structural properties of the proposed model. The proposed model is a particular case of the linear programming relaxation to the 0-1 Knapsack Problem (KP). In KP, there are rewards r_i and weights w_i , $i = 1, 2, \dots, m$, for the m items with a knapsack capacity c . The linear programming relaxation to KP can be solved in $O(m)$ time. To find the optimal solution, the items are sorted in non-increasing order of the ratio of the item reward to weight (i.e., $r_1/w_1 \geq r_2/w_2 \geq \dots \geq r_m/w_m$). This is defined as the *optimal knapsack sequence*. The knapsack is greedily packed in order, starting with item 1, until the knapsack's capacity is filled. The variables are one or zero for all items except the critical item s (where $s = \arg \min_j \{\sum_{i=1}^j w_i > c\}$). The equivalent model in terms of the KP has $m = (n + 1)$ items and capacity $c = B$. The rewards

are equal to the expected number of threats (for a particular classification) that yield k alarms, whereas the weights are equal to the expected number of containers (for a particular classification) that yield k alarms, $k = 0, 1, \dots, n$.

Define the reward for high-risk and high-background containers as

$$r_{HR,HB}^k = NP_{kA \cap HB \cap HR \cap T}, k = 0, 1, \dots, n.$$

Likewise, define the weight for high-risk, high-background containers as

$$w_{HR,HB}^k = N(P_{kA \cap HB \cap HR \cap T} + P_{kA \cap HB \cap HR \cap NT}), k = 0, 1, \dots, n,$$

The remaining rewards and weights are defined analogously. Therefore, the proposed model((2.1) – (2.6)) in can be rewritten as a knapsack problem:

$$\begin{aligned} Z_n = \max & \quad \sum_{k=0}^n (r_{HR,HB}^k x_{HR,HB}^k + r_{HR,LB}^k x_{HR,LB}^k + r_{LR,HB}^k x_{LR,HB}^k + r_{LR,LB}^k x_{LR,LB}^k) \\ \text{subject to} & \quad \sum_{k=0}^n (w_{HR,HB}^k x_{HR,HB}^k + w_{HR,LB}^k x_{HR,LB}^k + w_{LR,HB}^k x_{LR,HB}^k + w_{LR,LB}^k x_{LR,LB}^k) \leq B \\ & \quad 0 \leq x_{HR,HB}^k \leq 1, k = 0, 1, \dots, n \\ & \quad 0 \leq x_{HR,LB}^k \leq 1, k = 0, 1, \dots, n \\ & \quad 0 \leq x_{LR,HB}^k \leq 1, k = 0, 1, \dots, n \\ & \quad 0 \leq x_{LR,LB}^k \leq 1, k = 0, 1, \dots, n \end{aligned}$$

where Z_n above is equivalent to Z_n in (2.1).

This screening scenario uses a probability model based on Bayes Rule, where the two pre-screening layers defining the prior probabilities and the likelihood of number of alarms define the posterior probabilities. The prior probabilities that a high-risk, high-background cargo container is a threat is

$$\begin{aligned} P_{T|HR \cap HB} &= \frac{P_{HB \cap HR \cap T}}{P_{HB \cap HR}} = \frac{\sum_{k=0}^n P_{kA \cap HB \cap HR \cap T}}{\sum_{k=0}^n (P_{kA \cap HB \cap HR \cap T} + P_{kA \cap HB \cap HR \cap NT})} \\ &= \frac{\sum_{k=0}^n r_{HR,HB}^k}{\sum_{k=0}^n w_{HR,HB}^k}. \end{aligned}$$

The posterior probabilities are the conditional probabilities that a cargo container is a threat given that it yields k alarms and is classified as high-risk, high-background $P_{T|kA \cap HB \cap HR}$ (or

high-risk, low-background $P_{T|kA\cap LB\cap HR}$, low-risk, high-background $P_{T|kA\cap HB\cap LR}$, low-risk, low-background $P_{T|kA\cap LB\cap LR}$). Theorem 1 defines the posterior probabilities. Note that Theorem 1 does not rely on Assumptions 1 and 2.

Theorem 1 *The posterior probabilities $P_{T|kA\cap HB\cap HR}$, $P_{T|kA\cap LB\cap HR}$, $P_{T|kA\cap HB\cap LR}$, $P_{T|kA\cap LB\cap LR}$ are defined as the ratio of the reward to the weight, $r_{HR,HB}^k/w_{HR,HB}^k$, $r_{HR,LB}^k/w_{HR,LB}^k$, $r_{LR,HB}^k/w_{LR,HB}^k$, $r_{LR,LB}^k/w_{LR,LB}^k$, respectively, $k = 0, 1, \dots, n$.*

Proof. First consider high-risk, high-background cargo containers. The posterior probability that a high-risk, high-background cargo container yielding k alarms is a threat is

$$P_{T|kA\cap HB\cap HR} = \frac{P_{kA\cap HB\cap HR\cap T}}{P_{kA\cap HB\cap HR}} = \frac{P_{kA\cap HB\cap HR\cap T}}{P_{kA\cap HB\cap HR\cap T} + P_{kA\cap HB\cap HR\cap NT}} = \frac{r_{HR,HB}^k/N}{w_{HR,HB}^k/N}$$

The posterior probabilities for high-risk, low-background; low-risk, high-background; and low-risk, low-background cargo containers are computed in a similar manner. \square

It seems intuitive to select containers for secondary screening that yield more alarms rather than fewer alarms. However, the proposed model does not enforce such a policy. Theorem 2 indicates the conditions under which a container with a given prescreening classification yielding more alarms makes it more likely to be selected for secondary screening than a container with the same prescreening classification yielding fewer alarms. For each prescreening classification, the order that items are put into the knapsack (i.e., the order in which containers are selected for secondary screening) depends only on how the sensors work together. It does not take into account the prescreening, the underlying probability of a threat, or the proportion of containers classified as high-risk. Also, note that Theorem 2 does not depend on Assumptions 1 and 2.

Theorem 2 *High-risk, high-background containers that yield k alarms occur before high-risk, high-background containers that yield $k - 1$ alarms in the optimal knapsack sequence,*

$$\frac{r_{HR,HB}^k}{w_{HR,HB}^k} \geq \frac{r_{HR,HB}^{k-1}}{w_{HR,HB}^{k-1}}$$

only if

$$\frac{P_{kA\cap HR\cap HB|T}}{P_{(k-1)A\cap HR\cap HB|T}} \geq \frac{P_{kA\cap HR\cap HB|NT}}{P_{(k-1)A\cap HR\cap HB|NT}}.$$

Moreover, high-risk, low-background; low-risk, high-background; and low-risk, low-background containers that yield k alarms occur before containers that yield $k - 1$ alarms in the optimal knapsack sequence, only if

$$\frac{P_{kA \cap HR \cap LB|T}}{P_{(k-1)A \cap HR \cap LB|T}} \geq \frac{P_{kA \cap HR \cap LB|NT}}{P_{(k-1)A \cap HR \cap LB|NT}}; \quad \frac{P_{kA \cap LR \cap HB|T}}{P_{(k-1)A \cap LR \cap HB|T}} \geq \frac{P_{kA \cap LR \cap HB|NT}}{P_{(k-1)A \cap LR \cap HB|NT}};$$

$$\frac{P_{kA \cap LR \cap LB|T}}{P_{(k-1)A \cap LR \cap LB|T}} \geq \frac{P_{kA \cap LR \cap LB|NT}}{P_{(k-1)A \cap LR \cap LB|NT}},$$

respectively.

Proof. First, consider the high-risk, high-background containers. By definition,

$$\frac{r_{HR,HB}^k}{w_{HR,HB}^k} = \frac{N P_{kA \cap HB \cap HR \cap T}}{(N P_{kA \cap HB \cap HR \cap T} + N P_{kA \cap HB \cap HR \cap NT})} \geq$$

$$\frac{N P_{(k-1)A \cap HB \cap HR \cap T}}{(N P_{(k-1)A \cap HB \cap HR \cap T} + N P_{(k-1)A \cap HB \cap HR \cap NT})} = \frac{r_{HR,HB}^{k-1}}{w_{HR,HB}^{k-1}}.$$

Rearranging yields the desired result. The same approach can be taken for high-risk, low-background; low-risk, high-background; low-risk, low-background containers. \square

Corollary 1 illustrates when the conditions in Theorem 2 hold for the particular case when each sensor operates independently and identically and when the background prescreening layer is completely accurate (i.e., $P_{HB|M} = P_{LM|NM} = 1$). Although this assumption is optimistic, the information provided by shipping manifests can be used to identify NORM containers with a high degree of accuracy. It indicates that the single sensor true alarm probability must be higher than the single sensor false alarm probability in order for a cargo container yielding k alarms to occur earlier in the optimal knapsack sequence before a cargo container yielding $k - 1$ alarms, given the prescreening classification $k = 1, 2, \dots, n$.

Corollary 1 *When $P_{HB|M} = P_{LM|NM} = 1$ and sensors alarms are independently and identically distributed with alarm probabilities $P_{A|T \cap M}$ and $P_{A|NT \cap M}$, respectively, then*

$$\frac{r_{HR,HB}^k}{w_{HR,HB}^k} \geq \frac{r_{HR,HB}^{k-1}}{w_{HR,HB}^{k-1}}$$

only if $P_{A|T \cap M} \geq P_{A|NT \cap M}$. This applies to high-risk, low-background; low-risk, high-background; low-risk, low-background containers, given their associated alarm probabilities.

Proof. First, consider the high-risk, high-background containers. Using Assumptions 1 and 2 and the reasoning provided in (2.8), then $P_{kA \cap HR \cap HB|T} = P_{kA|M \cap T} P_{M|T} P_{HR|T}$. Then the condition in Theorem 2 simplifies to

$$\frac{P_{kA \cap HR \cap HB|T}}{P_{(k-1)A \cap HR \cap HB|T}} = \frac{P_{kA|M \cap T}}{P_{(k-1)A|M \cap T}} = \frac{C_k^n P_{A|T \cap M}^k (1 - P_{A|T \cap M})^{n-k}}{C_{k-1}^n P_{A|T \cap M}^{k-1} (1 - P_{A|T \cap M})^{n-k+1}} \geq$$

$$\frac{C_k^n P_{A|NT \cap M}^k (1 - P_{A|NT \cap M})^{n-k}}{C_{k-1}^n P_{A|NT \cap M}^{k-1} (1 - P_{A|NT \cap M})^{n-k+1}} = \frac{P_{kA|M \cap NT}}{P_{(k-1)A|M \cap NT}} = \frac{P_{kA \cap HR \cap HB|NT}}{P_{(k-1)A \cap HR \cap HB|NT}}.$$

Rearranging yields

$$\frac{P_{A|T \cap M}}{1 - P_{A|T \cap M}} \geq \frac{P_{A|NT \cap M}}{1 - P_{A|NT \cap M}},$$

and simplifying yields $P_{A|T \cap M} \geq P_{A|NT \cap M}$. The same approach can be taken for high-risk and low-background, low-risk and high-background, low-risk and low-background containers. \square

Note that several types of screening technologies (e.g., RPMs) may be more effective at identifying NORM containers than threat containers, and hence, the conditions in Corollary 1 may not hold. Therefore, a threshold policy may not be optimal even under simplifying assumptions, which sheds light on the challenges associated with port security system design. Section 6 illustrates this issue for a computational example.

2.5 Computational Example and Results

This section reports results for a computational example to explore the tradeoffs between the two layers of prescreening (i.e., β and γ), and the alarm rates associated with each sensor. The results not only indicate the likelihood of detecting nuclear material, they also indicate how to define a primary screening alarm, based on a container's prescreening classification and how many sensors yield an alarm. The analysis considers cargo containers screened by a series of n sensors that are independent and operate identically. For example, the number of alarms for threat and NORM containers are modeled as a Binomial random variable with parameters n and $P_{A|T \cap M}$. Although this independence assumption is not realistic, it sheds light on how a primary screening alarm can be defined in a multi-layered security system. McLay et al. (2010) illustrate that the highly dependent devices can be modeled using a single screening device.

The proposed model is analyzed for a hypothetical single security station over a time horizon of one year. Table 3.1 contains the base case input parameters, which remain constant unless

otherwise specified. It is assumed that $N = 1\text{M}$ containers enter the security station during the time horizon. The probability that a container is a threat is $P_T=1/N$, which is selected such that one threat is expected to pass through the security station. In the analysis, the objective function represents the expected number of threat containers that are selected for secondary screening (the number of true alarms). The expected number of threats in the system is $P_T N = 1$, and hence, 1.0 is an upper bound on the objective function value for the base case. Given that the probability of a threat is $1/N$, the objective function value captures the *detection probability*, i.e., the conditional probability that a threat is selected for secondary screening.

The fraction of containers yielding a particular number of primary alarms that are randomly selected for secondary screening defines the screening policy. For instance, $x_{HR,LB}^3 = 0.70$ is interpreted to mean that a high-risk, low-background container yielding three alarms has a probability of 0.70 of being selected for secondary screening and a probability of 0.30 of being cleared. The other variables are either zero or one, meaning no containers or all containers are selected for secondary screening, respectively.

The prescreening multiplier β determines the likelihood that a container with a threat is classified as high-risk for a given proportion of all containers classified as high-risk P_{HR} . McLay et al. (2010) report that $\beta = 10$ is realistic and that $\beta = 100$ is an upper bound for an excellent prescreening system. Since β is a function of P_{HR} , scenarios with a fixed value of P_{HR} are compared across different values of β . Of all containers, four-percent are assumed to be high-risk, which is consistent with what port authorities have reported (Lava 2008). The values of the prescreening multiplier considered are $\beta = 1, 10, 100$. When $\beta = 1$ the level of prescreening is *random*, which implies that a random proportion of containers are classified as high-risk, resulting in $P_{T|HR} = P_{T|LR}$.

The value of the prescreening multiplier γ determines the probability that a NORM container is classified as high-background for a given proportion of containers classified as high-background P_{HB} . Since γ depends on the container contents, which are readily available based on documentation available to ATS, it is assumed that $\gamma = 1000$. Moreover, it is assumed that fixed values of $P_{HB} = P_M = 0.025$ are considered based on publicly available data (Rooney 2005). The results are insensitive to γ , so the results for other values of γ are not reported.

The conditional probability that a threat is placed in a NORM container $P_{M|T}$ captures the likelihood that a threat would be hidden among NORM. Since there is considerable speculation

Table 2.1: Base case parameter values

Parameter	Value(s)
N	1,000,000
P_T	$1/N = 0.000001$
P_M	0.025
n	1,3,5
P_{HR}	0.04
P_{HB}	0.025
β	1, 10, 100
γ	1000
$P_{A T \cap M}$	0.95
$P_{A NT \cap M}$	0.95
$P_{A T \cap NM}$	0.5
$P_{A NT \cap NM}$	0.001
B	$0.02N$

regarding how terrorists would mask nuclear material, it is assumed that a threat is equally likely to be put in a NORM container or a non-NORM container, and hence, $P_{M|T} = 0.5$ in the base case. This allows for the greatest risk and highest variance in terms of what the defender expects. It is assumed that two-percent of containers can be selected for secondary screening ($B = 0.02N$). The single sensor alarm probabilities for NORM containers (both threat and non-threat) are set to $P_{A|T \cap M} = P_{A|NT \cap M} = 0.95$, which are consistent with the publicly reported estimates that NORM containers consistently result in RPM alarms (Rooney 2005). Likewise, the single sensor alarm probability for non-threat, non-NORM containers is set to $P_{A|NT \cap NM} = 0.001$, since there is no source of radiation in these containers. The single sensor alarm probability for threat, non-NORM containers depends on the source, the size of the source, and the amount of shielding. Publicly reported estimates for this alarm probability have widely varied, and hence, it is set to $P_{A|T \cap NM} = 0.5$ (Levi 2007, Cochran and McKinzie 2008). Note that Theorem 2 provides the conditions to guarantee a threshold policy for all prescreening classifications. However, none of the base case scenarios with $n > 1$ and $\beta = 10, 100$ guarantee a threshold policy for any of the four prescreening classifications when applying Theorem 2, although they may result in a threshold policy for the value of B selected. This illustrates the challenges associated with NORM alarms from an operational point of view.

Each cargo container is assumed to be scanned by a series of $n = 1, 2, 3, 4, 5$ sensors. Each sensor operates independently and identically. Figure 2.1 shows the objective function value Z_n

(i.e., the detection probability) as β varies from 1 to 100. It illustrates the importance of having many sensors. For example, when $n = 1$ and $\beta = 1$, the detection probability is 0.6304, and it increases to 0.7441 when $\beta = 100$. When $n = 5$ and $\beta = 1$, the detection probability is 0.8060 and it increases to 0.9480 when $\beta = 100$.

To consider the marginal improvement in the detection probability when adding one additional sensor, let $\Delta Z_n = Z_n - Z_{n-1}$, the change in the detection probability when one additional sensor is added to a system with $n - 1$ sensors (resulting in n sensors). Figure 2.2 illustrates ΔZ_n , $n = 2, 3, 4, 5$, as a function of β . The range of values for the marginal increase in the detection probability from one to two sensors (ΔZ_2) is narrow, ranging from 0.1050 to 0.1208. This indicates that adding a second sensor results in a relative constant improvement in the detection probability, regardless of prescreening intelligence. Note that this range is not as narrow for other values of $P_{A|NM \cap T}$ compared to the base case of $P_{A|NM \cap T} = 0.50$ (illustrated in Figure 2.2). Note that ΔZ_2 achieves its largest value of 0.1208 when $\beta = 77$, which suggests that adding a second sensor is relatively less beneficial when prescreening risk intelligence is extremely low (near $\beta = 1$) or extremely high (near $\beta = 100$) as compared to $\beta = 77$. This indicates that additional screening technologies are most beneficial when prescreening provides some guidance as to how they can best be used. In a similar fashion, the ranges of values for ΔZ_3 , ΔZ_4 , ΔZ_5 are also very narrow. Note that ΔZ_3 and ΔZ_4 increase with β . This suggests that adding a third or fourth sensor is most beneficial for systems with excellent prescreening risk intelligence. However, ΔZ_5 decreases with β . When $\beta \leq 5.2$, $\Delta Z_4 \leq \Delta Z_5$, which indicates that for low levels of prescreening risk intelligence, adding a fifth sensor is more beneficial than adding a fourth sensor. This surprising result suggests that many sensors can be used to mitigate the uncertainties that accompany low prescreening risk intelligence. This is examined in greater detail in Figure 2.3.

Figure 2.3 illustrates ΔZ_n , as a function of n , for $\beta = 1, 10, 100$ with n ranging from two to ten sensors. When $n > 5$, $\beta = 1$ results in the greatest marginal improvement in the detection probability as compared to $\beta = 10, 100$. For $\beta = 100$, ΔZ_n monotonically decreases with n . Note that ΔZ_n is not monotonically decreasing with n for $\beta = 1$ or $\beta = 10$, which indicates that adding a sixth sensor results in a greater marginal improvement in the detection probability than the fourth or fifth sensor. This highlights the benefits of a layered screening system, particularly when prescreening risk intelligence is low. All levels of β result in nearly identical values for ΔZ_5 , with

a marginal improvement in the detection probability of 0.015. However, this occurs only during the base case scenario of $P_{A|NM\cap T} = 0.50$.

To better understand the impacts of screening and costs, sensitivity analysis was performed for $P_{A|NM\cap T}$, since this is the most uncertain sensor operation characteristic. Figure 4 shows the detection probability as a function of the probability of a single sensor true alarm for NORM containers. Figure 2.4(a) illustrates the case with $n = 1$, Figure 2.4(b) illustrates the case with $n = 3$, and Figure 2.4(c) illustrates the case with $n = 5$. For $n = 1$, the detection probability increases linearly with the same slope for $P_{A|NM\cap T} \geq 0.50$ and for the $\beta = 1, 10, 100$ scenarios.

For $n = 3$, Figure 4 shows that the improvement in the detection probability from $\beta = 1$ to $\beta = 10$ is less than from $\beta = 10$ to $\beta = 100$. This suggests that efforts made to moderately improve prescreening intelligence over random may have a small improvement in security, whereas efforts to moderately improve good prescreening intelligence may have larger improvements in security. This large improvement in security when β increases from 10 to 100 is largely due to redefining the primary screening alarm, which illustrates the importance of how a primary screening alarm is defined. In Figure 2.4(a), the base case detection probability ($P_{A|NM\cap T} = 0.50$) for $\beta = 100$ is 0.7016 which is nearly identical to the $P_{A|NM\cap T} = 1.0$ and $\beta = 1$ scenario (with a detection probability of 0.7048). This suggests that excellent risk prescreening intelligence can in essence mitigate the risk associated with imperfect screening technologies. When $n = 3$, the $\beta = 100$ and $P_{A|NM\cap T} = 0.10$ scenario has a detection probability of 0.65, which is almost identical to the detection probability for the $\beta = 1$ and $P_{A|NM\cap T} = 0.50$ scenario. When $n = 5$ and $P_{A|NM\cap T} = 0.50$, both the $\beta = 1$ and $\beta = 10$ scenarios have higher detection probabilities as compared to the $P_{A|NM\cap T} = 0.10$ and $\beta = 100$ scenario, which suggests that the many sensors may mitigate the risks associated with low prescreening intelligence.

The optimal screening policies for screening cargo containers is not a threshold policy in all cases. Recall that the results of Theorem 2 (that guarantees a threshold policy) does not apply to any of the four prescreening classifications for the base case. However, a threshold policy may be observed for a given level of the budget. The values of β , γ , P_{HR} , and P_M result in $P_{HR\cap HB} = 0.001$, $P_{HR\cap LB} = 0.024$, $P_{LR\cap HM} = 0.039$, and $P_{LR\cap LM} = 0.936$. Figures 5 and 6 show the optimal screening policies as a function of $P_{A|T\cap NM}$ for each of the four risk classifications for the $n = 5$ scenarios for $\beta = 10$ and $\beta = 100$, respectively. In these figures,

an ‘X’ indicates that containers yielding exactly k alarms are selected for secondary screening, $k = 0, 1, \dots, n$ (i.e. if its associated variable is greater than zero). Note that Figure 2.5(a) indicates that all high-risk, high-background containers yielding at least one alarm are selected for secondary screening when $P_{A|T \cap NM} \leq 0.48$. Containers yielding two or more alarms are selected for secondary screening when $P_{A|T \cap NM} > 0.48$. Similar screening patterns exist for high-risk, low-background and low-risk, low-background containers. Figure 2.5(c) indicates that the optimal way to define a primary screening alarm is not a threshold policy for low-risk, high-background cargo containers for $P_{A|T \cap NM} \leq 0.82$, where containers yielding two, three or four alarms are selected for secondary screening, but containers yielding zero, one, or five alarms are not selected for secondary screening.

Next, consider the $\beta = 100$ scenarios in Figure 6. Figure 2.6(a) indicates that all high-risk, high-background containers are selected for secondary screening when $P_{A|T \cap NM} \leq 0.18$, which indicates that primary screening is essentially not necessary for these containers. Figure 2.6(c) shows a similar non-threshold policy to that illustrated in Figure 2.5(c). However, these figures differ in that no low-risk, high-background containers are selected for secondary screening when $P_{A|NM \cap T} \leq 0.18$ when $\beta = 100$. This is because improving prescreening risk intelligence combined with low screening technology accuracy limits containers from being selected to secondary screening. An additional non-threshold policy occurs for low-risk, low-background containers in Figure 2.5(d) when $P_{A|NM \cap T} \leq 0.16$ for four alarms, and $P_{A|NM \cap T} \leq 0.18$ for five alarms.

2.6 Conclusions

This chapter introduces a linear programming model for screening cargo containers for nuclear material at security stations throughout the United States using knapsack problem, reliability, and Bayesian probability models. The analysis provides a risk-based framework for determining how to define a primary screening alarm when screening cargo containers given limited screening resources. This highlights the operational challenges associated with designing effective security systems, given the presence of NORM alarms and motivates the need for additional analysis.

Analysis of this proposed model indicates that the optimal policy is not a threshold policy under reasonable assumptions. A computational example suggests that accurate risk prescreening intelligence is one of the most important factors for effective screening, particularly when sensors

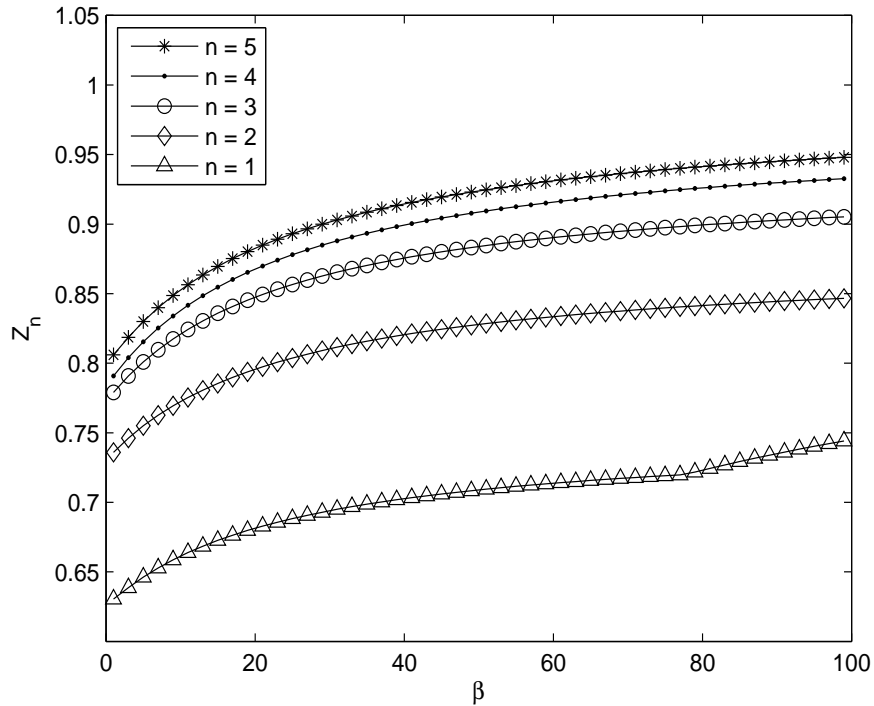


Figure 2.1: Detection probability Z_n as a function of β

are not effective at identifying non-NORM threats, which is the case with highly enriched uranium. Next-generation technology with high true alarm rates can mitigate some of the risk associated with low prescreening intelligence. It also suggests that a multiple layered prescreening system is most beneficial when β is low.

The proposed model investigates the issue of how to define a primary screening alarm given a set of screening devices, rather than depend on prespecified notions of how a primary screening alarm should be defined. The proposed model can be used as a general framework to determine how to design next-generation security screening system as well as define a primary screening alarm for any type of problem that relies on a series of screening devices or methods, risk assessments, and a limited secondary screening budget. It builds upon the approach taken by McLay et al. (2010) to investigate how to manage threats when sensor alarms depend on the cargo container contents.

There are several possible extensions. One extension is to consider this proposed model as one component in a larger access security system, with dependencies between the components. A

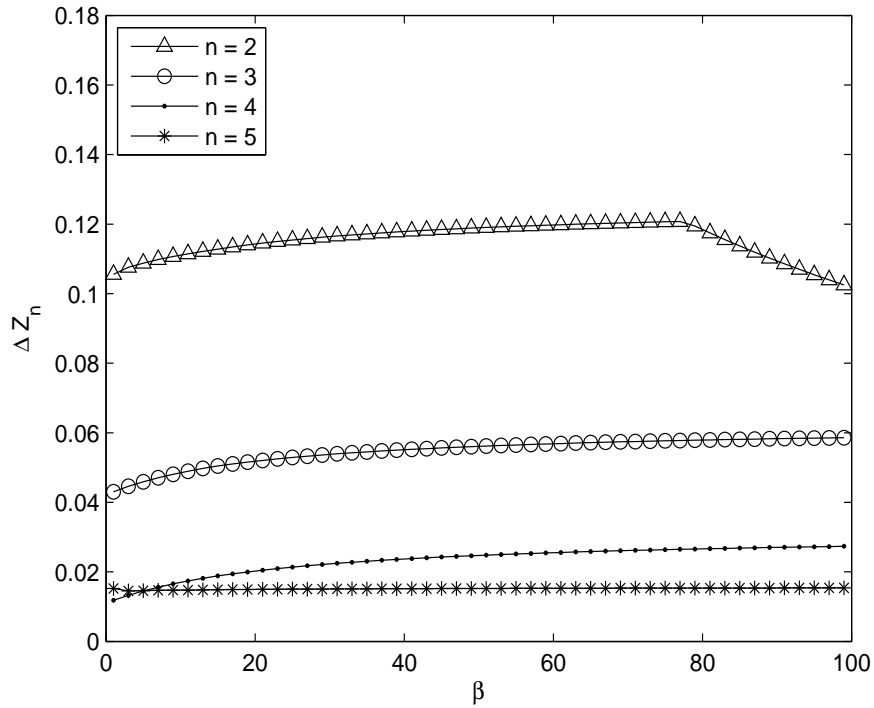


Figure 2.2: Marginal increase in the detection probability ΔZ_n as a function of β

second extension to this proposed model is to consider the impact of enforcing threshold policies. A third extension is to explore the impact of a portfolio of threats as well as complementary technologies to detect such threats (that detect alpha, beta, and gamma particles). Work is in progress to address these extensions.

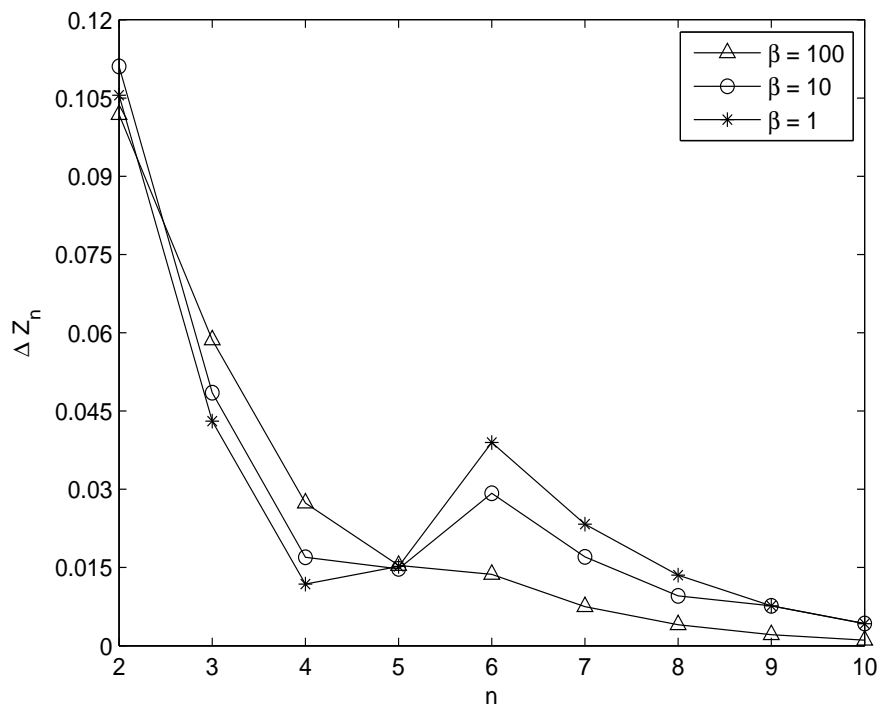
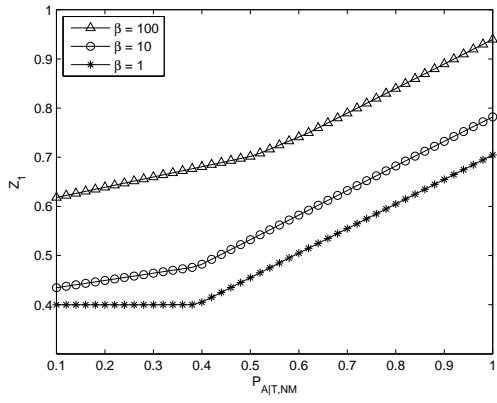
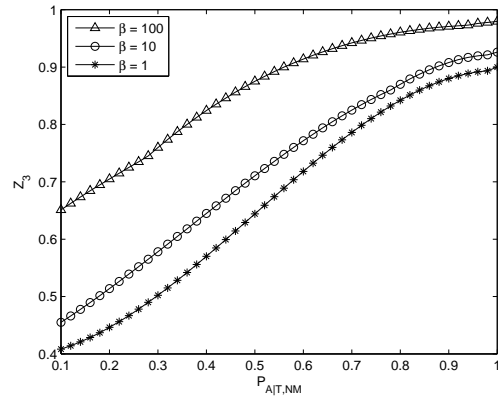


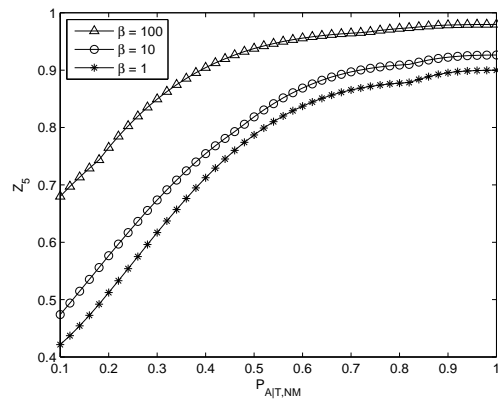
Figure 2.3: Marginal increase in the detection probability ΔZ_n as a function of n



(a) $n = 1$

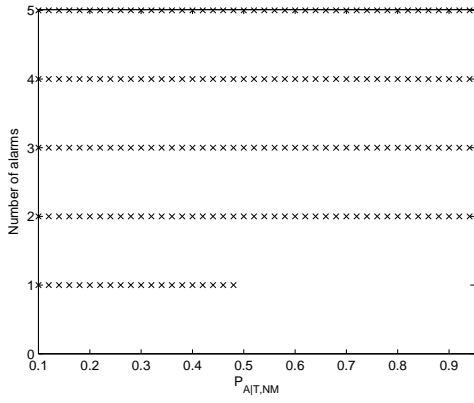


(b) $n = 3$

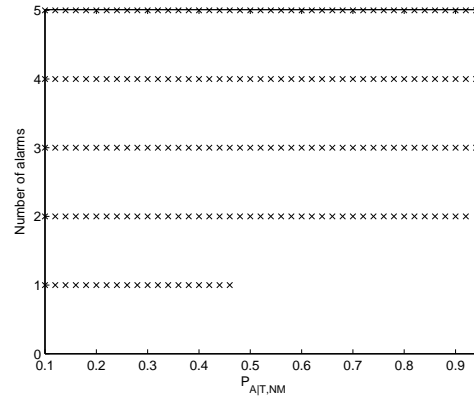


(c) $n = 5$

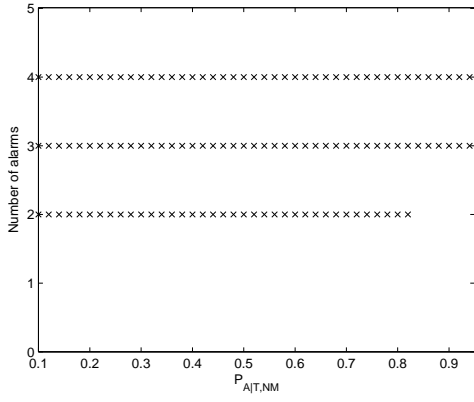
Figure 2.4: Detection probability as a function of $P_{A|NM \cap T}$ for $n = 1, 3, 5$



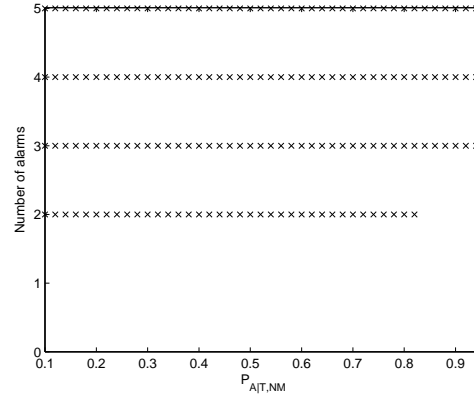
(a) HR,HB



(b) HR,LB

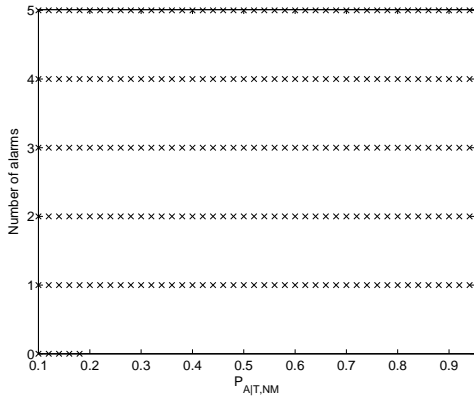


(c) LR,HB

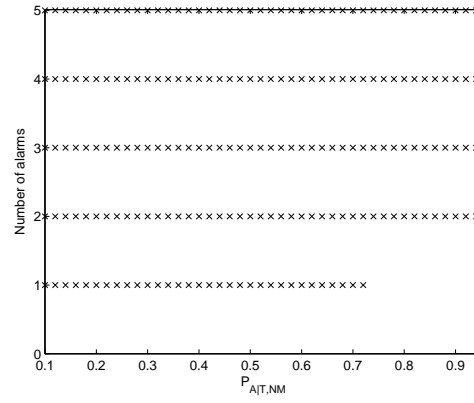


(d) LR,LB

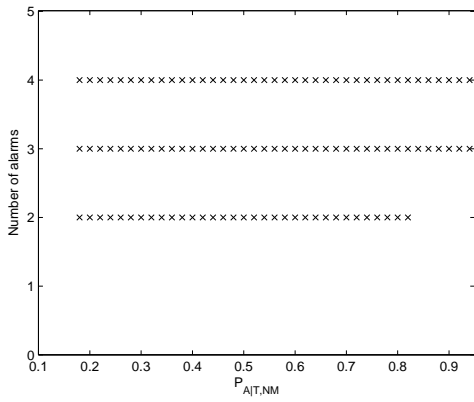
Figure 2.5: System alarm as a function of $P_{A|NM \cap T}$ for $\beta=10$ and $n = 5$



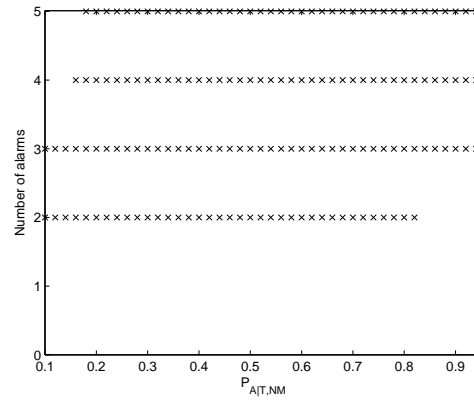
(a) HR,HB



(b) HR,LB



(c) LR,HB



(d) LR,LB

Figure 2.6: System alarm as a function of $P_{A|NM \cap T}$ for $\beta=100$ and $n = 5$

Chapter 3

Maritime Security and Bi-Level Screening Procedures

3.1 Introduction

The United States' ports consist of a multi-layer security screening system. Before containers depart from a foreign port, the Automated Targeting System (ATS) is used to prescreen and classify the risk level associated with the individual container, whether high- or low-risk (Strohm 2006). When a container arrives at a port, it goes through the primary screening procedures. Port security screening devices for nuclear or radioactive material are often radioactive isotope identification devices (RIIDs), radiation portal monitors (RPMs), and X-Ray imaging devices (McNicholas 2008). The first two devices focus on radiation emissions given off by radioactive and nuclear material. X-Ray devices can detect nuclear material that may be hidden within other contents of the container by imaging the inner contents of the containers. Primary screening can consist of any device or devices, and is the first of potential many layers of screening. A container either yields an alarm, saying a potential threat exists within, or the container will not yield an alarm, and may continue on its departure route. If the container yields a primary screening alarm, typically it is then sent to secondary screening. The devices are the same for secondary screening, yet they may have better detection capabilities if certain detection thresholds of the device are decreased. The outcome of yielding a secondary screening alarm will dictate if further screening is required, which may consist of physical inspection or K-9 units for the container. However, this tends to be not only time

consuming but costly, since it is labor-intensive.

Since this paper focuses on interdicting smuggled nuclear materials or weapons through ports security screening systems, it is important to note that multiple sources, including uranium or plutonium, can be used to create a WMD. Each source provides their own ‘radiation signature’, in which some of the current screening technologies are advantageous for detecting a smuggled WMD. Either type of source would produce a massive amount of destruction and death. Therefore, improving screening policies to detect, interdict or deter a terrorist group from a successful attack from a WMD is critical.

There are many approaches to help identify and analyze proposed procedures of preventing a nuclear attack from occurring. In this paper, primary and secondary screening procedures are analyzed through the use of linear programming models and decision analysis in hopes of detecting a nuclear threat. The linear programming model computes and analyzes primary and secondary screening decisions given a budget and different prescreening risk classifications of the containers. The model proposed can be applied to other transportation sectors, such as land and air, since the model is simplistic and arriving items, bags, or passengers follow a similar screening procedure to find a threat. Conclusions reveal the need to have accurate prescreening classifications. Sensitivity analysis is also performed for different costs associated with secondary screening and compares current technology to next-generation technology (which hypothetically should not only detect a threat, but be able to identify what it is) (GAO 2009). Results reveal as the secondary screening costs increase, decisions made for primary and secondary screening are altered, and the budget is allocated more towards the containers that are deemed a higher risk (and more likely to contain a threat). The next-generation technology analysis provides evidence that suggests the next-generation technology is not cost-effective and may not improve detection capabilities making it not worthwhile to employ.

McLay et al. (2010) provide a risk-based framework for incoming containers given a prescreening classification of either high- or low-risk. By using the framework, a system alarm is defined for the limited resources available for different levels of screening intelligence. The analysis reveals that intelligent screening is a necessity for effective screening. Chapter 2 extends McLay et al. (2010) by including a second prescreening classification of the container, high- or low-background. This classification takes into account the potential amount of naturally occurring

radioactive material (NORM) within a container. Since radiation of NORMs can often lead to false alarms, this classification adds a complexity to screening.

This chapter carefully analyzes primary and secondary screening decisions given prescreening by using a linear programming model. The scenarios explored include different sources used for nuclear weapons, hiding options, such as masking and/or shielding, multiple targets and interdiction probabilities. One strength of the model is the flexibility it offers as it is adaptable to other sectors of transportation. The model is easily updateable and provides a more concise procedure for screening. Also, sensitivity analysis provides more realistic outcomes, with respect to secondary screening costs and comparison of current technology and next-generation technology.

This chapter is organized as follows. Section 3.2 proposes a linear programming model that determines the primary and secondary screening decisions given a prescreening classification and budget. Section 3.3 states the structural framework for the model as a special case of the multiple choice knapsack problem. In Section 3.4, a computational example is provided along with results and sensitivity analysis. Section 3.5 provides conclusions, recommendations and future research.

3.2 NSP Model

In this section, terminology and parameters are introduced for the Nuclear Screening Problem (NSP), the proposed model, and the model is formally stated. NSP determines how to optimally screen items (e.g., cargo containers) at a single security station.

It is assumed that each item is classified into a risk group (due to prescreening), which determines its likelihood of containing a threat. The Automated Targeting System (ATS) is currently used to prescreen all cargo containers that enter the United States, and result in classifying the container into risk groups (Strohm 2006). All items undergo primary screening (determined by the model), which yields one of a given set of outcomes (which generalizes a binary response of alarm or clear). A subset of items is selected for secondary screening (inspection), subject to a screening budget. Thus, the model determines how to define primary and secondary screening alarms based on a risk-based approach. It implicitly determines where to deploy and how to use screening devices. The model is stated as an linear programming model.

The parameters for NSP are

- R = the set of risk groups, $i \in R$,

- T = the set of threat scenarios (with a subset of T representing non-threat (NT) scenarios),
- N = number of items (e.g., cargo containers, international aviation baggage, small vessels),
- J = the set of primary screening levels, each of which corresponds to a procedure for screening items using a set of screening devices, $j \in J$,
- O_j = the set of outcomes associated with primary screening, level j , $j \in J$, corresponding to the subset of primary screening devices that yield an alarm (based on j),
- a_{ij} = primary screening cost associated with risk group i and screening level j , $i \in R$, $j \in J$,
- b = inspection cost (secondary screening cost),
- B = the total screening budget,
- p_t = the probability of threat scenario t , with $\sum_{t \in T} p_t = 1$ and $P_t = \sum_{t \in T \setminus NT} p_t$,
- $p_{i|t}$ = the conditional probability of risk group i given threat scenario t (resulting in p_i probability that an item is classified in to risk group i after a large number of items have been screened with $\sum_i p_i = 1$),
- $p_{k|i \cap j \cap t}$ = the conditional probability of outcome k given risk group i , screening level j , and threat scenario t , $i \in R$, $j \in J$, $k \in O_j$, $t \in T$,
- $p_{A|i \cap j \cap k \cap t}$ = the conditional probability of a secondary screening alarm given risk group i , screening level j , outcome k , and threat scenario t , $i \in R$, $j \in J$, $k \in O_j$, $t \in T$
- $P_{I|t}$ = the conditional probability of interdiction I , given the threat scenario t , $t \in T$,
- $\beta = P_{T|HR}/P_{T|LR}$ = ratio of high-risk threats to low-risk threats.

The set of variables are:

- x_{ij} = proportion of containers with risk group i undergoing primary screening level j , $i \in R$, $j \in J$, where $0 \leq x_{ij} \leq 1$,
- y_{ijk} = proportion of containers with risk group i undergoing primary screening level j with outcome k to be selected for secondary screening, $i \in R$, $j \in J$, $k \in O_j$, where $0 \leq y_{ijk} \leq 1$.

The model is

$$\begin{aligned}
& \max \sum_{t \in T} \frac{1}{P_t} \left(\sum_{i \in R} \sum_{j \in J} \sum_{k \in O_j} \sum_{t \in T \setminus NT} (p_{A|i \cap j \cap k \cap t}) (p_{k|i \cap j \cap t}) (p_{i|t}) (p_t) (1 - P_{I|t}) y_{ijk} \right. \\
& \quad \left. + \sum_{t \in T \setminus NT} P_{I|t} P_t \right) \tag{3.1} \\
& \text{s.t. } N \sum_{i \in R} \sum_{j \in J} a_{ij} \sum_{t \in T \setminus NT} p_{i|t} p_t x_{ij} + N \sum_{i \in R} \sum_{j \in J} \sum_{k \in O_j} b \left(\sum_{t \in T \setminus NT} p_{k|i \cap j \cap t} p_{i|t} p_t \right) y_{ijk} \leq B,
\end{aligned}$$

$$\begin{aligned}
\sum_{j \in J} x_{ij} &= 1, \forall i \in R, \\
y_{ijk} &\leq x_{ij}, \forall i \in R, j \in J, k \in O_j, \\
0 &\leq x_{ij} \leq 1, \forall i \in R, j \in J, \\
0 &\leq y_{ijk} \leq 1, \forall i \in R, j \in J, k \in O_j,
\end{aligned}$$

The objective function of the model maximizes the interdiction probability. It takes into account the different threat scenarios, the probability of a primary screening alarm, and the level of primary screening being used. The term $\sum_t P_{I|t} P_t$ adds the exogenous interdiction probability, which will be described thoroughly in Section 3.4. The first set of constraints ensures cost of the primary and secondary screening decisions do not exceed the budget. The second set of constraints ensure that x_{ij} is one for any risk group classification. This allows for all containers to be accounted for. The last set of constraints ensures the proportion of containers sent to secondary screening y_{ijk} cannot exceed the proportion of containers sent to primary screening x_{ij} . The last two sets of constraints ensure that both x_{ij} and y_{ijk} are proportions, by making sure that they are between zero and one.

Since ATS is used to prescreen containers that enter the U.S., R is used to classify the risk group that is determined by ATS. The risk groups could consist of high- and low-risk classifications. The threat scenarios T could be based on the nuclear material, and the masking and/or shielding of the nuclear material. There is a subset of non-threats (NT), in which some containers could hold naturally occurring radioactive material (NORMs). The number of items N represent the number of containers that are screened at a specific port. This number can be changed based on what port is being used. The primary screening levels are represented by j , which include the use of different screening devices. O_j is the set of outcomes for j primary screening levels. For example, a container passes through 2 RPMs for primary screening. The set of outcomes O_j could be no device alarms, only the first device alarms, only the second device alarms, or both devices alarm, or the alarms could be defined differently, such as a threshold policy. The primary screening cost a_{ij} is based on the primary screening level that is used and the risk level of the container. The cost of secondary screening b is a constant for all containers that go through secondary screening, since the procedures for secondary screening are the same.

The utility of a successful attack u_t is determined through multiobjective decision analysis as a function related to the interdiction probability based on t , in which $u_t = 1 - P_{I|t}$. The total

screening budget is given as B . The probability of an individual threat scenario p_t . P_t , which is different from p_t , is defined as the sum of all threat scenarios including the NT threat scenarios.

3.3 Structural Framework

In this section, the nuclear screening model in Section 3.2 is reformulated as a particular case of the linear Multiple Choice Knapsack Problem (MCKP). The comparison sheds light on the structure to the optimal solution. First, we introduce the linear MCKP. It has m classes, and each class i is associated with a set of items N_i , $i = 1, 2, \dots, m$. Note that N_1, N_2, \dots, N_m are mutually exclusive, and hence, there are $\sum_{i=1}^m |N_i|$ total items. Associated with each item is a reward r_{ij} and a weight w_{ij} , $i = 1, 2, \dots, m$, $j \in N_i$. The total knapsack capacity is c . The objective of MCKP is to select one item in each class to add to the knapsack such that the total reward is maximized and the total weight is capacity feasible. The linear MCKP is formulated as a linear program.

$$\max \sum_{i=1}^m \sum_{j \in N_i} r_{ij} h_{ij} \quad (3.2)$$

$$\sum_{i=1}^m \sum_{j \in N_i} w_{ij} h_{ij} \leq c \quad (3.3)$$

$$\sum_{j \in N_i} h_{ij} = 1, \quad i = 1, 2, \dots, m \quad (3.4)$$

$$0 \leq h_{ij} \leq 1, \quad i = 1, 2, \dots, m, \quad j \in N_i \quad (3.5)$$

NSP can be formulated as a particular case of the linear MCKP. To see this, the objective of NSP seeks to identify a screening rule for each risk group, where a screening rule is defined by selecting a primary screening level as well as its outcomes that would lead to secondary screening. Therefore, the set of knapsack classes corresponds to the set of risk groups (i.e., $m = |R|$), and the items in each class correspond to a screening level and a combination of its outcomes (therefore, each $j \in N_i$ corresponds to a primary screening level $j' \in J$ and a set of primary screening outcomes $o_{j''} \in O_{j'}$). Note that there are $2^{|O_{j'}|}$ subsets of primary screening outcomes for each $j' \in J$, resulting in $|N_i| = \sum_{j' \in J} 2^{|O_{j'}|}$, $i = 1, 2, \dots, m$. It is assumed that O_j is bounded above by a constant in order to lead to prevent the introduction of an exponential number of variables, however we note this limitation. The knapsack capacity is $b = B$. The item rewards and weights are

$$r_{ij} = \frac{1}{P_t} \sum_{k \in O_{j''}} \sum_{t \in T \setminus NT} (1 - P_{I|t}) p_{A|i \cap j' \cap k \cap t} p_{k|i \cap j' \cap t} p_{i|t} p_t$$

and

$$w_{ij} = N b \sum_{k \in O_j} \sum_{t \in T \setminus NT} p_{A|i \cap j' \cap k \cap t} p_{k|i \cap j' \cap t} p_{i|t} p_t,$$

respectively. Note that the reward r_{ij} captures the utility of selecting containers for secondary screening based on their risk group i , primary screening level, and primary screening outcomes. The reward implicitly captures the interdiction term in the nuclear screening since adding a constant to the objective function value (the second term in the objective function in (3.1)) would not change the solution. The weight w_{ij} captures the cost associated with screening containers with risk group i with a given primary screening level and selecting a given subset of its outcomes for secondary screening.

Note that the MCKP formulation of NSP combines the primary screening levels and the primary screening outcomes, which link the NSP variables x_{ij} to the corresponding variables y_{ijk} , $k \in O_j$. Therefore, the resulting MCKP formulation implicitly assumes that $x_{ij} = y_{ijk}$ when y_{ijk} is non-zero, which implies that when containers yielding primary screening outcome $k \in O_j$ are selected for secondary screening, this decision is always deterministic rather than random. This assumption is not a limitation when assuming that all primary screening costs are non-negative (i.e., $a_{ij} \geq 0$, $i \in R$, $j \in J$), that secondary screening costs are positive $b > 0$, and that there is a primary screening level with zero cost (corresponding to no primary screening). This can be seen by noting that the x_{ij} variables have objective function coefficients of zero while the y_{ijk} variables have positive objective function coefficients. If $y_{ijk} > 0$ and $y_{ijk} < x_{ij}$, if then the objective function can be improved by reducing x_{ij} and increasing y_{ijk} until $y_{ijk} = x_{ij}$ (and screening any otherwise unscreened containers by the primary screening level with zero cost).

There are many well-known algorithms that can find solutions to the linear MCKP in linear time (see Kellerer et al. 2004) that shed light on the optimal policies for NSP. The solution to the linear MCKP has several properties, which are stated here as three results. They are applied to NSP.

The first result compares the rewards and weights of items within a single class (i.e., risk group) to determine which combinations of primary screening levels and their outcomes that will not be selected for secondary screening. Such items are said to be dominated. Recall that each item $j \in N_i$ corresponds to a primary screening level $j' \in J$ and a set of primary screening outcomes

$o_{j''} \in O_{j'}$. An item k is dominated by an item j , with both items j and k are in the same class i , if

$$w_{ij} \leq w_{ik} \text{ and } p_{ij} \geq p_{ik}.$$

Similarly, consider items j , k , and l in class i with

$$w_{ij} < w_{ik} < w_{il} \text{ and } p_{ij} < p_{ik} < p_{il}.$$

Item k is dominated by items j and l if

$$\frac{p_{il} - p_{ik}}{w_{il} - w_{ik}} \geq \frac{p_{ik} - p_{ij}}{w_{ik} - w_{ij}}.$$

Proposition 1 *If item j in class i dominated by another item, where item j corresponds to a primary screening level $j' \in J$ and a set of primary screening outcomes $o_{j''} \in O_{j'}$, then $h_{i,j} = 0$ and $y_{ij'k} = 0$ in NSP for all $k \in o_{j''}$).*

Proposition 1 indicates that dominated items always have values of zero in the optimal solution. This means that containers that yield the set of outcomes associated with the primary screening level (those associated with MCKP item j) will not be selected for secondary screening.

Proof. This is given by Corollary 11.2.2 in Kellerer et al. \square

Once the dominated items have been removed, without loss of generality, assume that the remaining items in each class are sorted such that their weights are non-decreasing. The next result describes the number of fractional solutions in NSP, where a fractional solution indicates which proportion of containers with a given risk group and yielding a given set of primary screening outcomes to randomly select for secondary screening. Proposition 2 indicates that in an optimal solution to NSP, containers in at most one risk group are randomly selected for secondary screening; the other risk groups select containers for secondary screening in a deterministic manner .

Proposition 2 *In an optimal solution to NSP, there are at most $2(\max 2^{|O_j|})$ fractional variables. If there are fractional variables, then they are in the same risk group.*

Proof. This is given by Corollary 11.2.3 in Kellerer et al. (2004) There are as many as $2(\max 2^{|O_j|})$ fractional variables in NSP that correspond to two fractional variables in the linear MCKP solution, since each variable in MCKP corresponds to one primary screening level and at most $2^{|O_j|}$ primary screening outcomes. \square

3.4 Computational Example and Results

This section introduces an illustrative computational example using the integer programming model from Section 3.2. The results compute and analyze the primary and secondary screening decisions for different levels of prescreening intelligence. Sensitivity analysis is performed for changes in secondary screening costs and the hypothetical detection capabilities of next-generation technology.

Table 3.1 summarizes the input parameters for the illustrative example. In this example, suppose there are 9,999 cargo containers entering a specific port daily, such as Norfolk, Virginia. There are two risk groups, high-risk and low-risk. Approximately 4% of all containers are assumed to be high-risk, which is agreeable with port authorities (Lava 2008). Therefore, low-risk containers are about 96% of all containers.

The levels of prescreening intelligence β are computed through the prescreening risk group classifications. This notion is taken from McLay et al. (2008). The ratio of high-risk threat containers to low-risk threat containers is therefore represented at β , in which $\beta = P_{T|HR}/P_{T|LR}$. Since threats must all be classified as either high- or low-risk, then $P_{HR|T} + P_{LR|T} = 1$. By use of Bayes Rules, then $P_{i|T} = \frac{\beta P_i}{1 - P_i + \beta P_i}$ for i representing the risk group classification. The levels of β range from 1 to 100, which correspond to random prescreening intelligence and excellent prescreening intelligence, respectively. There are 6 primary screening levels, (i.e., $|J| = 6$, which are comprised of all combinations of two RPMs and one X-Ray, where the RPMs are assumed to be indistinguishable. For instance, if only one device is being used to screen cargo, it could be one RPM or one X-Ray. It is assumed that the primary screening outcomes contain combinations of binary device outcomes. Therefore, the number of primary screening outcomes is at most $O_j = 2^{d_j}$, where d_j is the number of devices used by primary screening level j . Note that the devices are assumed to operate independently, but the alarm probabilities are not assumed to be independent. The outcome of one device does not depend on an outcome of another device, but rather, the alarm probabilities depend on the contents of the container. The cost for primary screening depends on the device(s) used for that primary screening level. If the container passes through 2 devices, either two RPMs or one X-Ray and one RPM, the costs associated will vary because of the differing costs between the two types of devices. The costs of primary screening level a_{ij} and secondary screening

b are subject to the budget B , which ranges from \$15K to \$50K USD.

Table 3.2 summarizes the 18 container scenarios, 16 of which are threat scenarios. The threat scenarios are comprised of the nuclear material, whether is it masked and/or shielded, and the target, 1 or 2. The probability of all threat scenarios is P_T , in which one threat is expected to exist. An example of a threat scenario is a container with a uranium-made weapon that is both masked and shielded and headed for target 2 (U-S-M-2). The other two container scenarios are non-threats, where one scenario corresponds to naturally occurring radioactive material (NORM) in a container and the other does not (non-NORM). Note that a target does not exist for either NT scenario, since there is no threat. Table 3.2 also contains the probability that any one container contains either a threat or non-threat scenario. Furthermore, the probability of detection by a specific device is given for all container scenarios. Since it is assumed that the primary screening devices operate independently, the probabilities can be multiplied for all combinations of the primary screening level. For example, if the primary screening level is two RPMs for U-S-M, and needed is the probability of both RPMs yielding an alarm, then one RPM alarm probability is multiplied by itself for a total of two RPMs alarming. To calculate a portion of the devices alarming, say one of two RPMs alarming, a probability tree was used to compute alarm probabilities based for each risk group.

Table 3.3 summarizes the primary screening decisions and outcomes and their abbreviations, and includes the costs a_{ij} for both risk groups. The primary screening cost a_{ij} changes depending on the primary screening level used. For example, a_{ij} for two RPMs is double the cost of one RPM.

To compute the input parameters, $P_{k|i \cap j \cap t}$ and $P_{A|i \cap j \cap t \cap k}$ for the NSP, a decision tree approach was used. The decision tree frames the modeling framework and was created and used to aid and visualize the model, not to solve NSP, since the model contains additional constraints. Figure 3.1 illustrates the decision and chance nodes of the model. The first chance node represents the prescreening classification of the container, whether it is high- or low-risk. The first decision node illustrates the primary screening level decision. The second chance node illustrates the different outcomes associated with the primary screening level that the container has passed through. Based on the outcomes, the decision to move the container through to secondary screening is made. From here, the next chance node illustrates whether or not a threat exists in the container. The

Table 3.1: Base case parameter values

Parameter	Value(s)
R	High, Low
N	9,999
Primary Screening Level j	Any combination of 2 RPMs and 1 XRay
P_T	$1/N \approx 0.0001$
$P_{i=HR}$	0.04
$P_{i=LR}$	0.96
β	1, 10, 100
B	\$15K- \$50K
b	100

last chance node reveals the results from the secondary screening, which indeed could contain a nuclear weapon, or confirm that the container does not contain a weapon.

A utility function is used to determine the likelihood of a successful attack, and is denoted as u_t , which is equivalent to $1 - P_{I|t}$. As the probability of interdiction decreases, the utility of success increases. It is important to note that there are two interdiction probabilities associated with the model, both of which affect the utility function. One interdiction probability, which will be referred to as the *overall interdiction probability* calculates the probability of detecting a threat in the system, which determines the utility value. The second interdiction probability, which will be referred to as the *exogenous interdiction probability*, is the probability that personnel not associated with screening (e.g. local law enforcement) interdicts the nuclear material as it travels to either target. For the computational example, target 1 refers to the port, and target 2 is another target away from the port. The exogenous interdiction probability will be denoted as $P_{I|2}$, since the model will only be considering the utility of success as a function specific to target 2. It is approached this way because if the goal is to attack the port, target 1, the detonation would more likely occur prior to any screening.

Figure 3.2 illustrates the overall interdiction probability as a function of the exogenous interdiction probability for $B = \$20K$. As the exogenous interdiction probability increases, the overall interdiction probability increases, with the greatest overall interdiction probabilities always occurring when $\beta = 100$. When $P_{I|2} = 0$, the difference in the overall interdiction probability between $\beta = 10$ and $\beta = 100$ is 0.0679, whereas the difference in the overall interdiction probability between $\beta = 1$ and $\beta = 10$ is 0.0191. When the exogenous interdiction probability is 0.9, the overall interdiction probability between $\beta = 10$ and $\beta = 100$ increases by 0.0374, and between $\beta = 1$

Table 3.2: Types of threats

Scenario Name	Container Scenario	Masked	Shielded	Target	p_t	1RPM Alarm Probabilities	1XRAY Alarm Probabilities
U-US-UM-1	Uranium	No	No	1	3.57×10^{-6}	0.95	0.75
U-US-UM-2	Uranium	No	No	2	3.57×10^{-6}	0.95	0.75
U-S-UM-1	Uranium	No	Yes	1	1.072×10^{-5}	0.50	0.99
U-S-UM-2	Uranium	No	Yes	2	1.072×10^{-5}	0.50	0.99
U-US-M-1	Uranium	Yes	No	1	7.15×10^{-6}	0.99	0.75
U-US-M-2	Uranium	Yes	No	2	7.15×10^{-6}	0.99	0.75
U-S-M-1	Uranium	Yes	Yes	1	7.15×10^{-6}	0.80	0.99
U-S-M-2	Uranium	Yes	Yes	2	7.15×10^{-6}	0.80	0.99
P-US-UM-1	Plutonium	No	No	1	3.57×10^{-6}	0.99	0.60
P-US-UM-2	Plutonium	No	No	2	3.57×10^{-6}	0.99	0.60
P-S-UM-1	Plutonium	No	Yes	1	7.15×10^{-6}	0.50	0.99
P-S-UM-2	Plutonium	No	Yes	2	7.15×10^{-6}	0.50	0.99
P-US-M-1	Plutonium	Yes	No	1	3.57×10^{-6}	0.99	0.60
P-US-M-2	Plutonium	Yes	No	2	3.57×10^{-6}	0.99	0.60
P-S-M-1	Plutonium	Yes	Yes	1	7.15×10^{-6}	0.95	0.99
P-S-M-2	Plutonium	Yes	Yes	2	7.15×10^{-6}	0.95	0.99
NT(Non-NORM)	Non-Threat (Non-NORM)	N/A	N/A	N/A	0.012699	0.005	0.02
NT(NORM)	Non-Threat (NORM)	N/A	N/A	N/A	0.987201	0.05	0.02

Table 3.3: Legend for primary screening levels and outcomes

Primary Screening Level j	PS Level Abbreviation	PS Outcomes k	PS Outcome Abbreviation	a_{ij}
No Device	Zero	No Alarms	None	0
1 RPM	1RPM	No Alarm RPM Alarm	None 1R	1
1 X-Ray	1XRAY	No Alarm XRAY Alarm	None 1X	20
2 RPMs	2RPM	No Alarm Either RPM Alarm Both RPMs Alarm	None 1R 2R	2
1 RPM and 1XRAY	1XRAY1RPM	No Alarm RPM Alarm XRAY Alarm Both XRAY and RPM Alarm	None 1R 1X 1X1R	21
2 RPMs and 1 XRAY	1XRAY2RPMS	No Alarm Either RPM Alarm Both RPMs Alarm XRAY Alarm Both XRAY and 1 RPM Alarm Both XRAY and 2 RPMs Alarm	None 1R 2R 1X 1X1R 1X2R	22

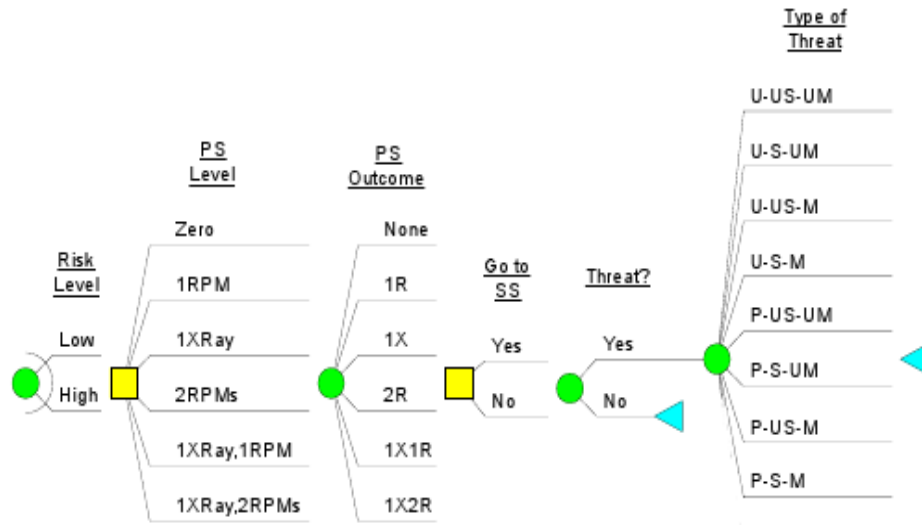


Figure 3.1: Decision Tree

and $\beta = 10$ the overall interdiction probability increases by 0.0105. It is important to note that for all exogenous interdiction probabilities, $\beta = 100$ always results in the greatest overall interdiction probabilities.

Figure 3.3 illustrates the overall interdiction probabilities as a function of budget when the exogenous interdiction probability is zero. At any level of the budget, it is clear that $\beta = 100$ screening capabilities surpass the other levels of β . At $B = \$30K$, $\beta = 1$ has nearly the same overall interdiction probability as $\beta = 10$, with values of 0.6217 and 0.6237, respectively. However, there is a greater relative difference in the overall interdiction probabilities at the ends of the budgets' range, say at $B = \$15K$ or $\$50K$. For $B = \$15K$, the difference in the overall interdiction probability between $\beta = 10$ and $\beta = 100$ is 0.0831, and between $\beta = 1$ and $\beta = 10$ the difference is 0.0197.

Figure 3.4 is similar to Figure 3.3, with different exogenous interdiction probabilities for the three levels of prescreening intelligence. The greatest relative difference between the exogenous interdiction probabilities for any prescreening intelligence level occurs when $B = \$15K$, and the smallest relative difference is when $B = \$50K$. The relative differences in the overall interdiction probability are always greater when $\beta = 1$ as compared to when $\beta = 100$. This indicates that $\beta = 1$ increases the overall interdiction probability more as the budget increases, even though $\beta = 100$ always results in a greater overall interdiction probability.

Tables 3.4, 3.5, and 3.6 illustrate the primary and secondary screening decisions for cargo containers as a function of the budget for the two prescreening classifications and $P_{I|2} = 0$. Note that when $\beta = 1$, the risk groups are classified as All, since the results are the same for the high- and low-risk classified containers. The column labeled PS Level j represents the physical screening decisions for the x_{ij} variables, and column x_{ij} is the proportion of containers that use PS Level j for a particular budget and prescreening classification. The Secondary Screening column summarizes the decisions for y_{ijk} , where PS Cleared represents the outcomes from going through PS Level j that will not continue to secondary screening. The PS Alarms column summarizes the screening decisions where the container has outcome k after using PS Level j and will continue to secondary screening. It is important to note that the values for y_{ijk} are not explicitly mentioned, but the outcomes that occur for a container that continues to secondary screening, y_{ijk} is the same value as x_{ij} . For example (from Table 3.4), when PS is ‘All, 1XRAY1RPM’, $B = \$35K$, and $x_{ij} = 0.019508$, then $y_{ijk} = 0.019508$ for the listed PS Alarms. Therefore, this proportion of containers hold tight to the constraints $y_{ijk} \leq x_{ij}$. It is interesting to note that for all results, these constraints all hold tight, meaning $y_{ijk} = x_{ij}$. As β increases for high-risk containers when $B = \$25K$, the primary screening decisions change from one RPM or two RPMs to strictly two RPMs to one X-Ray, one RPM or one X-Ray, 2 RPMs, whereas for low-risk containers, the primary screening decisions are one RPM or two RPMs when $\beta = 1$ or $\beta = 10$ and strictly one RPM when $\beta = 100$. This suggests that prescreening classification intelligence is crucial in determining the primary screening decisions, and it leads to allocating the budget in a manner that more effectively screens for high-risk containers.

Figures 3.5 and 3.6 illustrate the primary screening decisions for the high- and low-risk containers over the different values of β as a function of the budget. It is important to note that these figures only depict the primary screening decisions, and not the outcomes that are sent to secondary screening. These figures coincide with the data from Tables 3.4, 3.5, and 3.6. Since there are fractional values for some of the primary screening decisions, the only decisions illustrated are those in which the fractional values are greater than 0.50, which is defined as the *dominant primary screening decision*. For example, when $\beta = 10$ and $B = \$25K$, and the container is classified as low-risk, the dominant primary screening decision is two RPMs (which is illustrated). For $\beta < 5$ for high-risk containers, when $B < \$24K$, the primary screening decision is one RPM.

For the $B \geq \$24K$, the primary screening decision is two RPMs. As β increases, the dominant primary screening decision changes from one RPM to one X-Ray, two RPMs. Figure 3.6 illustrates the primary screening decisions for the low-risk containers. As β and the budget increase, the dominant primary screening decision changes from one RPM to two RPMs. When $\beta \geq 90$ and $B \leq \$16K$, the dominant primary screening decision is to use no devices. This suggests that when prescreening intelligence is high enough, its optimal to clear all low-risk containers from primary screening.

Figure 3.7 illustrates the similarities and differences in the primary screening decisions between the high- and low-risk containers over different β values as a function of the budget, synthesizing Figures 3.5 and 3.6. The light grey region illustrates where the dominant primary screening decisions are the same between high- and low-risk containers, whereas the dark grey region reveals where there are differences between the dominant primary screening decisions. For $\beta < 5$ and all budget amounts, the screening decisions are identical across both risk groups, and for $\beta \geq \$20K$, the primary screening decisions are always different. This reveals that β impacts the primary screening decisions by allocating the budget in order to more effectively screen the high-risk containers.

Sensitivity analysis was performed on the NSP parameters to understand how the primary and secondary screening decisions would change as a function of the secondary screening cost. Since the cost data was based on estimates available publicly, it is important to do sensitivity analysis for a better depiction of what real data might do. Also, next-generation technology is considered, in which the detection capabilities of the devices are better to distinguish between threats and non-threats. Specifically, the next-generation RPM device considered would have the same alarm probabilities for NORM threats and for NORM non-threats.

Figure 3.8 illustrates the overall interdiction probability as a function of the secondary screening cost for $B = \$20K$ and $P_{I2} = 0$. These results are compared to Figure 3.2, in which the cost of secondary screening is $b = 100$. As the cost of secondary screening increases, the overall interdiction probability decreases. The relative difference in the overall interdiction probability increases between all β as the secondary screening costs increase. As in Figure 3.2, the greatest relative difference in the overall interdiction probability as the cost of secondary screening increases is between $\beta = 10$ and $\beta = 100$ with a change in probability of 0.0651 and 0.1466, respectively.

Table 3.4: Primary and secondary screening for $\beta = 1$ and $P_{I|2} = 0$

Budget	Risk Group (i)	Primary Screening		Secondary Screening	
		PS Level (j)	x_{ij}	PS Cleared	PS Alarms
15000	All	Zero	0.001425	None	
		1RPM	0.998574	None	1R
20000	All	1RPM	0.749627	None	1R
		2RPM	0.250373	None	1R, 2R
25000	All	1RPM	0.414157	None	1R
		2RPM	0.585843	None	1R, 2R
30000	All	1RPM	0.078686	None	1R
		2RPM	0.921313	None	1R, 2R
35000	All	2RPM	0.980492	None	1R, 2R
		1XRAY1RPM	0.019508	None	1R, 1X, 1X1R
40000	All	2RPM	0.955006	None	1R, 2R
		1XRAY1RPM	0.044994	None	1R, 1X, 1X1R
45000	All	2RPM	0.929520	None	1R, 2R
		1XRAY1RPM	0.070480	None	1R, 1X, 1X1R
50000	All	2RPM	0.904034	None	1R, 2R
		1XRAY1RPM	0.095966	None	1R, 1X, 1X1R

Table 3.5: Primary and secondary screening for $\beta = 10$ and $P_{I|2} = 0$

Budget	Primary Screening			Secondary Screening	
	Risk Group (i)	PS Level (j)	x_{ij}	PS Cleared	PS Alarms
15000	High	2RPM	1	None	1R, 2R
	Low	Zero	0.084748	None	
	Low	1RPM	0.915251	None	1R
20000	High	2RPM	1	None	1R, 2R
	Low	1RPM	0.749781	None	1R
	Low	2RPM	0.250219	None	1R, 2R
25000	High	2RPM	1	None	1R, 2R
	Low	1RPM	0.414243	None	1R
	Low	2RPM	0.585757	None	1R, 2R
30000	High	2RPM	1	None	1R, 2R
	Low	1RPM	0.078704	None	1R
	Low	2RPM	0.921296	None	1R, 2R
35000	High	2RPM	0.531921	None	1R, 2R
	High	1XRAY1RPM	0.468079	None	1R, 1X, 1X1R
	Low	2RPM	1	None	1R, 2R
40000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.999823	None	1R, 2R
	Low	1XRAY1RPM	0.000177	None	1R, 1X, 1X1R
45000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.974337	None	1R, 2R
	Low	1XRAY1RPM	0.025663	None	1R, 1X, 1X1R
50000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.948850	None	1R, 2R
	Low	1XRAY1RPM	0.051149	None	1R, 1X, 1X1R

Table 3.6: Primary and secondary screening for $\beta = 100$ and $P_{I|2} = 0$

Budget	Primary Screening			Secondary Screening	
	Risk Group (i)	PS Level (j)	x_{ij}	PS Cleared	PS Alarms
15000	High	1XRAY1RPM	1	None	1R, 1X, 1X1R
	Low	Zero	0.632120	None	
	Low	1RPM	0.367880	None	1R
20000	High	1XRAY1RPM	1	None	1R, 1X, 1X1R
	Low	Zero	0.297943	None	
	Low	1RPM	0.702057	None	1R
25000	High	1XRAY1RPM	0.119794	None	1R, 1X, 1X1R
	High	1XRAY2RPM	0.880206	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	1RPM	1	None	
30000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	1RPM	0.669277	None	1R
	Low	2RPM	0.330723	None	1R, 2R
35000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	1RPM	0.333600	None	1R
	Low	2RPM	0.666400	None	1R, 2R
40000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.999842	None	1R, 2R
	Low	1XRAY1RPM	0.000157	None	1R, 1X, 1X1R
45000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.974356	None	1R, 2R
	Low	1XRAY1RPM	0.025644	None	1R, 1X, 1X1R
50000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.948869	None	1R, 2R
	Low	1XRAY1RPM	0.051131	None	1R, 1X, 1X1R

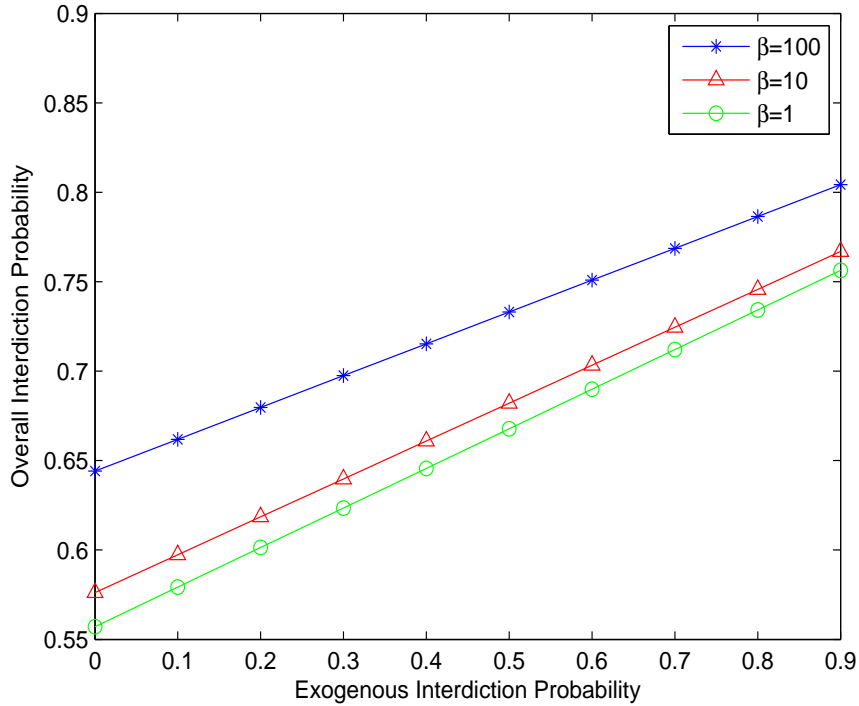


Figure 3.2: Overall interdiction probability as a function of the exogenous interdiction probability

The relative difference in the overall interdiction probability at $b = 25$ and $b = 300$ for $\beta = 1$ and $\beta = 10$ decrease by 0.0065 and 0.0401.

Figure 3.9 illustrates the overall interdiction probability between the current technology and the next-generation technology as a function of the budget for $\beta = 1, 10, 100$ and $P_{I|2} = 0$. The next-generation technology assumes that the RPM alarm probability (from Table 3.2 for NORM non-threats is equivalent to non-NORM non-threats, which is 0.005. As the budget increases, the relative differences between the two technologies decrease. The next-generation technology is denoted as NextGen. For $\beta = 1$, when $B = \$15K$, the difference between the next-generation technology and the current technology is 0.0192, whereas when the $B = \$50K$, the difference in the overall interdiction probability is 0.0004. Similarly to Figure 3.3, the highest overall interdiction probabilities exist when $\beta = 100$. However, the differences between the current and next-generation technologies for $\beta = 100$ range from 9.10×10^{-5} to 0.0029 for $B = \$50K$ and $B = \$20K$, respectively. The is evidence that with $\beta = 100$, the next-generation technology does not suggest a significant increase in the overall interdiction probability, and therefore, the need for

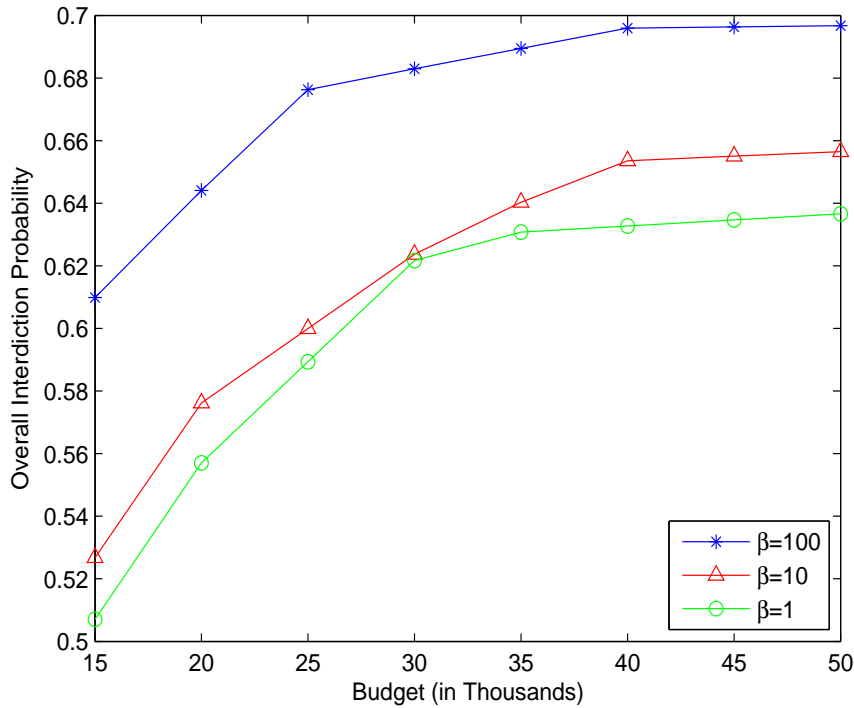


Figure 3.3: Overall interdiction probability as a function of the budget

the next-generation technology may not be worthwhile.

Figures 3.10 and 3.11 illustrate the dominant primary screening decisions for the three scenarios, $b = 25, b = 300$, and *NextGeneration*, $b = 100$, in which sensitivity analysis was performed as compared to the basecase ($b = 100$) from the primary results for different levels of β for high-risk and low-risk containers, respectively, as a function of the budget. Similar to Figures 3.5 and 3.6, as β increases the number of primary screening devices increases. However as the cost of secondary screening increases, the number of primary screening devices decrease. It is interesting to note that the dominant primary screening decision for high-risk containers is to screen with no primary screening devices when $\beta = 100$ and $b = 25$ for $B \geq \$35K$. For this particular scenario, these high-risk containers will all be sent to secondary screening without any primary screening. This is intuitive, since the cost of secondary screening is low, as compared to when $b = 300$, with a primary screening decision of one X-Ray, two RPMs. However, the optimal decision is to send the containers to secondary screening without using primary screening, which ultimately lowers the cost of the system. Note that in Figure 3.11, the dominant primary screening decisions for

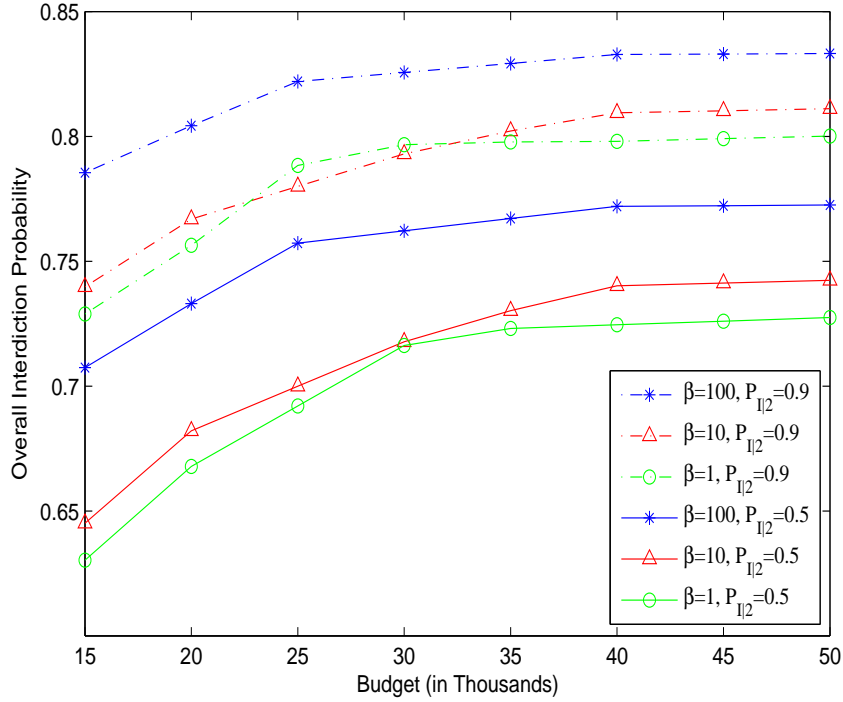


Figure 3.4: Overall interdiction probability as a function of the budget for different exogenous interdiction probabilities

$\beta = 1$ are equivalent to that of $\beta = 10$. Therefore, the prescreening intelligence is not critical for determining primary screening decisions. It is interesting to note that when $b = 300$, the dominant primary screening decisions for $\beta = 100$ change from using no primary screening devices to two RPMs to one RPM. This is due to allocating the scarce resources given the changes in the budget. For $\$25K \leq B \leq \$34K$, the only containers that continue to secondary screening for $j = 2$ RPMs, are the containers in which both RPMs alarm. Furthermore, when the budget is increased and the dominant primary screening decision is one RPM, the system allocates the budget more effectively, since the containers are low-risk, the secondary screening costs are large, and it costs less to screen with one RPM as compared to two RPMs. Ultimately, the low-risk containers receive less primary and secondary screening to allow for the budget to be allocated to the high-risk containers. In addition to changes in the secondary screening costs, an analysis was performed for next-generation technology. The results reveal that the next-generation technology (denoted as 'NG' in the figures) minimally changes the primary screening decisions as compared to $b = 100$ for high-risk

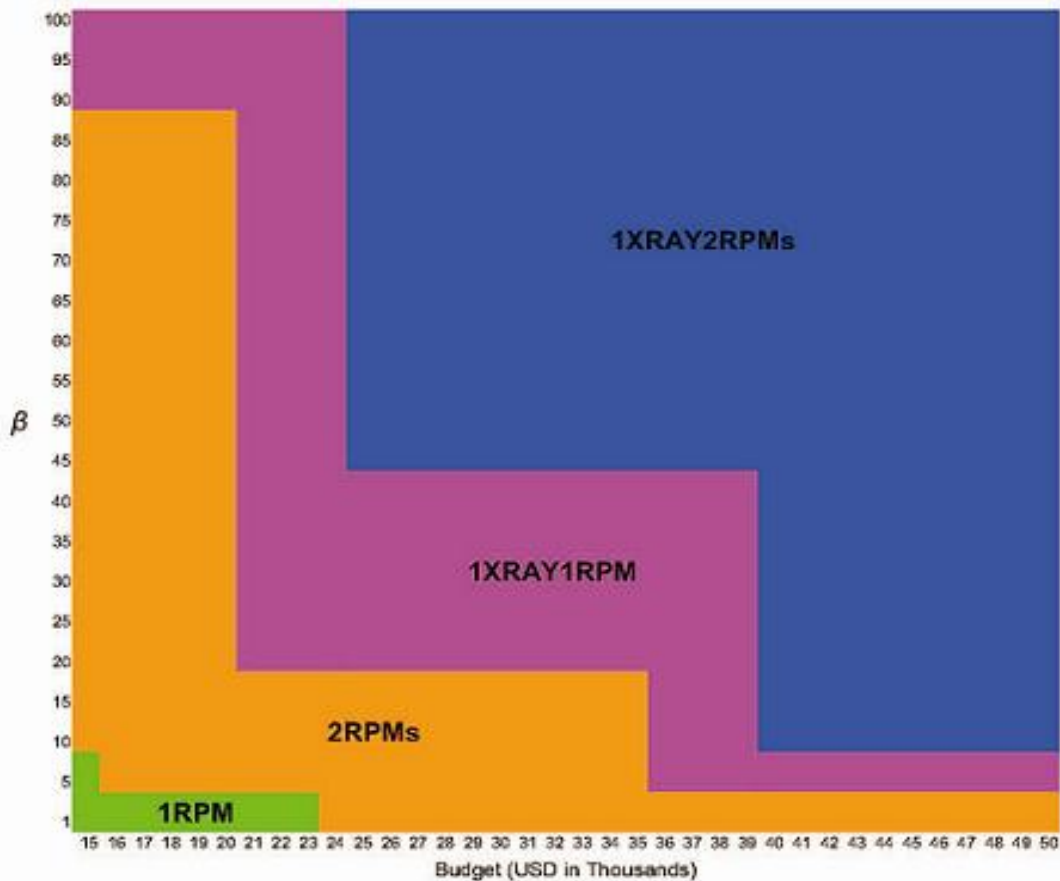


Figure 3.5: Primary screening decisions for high-risk containers

containers. This is evidence that the next-generation technology may not be cost effective when compared to the current technology capabilities. However, when $\beta = 100$ for low-risk containers, the next-generation technology would in fact be more cost effective than when $b = 25$ or $b = 100$, since no primary screening devices are used for a larger portion of the budget.

Similar to Tables 3.4, 3.5, and 3.6, the primary and secondary screening decisions for the next-generation are summarized in Tables 3.7, 3.8, and 3.9. The next-generation technology is applied to the all levels of prescreening intelligence for $P_{I|2} = 0$. The results reveal all primary screening decisions, not just the dominant primary screening decisions, as in Figures 3.10 and 3.11. Ultimately, the decisions for the x_{ij} only vary by the fractional value, not by the primary screening decisions themselves, which do not affect the dominant primary screening decision, except when the containers are low-risk and when $\beta = 100$.

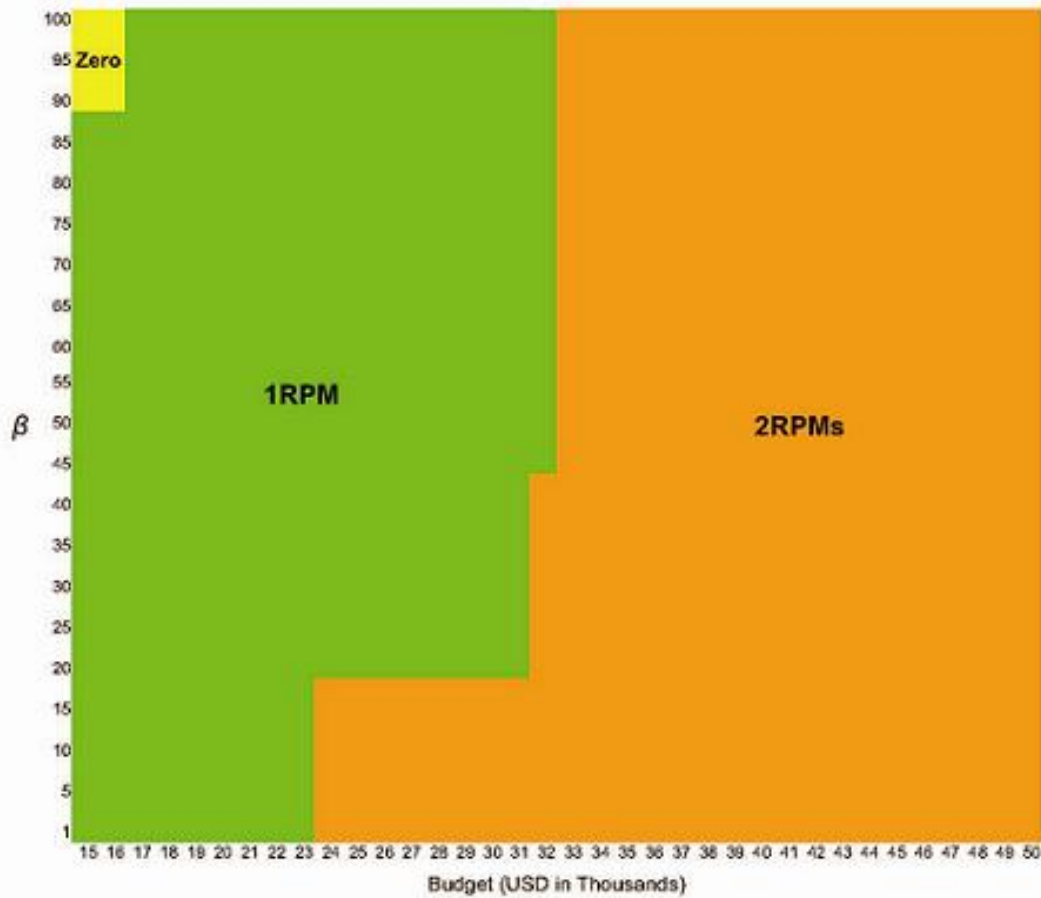


Figure 3.6: Primary screening decisions for low-risk containers

3.5 Conclusions

By using a linear programming model and decision analysis, primary and secondary screening decisions can be improved to be more likely to interdict a WMD, such as a nuclear weapon. The results provide strong evidence that prescreening intelligence is critical. This model also accounts for containers that have passed through the screening station, by using a utility function and incorporating exogenous interdiction. This captures containers that are cleared by screening and are interdicted after released from the screening station. As the exogenous interdiction probability increases, the overall probability of interdiction also increases. Sensitivity analysis was also performed on the costs of secondary screening and the hypothesized detection capabilities of next-generation technology. The results reveal that as the cost of secondary screening increases, the

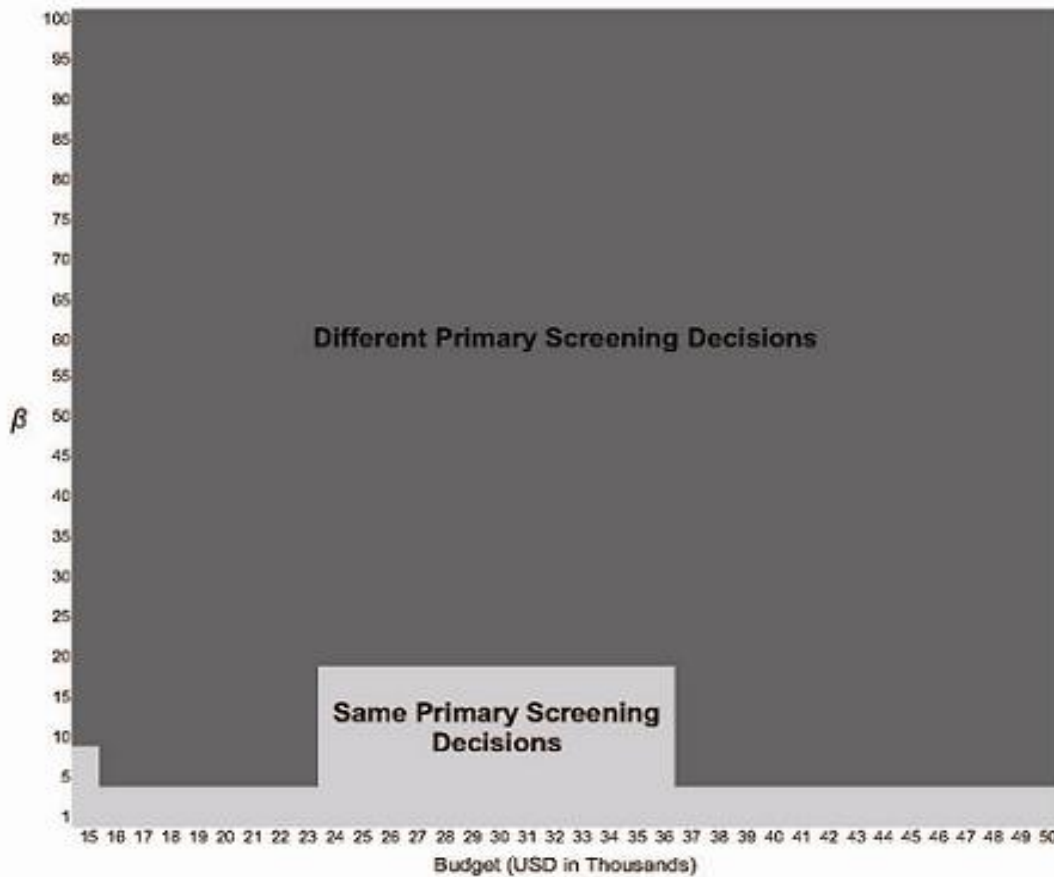


Figure 3.7: Similarities and differences between the primary screening decisions for high-risk and low-risk containers

budget is allocated mostly to the high-risk containers as the prescreening intelligence increases. Through the analysis of the current and next-generation technology, there is evidence that the next-generation technologies would not significantly improve the overall interdiction probability.

Future work will include weighting the targets, as to include the consequences of destruction or risk to particular locations, to determine more realistic outcomes. This would make stronger port security systems. In addition, the model could be extended to include a multiple station security scenario. Also, this model will be applied to different transportation sectors, such as aviation, in order to reveal the versatility of this model, as well as improve screening procedures. Another extension could include testing other technologies out there to see how well they perform. Work is in progress to address these extensions.

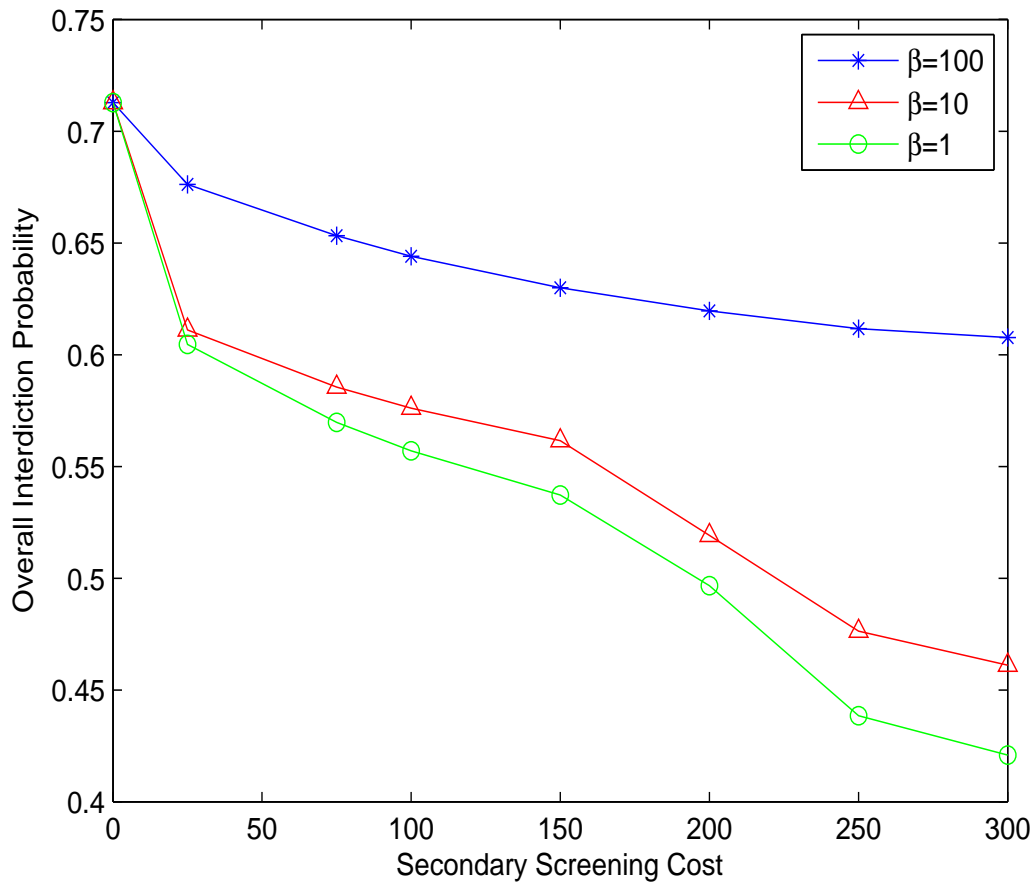


Figure 3.8: Overall interdiction probability for $\beta = 1, 10, 100$ as a function of secondary screening costs

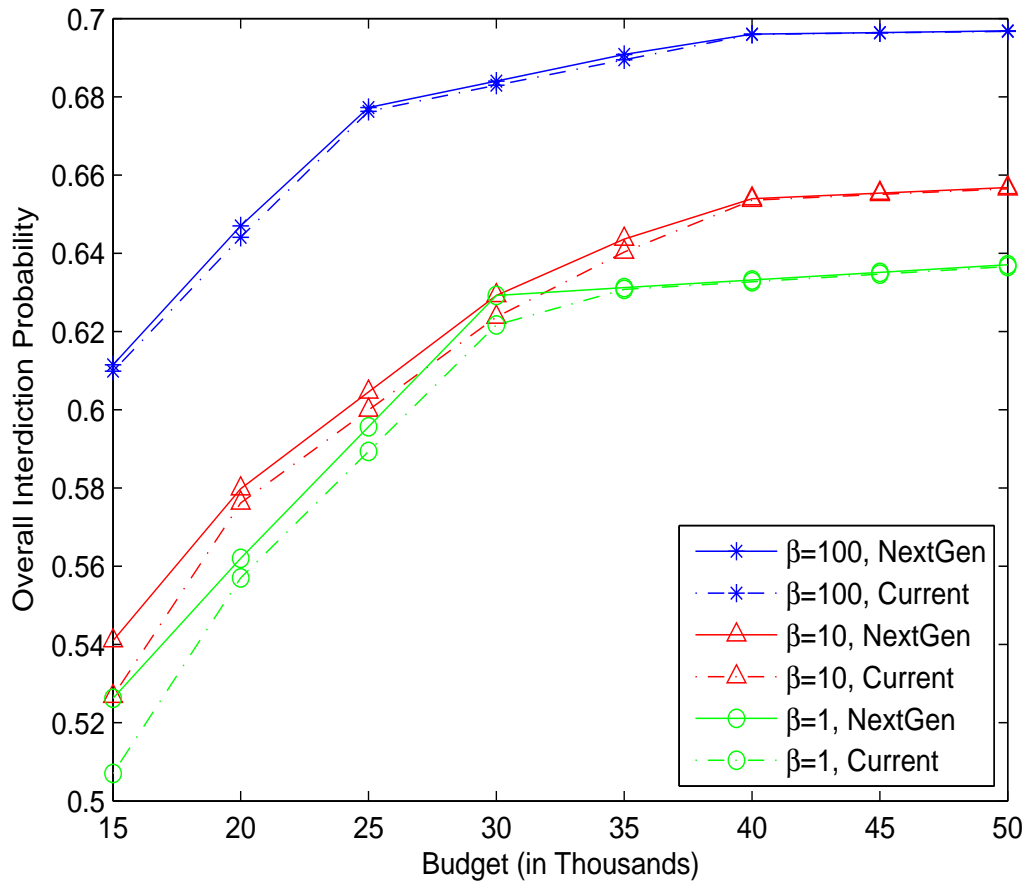


Figure 3.9: Overall interdiction probability for $\beta = 1, 10, 100$ as a function of budget for current and next-generation technology and $P_{I|2} = 0$

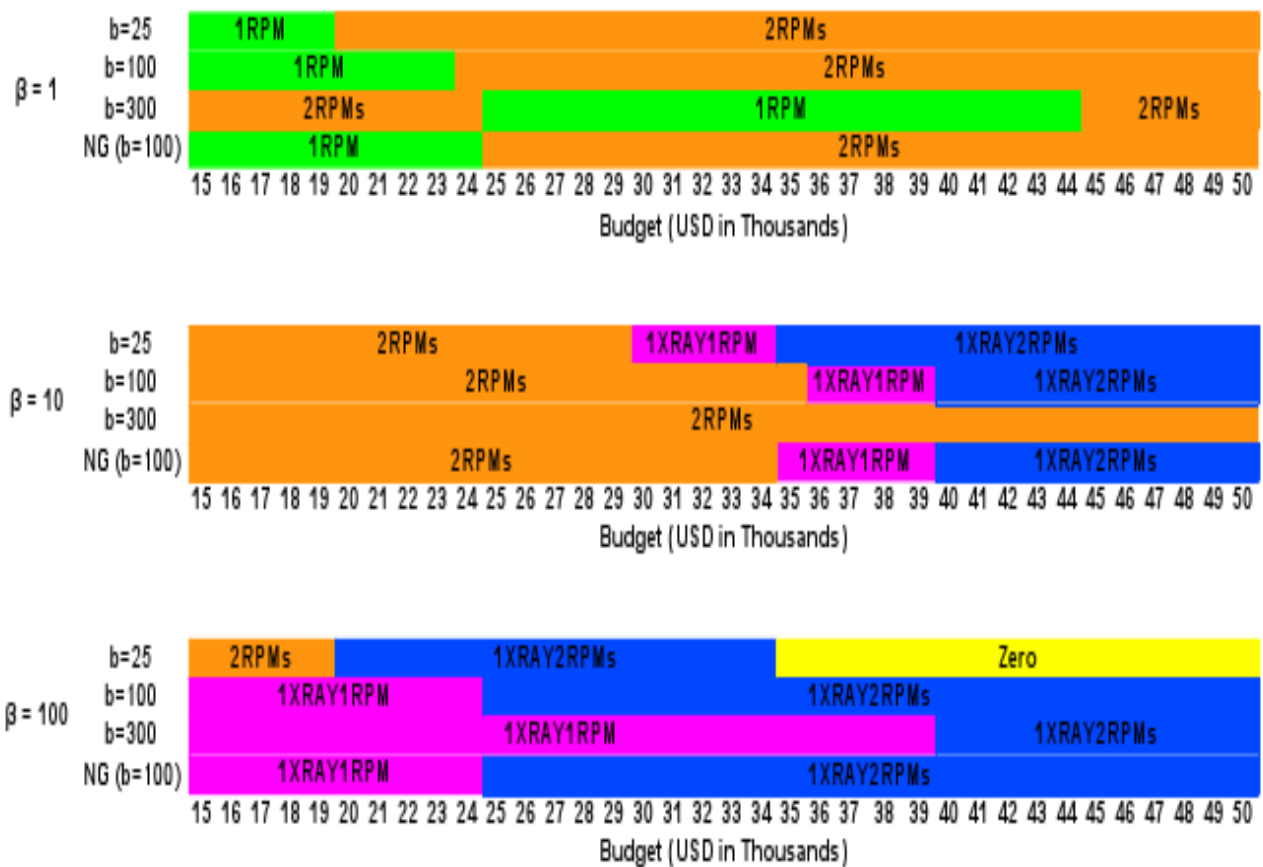


Figure 3.10: Sensitivity analysis for primary screening decisions for high-risk containers

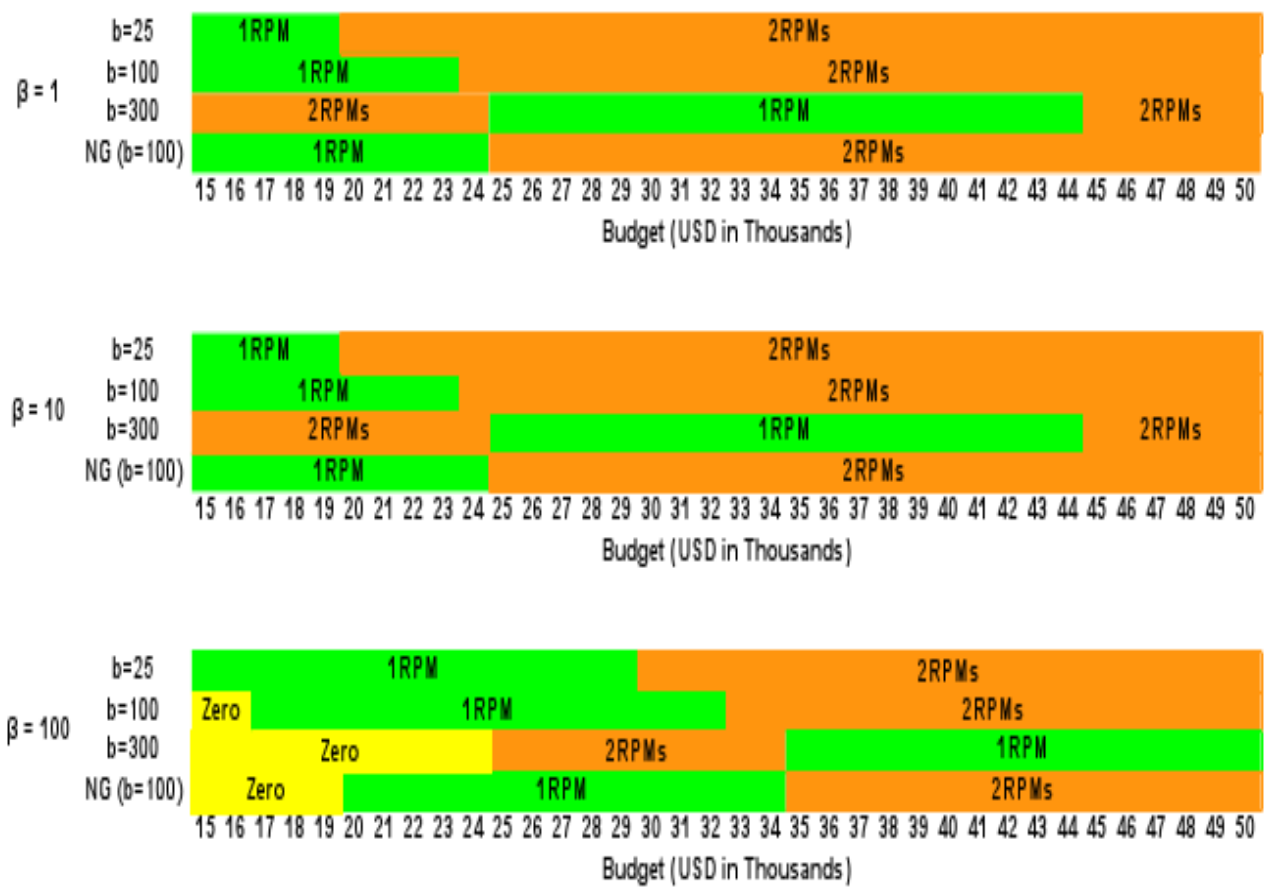


Figure 3.11: Sensitivity analysis for primary screening decisions for low-risk containers

Table 3.7: Primary and secondary screening for $\beta = 1$ and $P_{I|2} = 0$ for next-generation technology

Budget	Primary Screening			Secondary Screening	
	Risk Group (i)	PS Level (j)	x_{ij}	PS Cleared	PS Alarms
15000	All	Zero	0.005246	None	
		1RPM	0.994754	None	1R
20000	All	1RPM	0.697919	None	1R
		2RPM	0.302081	None	1R, 2R
25000	All	1RPM	0.348900	None	1R
		2RPM	0.651100	None	1R, 2R
30000	All	2RPM	1	None	1R, 2R
35000	All	2RPM	1	None	1R, 2R
40000	All	2RPM	0.990833	None	1R, 2R
		1XRAY1RPM	0.009167	None	1R, 1X, 1X1R
45000	All	2RPM	0.965424	None	1R, 2R
		1XRAY1RPM	0.034576	None	1R, 1X, 1X1R
50000	All	2RPM	0.940014	None	1R, 2R
		1XRAY1RPM	0.059986	None	1R, 1X, 1X1R

Table 3.8: Primary and secondary screening for $\beta = 10$ and $P_{I|2} = 0$ for next-generation technology

Budget	<i>Primary Screening</i>			<i>Secondary Screening</i>	
	Risk Group (<i>i</i>)	PS Level (<i>j</i>)	x_{ij}	PS Cleared	PS Alarms
15000	High	2RPM	1	None	1R, 2R
	Low	Zero	0.046745	None	
	Low	1RPM	0.953255	None	1R
20000	High	2RPM	1	None	1R, 2R
	Low	1RPM	0.698075	None	1R
	Low	2RPM	0.301925	None	1R, 2R
25000	High	2RPM	1	None	1R, 2R
	Low	1RPM	0.348977	None	1R
	Low	2RPM	0.651023	None	1R, 2R
30000	High	2RPM	0.999790	None	1R, 2R
	High	1XRAY1RPM	0.000210	None	1R, 1X, 1X1R
	Low	2RPM	1	None	1R, 2R
35000	High	2RPM	0.390048	None	1R, 2R
	High	1XRAY1RPM	0.609952	None	1R, 1X, 1X1R
	Low	2RPM	1	None	1R, 2R
40000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.993854	None	1R, 2R
	Low	1XRAY1RPM	0.006146	None	1R, 1X, 1X1R
45000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.968444	None	1R, 2R
	Low	1XRAY1RPM	0.031556	None	1R, 1X, 1X1R
50000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.943034	None	1R, 2R
	Low	1XRAY1RPM	0.056966	None	1R, 1X, 1X1R

Table 3.9: Primary and secondary screening for $\beta = 100$ and $P_{I|2} = 0$ for next-generation technology

Budget	<i>Primary Screening</i>			<i>Secondary Screening</i>	
	Risk Group (i)	PS Level (j)	x_{ij}	PS Cleared	PS Alarms
15000	High	1XRAY1RPM	1	None	1R, 1X, 1X1R
	Low	Zero	0.616568	None	
	Low	1RPM	0.383432	None	1R
20000	High	1XRAY1RPM	1	None	1R, 1X, 1X1R
	Low	Zero	0.269672	None	
	Low	1RPM	0.730328	None	1R
25000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	1RPM	0.963623	None	1R
	Low	2RPM	0.036377	None	1R, 2R
30000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	1RPM	0.614369	None	1R
	Low	2RPM	0.385631	None	1R, 2R
35000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	1RPM	0.265116	None	1R
	Low	2RPM	0.734884	None	1R, 2R
40000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.993878	None	1R, 2R
	Low	1XRAY1RPM	0.006122	None	1R, 1X, 1X1R
45000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.968468	None	1R, 2R
	Low	1XRAY1RPM	0.031532	None	1R, 1X, 1X1R
50000	High	1XRAY2RPM	1	None	1R, 1X, 2R, 1X1R, 1X2R
	Low	2RPM	0.943058	None	1R, 2R
	Low	1XRAY1RPM	0.056942	None	1R, 1X, 1X1R

Chapter 4

Aviation Security and Direct-to-Target Nuclear Attacks

4.1 Introduction

If a terrorist group acquires weapons-grade nuclear material abroad, it can be used to make a nuclear weapon and then be transported to the United States, where it will be presumably used to launch an attack. There are many ways to transport nuclear material or a nuclear weapon into the United States, including air, sea, and land border crossings. A weapon can be transported in a cargo container, in a small vessel, on a general aviation aircraft, on a train, or on a large cargo ship, to name a few modes of transportation. Much research has focused on screening for nuclear material in cargo containers at domestic and foreign ports and at land border crossings (Wein et al. 2007, Dimitrov et al. 2010, McLay et al. 2010).

Little attention has been paid to the critical role of commercial aviation security in nuclear attacks. Sweet (2009) claims that, “[f]rom a terrorist’s viewpoint, aircraft are a preferable target because of their international flavor and the likelihood the press will focus on the incident.” Due to the terrorist attacks and plots involving conventional weapons and commercial aviation, aviation security has evolved to include new procedures and technologies. These changes in aviation security have been designed to prevent or detect hijackings and explosions occurring, rather than to prevent a nuclear attack. However, commercial aircraft remain attractive targets for terrorists, as evidenced by the plan to detonate a bomb on an international commercial flight on December

24, 2009). While flights are routinely screened for conventional explosives, they are not routinely screened for nuclear material.

To address the threat of a nuclear attack, some baggage on incoming international flights are screened after they arrive at a U.S. hub airport, although most screening efforts are focused on international general aviation flights (rather than international commercial aviation flights) (Vojtech 2009; Sammon 2009). Thus, there are some current security procedures in place to detect nuclear weapons through intended security checkpoints once they have entered the United States. However, the system is vulnerable to modes of attack that breach the normal security procedures. This paper focuses exclusively on incoming international commercial aviation flights, since it has been hypothesized that a nuclear attack would involve smuggling in a nuclear weapon from abroad (Allison 2004). Thus, domestic flights and outgoing international flights are not considered.

Screening aviation baggage after it has arrived in the United States makes the system vulnerable to a *direct-to-target* attack. A direct-to-target attack uses an aircraft to transport a nuclear bomb directly to a particular target prior to border screening. In such an attack, terrorists would hijack the aircraft with the nuclear weapon and fly it to its intended target. The concept of a direct-to-target attack has not been explicitly considered in aviation security models. However, it has been indirectly considered in port security models, where some models assume that a nuclear weapon would be detonated at a U.S. port prior to security screening (which is normally performed at the exit lanes of a port), which is analogous to a direct-to-target attack (Fritelli 2005, Bakir 2008).

Currently baggage is screened for explosives that are used in conventional weapons at U.S. and foreign airports. Screening is a layered process consisting of different procedures and technologies, such as X-rays, Explosives Trace Detection (ETDs) and Explosives Detection Systems (EDSs). Radiation Isotope Identification Devices (RIIDs) are being used to screen for nuclear material in some bags within the United States (Sammon 2009). In the future, RIIDs could be used as part of the checked baggage screening system in foreign airports to screen baggage entering the U.S. in order to prevent a direct-to-target attack.

The objective of this chapter is to explore and analyze performance measures for using limiting screening devices to screen checked baggage at foreign airports prior in order to prevent a direct-to-target attack. To reach this objective, this chapter proposes and analyzes four performance measures that cover flights, passengers, baggage, and targets. This chapter proposes seven

discrete optimization models for comparing the performance measures and for evaluating baggage screening decisions. By comparing the proposed performance measures to the current proposed screening policy, that which maximizes the encounter probability—the proportion of bags that have been screened—the tradeoffs across the performance measures are analyzed. While the performance measures in this chapter by no means form a comprehensive set of all possible performance measures, they illustrate the unique challenge in evaluating performance when comparing conventional weapons to weapons of mass destruction (WMDs) such as nuclear weapons. The results suggest that focusing on the target in the WMD attack is important for utilizing baggage screening devices, since the scope of the attack is greatly increased in a WMD attack compared to a conventional weapon attack, and that the encounter probability is insufficient for protecting targets, flights, or passengers.

This chapter is organized as follows. A literature review is presented in Section 4.2. Performance measures are provided in Section 4.3. Section 4.4 introduces the parameters and the single objective models that each consider a single performance measure. Goal programming models are presented in Section 4.5 in order to evaluate the tradeoffs between the different performance measures. Section 4.6 describes three greedy heuristics that were used to identify near-optimal solutions to the models. Section 4.7 provides a computational example, analysis, and results from the greedy heuristics. Conclusions and future research are presented in Section 4.8.

4.2 Literature Review

At present, there are no papers that use operations research methodologies for preventing nuclear attacks that target commercial aviation flights. Several papers examine performance measures for screening passenger checked baggage for conventional weapons (explosives) as well as determine the need for baggage screening using cost-benefit analysis. The process of screening for nuclear material is similar to screening for any other threat. However, different devices are needed to detect the radiation emitted from nuclear material as well as to discover nuclear material that may be hidden within a radioactive source. The consequences of a nuclear attack, if successful, is much greater than any other type of attack (United States Homeland Security Council 2009).

Jacobson et al. (2001) use discrete optimization models to determine how to optimally deploy baggage screening security devices at a single checkpoint security screening station given

the risk of a conventional weapons attack using explosives placed in checked baggage. CAPPS (Computer-Aided Passenger Prescreening System) classifies passengers as either selectees or non-selectees. Prior to September 11, 2001, selectee baggage was routinely screened for explosives, whereas nonselectee baggage was not screened. However, there were often not enough baggage screening devices to screen all selectee baggage. To evaluate the baggage screening decisions, Jacobson et al. (2001) introduce three performance measures. The performance measures consist of Uncovered Flight Segments (UFS), Uncovered Passenger Segments (UPS), and Uncovered Baggage Segments (UBS). Note that a flight segment consists of a single, direct flight between two airports. If any selectees bags are not screened on a flight segment, then that flight segment is considered *uncovered*. The UFS measure captures the total number of uncovered flight segments. Likewise, the UPS measure captures the number of passengers on uncovered flight segments, and the UBS captures the number of bags on uncovered flight segments. These three performance measures are a proxy for determining how many flight segments are at risk. Jacobson et al. (2005a) use UFS, UPS, and UBS performance measures to analyze a multiple-station security system using knapsack problem models. Jacobson et al. (2005b) use the selectee or nonselectee paradigm to deploy and allocate baggage screening security devices as well as to determine which baggage to screen. They create integer programming models to determine how to optimally deploy a given set of devices to baggage screening devices when optimizing the UFS, UPS, and UBS performance measures.

Other papers analyze combinations and arrangements of different screening devices within aviation security. Sahin and Feng (2008) analyze a two-layer security system with different screening devices in order to evaluate risk and optimize the arrangement of screening devices. McLay et al. (2008) perform a cost-benefit analysis on two different screening devices for different passenger risk groups in order to minimize successful attacks. Candalino, Jr. et al. (2004) present a simulated annealing heuristic to evaluate and minimize the costs associated with an arrangement of baggage screening security devices. Kobza and Jacobson (1996, 1997) consider the design of security system architectures using reliability models in the context of aviation security baggage screening systems.

Bakir (2008) considers a direct-to-target attack in port security. In this paper, it assumed that a terrorist would smuggle a nuclear weapon into the United States in a cargo container entering the

United States on a truck at a land border crossing at the United States border with Mexico. The model assumes that the terrorist would detonate the nuclear weapon prior to screening operations at the port. However, the analysis does not consider the special requirements of a port attack nor does it compare a port attack to other types of attacks.

This chapter builds upon the existing research by extending the analysis of aviation security performance measures from considering attacks with conventional weapons to considering WMD attacks. This chapter proposes a new performance measure that considers covering targets and evaluates the tradeoffs between the performance measures, with a particular focus on existing performance measures that focus on device utilization. Covering targets becomes an important operational goal in addition to device utilization, since it is a proxy for minimizing the consequences associated with a direct-to-target attack using WMDs, such as nuclear weapons. This chapter analyzes the tradeoffs between covering targets and other performance measures.

4.3 Performance Measures

This section overviews five performance measures for evaluating checked baggage screening systems for detecting nuclear weapons. As noted earlier, these performance measures are not intended to be an exhaustive list. Rather, they are used to illustrate how screening devices are used in different ways according to each performance measure in order to identify robust performance measures.

The Department of Homeland Security (DHS) has proposed the Encounter Probability measure for evaluating baggage screening systems (Vojtech 2009). The Encounter Probability reflects the proportion of bags that have been screened. In order to be consistent with the remaining four performance measures, we rescale the Encounter Probability to capture the Encounter Number (EN) measure, which evaluates the total number of bags screened for nuclear material by baggage screening devices, such as RIIDs. The rationale is that to detect a threat, it must first be encountered via security screening (Vojtech 2009; Mullen 2009). Therefore, encountering a bag is a binary measure that is one if a bag is screened in any way and zero otherwise. The EN cannot ensure the safety of any particular flight or target, since it does not evaluate the quality of screening procedures.

The remaining four performance measures involve covering flight segments. For simplicity, we refer to each flight segment as a flight for the remainder of this paper. A flight is said to be *covered*

if all baggage on that flight is screened. While covering performance measures do not take the quality of screening into account, they are advantageous in that they can be directly compared to the EN measure. Covering a flight is used as a proxy for ensuring that the flight is protected against an attack. Three performance measures are used to analyze how bags are screened in terms of their corresponding flights, which were first proposed by Jacobson et al. (2001). The first performance measure, Flights Covered (FC), captures the number of covered flights. The rationale behind the FC measure is to screen bags in a way that covers the most flights. The second performance measure, Passengers Covered (PC), captures the number of passengers on covered flights. The rationale behind the PC measure is to screen bags in a way that maximizes the total number of passengers on covered flights. The third performance measure, Baggage Covered (BC), captures the number of bags on covered flights. The rationale behind the BC measure is to screen bags in a way that fully utilizes the baggage screening capacity while covering flights. The BC and the EN measures both reflect device utilization. They differ in that the BC measure captures the screened bags only on covered flights, whereas the EN measure captures all screened bags.

In a direct-to-target attack, the target is not only the aircraft, but rather it is a means to transport a nuclear weapon to a specific location where it would then be detonated. The consequences of the attack extends to the detonation site and surrounding area. The FC, PC, BC, and EN measures do not explicitly consider targets. To extend the performance measures proposed by Jacobson et al. (2001), the number of targets covered is proposed as a performance measure. The Targets Covered (TC) performance measure reflects the number of covered targets, where a target is covered if all flights to the target are covered. All of these performance measures except the EN measure are used in the discrete optimization models in Sections 4 and 5 to determine how to optimally use baggage screening devices. Goal programming is used to identify robust ways to use baggage screening devices across two performance measures, the TC measure and a second measure (either the FC, PC, or BC measures).

To illustrate the trade-offs between the performance measures, an example with four flights is discussed. Its parameters are located in Table 4.1. In this example, there is a single origin airport from which all flights depart. Each flight has an associated number of passengers and bags, and it is traveling to one of two targets, A or B. The origin airport can screen 30 bags. The FC measure can be at most two, any combination of two of the four flights. The PC measure can be at most 25

Table 4.1: Illustrative example parameters

	Number of Passengers	Number of Bags	Target
Flight 1	5	10	B
Flight 2	10	12	A
Flight 3	15	14	A
Flight 4	8	16	A

by covering flights 2 and 3. The BC measure can be at most 30 by covering flights 3 and 4, which uses the entire screening capacity. The TC measure can be at most one by screening flight 1, which covers target B. This suggests that using the different performance measures results in screening different sets of bags while covering different flights.

Note that the EN reflects the total number of bags that are screened. An EN of 30 corresponds to any combination of 30 of the 52 bags being screened across the four flights, of which there are $\binom{52}{30}$ ways to do so. However, there are only $\binom{4}{2} = 6$ ways to cover two of four flights, and 4 ways to cover one flight. This results in a probability of 3.7×10^{-14} of randomly covering any one or two flights when randomly screening 30 bags to fully utilize the total baggage screening capacity, according to the EN measure.

4.4 The Single Objective Discrete Optimization Models

This section introduces four discrete optimization models, each of which corresponds to the performance measures in Section 3 (except the EN measure). The EN measure is compared to each of the discrete optimization models when assuming that bags are randomly screened, to show the discrepancies between the EN measure and the other four performance measures. First, consider a set of international flights that originate at a set of origin airports. There is a station at each of the origin airports for screening airline baggage for nuclear material. It is important to note that using nuclear screening devices are distinct from explosives screening and that all checked baggage undergo screening for explosives but not necessarily for nuclear material. Note that bags that are screened are considered encountered and those that are not screened are considered not encountered. The single objective models include only direct flight segments between a foreign

airport and a domestic airport. When the flight arrives in the United States, it is assumed that no further screening takes place on checked bags. The input parameters for all of the models are as follows:

- n = total number of flights,
- m = number of origin airports (where screening occurs),
- d = number of targets (e.g., destination airports),
- F_i = set of outgoing flights that are screened at origin airport i , $i = 1, 2, \dots, m$, with F_1, F_2, \dots, F_m being mutually exclusive and exhaustive subsets of the set of flights $\{1, 2, \dots, n\}$,
- D_k = set of incoming flights to target k , $k = 1, 2, \dots, d$, with D_1, D_2, \dots, D_d being mutually exclusive and exhaustive subsets of the set of flights $\{1, 2, \dots, n\}$,
- w_j = the total number of bags on flight j , $j = 1, 2, \dots, n$,
- p_j = the total number of passengers on flight j , $j = 1, 2, \dots, n$,
- a_j = the expected cost to screen flight j , $j = 1, 2, \dots, n$,
- c_i = the screening capacity of station i (number of bags), $i = 1, 2, \dots, m$,
- B = screening budget.

All of the parameters are assumed to be deterministic. Without loss of generality, the number of bags reflects only those that are large enough to contain a nuclear weapon. In all of the models, flights are assumed to be screened only at their designated origin airport. Moreover, the security devices are allocated at stations at the origin airports, resulting in associated screening capacities at each origin airport. Also, the models explicitly address primary screening while implicitly addressing secondary screening. It is assumed that there are enough secondary screening resources at each of the origin airports for resolving primary screening alarms via secondary screening at each of the origins. All flights are assumed to have an associated target (given by D_1, D_2, \dots, D_d) based on its route or destination.

Four discrete optimization models are proposed for maximizing the four performance measures introduced in Section 3, the FC, PC, BC, and TC measures. The discrete optimization models are stated as integer programming models for simplicity. The first model is the Flight Coverage Problem (FCP), which maximizes the FC measure. The variables for FCP as well as the subsequent

models are given by x_1, x_2, \dots, x_n , where $x_j = 1$ if flight j is covered and 0 otherwise, $j = 1, 2, \dots, n$. FCP is stated as follows.

$$\begin{aligned}
\max \quad & z^{FCP} = \sum_{i=1}^m \sum_{j \in F_i} x_j & (4.1) \\
\text{subject to} \quad & \sum_{j \in F_i} w_j x_j \leq c_i, \quad i = 1, 2, \dots, m \\
& \sum_{j=1}^n a_j x_j \leq B \\
& x_j \in \{0, 1\}, \quad j = 1, 2, \dots, n.
\end{aligned}$$

The objective function in (4.1) maximizes the number of covered flights. The first set of constraints ensures that the total number of bags on the covered flights from each origin airport does not exceed the origin airport's screening capacity. The second constraint ensures that the cost of screening the bags on the covered flights does not exceed the budget. The third set of constraints ensures that the variables take on binary values.

The second discrete optimization model is the Passenger Coverage Problem (PCP), which maximizes the PC measure. PCP uses the same set of constraints and variables as (4.1) with an objective function given by

$$\max \quad z^{PCP} = \sum_{i=1}^m \sum_{j \in F_i} p_j x_j. \quad (4.2)$$

The third discrete optimization model is the Baggage Coverage Problem (BCP), which maximizes the BC measure. BCP uses the same set of constraints and variables as (4.1) with an objective function given by

$$\max \quad z^{BCP} = \sum_{i=1}^m \sum_{j \in F_i} w_j x_j. \quad (4.3)$$

The fourth discrete optimization model is the Target Coverage Problem (TCP), which maximizes the TC measure. The variables for the TCP are y_1, y_2, \dots, y_m , where $y_k = 1$ if target k is covered and 0 otherwise, $k = 1, 2, \dots, d$. When target k (given by D_1, D_2, \dots, D_d) is covered, all of the flights going to target k are covered. TCP is stated as follows:

$$\begin{aligned}
\max \quad z^{TCP} &= \sum_{k=1}^d y_k & (4.4) \\
\text{subject to} \quad & \sum_{k=1}^d \sum_{j \in F_i \cap D_k} w_j y_k \leq c_i, \quad i = 1, 2, \dots, m \\
& \sum_{k=1}^d \sum_{i=1}^m \sum_{j \in F_i \cap D_k} a_j y_k \leq B \\
& y_k \in \{0, 1\}, \quad k = 1, 2, \dots, d.
\end{aligned}$$

The TCP constraints and objective function are analogous to the constraints in FCP, PCP, and BCP. The first set of constraints ensures that the bags on the covered flights do not exceed the screening capacity at the origin airports. The second constraint ensures the cost of screening the bags on the covered flights does not exceed a given budget. The third set of constraints ensures that the y_k variables take on binary values.

All of these four discrete optimization problems are NP-hard. They are shown to be NP-hard through a polynomial transformation from the Two-Dimensional Knapsack Problem (2-KP) and the Multi-Dimensional Knapsack Problem (MDKP).

Theorem 3 *FCP is NP-hard.*

Proof. Cardinality 2-KP is a particular case of FCP when $m = 1$ and $F_1 = \{1, 2, \dots, n\}$ (Kellerer et al. 2004, pg. 53). In this case, there are n items corresponding to the n flights, each of which has weights w_j and a_j , $j = 1, 2, \dots, n$. The two knapsack capacities are c_1 and B . FCP is solvable in polynomial time when $B \geq \sum_{j=1}^n a_j$ or when each a_j is a multiple of w_j , $j = 1, 2, \dots, n$, with a common multiple across all flights. In the former case, the screening operations at the m origin airports are independent. In both cases, the optimal solution is to greedily cover flights from the smallest to largest w_j values at each of the origin airports. \square

Theorem 4 *PCP is NP-hard.*

Proof. Follows from FCP being NP-hard. PCP degenerates into m independent 0-1 knapsack problems when $B \geq \sum_{j=1}^n a_j$, each of which corresponds to the screening operations at one of the m origin airports. However, this particular case of PCP is not NP-hard in the strong sense. \square

Theorem 5 *BCP is NP-Hard.*

Proof. Follows from FCP being NP-hard. BCP degenerates into m independent subset-sum problems when $B \geq \sum_{j=1}^n a_j$, each of which corresponds to the screening operations at one of the m origin airports. However, this particular case of BCP is not NP-hard in the strong sense. \square

Theorem 6 *TCP is NP-Hard.*

Proof. Multi-Dimensional Knapsack Problem (MDKP) is a particular case of TCP (Kellerer et al. 2004, pg. 53). In this case, there are d items corresponding to the targets and $m+1$ knapsacks, with the first m knapsacks corresponding to the capacities at the origin airports and the last knapsack corresponding to the budget constraint. Each item has weights $W_{ik} = \sum_{j \in F_i \cap D_k} w_j$ for knapsacks $i = 1, 2, \dots, m$ (the sum of the baggage going from origin airport i to target k) and $W_{(m+1)k} = \sum_{i=1}^m \sum_{j \in F_i \cap D_k} a_j$ for knapsack $m+1$ (the total cost to screen the baggage going to target k), $k = 1, 2, \dots, d$. The $m+1$ knapsack capacities are c_1, c_2, \dots, c_m and B . Note that this problem remains NP-hard even when $B \geq \sum_{j=1}^n a_j$ (MDKP is a particular case of the resulting problem with m knapsacks) and when there is one origin airport (Cardinality 2-KP is a particular case of the resulting problem). \square

One of the goals of this chapter is to compare the screening policy when maximizing the encounter number (EN) to the screening policies when maximizing the other four performance measures. To make this comparison, note that the EN reflects the total number of bags that are screened, regardless of whether they cover targets or flights. Consider bags to be randomly screened when bags at the same origin airport are equally likely to be screened, depending on the origin airport's capacity and assigned budget. Under such a scenario, it is possible to cover several flights and targets. However, due to the large number of ways to screen bags and the relatively few ways to cover flights and targets, it is unlikely that maximizing the EN measure by screening as many bags as possible simultaneously covers flights and targets.

To see that maximizing the EN measure covers few flights, assume that each bag at an origin airport is equally likely to be screened given the capacity level and budget level allocated to the origin (i.e., the fraction of b used to screen bags at origin airports). Then the probability of covering flights resulting in non-zero FC, PC, BC and TC measures can be determined. The probability of covering a flight is determined by computing the number of solutions resulting in a specific FC, PC, BC, and TC measures over the total number of ways to screen bags. There are several methods

for determining the number of solutions resulting in specific FC, PC, BC, and TC measures that utilize the knapsack problem substructures. Lambe (1974), Padberg (1971), and Begeed-Dov (1972) provide lower and upper bounds for the number of feasible solutions for the Integer Knapsack Problem. Since these methods assume that items can be added to the knapsack multiple times they must be adapted to consider the 0-1 Knapsack Problem structure used by the models in this paper (where flights and targets can be covered only once), resulting in a smaller number of feasible solutions. Dyer (2003) provides an algorithm for approximately counting the number of feasible solutions in the 0-1 Knapsack Problem and MDKP. As the example in Section 3 illustrates, this number is likely to be extremely small compared to the total number of ways to screen bags.

The total number of ways to screen bags is computed as follows for two particular cases. First consider the budget B being sufficiently large such that the entire screening capacity can be used at each origin airport and assume that $c_i \leq \sum_{j \in F_i} W_i$, $i = 1, 2, \dots, m$. Then the total number of ways to screen exactly $EN = \sum_{i=1}^m c_i$ bags is

$$\prod_{i=1}^m \binom{W_i}{c_i},$$

where $W_i = \sum_{j \in F_i} w_i$, $i = 1, 2, \dots, m$.

For general levels of the budget B , consider the particular case when the cost to screen each bag is a constant value q , such that $EN = \lfloor B/q \rfloor$ total bags can be screened across the m origins, resulting in $a_j = qw_j$, $j = 1, 2, \dots, n$. Let I denote the set of unique budget allocations to the origins, with (b_1, b_2, \dots, b_m) denoting a single group of budget allocations to the m origins that fully utilizes the budget EN (i.e., $\sum_{i=1}^m b_i = EN$). Then, the total number of ways to screen $EN = \sum_{i=1}^m b_i$ bags is

$$\sum_{(b_1, b_2, \dots, b_m) \in I} \prod_{i=1}^m \binom{W_i}{b_i}. \quad (4.5)$$

Note that I is potentially a large set, and its size can be determined by inclusion-exclusion as follows (Brualdi 1999). Let S denote the set of ways in which EN bags to be screened are allocated to the origins, and let A_i denote the set of ways in which the budget allocation to origin i exceeds its capacity (i.e., $b_i > c_i$), then

$$|I| = |\bar{A}_1 \cap \bar{A}_2 \cap \dots \cap \bar{A}_m| = |S| - \sum_i |A_i| + \sum_{i \neq j} |A_i \cap A_j| + \dots + (-1)^m |A_1 + A_2 + \dots + A_m|.$$

Then,

$$|S| = \binom{EN + m - 1}{EN}.$$

The values of A_i , $i = 1, 2, \dots, m$, depend on EN and c_i , with

$$\begin{aligned} |A_i| &= \binom{m - 1 + EN - (c_i + 1) - 1}{m - 1}, \text{ if } EN > c_i, \text{ and} \\ |A_i| &= 0 \text{ otherwise,} \end{aligned}$$

and for $i \neq j$,

$$\begin{aligned} |A_i \cap A_j| &= \binom{m - 1 + EN - (c_i + 1) - (c_j + 1) - 1}{m - 1}, \text{ if } EN > c_i + c_j, \text{ and} \\ |A_i \cap A_j| &= 0 \text{ otherwise,} \end{aligned}$$

and so on. This analytical method for determining the total number of ways to screen bags is intractable, and hence, it is impractical to use. However, it suggests that the EN may be ill-suited for covering flights and targets, since there is an extremely large number of solutions that correspond to a constant EN value, few of which cover flights and targets.

4.5 Goal Programming Models

Goal programming models are used to explore the trade-offs between multiple objectives. Non-preemptive goal programming models are introduced in this section, where the two objectives are considered to be of equal importance (Hillier and Lieberman 1990, p. 1009). The three goal programming models simultaneously consider two objectives, which consist of maximizing the FC, PC, or BS measures with the goal of maintaining a minimum number of covered targets, given budget and screening capacity constraints.

The first goal programming model is the Target Coverage Flight Coverage Problem (TCFCP). The goal is to maximize the number of flights covered while maintaining a minimum number of covered targets. The parameter T denotes the minimum number of targets to be covered. The variables are x_1, x_2, \dots, x_n , and y_1, y_2, \dots, y_d , and they are defined in the same way as in the single objective discrete optimization models in Section 4. TCFCP is formally stated as an integer programming model.

$$\max z^{TCFCP} = \sum_{i=1}^m \sum_{j \in F_i} x_j \quad (4.6)$$

$$\begin{aligned}
\text{subject to} \quad & \sum_{k=1}^d y_k \geq T \\
& \sum_{j \in F_i} w_j x_j \leq c_i, \quad i = 1, 2, \dots, m \\
& \sum_{j=1}^n a_j x_j \leq B \\
& y_k \leq x_j, \quad j \in D_k, \quad k = 1, 2, \dots, d \\
& x_j \in \{0, 1\}, \quad j = 1, 2, \dots, n \\
& y_k \in \{0, 1\}, \quad k = 1, 2, \dots, d.
\end{aligned}$$

The objective function in (4.6) maximizes the number of covered flights, which captures the FC measure. The first constraint ensures that the minimum number of covered targets are covered. The second set of constraints ensures that the total number of bags on the covered flights from each origin airport does not exceed the origin airport's screening capacity. The third constraint ensures that the cost of screening the bags on the covered flights does not exceed the budget. The fourth set of constraints ensures that in order for a target to be covered, all flights to that target are covered. The final two sets of constraints ensure that the x_j and y_k variables are binary.

The second goal programming model is the Target Coverage Passenger Coverage Problem (TCPCP), which maximizes the number of passengers on the covered flights while maintaining a minimum number of covered targets. TCPCP uses the same variables and constraints as in (4.6), but its objective function maximizes the PC measure:

$$\max \quad z^{TCPCP} = \sum_{i=1}^m \sum_{j \in F_i} p_j x_j. \quad (4.7)$$

The third goal programming model is the Target Coverage Baggage Coverage Problem (TCBCP), which maximizes the number of bags on the covered flights while maintaining a minimum number of covered targets. TCBCP uses the same variables and constraints as in (4.6), but its objective function maximizes the BC measure:

$$\max \quad z^{TCBCP} = \sum_{i=1}^m \sum_{j \in F_i} w_j x_j. \quad (4.8)$$

It should be noted that the discrete optimization model in (4.6) can be used to optimally allocate resources according to the TC measure when lifting the assumption that the set of incoming flights are mapped to a single target. The formulation for TCP considered in (4.4) cannot be used when the

sets of flights associated with the targets D_1, D_2, \dots, D_d are overlapping. Lifting this assumption is useful when defining targets based on geographical areas that do not necessarily correspond to destination airports in the United States. For example, a flight with a destination airport of Detroit could be used for the targets of Cleveland, Buffalo, and Boston in addition to Detroit. Thus, by changing the objective function in (4.6) to $\sum_{k=1}^d y_k$ as in (4.4) and by omitting the first constraint, the model in (4.6) can be used as an alternative, generalized TCP formulation.

4.6 Greedy Heuristics

Exact algorithms, such as branch and bound, can result in long computation times when solving the integer programming models, since all of the discrete optimization models are NP-hard. Several greedy heuristics are applied to the models introduced in Sections 4.4 and 4.5 in order to identify near-optimal solutions in a reduced amount of CPU time. Greedy heuristics for multi-dimensional knapsack problem are used, since all of the discrete optimization models considered have a multidimensional knapsack sub-structure. Note that all of the greedy heuristics considered define an *efficiency* for each flight, $e_j, j = 1, 2, \dots, n$, or each target $e_k, k = 1, 2, \dots, d$. As with knapsack greedy heuristics, the flights or targets for all greedy heuristics introduced in this section are covered in non-increasing order of their efficiency values.

First, the two greedy heuristics used for the single objective models are introduced, which adapt the Dobson heuristic and the Toyoda and Senju heuristic (Kellerer et al. 2004 pg. 257) to the single objective discrete optimization models. The reader is referred to Dobson (1982) and Toyoda and Senju (1968) for more details.

The efficiency values of the Dobson heuristic consist of a ratio, whose numerator is the objective function coefficient and whose denominator consists of the sum of the coefficients across all of the constraints. The Dobson heuristic efficiency values for FCP, PCP, and BCP are

$$e_j = \frac{1}{w_j + a_j},$$

$$e_j = \frac{p_j}{w_j + a_j},$$

$$e_j = \frac{w_j}{w_j + a_j},$$

respectively, $j = 1, 2, \dots, n$. The Dobson heuristic efficiency values for TCP are

$$e_k = \frac{1}{\sum_{i=1}^{m+1} W_{ik}}, \quad k = 1, 2, \dots, d.$$

Recall that $W_{ik} = \sum_{j \in F_i \cap D_k} w_j$ for knapsacks $i = 1, 2, \dots, m$ (the sum of the baggage going from origin airport i to target k) and $W_{(m+1)k} = \sum_{i=1}^m \sum_{j \in F_i \cap D_k} a_j$ for knapsack $m+1$ (the total cost to screen the baggage going to target k), $k = 1, 2, \dots, d$.

The efficiency values of the Toyoda and Senju heuristic capture the relative contribution of the constraints. Let h_{ij} denote the excess capacity in knapsack i (either a capacity or the budget constraint) if all flights are covered. For FCP, PCP, and BCP,

$$h_{ij} = \max\{0, \sum_{j \in F_i} w_j - c_i\}, \quad i = 1, 2, \dots, m$$

and

$$h_{(m+1)j} = \max\{0, \sum_{j=1}^n a_j - B\}.$$

If there is not some i ($i = 1, 2, \dots, m$) for each $j = 1, 2, \dots, n$ such that $h_{ij} > 0$, then Toyoda and Senju heuristic efficiency value $e_j = 0$, $j = 1, 2, \dots, n$. Otherwise, the Toyoda and Senju heuristic efficiency values for FCP, PCP, and BCP are

$$e_j = \frac{1}{\sum_{i=1}^m (w_j h_{ij}) + a_j h_{(m+1)j}},$$

$$e_j = \frac{p_j}{\sum_{i=1}^m (w_j h_{ij}) + a_j h_{(m+1)j}},$$

$$e_j = \frac{w_j}{\sum_{i=1}^m (w_j h_{ij}) + a_j h_{(m+1)j}},$$

respectively, $j = 1, 2, \dots, n$. Note that the Toyoda and Senju heuristic efficiency values are updated after each new flight is covered, which change the remaining capacities c_1, c_2, \dots, c_m and the remaining budget B .

For TCP, let h_{ik} denote the excess capacity in knapsack i (either a capacity or the budget constraint) if all targets are covered,

$$h_{ik} = \max\{0, \sum_{k=1}^d W_{ik} - c_i\}, \quad i = 1, 2, \dots, m,$$

and

$$h_{(m+1)k} = \max\{0, \sum_{k=1}^d W_{(m+1)k} - B\}.$$

If there is not some i ($i = 1, 2, \dots, m+1$) for each $k = 1, 2, \dots, d$ such that $h_{ij} > 0$, then the Toyoda and Senju heuristic efficiency value $e_k = 0$. Otherwise, the Toyoda and Senju heuristic efficiency values for TCP are

$$e_k = \frac{1}{\sum_{i=1}^{m+1} W_{ik} h_{ik}},$$

$k = 1, 2, \dots, d$. Note that the Toyoda and Senju heuristic efficiency values are updated after each new target is covered, which change the remaining capacities c_1, c_2, \dots, c_m and the remaining budget B .

Next, consider the goal programming models (TCFCP, TCPCP, TCBCP). The greedy heuristics are applied in two stages. In the first phase, a greedy heuristic for TCP is used to cover targets until T targets are covered (the minimum number of targets to be covered). The three greedy heuristics for TCP include the Dobson heuristic and the Toyoda and Senju heuristic (introduced earlier) as well as the Scaled Dobson heuristic, whose efficiency values are

$$e_k = \frac{1}{\sum_{i=1}^m (W_{ik}/c_i) + W_{(m+1)k}/B}, \quad k = 1, 2, \dots, d.$$

Note that $e_k > 0$, $k = 1, 2, \dots, d$, unless the budget and all origin capacities are zero. The efficiency values are updated after each new target is covered, which change the remaining capacities c_1, c_2, \dots, c_m and the remaining budget B .

In the second stage, one of two greedy heuristics are used (the Dobson heuristic and the Toyoda and Senju heuristic) that correspond to the objective that maximizes either the FC, PC, or BC measure. The adjusted budget and capacities (e.g., after adjusting for the flights that were covered in order to cover T targets) are used to compute the efficiency values for the FCP, PCP, and BCP heuristics. Thus, six greedy heuristics are applied to each goal programming model, each of which uses one of three heuristics for the first stage and one of two heuristics for the second stage. A comparison of all of the greedy heuristics is performed in the next section.

4.7 Computational Results

This section introduces and analyzes a real-world example using the discrete optimization models from Sections 4.4 and 4.5. The results shed light on how bags are screened according to different performance measures while maintaining identical EN measures.

A set of flights was constructed using all flights for a single day between eighteen international airports ($m = 18$) and fifteen domestic airports, resulting in a total of $n = 469$ flights. Table 4.2 reports the airports used and their corresponding airport codes. The data gathered include the type of aircraft used for a particular flight, the number of seats available, and the available cargo capacity. The example defines each target as one of the destination airports, resulting in ($d = 15$) targets.

The number of passengers aboard a flight was determined by using the seating capacity of the aircraft. It is assumed that the enplanement rate on a flight is uniformly distributed between 0.80 and 0.92, to be consistent with the average enplanement rate of 0.86 (United States Department of Transportation 2009). The number of bags on each flight is assumed to be a function of the number of passengers on the flight. The average number of bags per passenger is 1.45 (U.S. Transportation Security Administration 2009). Therefore, it is assumed that the number of bags per passenger on each flight is uniformly distributed between 1.3 and 1.6. It is assumed that all bags are large enough to contain a nuclear weapon. This assumption can be lifted by redefining the set of bags as only those that are large enough to contain a nuclear weapon. The size and weight of nuclear weapons varies according to the source used and its construction, and some nuclear devices, such as those made from a plutonium source, are small enough to fit in checked baggage. The numbers of passengers and bags on each flight were randomly generated and then interpreted as known, deterministic values. Table 4.3 summarizes the total number of flights, passengers, and bags between origin airports and destination airports. For instance, there are a total of 745 passengers with 1117 bags flying from Amsterdam to Atlanta across 3 flights.

The other parameters include the screening capacities at each of the origin airports, the budget, and the cost to cover each flight. The screening capacities for the origin airports are based on many factors, such as the type of the baggage screening device and the number available. It is assumed that the total screening capacity at an origin airport is $c_i = C \times v_i$, where v_i is the number of screening devices for that airport and C is the screening capacity per device. Four values for the capacity per device are considered, with $C = 1000, 1250, 2000, 2500$ bags per day based on ten hours of operation per day. The number of screening devices at each origin airport is determined by the number of bags that depart from that airport divided by 5000 and rounded up to ensure that there would be at least one device per airport. This results in $v_i = \left\lceil \frac{\sum_{j \in F_i} w_j}{5000} \right\rceil$. The budget B

ranges from \$25K to \$300K US per day in increments of \$25K. The expected cost of screening a flight a_j includes the purchase price, installation price, cost of modifications, and the lifetime for the machine, in addition to an employee's salary and the number of employees for each machine. The cost was estimated to be $q = \$2$ US per bag. This results in $a_j = q w_j, j = 1, 2, \dots, n$.

A Monte Carlo simulation with 100,000 replications was performed in order to analyze the impact of random screening. The simulation was run using Matlab 7.9.0.529 (R2009b). To reflect a best case scenario, the capacity per device used is $C = 2500$ across all budget values. The EN is computed by taking the minimum of either the total number of bags leaving the origin airport or the screening capacity at the origin airport. Therefore, the largest possible value of EN in each replication when the budget is unlimited is

$$EN = \sum_{i=1}^m \min\{c_i, \sum_{j \in F_i} w_j\} = 91,455.$$

For general values of the budget when the budget is not sufficient to screen all of the bags, the baggage screening capacity at each origin is reduced proportionally until the resulting capacities would lead to budget feasible solutions.

At each origin airport, c_i of the $\sum_{j \in F_i} w_j$ bags are randomly screened with equal likelihood unless $c_i \geq \sum_{j \in F_i} w_j$, in which case all bags are screened. In all replications with an unlimited budget, the FC measure is 10 flights, since the screening capacity at several origins was large enough to screen all departing flights. This would mean that 0.0213 of the flights are covered if screening is random and the devices are fully utilized, thus maximizing the EN. The PC, BC, and TC measures are 2782, 3955, and 0, respectively, in all replications. The best Monte Carlo simulation values across 100,000 replications are compared to the optimal solution values later in this section.

4.7.1 Optimization Results

The optimization models were solved using CPLEX 10.0 on a Linux Opteron 2.6 GHz processor with 4 GB RAM. For the models involving the FC measure, FCP, TCP, and TCFCP, the CPU time required less than one second for each instance. For PCP and TCPCP, nearly all instances required less than a minute of CPU time, although several instances required 24 hours of CPU time. For BCP and TCBCP, most instances required less than five minutes of CPU time. However, significant number of instances required between one to seven days. Three instances of TCBCP

did not complete within seven days. The long CPU times for several of the models motivates the application of greedy heuristics. Long CPU times were not unexpected, since the problem parameters for the discrete optimization models with the PC and BC measures result in strongly correlated knapsack problem instances, which often lead to long CPU times in branch and bound algorithms (Martello et al. 2000).

Figure 1 illustrates the optimal solution values for FCP, PCP, BCP, and TCP for all capacity levels. Figure 4.1(a) shows that FCP obtains a solution value of 322 covered flights when $C = 2500$ and $B \geq \$175K$. This can be compared to the Monte Carlo simulation results that cover ten flights, which reveals substantial improvement in covered flights. Figure 4.1(b) illustrates that the greatest number of passengers on covered flights is 64,686 when $C = 2500$ and $B \geq \$200K$. This can be compared a PC measure of 2,782 in the Monte Carlo simulation. Figure 4.1(c) illustrates that when $C = 2500$ and $B \leq \$75K$, the BC measure is the same for all screening capacities. The largest BC measure occurs when $C = 2500$ and $B \geq \$200K$, resulting in 91,010 bags on covered flights. Figure 4.1(d) shows the TC values, in which up to ten targets could be covered when $C = 2500$ and $B \geq \$125K$. Note that when the budget is less than $\$50K$, FCP is the only model with different objective function values across the four capacity levels.

Figure 2 illustrates the same information as in Figure 1 in terms of the EN value rather than the budget. It illustrates how the same EN value can lead to different FC, PC, BC, and TC measure values, depending on the device capacities, and it highlights the need to consider additional performance measures for determining how to use limited screening capacities.

Figure 3 illustrates the corresponding FC, PC, BC, and TC measure values when solving FCP, PCP, BCP, and TCP when $C = 2500$. Note that these solution values are not necessarily optimal, except when the performance measure matches the problem considered (e.g., the TC measure and TCP). Figure 4.3(a) illustrates the FC measure for FCP, PCP, BCP, TCP, and EN (from the Monte Carlo simulation results). At most 322 flights can be covered when solving the FCP, whereas TCP results in FC measure values of at most 225. All discrete optimization models exceed the EN by at least 215 flights when $B \geq \$150K$. Figure 4.3(b) illustrates the PC measure across all models. The PC measure reaches a value of 64,686 when solving PCP, whereas the TCP results in a PC measure of at most 47,429. Note that all proposed models exceed the corresponding EN by at least 44,647 passengers on covered flights. Figure 4.3(c) illustrates the BC measure across all

models. The BC measure is at most 91,010 when solving BCP. The BC measure indicates that the FCP, PCP, BCP, and TCP fully utilize the available screening capacity for $B \leq \$125K$, which suggests that all performance measures are robust in terms of the BC measure. All of the discrete optimization model BC values exceed the EN by at least 87,055 bags when $B \geq \$150K$. Figure 4.3(d) illustrates the TC measure across all models. The PCP and EN result in TC values of zero across all levels of the budget. BCP covers at most two targets, FCP covers at most one target, and PCP and EN cover no flights. This suggests that the FC, PC, BC, and EN measures are not robust in terms of covering targets, and it motivates a comparison of the tradeoffs between the TC measure and the FC, PC, and BC measures in the goal programming models.

Figure 4 illustrates the tradeoff between the FC, PC, BC measures and the TC measures as reflected in the TCFCP, TCPCP, and TCBCP solutions with $C = 2500$ and $B = \$275K$. In Figure 4.4(a), 322 flights can be covered while ensuring that up to four targets are covered and that 312 flights can be covered while ensuring that ten targets are covered. Figure 4.4(b) suggests that ten targets can be covered with a reduction of 997 passengers not on covered flights, which is a relative reduction of 0.015 in the PC measure as compared to when no targets are covered. Three TCBCP instances did not complete after one week of CPU time (for $T = 8, 9, 10$). The best integer BC values are reported, indicated by the circled values in Figure 4.4(c), which are not necessarily optimal. Figure 4.4(c) suggests that ten targets can be covered with a reduction of 22 bags not on covered flights, which is a relative reduction of 2.4×10^{-4} in the BC measure as compared to when no targets are covered. These results suggest that the EN measure is not adequate for evaluating baggage screening systems, since many targets and flights can be covered with identical EN values.

Figure 5 illustrates the optimal solution values for TCFCP, TCPCP, and TCBCP when $C = 1250, 2500$ for $T = 0, 7, 10$ as a function of EN. Note that it is not possible to cover ten targets in any of the models when $C = 1250$. In Figure 4.5(a), when $C = 1250$ and $B \geq \$100K$, there is a difference of at most 21 in the FC measure when comparing $T = 0$ to $T = 7$. When $C = 2500$ and $B \geq \$175K$, there is a difference of at most 10 in the FC measure when comparing $T = 0$ to $T = 7$. Figure 4.5(b) illustrates similar results when comparing PC measure and the number of covered targets across both values of C . For example, when $C = 2500$, there is a difference of 100 passengers when comparing $T = 0$ to $T = 10$ with $B = \$275K$. Figure 4.5(c) also yields similar results when comparing BC measure and the number of covered targets across both values of C .

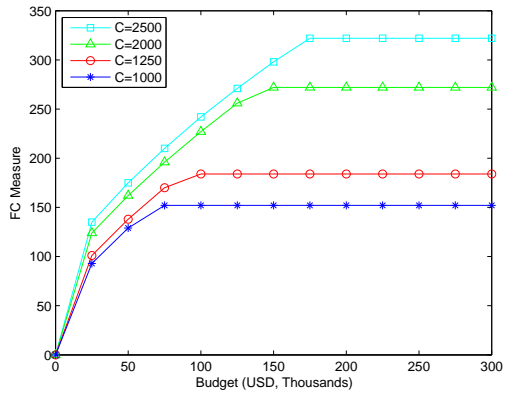
Table 4.2: Airport codes of origins and targets

<i>ORIGIN</i>	<i>TARGET</i>
Amsterdam-AMS	Atlanta-ATL
Beijing-PEK	Baltimore-BWI
Brussels-BRU	Boston-BOS
Frankfurt-FRA	Chicago-ORD
Hong Kong-HKG	Cincinnati-CVG
London-LHR	Denver-DEN
Madrid-MAD	Detroit-DTW
Manila-MNL	Houston-IAH
Montreal-YUL	Los Angeles-LAX
Moscow-DOM	Minneapolis-MSP
Mumbai-BOM	New York-JFK
Munich-MUC	Newark-EWR
Paris-CDG	San Francisco-SFO
Rome-FCO	Seattle-SEA
Shanghai-SHA	Washington D.C.-IAD
Tokyo-NRT	
Toronto-YYZ	
Zurich-ZRH	

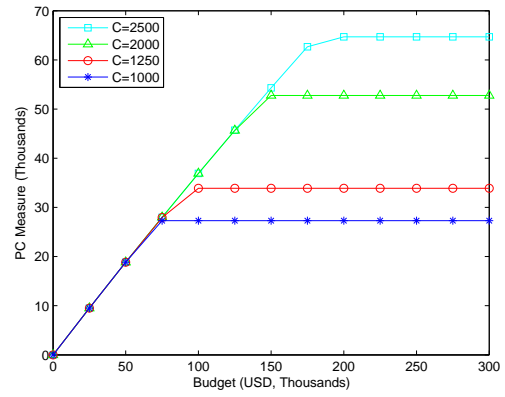
When $C = 2500$, there is no difference in the BC measure between the $T = 0$ to $T = 10$ scenarios across all values of B , which indicates that the baggage screening capacity is fully utilized for all scenarios. Similar to Figure 2, the TCFCP result in the greatest relative difference in the FC measure when compared to the relative differences in the PC measure (from TCPCP) or in the BC measure (from TCBCP). This implies that the tradeoff in the PC or BC values when moving from $T = 0$ to $T = 7$ (or 10) is much lower than the tradeoff in the FC values.

Table 4.3: Total number of flights, passengers, and bags from origins to targets

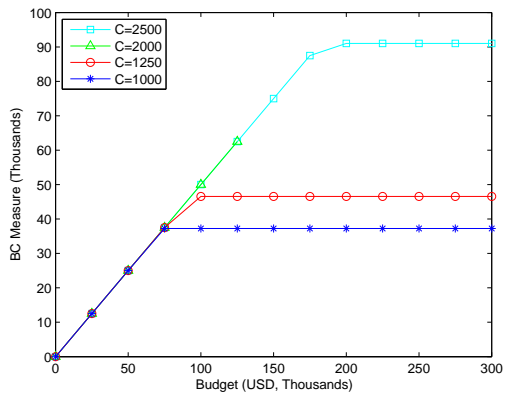
ORIGIN		TARGET														
		ATL	BWI	BOS	ORD	CVG	DEN	DTW	IAH	LAX	MSP	JFK	EWB	SFO	SEA	IAD
AMS	Flights	3	0	1	1	0	0	4	3	1	3	4	3	1	1	2
	Passengers	745	0	283	252	0	0	1227	1017	278	753	977	722	382	273	470
	Bags	1117	0	397	401	0	0	1812	1568	420	1080	1465	1034	606	392	633
PEK	Flights	0	0	0	1	0	0	0	0	1	0	1	1	2	1	1
	Passengers	0	0	0	330	0	0	0	0	278	0	400	336	589	251	335
	Bags	0	0	0	448	0	0	0	0	382	0	578	506	806	399	523
BRU	Flights	0	0	0	1	1	0	0	0	0	0	3	2	0	0	1
	Passengers	0	0	0	318	178	0	0	0	0	0	589	473	0	0	308
	Bags	0	0	0	496	249	0	0	0	0	0	872	668	0	0	439
FRA	Flights	2	0	2	6	2	1	2	1	2	0	4	4	3	1	4
	Passengers	539	0	580	1923	519	307	525	279	699	0	1341	1482	1167	283	1321
	Bags	800	0	828	2791	754	477	790	394	1042	0	1981	2208	1598	415	1870
HKG	Flights	0	0	0	1	1	0	0	0	2	0	2	1	4	0	0
	Passengers	0	0	0	414	399	0	0	0	765	0	785	343	1503	0	0
	Bags	0	0	0	620	560	0	0	0	1047	0	1191	475	2177	0	0
LHR	Flights	3	1	7	12	9	2	0	5	7	1	18	7	5	2	9
	Passengers	896	263	1819	3487	2574	671	0	1667	2513	240	6015	2004	1847	712	2854
	Bags	1210	352	2611	5211	3770	966	0	2387	3710	382	8648	2961	2859	977	4161
MAD	Flights	1	0	0	1	0	0	0	0	0	0	3	1	0	0	1
	Passengers	250	0	0	320	0	0	0	0	0	0	827	178	0	0	304
	Bags	377	0	0	455	0	0	0	0	0	0	1161	278	0	0	434
MNL	Flights	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
	Passengers	0	0	0	0	0	0	0	0	422	0	0	0	385	0	0
	Bags	0	0	0	0	0	0	0	0	657	0	0	0	523	0	0
YUL	Flights	3	0	5	13	2	0	4	0	1	1	6	8	1	0	5
	Passengers	183	0	220	743	120	0	260	0	110	66	297	395	101	0	321
	Bags	263	0	316	1076	180	0	377	0	153	93	431	568	133	0	479
DVG	Flights	1	0	0	1	0	0	0	0	1	0	2	0	0	0	1
	Passengers	249	0	0	254	0	0	0	0	233	0	493	0	0	0	233
	Bags	350	0	0	394	0	0	0	0	320	0	685	0	0	0	318
BOM	Flights	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	Passengers	267	0	0	0	0	0	0	0	0	0	246	0	0	0	0
	Bags	383	0	0	0	0	0	0	0	0	0	325	0	0	0	0
MUC	Flights	1	0	1	2	0	0	0	0	0	0	2	1	1	0	2
	Passengers	265	0	258	678	0	0	0	0	0	0	383	309	332	0	578
	Bags	381	0	343	939	0	0	0	0	0	0	607	403	483	0	841
CDG	Flights	4	0	2	2	2	0	1	2	3	1	7	6	1	1	4
	Passengers	1094	0	641	573	503	0	239	584	970	240	2067	1208	401	228	1225
	Bags	1549	0	967	890	673	0	329	796	1330	328	3061	1797	594	336	1811
FCO	Flights	1	0	0	2	0	0	0	0	0	0	6	2	0	0	2
	Passengers	267	0	0	508	0	0	0	0	0	0	1480	434	0	0	549
	Bags	358	0	0	733	0	0	0	0	0	0	2133	654	0	0	786
SHA	Flights	1	0	0	2	0	0	0	0	0	0	1	1	1	0	0
	Passengers	331	0	0	692	0	0	0	0	0	0	335	337	380	0	0
	Bags	480	0	0	1055	0	0	0	0	0	0	529	480	607	0	0
NRT	Flights	1	0	0	4	0	0	1	1	7	1	4	1	5	2	2
	Passengers	398	0	0	1504	0	0	411	302	2406	404	1317	330	1602	269	630
	Bags	605	0	0	2182	0	0	607	447	3601	595	1893	442	2303	849	914
YYZ	Flights	12	4	11	25	5	4	8	7	4	5	7	22	3	1	5
	Passengers	768	130	619	3474	304	386	515	347	518	378	555	1558	340	93	276
	Bags	1106	189	888	4956	447	552	738	492	739	542	773	2220	514	140	381
ZRH	Flights	1	0	0	2	0	0	0	0	1	0	3	2	0	0	1
	Passengers	257	0	0	531	0	0	0	0	249	0	770	319	0	0	256
	Bags	402	0	0	774	0	0	0	0	349	0	1163	454	0	0	339



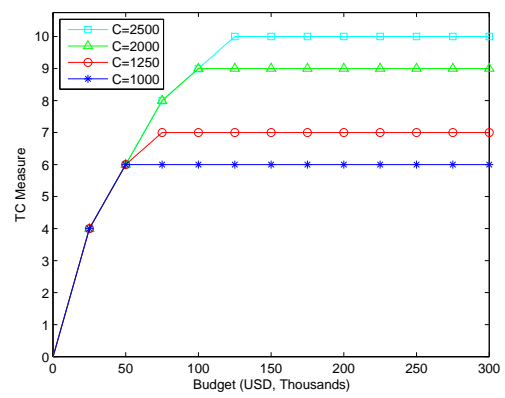
(a) FCP



(b) PCP

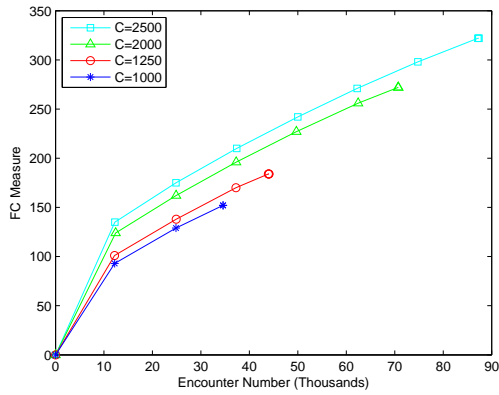


(c) BCP

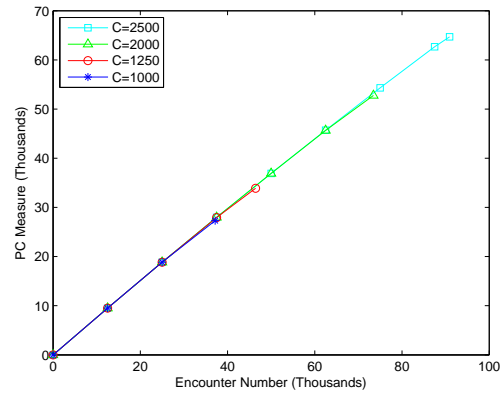


(d) TCP

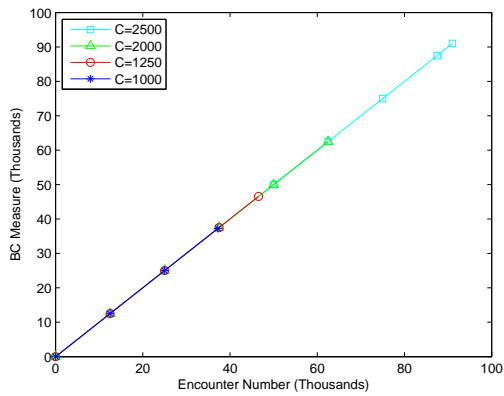
Figure 4.1: Optimal solution values for FCP, PCP, BCP, and TCP as a function of budget



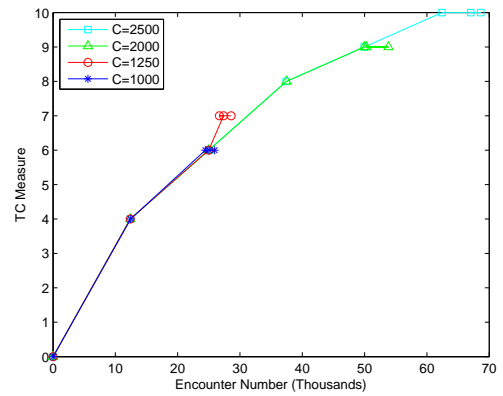
(a) FCP



(b) PCP

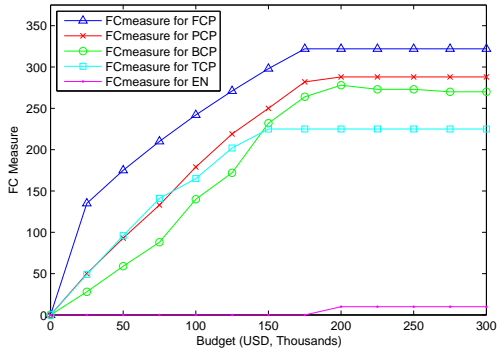


(c) BCP

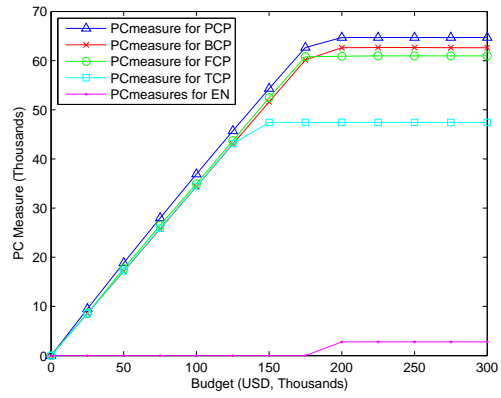


(d) TCP

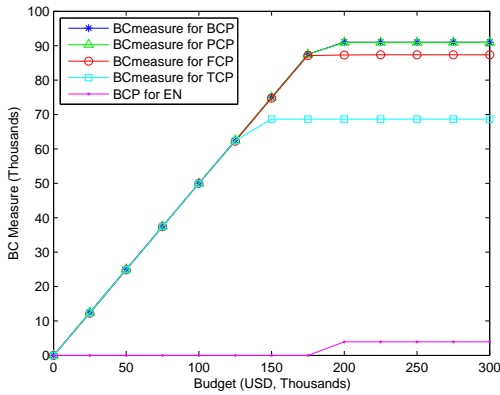
Figure 4.2: Optimal solution values for FCP, PCP, BCP, and TCP as a function of the EN



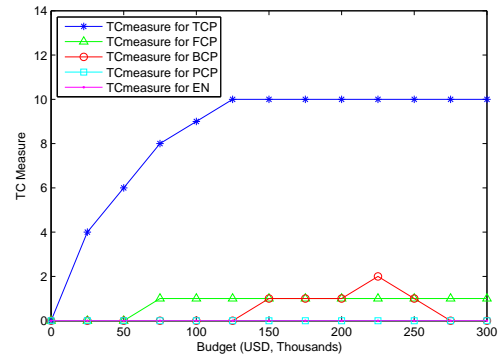
(a) FC Measure



(b) PC Measure

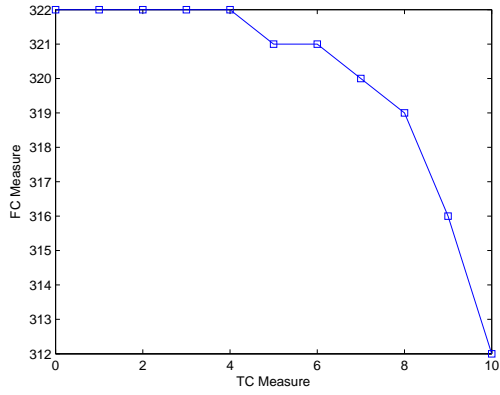


(c) BC Measure

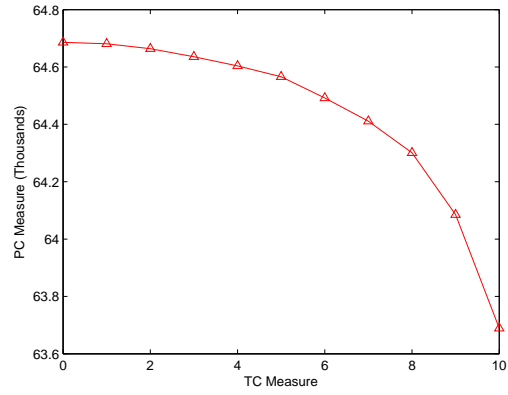


(d) TC Measure

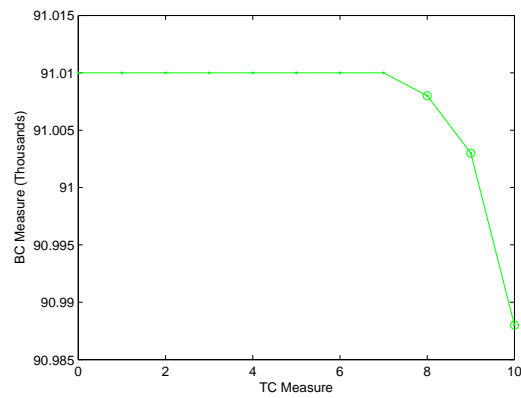
Figure 4.3: Corresponding FC, PC, BC, and TC measures for FCP, PCP, BCP, TCP and EN as a function of budget for $C = 2500$



(a) TCFCP

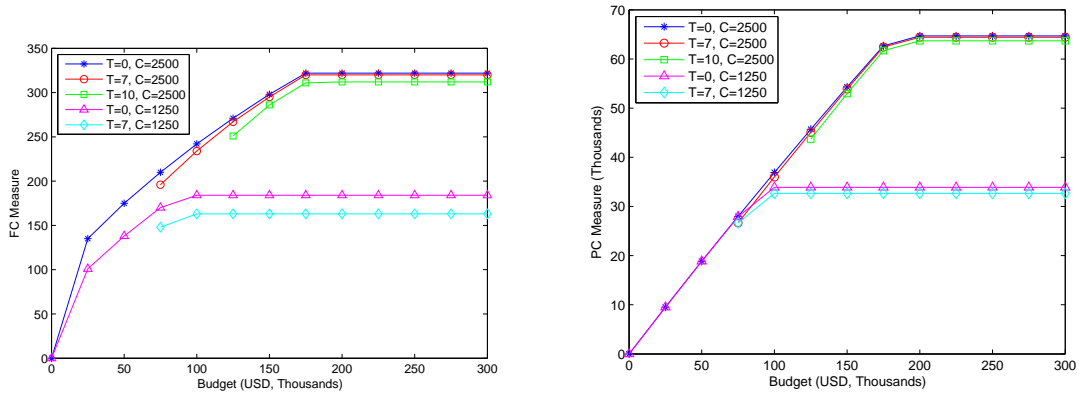


(b) TCPCP



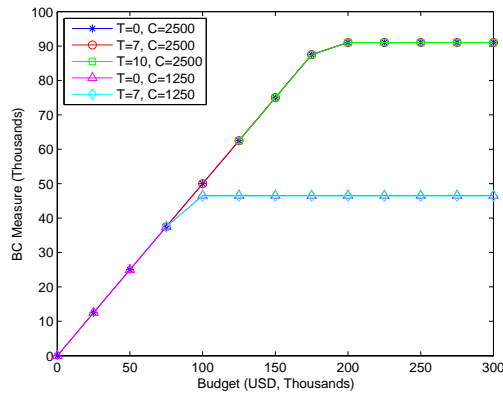
(c) TCBCP

Figure 4.4: TCFCP, TCPCP, and TCBCP for $C = 2500$ and $B = \$275K$ USD



(a) TCFCP

(b) TCPCP



(c) TCBCP

Figure 4.5: TCFCP, TCPCP, and TCBCP optimal solution values as a function of budget for $C = 1250, 2500$ and $T = 0, 7, 10$

4.7.2 Greedy Heuristics Results

The long CPU times to solve several PCP, BCP, TCPCP, and TCBCP motivate the use of greedy heuristics introduced in Section 6 to efficiently identify near-optimal solutions. All heuristics were run using Matlab 7.9.0.529 (R2009b) on an Intel Xeon X5365 3 GHz processor with 3.25 GB of RAM. The results are illustrated for the scenarios when $C = 2500$.

Tables 4.4 – 4.7 summarize the solution values for each performance measure from the Dobson heuristics and Toyoda & Senju (T & S) heuristics. Table 4.4 reports the values of the FC measure using the Dobson heuristic and the Toyoda and Senju heuristic for FCP, PCP, BCP, and TCP. For convenience, the optimal solution values for FCP are listed. Heuristic values that match the optimal solution values are in boldface. Both FCP heuristics identify the optimal solutions for all values of B . The PCP heuristics identify solutions whose FC values are at least a factor of 0.370 of the optimal solution values, the BCP heuristics identify solutions whose FC values are at least a factor of 0.227 of the optimal solution values, and the TCP heuristics identify solutions whose FC values are at least a factor of 0.244 of the optimal solution values. The PCP, BCP, and TCP heuristics all fail to cover many flights.

Table 4.5 reports the values of the PC measure using the Dobson heuristics and the Toyoda and Senju heuristics for FCP, PCP, BCP, and TCP. The PCP Dobson heuristic identifies one optimal solution (when $B = \$25K$), and the PCP Toyoda and Senju heuristic does not identify any optimal solutions. The PCP Dobson heuristic and the Toyoda and Senju heuristics identify PC solutions whose values are at least a factor of 0.990 of the optimal solution values. The FCP heuristics identify solutions whose PC values are at least a factor of 0.904 of the optimal solution values, the BCP heuristics identify solutions whose PC values are at least a factor of 0.893 of the optimal solution values, and the TCP heuristics identify solutions whose PC values are at least a factor of 0.590 of the optimal solution values.

Table 4.6 reports the values of the BC measure using the Dobson heuristics and the Toyoda and Senju heuristics for FCP, PCP, BCP, and TCP. The BCP Dobson heuristic identifies two optimal BCP solutions (when $B = \$100K, \$150K$), and the BCP Toyoda and Senju heuristic identifies one optimal solution (when $B = \$100K$). The BCP Dobson heuristic and the Toyoda and Senju heuristics identify solutions to BCP whose values are at least a factor of 0.980 of the optimal

solution value. The FCP heuristics identify solutions whose BC values are at least a factor of 0.957 of the optimal solution values, the PCP heuristics identify solutions whose BC values are at least a factor of 0.985 of the optimal solution values, and the TCP heuristics identify solutions whose BC values are at least a factor of 0.608 of the optimal solution values. The FCP, PCP, and BCP heuristics tend to nearly fully-utilize the baggage screening capacity.

Table 4.7 reports the values of the TC measure using the Dobson heuristics and the Toyoda and Senju heuristics for FCP, PCP, BCP, and TCP. It reveals that both the TCP Dobson heuristic and the TCP Toyoda and Senju heuristic identify the optimal TCP solutions for all scenarios considered. The PCP heuristics do not cover any targets, whereas both FCP heuristics cover at most one target, and both BCP heuristics cover at most four targets. The results of Tables 4.4 – 4.7 suggest that the FCP, PCP, and BCP heuristics are effective in covering targets, and that the TCP heuristics are not effective for covering flights, passengers, or bags.

The Dobson heuristics identify solutions whose values are at least as good as the their corresponding Toyoda and Senju heuristic solution values in all but two scenarios across all four models considered (BCP with $B = \$25K$, $\$175K$), which suggests that despite its simplicity, the Dobson heuristic is more effective for FCP, PCP, BCP, and TCP.

Tables 4.8, 4.9, and 4.10 summarize the solution values identified by the heuristics used for the three goal programming models when $C = 2500$, $B = \$275K$, and T varies from 1 to 10. All heuristics cover T targets in the scenarios considered, and hence, only the objective function values are reported (either FC, PC, or BC values). Table 4.8 reports the FC values for all of the heuristics. The TCFCP heuristics identify FC solution values that are at least 0.985 of the optimal solution values, whereas the TCPCP heuristics identify FC solution values that are at least 0.898 of the optimal solution value, and the TCBCP heuristics identify FC solution values that are at least 0.811 of the optimal solution values. A heuristic *dominates* the other heuristics if it identifies solution values whose values are at least as good as the those identified by the other heuristics for all scenarios considered (in this case, across all values of T). None of the six TCFCP heuristics dominate the other heuristics in terms of the FC values. However, each of the three TCFCP heuristics that use the Toyoda and Senju heuristic in the FC phase dominate other heuristics in all but two or fewer scenarios, which suggests that the Toyoda and Senju heuristic in the FC phase identifies better solution values than the Dobson heuristic in the FC phase.

Table 4.9 reports the PC values for the heuristics. The TCPCP heuristics identify PC solution values that are at least 0.983 of the optimal solution values, whereas the TCFCP heuristics identify PC solution values that are at least 0.930 of the optimal solution value and the TCBCP heuristics identify PC solution values that are at least 0.926 of the optimal solution values. None of the six TCPCP heuristics dominate the other heuristics in terms of the PC values.

Table 4.10 reports the BC values for the heuristics. The TCBCP heuristics identify solution values that are at least 0.955 of the optimal BC solution values, whereas the TCFCP heuristics identify BC solution values that are at least 0.945 of the optimal solution values, and the TCPCP heuristics identify BC solution values that are at least 0.978 of the optimal solution values. The TCBCP heuristic that uses the Scaled Dobson heuristic in the TC phase and then the Toyoda and Senju heuristic in the BC phase dominates all other TCBCP heuristics. The three TCBCP heuristics that use the Toyoda and Senju heuristics in the BC phase identify significantly better BC values than the corresponding TCBCP heuristic that uses the Dobson heuristic in the BC phase. This observation mirrors that of the TCFCP heuristics (and, to a lesser extent, of the TCPCP heuristics), which suggests that the Toyoda and Senju heuristic for the second phase is more effective than using the Dobson heuristic. We also note that the three TCPCP heuristics that use the Dobson heuristic in the BC phase outperform the corresponding TCBCP heuristic in terms of the BC solution values they identify in all thirty instances. The TCPCP heuristics that use the Toyoda and Senju heuristic in the BC phase outperform the corresponding TCBCP heuristics in terms of the BC solution values they identify in fourteen of the thirty instances. This observation is surprising, since it suggests that heuristics aimed at covering the most passengers more fully utilize the baggage screening capacity than heuristics designed to fully utilize the baggage screening capacity. This is largely the result of the TCBCP having a Subset Sum Problem, where the efficiency values are identical across many flights.

Table 4.4: FCP, PCP, BCP, and TCP heuristic results for FC measure

Budget (in USD)	Optimal FC Values	FCP		PCP		BCP		TCP	
		Dobson	T & S	Dobson	T & S	Dobson	T & S	Dobson	T & S
25000	135	135	135	50	53	31	32	33	33
50000	175	175	175	93	101	61	85	72	72
75000	210	210	210	133	144	89	111	123	123
100000	242	242	242	181	183	114	144	158	158
125000	271	271	271	225	227	169	208	186	186
150000	298	298	298	254	256	198	237	186	186
175000	322	322	322	285	284	263	264	186	186
200000	322	322	322	290	290	268	268	186	186
225000	322	322	322	290	290	268	268	186	186
250000	322	322	322	290	290	268	268	186	186
275000	322	322	322	290	290	268	268	186	186
300000	322	322	322	290	290	268	268	186	186

Table 4.5: FCP, PCP, BCP, and TCP heuristic results for PC measure

Budget (in USD)	Optimal PC Values	FCP		PCP		BCP		TCP	
		Dobson	T & S	Dobson	T & S	Dobson	T & S	Dobson	T & S
25000	9518	8603	8603	9518	9444	8497	8560	6264	6264
50000	18842	17698	17708	18841	18675	17076	17298	13635	13635
75000	27980	26491	26574	27975	27743	25701	25750	22651	22651
100000	36918	34923	35106	36915	36664	34267	34364	29162	29162
125000	45690	43692	43695	45686	45470	42999	42974	38187	38187
150000	54307	52382	52417	54302	53927	51625	51561	38187	38187
175000	62673	60766	60766	62641	62420	60082	60065	38187	38187
200000	64685	60766	60766	64047	64047	61372	61372	38187	38187
225000	64685	60766	60766	64047	64047	61372	61372	38187	38187
250000	64685	60766	60766	64047	64047	61372	61372	38187	38187
275000	64686	60766	60766	64047	64047	61372	61372	38187	38187
300000	64686	60766	60766	64047	64047	61372	61372	38187	38187

Table 4.6: FCP, PCP, BCP, and TCP heuristic results for BC measure

Budget (in USD)	Optimal BC Values	FCP		PCP		BCP		TCP	
		Dobson	T & S	Dobson	T & S	Dobson	T & S	Dobson	T & S
25000	12500	12242	12242	12499	12488	12482	12494	9064	9064
50000	25000	24846	24913	24997	24993	24996	24985	19801	19801
75000	37500	37393	37460	37495	37486	37498	37484	32784	32784
100000	50000	49923	49974	49995	49976	50000	50000	42165	42165
125000	62500	62136	62193	62498	62499	62489	62395	55368	55368
150000	75000	74600	74685	74993	74995	75000	74961	55368	55368
175000	87500	87090	87090	87462	87231	87330	87403	55368	55368
200000	91010	87090	87090	89664	89664	89232	89232	55368	55368
225000	91010	87090	87090	89664	89664	89232	89232	55368	55368
250000	91010	87090	87090	89664	89664	89232	89232	55368	55368
275000	91010	87090	87090	89664	89664	89232	89232	55368	55368
300000	91010	87090	87090	89664	89664	89232	89232	55368	55368

Table 4.7: FCP, PCP, BCP, and TCP heuristic results for TC measure

Budget (in USD)	Optimal TC Values	FCP		PCP		BCP		TCP	
		Dobson	T & S	Dobson	T & S	Dobson	T & S	Dobson	T & S
25000	4	0	0	0	0	0	0	4	4
50000	6	0	0	0	0	0	0	6	6
75000	8	1	1	0	0	0	0	8	8
100000	9	1	1	0	0	0	0	9	9
125000	10	1	1	0	0	0	0	10	10
150000	10	1	1	0	0	0	3	10	10
175000	10	1	1	0	0	3	4	10	10
200000	10	1	1	0	0	4	4	10	10
225000	10	1	1	0	0	4	4	10	10
250000	10	1	1	0	0	4	4	10	10
275000	10	1	1	0	0	4	4	10	10
300000	10	1	1	0	0	4	4	10	10

Table 4.8: FC measure for TCFCP, TCPCP, and TCBCP heuristics for B = \$275K

			Number of Targets									
			1	2	3	4	5	6	7	8	9	10
	TC Heuristic	FC Heuristic	FC	FC	FC	FC	FC	FC	FC	FC	FC	FC
TCFCP	DOBSON	DOBSON	322	320	319	319	318	317	316	316	315	309
		T & S	322	321	320	320	319	318	317	317	316	310
	S. DOBSON	DOBSON	322	320	319	319	318	318	316	316	315	308
		T & S	322	321	320	320	319	319	317	317	316	308
	T & S	DOBSON	322	320	319	319	318	317	316	316	315	309
		T & S	322	321	320	320	319	318	317	317	316	310
TCPCP	DOBSON	DOBSON	292	290	290	289	288	289	291	292	294	292
		T & S	292	289	290	289	288	289	291	292	294	292
	S. DOBSON	DOBSON	292	290	290	289	290	291	289	292	294	290
		T & S	292	289	290	289	290	291	289	292	294	290
	T & S	DOBSON	292	290	290	289	288	289	291	292	294	292
		T & S	292	289	290	289	288	289	291	292	294	292
TCBCP	DOBSON	DOBSON	268	261	262	262	262	268	268	273	274	273
		T & S	268	267	268	268	268	274	274	279	279	278
	S. DOBSON	DOBSON	268	261	262	262	263	265	273	273	274	276
		T & S	268	267	268	268	269	271	279	279	279	280
	T & S	DOBSON	268	261	262	262	262	268	268	273	274	273
		T & S	268	267	268	268	268	274	274	279	279	278
TCFCP	Optimal FC Values		322	322	322	322	321	321	320	319	316	312

Table 4.9: PC measure for TCFCP, TCPCP, and TCBCP heuristics for B = \$275K

			Number of Targets									
			1	2	3	4	5	6	7	8	9	10
	TC Heuristic	PC Heuristic	PC	PC	PC	PC	PC	PC	PC	PC	PC	PC
TCFCP	DOBSON	DOBSON	60766	60289	60100	60196	60413	60460	60581	60627	60772	60706
		T & S	60766	60711	60522	60618	60835	60882	61003	61049	61194	61128
	S. DOBSON	DOBSON	60766	60289	60100	60196	60226	60266	60438	60627	60772	60830
		T & S	60766	60711	60522	60618	60648	60688	60860	61049	61194	60830
	T & S	DOBSON	60766	60289	60100	60196	60413	60460	60581	60627	60772	60706
		T & S	60766	60711	60522	60618	60835	60882	61003	61049	61194	61128
TCPCP	DOBSON	DOBSON	64043	64021	63901	63763	63533	63677	63583	63342	63388	62820
		T & S	64043	63845	63901	63763	63533	63677	63583	63342	63388	62820
	S. DOBSON	DOBSON	64043	64021	63901	63763	63735	63616	63580	63342	63388	62600
		T & S	64043	63845	63901	63763	63735	63616	63580	63342	63388	62600
	T & S	DOBSON	64043	64021	63901	63763	63533	63677	63583	63342	63388	62820
		T & S	64043	63845	63901	63763	63533	63677	63583	63342	63388	62820
TCBCP	DOBSON	DOBSON	61372	59868	59913	60082	60077	60100	60100	60101	60358	59998
		T & S	61372	61415	61460	61629	61624	61647	61647	61648	61648	61288
	S. DOBSON	DOBSON	61372	59868	59913	60082	60087	60074	60101	60101	60358	60383
		T & S	61372	61415	61460	61629	61634	61621	61648	61648	61648	61424
	T & S	DOBSON	61372	59868	59913	60082	60077	60100	60100	60101	60358	59998
		T & S	61372	61415	61460	61629	61624	61647	61647	61648	61648	61288
TCPCP	Optimal PC Values		64681	64664	64636	64604	64566	64492	64411	64301	64085	63689

Table 4.10: BC measure for TCFCP, TCPCP, and TCBCP heuristics for B = \$275K

			Number of Targets									
			1	2	3	4	5	6	7	8	9	10
	TC Heuristic	BC Heuristic	BC	BC	BC	BC	BC	BC	BC	BC	BC	BC
TCFCP	DOBSON	DOBSON	87090	86373	86009	86105	86395	86439	86600	86738	87015	87129
		T & S	87090	87030	86666	86762	87052	87096	87257	87395	87672	87786
	S. DOBSON	DOBSON	87090	86373	86009	86105	86080	86215	86485	86738	87015	87587
		T & S	87090	87030	86666	86762	86737	86872	87142	87395	87672	87587
	T & S	DOBSON	87090	86373	86009	86105	86395	86439	86600	86738	87015	87129
		T & S	87090	87030	86666	86762	87052	87096	87257	87395	87672	87786
TCPCP	DOBSON	DOBSON	89665	89664	89502	89355	89038	89338	89327	89144	89526	89302
		T & S	89665	89385	89502	89355	89038	89338	89327	89144	89526	89302
	S. DOBSON	DOBSON	89665	89664	89502	89355	89362	89302	89317	89144	89526	89120
		T & S	89665	89385	89502	89355	89362	89302	89317	89144	89526	89120
	T & S	DOBSON	89665	89664	89502	89355	89038	89338	89327	89144	89526	89302
		T & S	89665	89385	89502	89355	89038	89338	89327	89144	89526	89302
TCBCP	DOBSON	DOBSON	89232	86955	87001	87194	87167	87169	87169	87174	87576	87152
		T & S	89232	89259	89305	89498	89471	89473	89473	89478	89478	89054
	S. DOBSON	DOBSON	89232	86955	87001	87194	87188	87188	87174	87174	87576	87715
		T & S	89232	89259	89305	89498	89492	89492	89478	89478	89478	89268
	T & S	DOBSON	89232	86955	87001	87194	87167	87169	87169	87174	87576	87152
		T & S	89232	89259	89305	89498	89471	89473	89473	89478	89478	89054
TCBCP	Optimal BC Values		91010	91010	91010	91010	91010	91010	91010	(91008)	(91003)	(90988)

4.8 Conclusions

This chapter analyzes alternative performance measures to the encounter number (or encounter probability) when examining how to deploy and use baggage screening devices for detecting nuclear WMDs in aviation baggage on international commercial aviation flights. Seven discrete optimization models are formulated and solved for protecting against a direct-to-target attack, including three goal programming models that balance the protection of targets with the protection of flights, passengers, and device utilization. Although the performance measures proposed here are by no means exhaustive, they demonstrate the importance of focusing on potential targets when

designing systems to protect against WMDs and motivate the need to use advanced analytical techniques to create new aviation security performance measures for WMD attacks.

The encounter number (i.e., a measure of device utilization) is inadequate for covering targets and flights. The analysis of a real-world example illustrates that given identical values of the encounter numbers, it is possible to screen baggage in such a way that it covers many flights, passengers, and targets. The example suggests that the flights covered (FC) performance measure is robust across the PC and BC measures (not across the TC measure). The goal programming models show that there are few tradeoffs between covering targets and the FC, PC, or BC measures. In particular, the TCBCP results show that up to ten targets could be covered while almost fully utilizing the screening device capacity. A detailed analysis of several of greedy heuristics illustrate that near-optimal solutions to the models can be identified quickly.

Future research directions include using risk-based models to take passenger and cargo pre-screening into account, as well as associating a risk factor for the targets for weighing the importance of each target. All of the performance measures considered in this paper focus on coverage, a binary measure. One extension would be to consider non-binary performance measures that reflect the quality of the screening performed as well as the impact of false alarms. Another extension would be identify performance measures that balance multiple types of attacks rather than solely focusing on a direct-to-target attack.

Chapter 5

Conclusions

The fight against terrorism has become an increased focus for homeland security. It is known that terrorist groups are trying to obtain nuclear material, in hopes of creating a WMD, with the intention of detonating it on American soil. In order for a terrorist group to get the WMD or nuclear material into the United States, the terrorist group must smuggle it in through the borders. There are many avenues for terrorists to use in order to achieve smuggling in the nuclear weapon or nuclear material. Two of these avenues are through ports and aircrafts, in which there are many vulnerabilities within each. Homeland security is currently implementing new technology and screening procedures to reduce the vulnerabilities within these sectors. The focus of this thesis is to use operations research methodologies, such as linear programming models, to make screening decisions for these sectors more effective and efficient, in hopes at detecting smuggled nuclear material or WMD.

Chapter 2 illustrates a more granulated risk-based system for prescreening classifications of cargo containers arriving at the United States ports. Instead of only classifying a container by high-risk and low-risk, analysis is provided that uses the classification of high-background and low-background. The second classification is based on radiation that is emitted from a container, in which the goal is to reduce the number of false alarms in the system due to NORM containers. This chapter also uses a multi-layered screening approach. Furthermore, the proposed model applies the knapsack problem to a linear programming model given resource limitations to analyze the tradeoffs between prescreening intelligence and the efficacy of the radiation detectors. Results suggest that prescreening intelligence is crucial and the more accurate the prescreening

intelligence, the layers of screening needed can be reduced. This would lead to a more efficient screening system as well as being more cost effective.

Chapter 3 extends some ideas from Chapter 2, including greater detail about specific detection devices and the risk-based prescreening intelligence, as well as determining optimal primary and secondary screening decisions. This chapter uses a linear programming model, in addition to decision analysis, to provide a simplistic and versatile approach that could be applied to other avenues of transportation. As the prescreening intelligence increases, or becomes more accurate, the optimal primary and secondary screening decisions become more focused on high-risk classified containers. Sensitivity analysis is performed on the costs on secondary screening and a comparison between current technology and next-generation technology is analyzed. Results suggest that as the cost of secondary screening increases and prescreening intelligence is high, a more in-depth level of primary screening will be used for high-risk containers, whereas when the secondary screening cost is low, containers, namely high-risk containers, will skip primary screening and will only be screened in the secondary screening level. In the comparison of current and next-generation technology, results suggest that next-generation technology may not be worthwhile to employ since the detection capabilities are not a significant improvement and the costs are similar for both technologies.

Chapter 4 applies a linear programming using the knapsack problem to screening procedures in commercial aviation security. Performance measures, such as covering flights, passengers, baggage, and targets are evaluated with regards to screening. Tradeoffs between the performance measures are analyzed and compared to the current proposed screening policies for flights incoming to the United States from a foreign location. Results suggest that a minimal expense, flights and targets can be covered, ensuring safety to those flights and targets. Greedy heuristics are performed and analyzed against the optimal solutions. These reveal that certain greedy heuristics can be applied and result in the optimal solution values, with a better solving time. By using the operations research methodologies, as described throughout this thesis, many improvements to screening procedures can be achieved. However, there is always a constant need for improving these systems, since terrorist groups are evolving and adapting to the current procedures and coming up with new ways to skirt around our security. Future ideas for research include using more decision analysis and behavioral analysis to try and determine what terrorist groups may do before they are success-

ful. Also, another expansion could include creating a network of defenses while using multiple modes of transportation. For example, if aviation and maritime security systems were analyzed using the same system, to ensure a constant defense. This would help with the notion of simultaneous attacks being prevented or at least deterred.

References

- Allison, G. (2004). *Nuclear Terrorism: The Ultimate Preventable Catastrophe*. New York, NY: Henry Hold and Company.
- Bakir, N. O. (2008). A decision tree model for evaluating countermeasures to secure cargo at United States Southwestern ports of entry. *Decision Analysis* 5(4), 230–248.
- Bakir, N. O. (2010). A stackelberg game model for resource allocation in cargo container security. *Annals of Operations Research*. forthcoming.
- Beged-Dov, A. G. (1972, January). Lower and upper bounds for the number of lattice points in a simplex. *SIAM Journal of Applied Mathematics* 22(1), 106–108.
- Boros, E., L. Fedzhora, P. B. Kantor, K. Saeger, and P. Stroud (2009, April). Large scale LP model for finding optimal container inspection strategies. *Naval Research Logistics* 56(5), 404–420.
- Brualdi, R. A. (1999). *Introductory Combinatorics* (3rd ed.). Upper Saddle River, NJ: Prentice-Hall, Inc.
- Candalino, Jr., T. J., J. E. Kobza, and S. H. Jacobson (2004). Designing optimal aviation baggage screening strategies using simulated annealing. *Computers and Operations Research* 31, 1753–1767.
- Cochran, T. B. and M. G. McKinzie (2008). Detecting nuclear smuggling: Radiation monitors at U.S. ports cannot reliably detect highly enriched uranium, which onshore terrorists could assemble into a nuclear bomb. *Scientific American* 298(4).
- Dimitrov, N. B., D. P. Michalopolous, D. P. Morton, M. V. Nehme, F. Pan, E. Popova, E. A. Schneider, and G. G. Thoreson (2010). Network deployment of radiation detectors with physics-based detection probability calculations. *Annals of Operations Research* (to appear).

- Dobson, G. (1982). Worst-case analysis of greedy heuristics for integer programming with non-negative data. *Mathematics of Operations Research* 7, 515–531.
- Dyer, M. (2003). *Proceedings of the Thirty-Fifth Annual ACM Symposium on Theory of Computing*, Chapter Approximate counting by dynamic programming, pp. 693 – 699. San Diego, CA.
- Fritelli, J. F. (2005). Port and maritime security: Background issues for congress. CRS Report for Congress, Congressional Research Service, The Library of Congress, RL31733.
- Gaukler, G., Y. Ding, and C. Li (2009, October 11–14). A systems concept for detecting nuclear material, INFORMS annual meeting. San Diego, CA.
- Goldberg, N., J. Word, E. Boros, and P. Kantor (2008). Optimal sequential inspection policies. Rutcor Research Report RRR 14, October 1, 2008, Rutgers University, Piscataway, New Jersey.
- Hillier, F. S. and J. Lieberman (1990). *Introduction to Operations Research* (5th ed.). New York: McGraw-Hill, Inc.
- Huizenga, D. (2005, June 21). Detecting nuclear weapons and radiological material: How effective is available technology? Statement before the Subcommittee on Prevention of Nuclear and Biological Attacks and Subcommittee on Emergency Preparedness, Science and Technology, The House Committee on Homeland Security.
- International Atomic Energy Agency (2007). IAEA illicit trafficking database. Fact Sheet. Vienna, Austria.
- Jacobson, S. H., L. A. McLay, J. E. Kobza, and J. M. Bowman (2005a). Modeling and analyzing multiple station baggage screening security system performance. *Naval Research Logistics* 52(1), 30–45.
- Jacobson, S. H., L. A. McLay, J. L. Virta, and J. E. Kobza (2005b). Integer programming models for the deployment of airport baggage screening security devices. *Optimization and Engineering* 6(3), 339–359.
- Jacobson, S. H., J. E. Virta, J. M. Bowman, J. E. Kobza, and J. J. Nestor (2001). Modeling aviation baggage screening security systems: A case study. *IIE Transactions* 35(3), 259–

269.

- Kantor, P. and E. Boros (2010). Deceptive detection methods for effective security with inadequate budgets: The testing power index. *Risk Analysis* 30(4), 663–673.
- Kellerer, H., U. Pferschy, and D. Pisinger (2004). *Knapsack Problems*. Berlin, Germany: Springer-Verlag.
- Kobza, J. E. and S. H. Jacobson (1996). Addressing the dependency problem in access security system architecture design. *Risk Analysis* 16(6), 801–812.
- Kobza, J. E. and S. H. Jacobson (1997). Probability models for access security system architectures. *Journal of the Operational Research Society* 48(3), 255–263.
- Koucky, M. (2003). Exact reliability formula and bounds for general k -out-of- n systems. *Reliability Engineering and System Safety* 82, 229 – 231.
- Lambe, T. A. (1974). Bounds on the number of feasible solutions to a knapsack problem. *SIAM Journal on Applied Mathematics* 26(2), 302–305.
- Lava, J. (2008). U.S. Customs and Border Protection. DIMACS/DyDAn/LPS Workshop on Port Security/Safety, Inspection, Risk Analysis, and Modeling, Piscataway, NJ, November 17-18, 2008.
- Levi, M. (2007). *On Nuclear Terrorism*. Cambridge, Mass.: Harvard University Press.
- Martello, S., D. Pisinger, and P. Toth (2000). New trends in exact algorithms for the 0-1 knapsack problem. *European Journal of Operational Research* 123, 325–332.
- McLay, L. A., S. H. Jacobson, and J. E. Kobza (2008). The tradeoff between technology and prescreening intelligence in checked baggage screening for aviation security. *Journal of Transportation Security* 1(2), 107 – 126.
- McLay, L. A., J. D. Lloyd, and E. Niman (2010). Interdicting nuclear material on cargo containers using knapsack problem models. *Annals of Operations Research* to appear.
- McNicholas, M. (2008). *Maritime Security*. New York: Elsevier, Inc.
- Morton, D. P., F. Pan, and K. J. Saeger (2007). Models for nuclear smuggling interdiction. *IIE Transactions* 39, 3–14.

- Mullen, M. (2009, April 6–9). Welcome address, Academic Research Initiative Grantees Conference. Washington, D.C.
- Padberg, M. W. (1971). A remark on “An inequality for the number of lattice points in a simplex”. *SIAM Journal of Applied Mathematics* 20(4), 638–641.
- Ramirez-Marquez, J. E. (2008). Port-of-entry safety via the reliability optimization of container inspection strategy through an evolutionary approach. *Reliability Engineering and System Safety* 93, 1698–1709.
- Rooney, B. (2005, June 21). Detecting nuclear weapons and radiological material: How effective is available technology? Statement before the Subcommittee on Prevention of Nuclear and Biological Attacks and Subcommittee on Emergency Preparedness, Science and Technology, The House Committee on Homeland Security.
- Royal Society, The (2008). Detecting nuclear and radiological materials. RS policy document 07/08, The Royal Society, London, UK.
- Sahin, H. and Q. M. Feng (2008). A mathematical framework for sequential passenger and baggage screening to enhance aviation security. *Computers & Industrial Engineering* 57, 148–155.
- Sammon, J. (2009). TSA testimony on general aviation status. Statement, United States Department of Homeland Security Transportation Security Administration before the Subcommittee on Transportation Security and Infrastructure Protection, Committee on Homeland Security, United States House of Representatives, July 15, 2009.
- Senju, S. and Y. Toyoda (1968). An approach to linear programming with 0-1 variables. *Management Science* 15, 196–207.
- Strohm, C. (2006, April 5). Investigators call cargo security program unreliable. *GovExec.com*. available at www.govexec.com/dailyfed/0406/040506cdam3.htm, accessed on November 22, 2006.
- Sweet, K. M. (2009). *Aviation and Airport Security: Terrorism and Safety Concerns* (2nd ed.). Boca Raton, FL: Taylor & Francis Group, LLC.

- United States Customs and Border Protection (2007). Secure Freight with CSI, Megaports. Fact sheet, Washington, D.C.
- United States Department of Transportation (2009, November). U.S. air carrier traffic statistics through September 2009. Research and Innovative Technology Administration, Bureau of Transportation Statistics. Washington, D.C.
- United States Government Accountability Office (2009, November). Recent testing raises issues about the potential effectiveness of advanced radiatoin detection portal monitors. Gao-10-252t, Government Accountability Office, Washington, D.C.
- U.S. Homeland Security Council, Interagency Policy Coordination Subcommittee for Preparedness and Response to Radiological and Nuclear Threats (2009). Planning guidance for response to a nuclear detonation. Report, Executive Office of the President.
- U.S. Transportation Security Administration (2009, January). *Planning Guidelines and Design Standards for Checked Baggage Inspection Systems* (2nd ed.). Washington, D.C.
- Vojtech, R. (2009, January 7). Personal Interview. Department of Homeland Security, Washington, D.C.
- Wein, L. M., Y. Liu, Z. Cao, and S. E. Flynn (2007). The optimal spatiotemporal deployment of radiation portal monitors can improve nuclear detection at overseas ports. *Science and Global Security* 15, 211–233.
- Wein, L. M., A. H. Wilkins, M. Baveja, and S. E. Flynn (2006). Preventing the importation of illicit nuclear materials in shipping containers. *Risk Analysis* 26(5), 1377–1393.

Vita

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