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Cooperative Wideband Spectrum Sensing Based on Joint Sparsity

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A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science at Virginia Commonwealth University

By

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ABSTRACT

COOPERATIVE WIDEBAND SPECTRUM SENSING BASED ON JOINT SPARSITY

By Ghazaleh Jowkar, Master of Science

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science at Virginia Commonwealth University

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In this thesis, the problem of wideband spectrum sensing in cognitive radio (CR) networks using sub-Nyquist sampling and sparse signal processing techniques is investigated. To mitigate multi-path fading, it is assumed that a group of spatially dispersed SUs collaborate for wideband

spectrum sensing, to determine whether or not a channel is occupied by a primary user (PU). Due to the underutilization of the spectrum by the PUs, the spectrum matrix has only a small number of non-zero rows. In existing state-of-the-art approaches, the spectrum sensing problem was solved using the low-rank matrix completion technique involving matrix nuclear-norm minimization. Motivated by the fact that the spectrum matrix is not only low-rank, but also sparse, a spectrum sensing approach is proposed based on minimizing a mixed-norm of the spectrum matrix instead of low-rank matrix completion to promote the joint sparsity among the column vectors of the spectrum matrix. Simulation results are obtained, which demonstrate that the proposed mixed-norm minimization approach outperforms the low-rank matrix completion based approach, in terms of the PU detection performance. Further we used mixed-norm minimization model in multi time frame detection. Simulation results shows that increasing the number of time frames will increase the detection performance, however, by increasing the number of time frames after a number of times the performance decrease dramatically.

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

With an ever-increasing number of wireless users and devices, the radio frequency spectrum becomes a more and more scarce resource. On the other hand, a large percentage of spectrum resources are underutilized by the licensed primary users (PUs). Therefore, the cognitive radio (CR) system has the potential to take full advantage of the underutilized spectrum resources by allowing unlicensed usage of vacant spectrum. For CR systems, spectrum sensing is a key step to detect spectrum holes/vacancies which can be used by secondary users (SUs) without causing any interference to PUs.

We focused our research on wideband spectrum sensing in CR networks using sub-Nyquist sampling and sparse signal processing techniques. To mitigate multi-path fading, we assume that a group of spatially dispersed SUs collaborate for wideband spectrum sensing, to determine the spectrum holes and identify potential transmission opportunities for SUs. In some state-of-the-art approaches [1,2], multiple spatially dispersed SUs have been used to mitigate wireless fading effects, and the low-rank matrix completion technique involving convex optimization has been applied to reconstruct a low-rank spectrum matrix, and determine whether or not a certain channel has been occupied by a PU. The spectrum is usually under-utilized, and the spectrum matrix has the spectrum vectors at different SUs as its columns. As a result, the spectrum matrix has only a small number of non-zero rows, meaning that it is low-rank. To reduce the burden on the analog-digital converter and the sensing cost, sub-Nyquist sampling and compressive sensing have been applied.

Due to the underutilization of the spectrum resource, we found that the spectrum matrix is not only low-rank, but also sparse. This motivates us to propose a spectrum sensing approach based on minimizing a l_2 / l_1 mixed-norm of the spectrum matrix to promote joint sparsity among the columns of the spectrum matrix, instead of low-rank matrix completion. We investigated the performance of our model by performing detection in multiple time frames using mixed-norm minimization model. Experiment results based simulation demonstrate that the proposed new approach outperforms the low-rank matrix completion based approach in higher SNRs, Also, the Detection performance will increase by increasing the number of time frames through the comparison of the receiver operating characteristic (ROC) curves.

1.2 THESIS STRUCTURE

In Chapter Two, we will give general background on cognitive radio networks and spectrum sensing, and an overview on cooperative spectrum sensing. Then an overview of low rank matrix completion model and joint sparse matrix reconstruction will be provided.
In Chapter Three we will go through the system model and discussion of the problem and our solutions to the problem using low rank matrix completion and mixed norm matrix reconstruction models. We compare the results of two proposed model and at the end we explain the system model on multi time frame detection using mixed norm minimization. In Chapter four we will give a brief conclusion and we will review our future work in using Mixed Norm Matrix Completion model based sequential detection, we will discuss our expected results and our current results.

CHAPTER TWO

BACKGROUND

2.1 COGNITIVE RADIO

National regulatory bodies such as FCC are in control of usage of radio spectrum resources and the regulation of radio emissions. FCC assigns spectrum to licensed users or *primary users* on a long-term basis for large geographical regions. However, due to inefficient usage of the limited spectrum, a large portion of the assigned spectrum remains under-utilized. Therefore, the development of dynamic spectrum access techniques is becoming necessary. Dynamic access techniques refer to the case where non-licensed users or the *secondary users*, are allowed to temporarily use the unused part of the licensed spectrum. Cognitive radio is the next generation communication network solution, also known as dynamic spectrum access (DSA) networks, to make the use of spectrum more efficient in an opportunistic way without interfering with the primary users.

Cognitive radio is an intelligent wireless communication system which uses its cognitive capability to become aware of its surrounding environment and by learning from environment can identify the available spectrum and adapt its internal states to achieve the optimal performance. Cognitive radio should adaptively modify its state and spectrum access to assure that primary user reclaims spectrum usage right. In this chapter recent research on cognitive radios will be reviewed. We overview the basics of cognitive radio technology, architecture, and its applications, and we talk about spectrum sensing, types of detection methods, and cooperative spectrum sensing. Finally, we discuss low rank matrix completion and joint sparse matrix reconstruction models as two reconstruction methods for spectrum sensing.

2.2 *COGNITIVE RADIO BASICS:*

Cognitive radio (CR) is the next generation of communications and networking that can adapt its operating parameters to utilize the limited network resources in a more efficient and flexible way. Two major functionalities of CRs are *cognitive capability* and *reconfigurability*. Before adapting their operating parameters CRs use their *cognitive capability* to gather information about the channel and make a decision accordingly. *Cognitive capability* is the ability of the cognitive radio transceiver to gather information from radio environment, and accordingly decide which spectrum band(s) to be used and the best transmission method to be adopted. *Reconfigurability* is the use of the information from the radio environment and change of CRs parameters to achieve optimal performance.

A typical duty cycle of CR includes:

- Spectrum sensing

Spectrum sensing is the ability of a CR to measure the activities of the radio transmissions over different spectrum bands and to capture the parameters related to such bands (e.g., power levels, user activities, etc.). Spectrum sensing is one of the most critical functions of a cognitive radio as it provides the awareness of the spectrum usage in the surrounding environment. Existing spectrum sensing techniques focuses on detecting the activities of the primary users. Such methods are based on matched filter detection, energy detection, feature detection, and interference temperature measurement, respectively.

- Spectrum Analysis

Spectrum analysis is to infer if a primary user is occupying the band at a certain time and geographic area. Such a definition covers only three dimensions of the spectrum space: frequency, time, and space. Other dimensions of a given spectrum can be exploited.

- Spectrum Access Decisions

The last step of the cognition cycle of a cognitive radio is to decide the set of transmission actions to be taken based on the outcome of the spectrum sensing and analysis procedures. More specifically, a cognitive radio utilizes the information gathered regarding the spectrum bands identified as available spectral opportunities to define the radio transceiver parameters for the upcoming transmission(s) over such frequency bands. The set of transceiver parameters to be decided depends on the underlying transceiver architecture.

2.3 *NETWORK STRUCTURE*

In a CR network architecture, since secondary users who are not authorized with spectrum usage rights can utilize the temporally unused licensed bands owned by the primary users, the components include both a secondary network and a primary network.

A secondary network refers to a network composed of a set of secondary users with/without a secondary base station. Secondary users can only access the licensed spectrum when it is not occupied by a primary user. The opportunistic spectrum access of secondary users is usually coordinated by a secondary base station, which is a fixed infrastructure component serving as a hub of the secondary network. Both secondary users and secondary base stations are equipped with CR functions.

A primary network is composed of a set of primary users and one or more primary base stations. Primary users are authorized to use certain licensed spectrum bands under the coordination of primary base stations. Their transmission should not be interfered by secondary networks. Primary users and primary base stations are in general not equipped with CR functions. Therefore, if a secondary network share a licensed spectrum band with a primary network, besides detecting the spectrum white space and utilizing the best spectrum band, the secondary network is required to immediately detect the presence of a primary user and direct the secondary transmission to another available band so as to avoid interfering with primary transmission.

Since CRs are able to sense, detect, and monitor the surrounding RF environment such as interference and access availability, and reconfigure their own operating characteristics to best match outside situations, cognitive communications can increase spectrum efficiency and support higher bandwidth service. Moreover, the capability of real-time autonomous decisions for efficient spectrum sharing also reduces the burdens of centralized spectrum management. As a result, CRs can be employed in many applications.

As a CR can recognize spectrum availability and reconfigure itself for much more efficient communication, this provides public safety personnel with dynamic spectrum selectivity and reliable broadband communication to minimize information delay. Moreover, CR can facilitate interoperability between various communication systems. Through adapting to the requirements and conditions of another network, the CR devices can support multiple service types, such as voice, data, video, etc.

***2.4** SPECTRUM SENSING ANALYSIS:*

As mentioned, spectrum sensing detects the primary user's activity based on the local measurements of secondary users. The following are the most common spectrum sensing techniques:

1) Energy Detector: Ease of implementation and no need of any prior knowledge of primary user's signal have made energy detection the most common type of spectrum sensing.

$$\begin{aligned} H_0 &: y(t) = n(t), \\ H_1 &: y(t) = hx(t) + n(t) \end{aligned} \tag{2.1}$$

in which $x(t)$ is the primary user's signal received at the local receiver of a secondary user, $n(t)$ is the additive white Gaussian noise, h is the channel gain from the primary user's transmitter to the secondary user's receiver. H_0 is a null hypothesis, meaning there is no primary user present in the band, H_1 means the primary user's presence.

The detection statistic of the energy detector is the average (or total) energy of N observed samples,

$$T = \frac{1}{N}\sum_{t=1}^{N} | \mathrm{y(t)} |^2 \tag{2.2}$$

By comparing the detection statistic T, with a predetermined threshold λ the decision on occupancy of the channel is made.

The performance of the detector is characterized by two probabilities:

- The probability of false alarm P_F (the probability that the hypothesis test decides H_1 while it is H_0)

$$P_F = P_r(T > \lambda \mid H_0) \tag{2.3}$$

- The probability of detection P_D (the probability that the test correctly decides H1).

$$P_D = P_r(T > \lambda \mid H_1) \tag{2.4}$$

A good detector should ensure a high detection probability and a low false alarm, or it should optimize the spectrum usage efficiency.

The region of convergence (ROC) curve is typically used to show the relationship between P_F and P_D. Cognitive radio with more efficient detection will have a ROC curve closer to the up-left corner and further away from the 45-degree line.

Choosing a right detection approach has an important role in minimizing spectrum sensing error, improving the spectrum utilization, and protecting the PU from interference from the SUs. By utilizing the spectrum sensing error function an optimal adaptive threshold level can be developed [9-10].

Besides its low computational and implementation complexity and short detection time, there are some challenges in designing a good energy detector.

- Noise power might change over time and precise measurement of it can be difficult in real time. The detection threshold depends on the noise power and in the cases where the noise power is very high (low signal-to-noise ratio (SNR)), reliable identification of a primary user is even impossible[8].

- An energy detector determines primary user's presence only by comparing the received signal energy with a threshold. As a result, it cannot differentiate the primary user from other unknown signal sources, a situation that can trigger false alarm frequently.

*2) Feature Detector: (*cyclostationary features*)* There are specific features associated with the information transmission of a primary user. For instance, the statistics of the transmitted signals in many communication paradigms are periodic because of the inherent periodicities such as the modulation rate, carrier frequency, etc. Such features are usually viewed as the cyclostationary features, based on which a detector can distinguish cyclostationary signals from stationary noise. In a more general sense, features can refer to any intrinsic characteristics associated with a primary user's transmission, as well as the cyclostationary features.

For example, center frequencies and bandwidths extracted from energy detection can also be used as reference features for classification and determining a primary user's presence. In this section, we will introduce the cyclostationary feature detection followed by a generalized feature detection.

Cyclostationary feature [6]: as in most communication systems, the transmitted signals are modulated signals coupled with sine wave carriers, pulse trains, hopping sequences, or cyclic prefixes, while the additive noise is generally wide-sense stationary (WSS) with no correlation. Cyclostationary feature detectors can be utilized to differentiate noise from primary users' signal and distinguish among different types of transmissions and primary systems.

Unlike energy detector which uses time-domain signal energy as test statistics, a cyclostationary feature detector performs a transformation from the time-domain into the frequency feature domain and then conducts a hypothesis test in the new domain. Cyclic autocorrelation function (CAF) of the received signal y(t) is defined by,

$$R_y^{\alpha} = E[y(t+\tau)y^*(t-\tau)e^{j2\pi\alpha t}] \tag{2.5}$$

Where $E[.]$ is the expectation operation, * denotes complex conjugation, and α is the cyclic frequency. Since periodicity is a common property of wireless modulated signals, while noise is WSS, the CAF of the received signal also demonstrates periodicity when the primary signal is present. If we can represent the CAF using its Fourier series expansion, we will have the cyclic spectrum density (CSD) function, expressed as,

$$S(f,\alpha) = \sum_{\tau=-\infty}^{\infty} R_y^{\alpha}(\tau)e^{-j2\pi f\tau} \tag{2.6}$$

The CSD function have peaks when the cyclic frequency α equals to the fundamental frequencies of the transmitted signal $x(t)$, i.e. $\alpha = (k/T_x)$, with T_x is the period of $x(t)$. Under H_0 the CSD function does not have any peaks since the noise is non-cyclostationary. A peak detector or a generalized likelihood ratio test can be further used to distinguish between the two hypotheses. Different primary communication systems using different air interfaces (modulation, multiplexing, coding, etc.) can also be differentiated by their different properties of cyclostationarity.

However, when frequency-division multiplexing (FDM) becomes the air interface, identification of different systems may become an issue, since the features due to the nature of OFDM signaling are likely to be close or even identical. To address this problem, particular features need to be introduced to OFDM-based communications. The OFDM signal is configured before transmission so that its CAF outputs peaks at certain pre-chosen cycle frequencies, and the difference in these frequencies is used to distinguish among several systems under the same OFDM air interface.

Compared to energy detectors that are prone to high false alarm probability due to noise uncertainty and unable to detect weak signals in noise, cyclostationary detectors become good alternatives because they can differentiate noise from primary users' signal and have better detection robustness in low SNR regime.

Generalized feature detection refers to detection and classification that extracts more feature information other than the cyclostationarity due to the modulated primary signals, such as the transmission technologies used by a primary user, the amount of energy and its distribution across different frequencies, channel bandwidth and its shape, power spectrum density, center frequency, idle guard interval of OFDM, FFT-type of feature, etc. By matching the features extracted from the received signal to the *a priori* information about primary users' transmission characteristics, primary users can be identified.

Location information of the primary signal is also an important feature that can be used to distinguish a primary user from other signal sources.

3) Matched Filtering and Coherent Detection: If secondary users have information about a primary user' signal *a priori*, then the optimal detection method is the matched filter, since a matched filter can correlate the already known primary signal with the received signal to detect the presence of the primary user and thus maximize the SNR in the presence of additive stochastic noise. The merit of matched filtering is the short time it requires to achieve a certain detection performance such as low probabilities of missed detection and false alarm, since a matched filter needs less received signal samples. However, the required number of signal samples also grows as the received SNR decreases, so there exists a SNR wall for a matched filter. In addition, its implementation complexity and power consumption is too high, because the matched filter needs receivers for all types of signals and corresponding receiver algorithms to be executed.

Matched filtering requires perfect knowledge of the primary user's signal, such as the operating frequency, bandwidth, modulation type and order, pulse shape, packet format, etc. If wrong information is used for matched filtering, the detection performance will be degraded a lot.

Even though perfect information of a primary user's signal may not be attainable, if a certain pattern is known from the received signals, coherent detection (a.k.a. waveform-based sensing) can be used to decide whether a primary user is transmitting or not. [16]

4) Other Techniques: There are several other spectrum sensing techniques proposed in recent literature, and some of them are variations inspired by the above-mentioned sensing:

- *Statistical Covariance-Based Sensing*: The difference of statistical covariance matrices of the received signal and noise is used to differentiate the desired signal component from background noise [11-12]. *Filter-Based Sensing*: filter banks are used for multicarrier communications in CR networks, and spectrum sensing can be performed by only measuring the signal power at the outputs of subcarrier channels with virtually no computational cost [13] .
- *Fast Sensing*: Quickest detection performs a statistical test to detect the change of distribution in spectrum usage observations as quickly as possible. The unknown parameters after a primary user appears can be estimated using the proposed successive refinement, which combines both generalized likelihood ratio and parallel cumulative sum tests.
- *Learning/Reasoning-Based Sensing*: optimal detection strategy is obtained by solving a Markov decision process (MDP).

2.5 COOPRATIVE SENSING

The performance of spectrum sensing is limited by noise uncertainty, shadowing, and multi-path fading effect. In Low SNR cases, a hidden primary user problem occurs where secondary users cannot detect the primary transmitter, when the primary user is occupying the channel, therefore the primary user will be interfered. To solve this issue the advantage of the independent fading channels (i.e., spatial diversity) and multiuser diversity has been considered and cooperative spectrum sensing is proposed to improve the reliability of spectrum sensing, increase the detection probability to better protect a primary user, and reduce false alarm to utilize the idle spectrum more efficiently.

Centralized cooperative spectrum sensing: a central controller, e.g., a secondary base station, collects local observations from multiple secondary users, decides the available spectrum channels using some decision fusion rule, and informs the secondary users which channels to access.

Distributed cooperative spectrum sensing: secondary users exchange their local detection results among themselves without requiring a backbone infrastructure with reduced cost. Relays can also be used in cooperative spectrum sensing, where the cognitive users operating in the same band help each other relay information using amplify-and-forward protocol.

Challenges on cooperative spectrum sensing come from the limitation of the secondary users. Since SRs can be low-cost devices only equipped with a limit amount of power, they cannot employ very complicated detection hardware with high computational complexity. In wideband cooperative sensing, multiple secondary users have to scan a wide range of spectrum channels and share their detection results. This results in a large amount of sensory data exchange, high energy consumption, and an inefficient data throughput.

1) User Selection: Due to secondary users' different locations and channel conditions involving all the secondary users in spectrum sensing is not efficient, and cooperating more efficient approach is to select only a group of users who have higher SNR of the received primary signal.

Since detecting a primary user costs battery power of secondary users, and shadow fading may be correlated for nearby secondary users, an optimal selection of secondary users for cooperative spectrum sensing is desirable. If a secondary user cannot distinguish between the transmissions of a primary user and another secondary user, it will lose the opportunity to use the spectrum. The presence/absence of possible interference from other secondary users is the main reason of the uncertainty in primary user detection, and coordinating with nearby secondary users can greatly reduce the noise uncertainty due to shadowing, fading, and multi-path effects. A good degree of coordination should be chosen based on the channel coherent times, bandwidths, and the complexity of the detectors.

2) Decision Fusion: Different decision fusion rules for cooperative spectrum sensing have been studied in the literature. An optimal way to combine the received primary signal samples in space and time is to maximize the SNR of local energy detectors. In general, cooperative sensing is coordinated over a separate control channel, so a good cooperation scheme should be able to use a small bandwidth and power for exchanging local detection results while maximizing the detection reliability. An efficient linear cooperation framework for spectrum sensing is proposed in [7], where the global decision is a linear combination of the local statistics collected from individual nodes using energy detection. Compared to the likelihood ratio test, the proposed method has lower computational complexity, closed-form expressions of detection and false alarm probabilities, and comparable detection performance.

3) Efficient Information Sharing: In order to coordinate the cooperation in spectrum sensing, a lot of information exchange is needed among secondary users, such as their locations, estimation of the primary user's location and power, which users should be clustered into a group, which users should perform cooperative sensing at a particular time epoch, and so on. Such a large amount of information exchange brings a lot of overhead to the secondary users, which necessitates an efficient information sharing among the secondary users.

In order to reduce the bandwidth required by a large number of secondary users for reporting their sensing results, only users with reliable information will send their local observations, i.e., one-bit decision 0 or 1, to the common receiver.

4) Distributed Cooperative Sensing: Cooperative spectrum sensing has been shown to be able to greatly improve the sensing performance in CR networks. However, if cognitive users belong to different service providers, they tend to contribute less in sensing in order to increase their own data throughput. Using replicator dynamics, the evolutionary game modeling provides an excellent means to address the strategic uncertainty that a user may face by exploring different actions, adaptively learning during the strategic interactions, and approaching the best response strategy under changing conditions and environments.

2.6 LOW RANK MATRIX COMPLETION:

In this section, we introduce low rank matrix completion model and the properties of a low rank measurement matrix. In the next chapter, we use the low rank properties of the measurement matrix formed by measurement vectors from multiple cooperative CRs. Capitalizing on such a nice property, we then develop a multiple reaction monitoring (MRM)based cooperative support detection algorithm. To perform cooperative support detection from multiple measurements we make an important observation that these measurement vectors permit sparse representations due to low spectrum utilization of the primary system, and that these sparse representations jointly possess a desired low-rank property.

2.6-1 MOTIVATION

We have an n_1 by n_2 array of real numbers and that we are interested in knowing the value of each of the $n_1\ n_2$ entries in this array. However, we only get to see a small number of the entries so that most of the elements about which we wish information are simply missing.

Now the question is if we are able to reconstruct the matrix from the existing entries? This problem is now known as the matrix completion problem. In mathematical terms, the problem may be posed as follows:

We have a data matrix $M \in R^{n_1 \times n_2}$ which we would like to know as precisely as possible. However, the only information available about M is a sampled set of entries $M_{ij}, (\mathrm{i, j}) \in \Omega$, where Ω is a subset of the complete set of entries $[n_1] \times [n_2]$. (Here and in the sequel, $[n]$ denotes the list $\{1, \ldots, n\}$.) In order to reconstruct the matrix M from its partial entries a few assumptions about the matrix M is needed.

2.6-2 Model description

Here, we are concerned with the theoretical underpinnings of matrix completion and more specifically in quantifying the minimum number of entries needed to recover a matrix of rank r exactly. This number generally depends on the matrix we wish to recover.

Let us assume that the unknown rank-r matrix M is $n \times n$. Then it is not hard to see that matrix completion is impossible unless the number of samples m is at least $2nr - r^2$, as a matrix of rank r depends on these many degrees of freedom. The singular value decomposition (SVD),

$$M = \sum_{k \in [\mathrm{r}]} \sigma_k u_k v_k^* \qquad (2.7)$$

Where $\sigma_1, ..., \sigma_r \geq 0$ are the singular values, and the singular vectors $u_1, ..., u_r \in R^{n_1} = R^n$ and $u_1, ..., u_r \in R^{n_2} = R^n$ are two sets of orthonormal vectors, is useful to reveal these degrees of freedom. Informally, the singular values $\sigma_1 \geq ... \geq \sigma_r$ depend on r degrees of freedom, the left singular vectors u_k on $(\mathrm{n}-1) + (\mathrm{n}-2) + ... + (\mathrm{n}-r) = nr - r(\mathrm{r}+1)/2$ degrees of freedom, and similarly for the right singular vectors v_k. If $m < 2nr - r^2$, no matter which entries are available, there can be an infinite number of matrices of rank at most r with exactly the same entries, and so exact matrix completion is impossible. In fact, if the observed locations are sampled at random, we will see later that the minimum number of samples is better thought of as being on the order of $nr \log n$ rather than nr.

let $P_\Omega : R^{n \times n} \to R^{n \times n}$ be the orthogonal projection onto the subspace of matrices which vanish outside of Ω ($(i, j) \in \Omega$ if and only if M_{ij} is observed) that is, $Y = P_\Omega(X)$ is defined as,

$$Y_{ij} = \begin{cases} X_{ij}, & (i,j) \epsilon \Omega \\ 0, & otherwise \end{cases} \tag{2.8}$$

so that the information about M is given by $P_\Omega(X)$ .The matrix M can be, in principle, recovered from $P_\Omega(X)$ if it is the unique matrix of rank less or equal to r consistent with the data. In other words, if M is the unique solution to,

$$\begin{array}{ll} \text{minimize} & \text{rank}(X) \\ \text{subject to} & P_\Omega(X) = P_\Omega(M) \end{array} \tag{2.9}$$

Knowing when this happens is a delicate question which shall be addressed later. For the moment, note that attempting recovery via rank minimization is not practical as rank minimization is in general an NP-hard problem for which there are no known algorithms capable of solving problems in practical time once, say, $n \geq 10$.

In [4], it was proved that:

1) matrix completion is not as ill-posed as previously thought.

2) exact matrix completion is possible by convex programming.

The author of [4] proposed recovering the unknown matrix by solving the nuclear norm minimization problem,

$$\begin{array}{ll} \text{minimize} & \| X \|_* \\ \text{subject to} & P_\Omega(X) = P_\Omega(M), \end{array} \tag{2.10}$$

where the nuclear norm $\| X \|_*$ of a matrix X is defined as the sum of its singular values,

$$\| X \|_* := \sum_i \sigma_i(X) \tag{2.11}$$

It is proved that if Ω is sampled uniformly at random among all subset of cardinality m and M obeys a low coherence condition which we will review later, then with a large probability, the unique solution to nuclear norm minimization problem is exactly M, provided that the number of samples obeys,

$$m \geq Cn^{6/5} r \log n \tag{2.12}$$

(To be completely exact, there is a restriction on the range of values that r can take on). The number of samples per degree of freedom is not logarithmic or polylogarithmic in the dimension, and one would like to know whether better results approaching the $nr \log n$ limit are possible. [4] provides a positive answer. In details, this work develops many useful matrix models for which nuclear norm minimization is guaranteed to succeed as soon as the number of entries is of the form $nrpoly \log(n)$.

2.7 JOINTLY SPARSE SIGNALS AND MIXED NORM MINIMIZATION MATRIX RECONSTRUCTION

Over the last few years, sparsity has emerged as a general principle for signal modeling. Many signals of interest often have sparse representations, meaning that the signal is well approximated by only a few nonzero coefficients in a specific basis. Compressive sensing (CS) has recently emerged as an active research area which aims to recover sparse signals from measurement data [14-15].

In the basic CS, the unknown sparse signal is recovered from a single measurement vector, this is referred to as a single measurement vector (SMV) model. In our study, we consider the problem of finding sparse representation of signals from multiple measurement vectors, which is known as the MMV model. In the MMV model, signals are represented as matrices and are assumed to have the same sparsity structure. Specifically, the entire rows of signal matrix may be 0.

Most sparsity based approaches start by expanding signals on a given waveform family (basis, frame, dictionary . . .), and process the coefficients of the expansion individually. Therefore, an assumption on the coefficients independence is implicitly done.

Sparse expansion methods explicitly introduce a notion of structured sparsity. Our approach is based on mixed norms, which may be introduced whenever signal expansions on doubly labeled families are considered. S is a sparse expansion of signal α .

$$S = \sum_{i,j} \alpha_{ij} \varphi_{ij} \tag{2.13}$$

Where $\{\varphi_{ij}\}$ are the waveforms of a given basis or frame.

Considering the mixed norm ℓ_{pq} ,

$$\| \alpha \|_{pq} = \left(\sum_i \left(\sum_j | \alpha_{ij} |^p \right)^{q/p} \right)^{1/q} \tag{2.14}$$

We shall be mainly concerned with the regression problem,

$$\min_{\alpha} \left[\left\| s - \sum_{i,j} \alpha_{ij} \varphi_{ij} \right\|_2^2 + \lambda \| \alpha \|_{pq}^q \right] \tag{2.15}$$

With $\lambda > 0$ a fixed parameter.

When $\{\varphi_{ij}\}$ is a basis, we give practical estimates for the regression coefficients α_{ij}, obtained by generalized soft thresholding. This former case is well adapted when the observation of the signal is noisy.

CAHPTER THREE

SYSTEM MODEL AND SOLUTION

3.1 DISCUSSION OF THE PROBLEM

When channel state information (CSI) from PU transmitters to CR receivers is available, the CRs can jointly estimate the common transmitted spectrum of the primary system from their individually received measurement vectors, which is the widely studied *cooperative estimation* problem. However, when the CSI is unavailable, CRs can only decide the spectrum occupancy of the PU systems, indicated by the nonzero support of the

Transmitted spectrum. This becomes a *cooperative support detection* problem, which is more challenging than cooperative estimation. In the cooperative multiple nodes, the signals received at SUs exhibit a sparsity property that yields a low-rank spectrum matrix of compressed measurements at the fusion center. We propose an approach to take advantage the sparsity property of the spectrum matrix at the fusion center.

With Adopting a system model from [1], let us assume that a wideband PU system spans over a total of B Hz, and the overall frequency band is divided into N non-overlapping bins of equal bandwidth B/N Hz, which are termed as channels and indexed by $n \in [0, 1, ..., N-1]$. There are J spatially distributed CRs that cooperate during the sensing stage and are indexed by $j \in [1, 2. . . J]$. Each CR senses only a small spectrum segment of bandwidth $M(B/N)$, so that the Nyquist sampling rate per CR is reduced by M/N, compared to that for monitoring the entire wideband spectrum. Further, it is assumed that the J CRs monitors different yet overlapping segments of the entire spectrum. Namely, the j th CR monitors M channels with channel indices from $(j-1)\Delta$ to $(j-1)\Delta + M - 1$, where $\Delta > 0$ is an integer denoting the shift between the channel assignments of two adjacent CRs. When $1 \le \Delta \le M$, and $(J-1)\Delta + M \ge N$, each channel is guaranteed to be covered by at least one CR. A scenario for $\Delta = 1$ and $M = 4$ is illustrated in Figure 1.

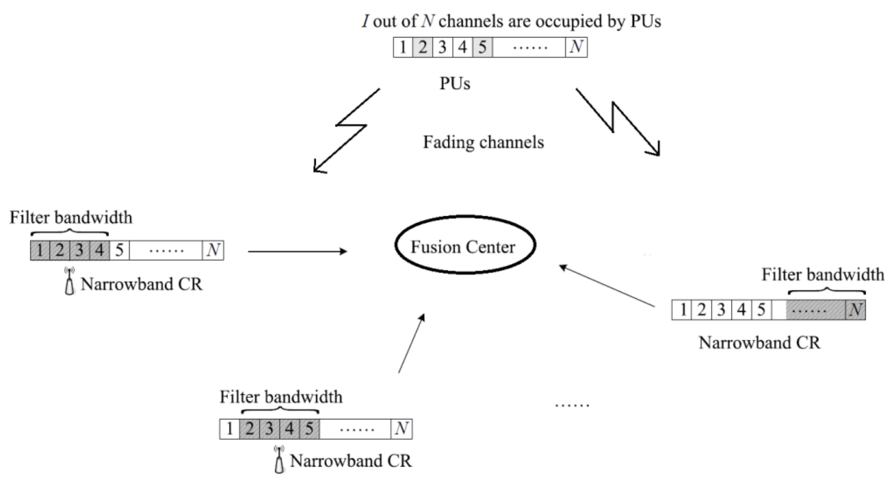

Figure 1. A cooperative spectrum sensing system with multiple CRs

Let s_f denote the unknown spectrum of the wideband signals transmitted by the PU. The sparsity of the transmitted spectrum is $||\boldsymbol{S_f}||_0 = \boldsymbol{I}$, which is the l_0-norm of the spectrum vector and measures the size of the nonzero support of $\boldsymbol{S_f}$. Let us assume that at the j th CR, we have a spectrum vector which is a faded version of $\boldsymbol{S_f}$,

$$\mathbf{r}_j = \mathbf{H}_j \mathbf{s}_f \tag{3.1}$$

where $\mathbf{H}_j \in C^{N\times N}$ is a diagonal channel matrix, whose diagonal elements are the independent fading coefficients of the corresponding channels.

Note that in the cooperative spectrum sensing system, each CR only monitors M out of N channels, and the actual received spectrum after passing through a selective filter becomes,

$$\mathbf{r}_{s,j} = \mathbf{B}_j \mathbf{r}_j \tag{3.2}$$

where $\mathbf{B}_j \in \{0,1\}^{M\times N}$ is the channel selection matrix of the j th CR. $\mathbf{B}_j$ is obtained from a $N\times N$ identity matrix by keeping only those M rows corresponding to the channel subset of the j th CR.

When Nyquist-rate sampling is adopted at each CR, the j th CR collects discrete-time sample vector $\mathbf{x}_j$ in the form of,

$$\mathbf{x}_j = \mathbf{F}^{-1}\mathbf{r}_{s,j} \tag{3.3}$$

where $\mathbf{F}$ is the square discrete Fourier transform (DFT) matrix. When compressive sensing is used, $\mathbf{x}_j$ can be pre-multiplied by a random sensing matrix $\mathbf{\Phi}_j \in C^{K\times M}$ to collect compressive linear projections from the filtered waveform $\mathbf{x}_j$ [3], where K/M is the compression ratio. In the presence of channel noise, the compressed sample vector at the j th CR can be modeled as,

$$\mathbf{x}_j = \mathbf{\Phi}_j \mathbf{F}^{-1}\mathbf{r}_{s,j} + \mathbf{w}_j \tag{3.4}$$

$\mathbf{x}_j$ is a $K\times 1$ vector, which corresponds to a sampling rate of $(K/M)(MB/N) = KB/N$, and can be generated by an analog sampler [3].

By defining $\mathbf{A}_j = \mathbf{\Phi}_j \mathbf{F}^{-1}\mathbf{B}_j$, (3.4) can be re-written as,

$$\mathbf{x}_j = \mathbf{A}_j \mathbf{r}_j + \mathbf{w}_j \tag{3.5}$$

Our goal is to infer the binary occupancy state of each channel, defined as a spectrum state vector $\mathbf{d}_f \in \{0,1\}^N$, $\mathbf{d}_f[i]=1$ when $\mathbf{s}_f[i]$ is nonzero; otherwise, $\mathbf{d}_f[i]=0$. Therefore, the goal is to find the support of the spectrum vector $\mathbf{s}_f$, when multiple measurements $\left\{\mathbf{x}_j\right\}_{j=1}^{J}$ are available.

3.2 METHODOLOGY

3.2-1 LOW-RANK MATRIX COMPLETION BASED SPECTRUM SENSING

A low-rank matrix completion based spectrum sensing approach was proposed in [1]. This work was motivated by the observation that only a small percentage of the channels are occupied by the PUs. As a result, if one defines the spectrum matrix as,

$$\mathbf{R}_f = \left[\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_J\right] \tag{3.6}$$

Then there will be only a small number of nonzero rows in $\mathbf{R}_f$, making it a low-rank matrix.

First, let us assume that all the measurements $\left\{\mathbf{x}_j\right\}_{j=1}^{J}$ are stacked as a single $(JK)\times 1$ vector $\mathbf{x}_t = [\mathbf{x}_1^T, \cdots, \ \mathbf{x}_J^T]^T$, and all the measurement noise vectors $\left\{\mathbf{w}_j\right\}_{j=1}^{J}$ are stacked as a single $(JK)\times 1$ vector $\mathbf{w}_t = [\mathbf{w}_1^T, \cdots, \ \mathbf{w}_J^T]^T$. Next, the spectrum matrix $\mathbf{R}_f$ is vectorized column-wise, namely,

$$\mathbf{r}_f = vec(\mathbf{R}_f) = [\mathbf{r}_1^T, \cdots, \ \mathbf{r}_J^T]^T \tag{3.7}$$

Further let us define $\tilde{\mathbf{A}} = diag\{\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_J\}$, which is a block diagonal matrix with the diagonal blocks consist of $\{\mathbf{A}_j\}_{j=1}^{J}$. With these notations and considering the low-rank property of $\mathbf{R}_f$, $\mathbf{R}_f$ can be estimated based on all the measurements by solving the following matrix rank minimization problem:

$$\min_{\mathbf{R}_f} \; Rank\left(\mathbf{R}_f\right) + \lambda \| \mathbf{x}_t - \tilde{\mathbf{A}} vec\left(\mathbf{R}_f\right) \|_2^2 \qquad (3.8)$$

The second term in (3.8) penalizes the model fitting error, and λ is the Lagrangian parameter which provides the relative emphases on the low-rank property of $\mathbf{R}_f$ and the tolerance on measurement model errors.

Note that (3.8) is an intractable optimization problem since the combinatorial nature of the rank of a matrix. Therefore, rank function can be replaced by its convex surrogate [4], the nuclear norm function, denoted as $\|.\|_*$. The nuclear norm of a matrix is the sum of all the singular values of the matrix. As a result, the optimization problem in (8) becomes

$$\min_{\mathbf{R}_f} \left\|\mathbf{R}_f\right\|_* + \lambda \| \mathbf{x}_t - \tilde{\mathbf{A}} vec\left(\mathbf{R}_f\right) \|_2^2 \qquad (3.9)$$

In [1], it was shown that a spectrum sensing approach based on the nuclear norm minimization provides very good detection performance.

3.2-2 SPECTRUM SENSING BASED ON MIXED-NORM MINIMIZATION

Taking a closer look at (3.6), one can find that $\mathbf{R}_f$ has a small number of non-zero rows, implying that it is not only low-rank but also sparse, which means it has a small number of non-zero elements. More particularly, the columns in $\mathbf{R}_f$, namely $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_J$, share the same support, and are jointly sparse. This motivates us to explore spectrum sensing algorithm based on matrix mixed-norm minimization.

We are particularly interested in l_2 / l_1 norm of a matrix, defined as,

$$\| \mathbf{A} \|_{2,1} = \sum_i \sum_j \sqrt{| \mathrm{A}_{i,j} |^2} \tag{3.10}$$

Which is the sum of the l_2 norms of the rows of matrix $\mathbf{A}$. Minimizing the l_2 / l_1 norm of a matrix will promote the joint-sparsity among its columns. Here, replacing the nuclear norm in (3.9) with the l_2 / l_1 norm, we have the following convex optimization problem,

$$\min_{\mathbf{R}_f} \left\| \mathbf{R}_f \right\|_{2,1} + \lambda \| \mathbf{x}_t - \tilde{\mathbf{A}} vec\left(\mathbf{R}_f \right) \|_2^2 \tag{3.11}$$

3.3 SPECTRUM SENSING DECISION MAKING

Once the estimated spectrum matrix $\hat{\mathbf{R}}_f$ is found by solving (3.9) or (3.11), the fusion center can make a decision on whether or not a particular channel has been occupied by a PU. More specifically, the fusion center first calculates the energy in the i th channel, averaged over J CRs, which is then compared to a threshold to make a decision:

$$\hat{\mathbf{d}}_f[i] = \left(\frac{1}{J} \sum_{j=1}^{J} | \hat{\mathbf{r}}_j[i] |^2 \geq \eta^2 \right) \tag{3.12}$$

3.4 NUMERICAL RESULTS FOR SIGNALS IN SINGLE TIME FRAME

In the simulations, we choose the following parameters for the cooperative spectrum sensing network: N = 20, I = 2, J = 20, M = 4, K=5, $\Delta = 1$. So as illustrated in Figure. 1, on the average each channel is monitored by 4 CRs. Both the nuclear norm minimization problem as described in (3.9) and the l_2 / l_1 mixed norm minimization problem defined in (3.11) are solved by the CVX package [5], with the Lagrangian coefficient λ being set as 5 in both optimization problems, respectively. The signal-to-noise ratio (SNR) is defined to be the total signal power over the entire spectrum, normalized by the power of the white noise.

Given the true channel state vector $\mathbf{d}_f$, the probabilities of detection and false alarm are defined as,

$$P_d = E\left\{\frac{\mathbf{d}_f^T(\hat{\mathbf{d}}_f = \mathbf{d}_f)}{\mathbf{1}^T\mathbf{d}_f}\right\} \tag{3.13}$$

$$P_f = E\left\{\frac{(\mathbf{1}-\mathbf{d}_f)^T(\hat{\mathbf{d}}_f \neq \mathbf{d}_f)}{N-\mathbf{1}^T\mathbf{d}_f}\right\} \tag{3.14}$$

Respectively, where **1** denotes an all-one vector.

The ROC curves for the two approaches based on nuclear norm minimization and l_2 / l_1 mixed norm minimization are obtained from 1000 Monte-Carl trials and provided in Figure 2 for different SNR values. It is clear that as SNR increases, the performance for detecting the PUs is improved. Further, the approach based on matrix mixed norm minimization provides a better detection performance in higher SNR, since it takes advantage of the sparse property of the spectrum matrix, instead of merely its low-rank property. However, as SNR decrease we observe that increasing the noise level has more effect on sparsity of a matrix than its low rank property. As we can see from the result, by reducing SNR from 10 dB to 0 dB, the matrix rank minimization approach has better detection performance in lower P_f values, but as P_f increases the mixed norm approach shows a better result. At lower SNR such as -5 dB, both approaches have approximately the same detection performance.

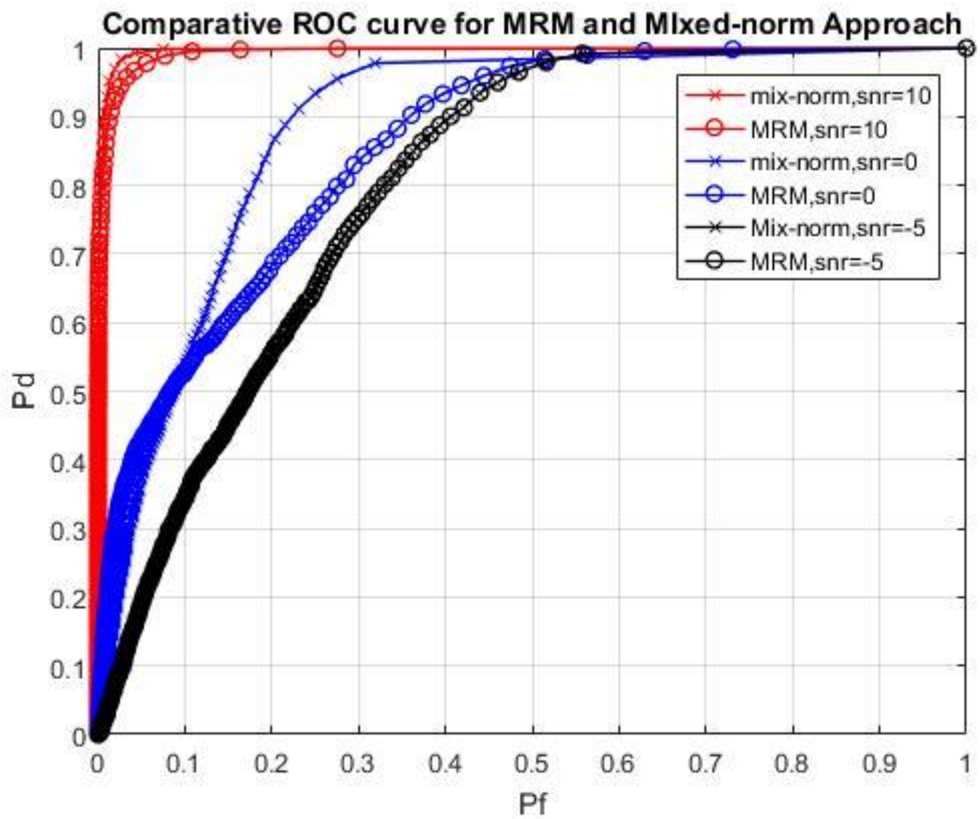

Figure 2. ROC curves for spectrum sensing approaches based on nuclear norm minimization and mixed l_2 / l_1 norm minimization.

3.5 Spectrum Sensing over Multiple Time Frames

So far, we have assumed that the occupied channels are sparse, now we want to add the time dimension to our assumptions and detect the PUs using signals over multiple time-frames.

Let us assume that channel occupancy remains the same at each time frame, however the energy of the occupied sub-channels will change over the time. We start with the following assumptions to model our new system.

Fading in the channel,

- is not changing over time
- over frequency it is changing independently
- the changes over the space are not our concerns

With these assumptions, over time the same number of channels are occupied but their energy will change. We will add time index to our model meaning that we study the channel over multiple time frames.

3.6 SYSTEM MODEL

Let us add a time index k to the unknown spectrum of the wideband signal transmitted by the PU at time k , namely $\boldsymbol{S_{fk}}$. Define the fading coefficient matrix at time k and sensor j as $\boldsymbol{H_{jk}}$, then (3.1) becomes,

$$r_{jk} = \boldsymbol{H_{jk}} \, \boldsymbol{S_{fk}} \tag{4.1}$$

Since fading is not changing over time we can rewrite the equation as follows,

$$r_{jk} = H_j S_{fk} \tag{4.2}$$

At sensor j, using time-invariant channel selection matrix B_j and random sensing matrix

φ_j, then we have the compressed sample vector at the j th CR as,

$$\begin{aligned} X_{jk} &= \varphi_j F^{-1} B_j r_{jk} + W_{jk} \\ &= A_j r_{jk} + W_{jk} \end{aligned} \tag{4.3}$$

Where W_{jk} is an additive noise at time k and CR j.

3.7 METHODOLOGY: MIXED NORM MINIMIZATION BASED SOLUTION

Now if we define spectrum matrix at each time frame as follows,

$$R_{fk} = [\mathrm{r}_{1k}, \mathrm{r}_{2k}, \ldots, \mathrm{r}_{jk}] \tag{4.4}$$

Let us assume that we have z number of time frames, let us stack all the spectrum matrices $\{R_{fk}\}_{k=1}^{Z}$ in one large matrix RF.

$$RF = [\mathrm{R}_{f1}, \mathrm{R}_{f2}, \ldots, \mathrm{R}_{fZ}] \tag{4.5}$$

Let us assume that all the measurements $\{\mathrm{x}_{jk}\}_{j=1}^{J}$ are stacked as a single $(JK)\times 1$ vector $X_{tk} = [\mathrm{X}_{1k}{}^{T}, \mathrm{X}_{2k}{}^{T}, \ldots, \mathrm{X}_{Jk}{}^{T}]^{T}$ and all the measurement noise vectors $\{\mathrm{W}_{jk}\}_{j=1}^{J}$ are stacked

As a single $(JK)\times 1$ vector $W_{tk}=[W_{1k}{}^{T},W_{2k}{}^{T},\ldots,\mathrm{W}_{Jk}{}^{T}]^{T}$. Now we wat to vectorize all the stacked vectors of $\{\mathrm{x}_{tk}\}_{k=1}^{Z}$ to a single vector of $(JKZ)\times 1$ and the same for stacked vectors of noise $\{\mathrm{W}_{tk}\}_{k=1}^{Z}$.

$$XT=[\mathrm{X}_{t1}{}^{T},\mathrm{X}_{t2}{}^{T},\ldots,\mathrm{X}_{tZ}{}^{T}]^{T} \tag{4.6}$$

$$WT=[W_{t1}{}^{T},W_{t2}{}^{T},\ldots,\mathrm{W}_{tZ}{}^{T}]^{T} \tag{4.7}$$

Next, we want to vectorize the spectrum matrix, we start by vectorizing $\{R_{fk}\}_{k=1}^{Z}$ to a $(NJZ)\times 1$ vector $r_{fk}=Vec(\mathrm{R}_{fk})=[r_{1k}{}^{T},r_{2k}{}^{T},\ldots,r_{Jk}{}^{T}]^{T}$ then we stack all $\{\mathrm{r}_{fk}\}_{k=1}^{Z}$ to a single vector,

$$Vec(\mathrm{RF})=[r_{f1}{}^{T},r_{f2}{}^{T},\ldots,r_{fZ}{}^{T}]^{T} \tag{4.8}$$

With these notations and considering its joint-sparse property, RF can be estimated based on all the measurements by solving the following matrix mixed norm minimization problem,

$$\min_{RF} \quad \| RF \|_{2,1} + \lambda \| \mathrm{XT} - \tilde{\mathrm{A}}\mathrm{Vec(RF)} \|_2^2 \tag{4.9}$$

Where $\tilde{\mathrm{A}}$ is the diagonal matrix of $\{\{A_{jk}\}_{j=1}^{J}\}_{k=1}^{Z}$. However, we know that A_{jk} does not change over time. Therefore, $\tilde{\mathrm{A}}$ will be as follows,

$$\tilde{\mathbf{A}}=diag\{\underset{k=1}{\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_J},\underset{k=2}{\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_J},\ldots\ldots,\underset{k=Z}{\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_J}\} \tag{4.10}$$

3.8 Numerical Results for Signals over Multiple Time Frames

In the simulations, with the same parameters for the cooperative spectrum sensing network as before, the ROC curves for the approach based on l_2 / l_1 mixed norm minimization are obtained from 1000 Monte-Carl trials and provided in Figure 3 for different number of time frames. It is shown that as number of time frames increases from one to two, the performance for detecting the PUs is improved.

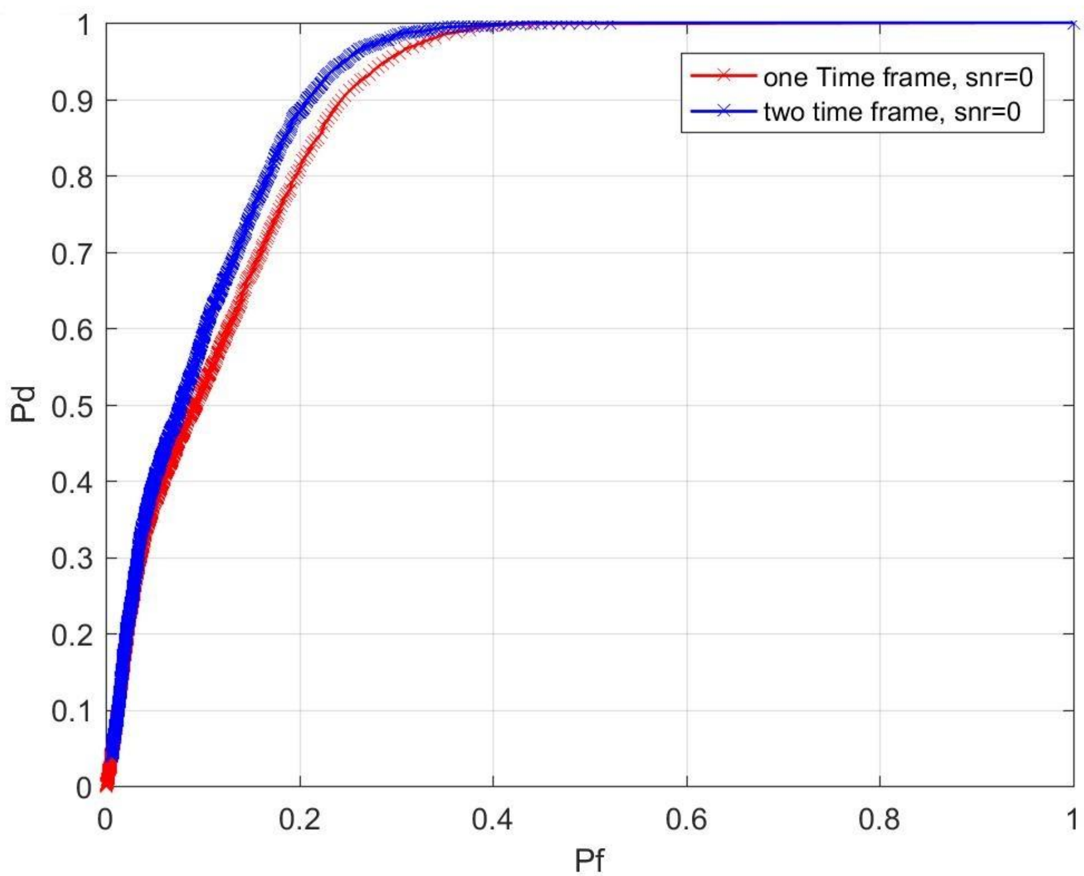

Figure 3. ROC curves for spectrum sensing based on mixed l_2 / l_1 norm minimization approach.

CHAPTER 5

Conclusion

In this thesis, the problem of wideband spectrum sensing in CR networks using sub-Nyquist sampling and sparse signal processing techniques was investigated. To mitigate multi-path fading, we assumed that a group of spatially dispersed SUs collaborate for wideband spectrum sensing, to determine whether or not a channel is occupied by PUs. Due to the underutilization of the spectrum by the PUs, the spectrum matrix has only a small number of non-zero rows. In some existing state-of-the-art approaches, the spectrum sensing problem was solved using the low-rank matrix completion technique involving matrix nuclear norm minimization. Motivated by the observation that the spectrum matrix is not only low-rank, but also sparse, we proposed a spectrum sensing approach based on minimizing the l_2 / l_1 mixed-norm of the spectrum matrix to promote joint sparsity among the spectrum matrix's columns, instead of low-rank matrix completion. Experiment results based simulation showed that the proposed new approach outperforms the low-rank matrix completion based approach, through the comparison of the ROC curves. In practice channels are steady over time. Therefore, by adding time index, we proposed a spectrum sensing approach based on mixed l_2 / l_1 norm minimization over multiple time frames. In our model, we assumed that channel occupancy remains the same at each time frame but the energy of the occupied sub-channels will change over the time. We showed that increasing the number of time frames from one to two will improve the detection performance. However, our observation showed that increasing the number of time frames from two to three will have less efficient detection. This issue guides us to our next step of the study and in our future works we plan to research and explain this phenomenon.

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