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DEVELOPING CONCEPTUAL UNDERSTANDING AND PROCEDURAL FLUENCY IN ALGEBRA FOR HIGH SCHOOL STUDENTS WITH INTELLECTUAL DISABILITY

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DEVELOPING CONCEPTUAL UNDERSTANDING AND PROCEDURAL FLUENCY IN ALGEBRA FOR HIGH SCHOOL STUDENTS WITH INTELLECTUAL DISABILITY

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Virginia Commonwealth University.

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Dedication

All my life, my family has encouraged me to continue my education. This work is dedicated to Catherine, Zachary, Sarah, Paul, Karen, George, Tim, Melissa, Bonnie, Stanley, Edna, Paulette, Bill, Stanley, Marianne, Louis, Theresa, Linda (Sarah), Robert, Shelia, Patricia, John, Michael, Al, Helen, Andrea, Theresa, Suzie, Jim, Anne, Rick, John, Terry, John, Maxine, Barbara, Howard, Debbie, Dan, Yitzhak, Mordechai, Shalom, and Asher. Thank-you for your help.

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Abstract

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Teaching students with Intellectual Disability (ID) is a relatively new endeavor. Beginning in 2001 with the passage of the No Child Left Behind Act, the general education curriculum integrated algebra across the K-12 curriculum (Kendall, 2011; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and expansion of the curriculum included five intertwined skills (productive disposition, procedural fluency, strategic competence, adaptive reasoning, and conceptual understanding) (Kilpatrick, Swafford, & Findell, 2001). Researchers are just beginning to explore the potential of students with ID with algebra (Browder, Spooner, Ahlgrim-Delzell, Harris & Wakeman, 2008; Creech-Galloway, Collins, Knight, & Bausch, 2013; Courtade, Spooner, Browder, & Jimenez, 2012;

Göransson, Hellblom-Thibblin, & Axdorph, 2016). Most of the research examines the development of procedural fluency (Göransson et al., 2016) and few researchers have explored high school level skills.

Using a single-case multiple-baseline across participants design, the study proposes to teach two algebra skills to six high school students with ID, creating an equation ($y=mx+b$) from a graph of a line and creating a graph from an equation. The six high school students with ID will be recruited from a school district in central Virginia. The intervention package modeled after Jimenez, Browder, and Courtade (2008), included modeling, templates, time delay prompting, and a task analysis. Results showed that all six individuals improved performance during intervention for the target skills over baseline; results also indicated that in three out of the six cases some generalization to the inverse skill occurred without supplemental intervention. The ability of individuals with ID to generalize the learning without intervention provides some evidence that individuals with ID are developing conceptual understanding while learning procedural fluency.

Chapter I

Introduction

Historically, challenging academic curricula provided limited access to students from certain populations including those with disabilities, those from minority communities, and those with economic disadvantages (Johnson, Galow, & Allenger, 2013; Kress, 2005; Moses, Kamii, Swap, & Howard, 1989). When Congress passed the No Child Left Behind Act (2001; P.L. 107-110), states were asked to test, monitor, and publicly report performance data on at-risk groups (students with disabilities, with economic disadvantages, or from minority groups). This legislation increased access to the general education academic curriculum, and subsequently, raised the academic expectations for students with disabilities in the areas of reading and mathematics. As a result, students with disabilities have struggled to master the new academic standards (Johnson et al., 2013).

Grade-level academic instruction for students with Intellectual Disability (ID) is a relatively new undertaking. Until the 1950's, most individuals with ID were excluded from public schools, and education took place on collective farms or institutions if at all (Trent, 1994). Academic research into instruction was limited to the area of reading and arithmetic (adding, subtracting, multiplying and dividing numbers) (Kirk, 1955), and researchers remained skeptical of students' basic academic abilities through the end of the twentieth century (Ayres, Lowrey, Douglas, Sievers, 2012; Connolly, 1973; Trent, 1994). Legislation (Education of All Handicapped Children Act, 1975, P. L. 94-142 and its respective reauthorizations, Individuals with Disabilities Education Act of 1990, P.L. 101-476; 1997, P.L.101-476; 2004, P.L. 108-446) stressed the need for students with ID to access public education, and more recent legislation (IDEA, 2004; NCLB, 2001) required students to access the general education curriculum using general education reading and math standards. Congress reasserted the ethic with the passage of the Every Student Succeeds Act of 2015 (ESSA, 2016, P.L. 114-95). ESSA strengthens protections for students. The law affirms students' right to work towards an academic high school diploma and to participate in the standardized assessment process. Through ESSA, Congress challenges states to increase the number of students with ID who graduate with general education diplomas.

Teachers and scholars remain skeptical that students with ID can earn the general education diploma because the general education diploma (for many states) requires students to learn algebra (Ayres, Lowrey, Douglas, Sievers, 2011; Johnson et al., 2013, Loveless, 2008). To earn a high school diploma, students must participate in algebra (Kilpatrick, Swafford, & Findell, 2001; National Governors Association Center for Best Practices & Council of Chief State School Officers, NGACBP & CCSSO, 2010). Again, the inclusion of students with ID in the secondary academic curricula, particularly in the area of algebra, is a relatively new phenomenon (Ayres et al., 2012; Browder, Spooner, Ahlgrim-Delzell, Harris & Wakeman, 2008; Jimenez, Browder, &

Courtade, 2008; Monari Martinez & Pelligrini, 2010; Rodriguez, 2016). Prior to the implementation of NCLB, (2001) students with ID participated in functional curricula designed to help prepare students for work or independent life (Ayres et al., 2011; Trent, 1994). Although the assumption remained unsubstantiated, those in the field of special education historically felt that including students with ID in academic curricula was impractical because the belief was that students were incapable of learning algebra (Ayres et al., 2012; Connolly, 1973; Courtade, Spooner, Browder, & Jimenez, 2012; Kirk, 1955; Kirk & Johnson, 1951). However, scholars outside of the field of special education began to view algebra as an essential prerequisite needed for individuals to participate fully in society (Kilpatrick et al., 2001; Kress, 2005; Moses et al., 1989).

Some members of the general education community viewed access to algebra as a civil right (Kress, 2005; Moses et al., 1989). Algebra knowledge was considered essential for high school students in order to access the science and technology curriculum found in the high school science and vocational classes (Kendall, 2011; Kilpatrick et al., 2001; Kress, 2005; Moses et al., 1989). Employers for entry level jobs required new employees to understand algebra skills (Rosenbaum & Binder, 1997), and without basic level algebra skills, students were excluded from employment opportunities related to vocations such as the construction, data entry, or warehousing. Even in daily life, algebra can be identified as an independent living skill needed for banking (Kilpatrick et al., 2001; Rodriguez, 2016).

To address civil rights concerns, the general education algebra curriculum changed, and the curricula expanded to include five intertwined skills (Figure 1). The cognitive processes associated with higher level algebra curricula include (a) working through the steps of a problem (procedural fluency), (b) recognizing the importance of mathematics (productive disposition), (c) applying multiple procedures to solve a problem (conceptual understanding), (d) choosing the best tools to solve problems (strategic competence), and (e) applying logic to solve or explain problems (adaptive reasoning) (Kilpatrick et al., 2001).

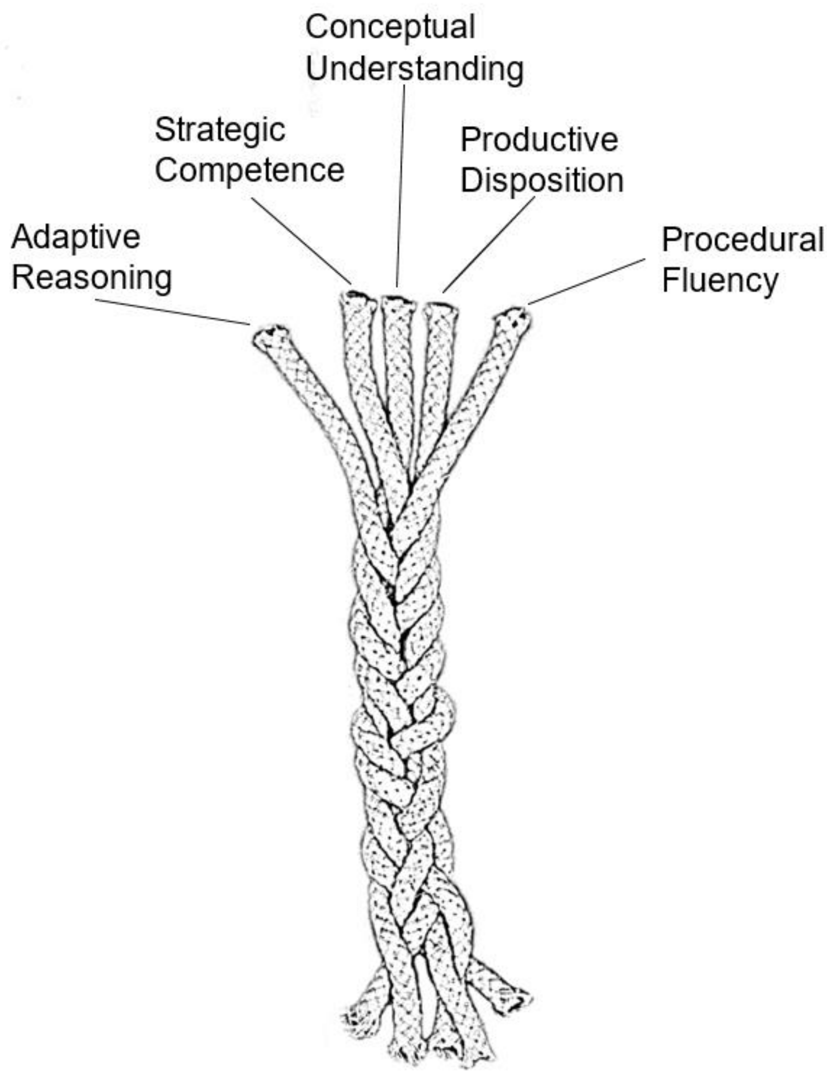

Figure 1. The Braided Conceptual Model of Mathematical Cognitive Processes
from Kilpatrick et al., (2001) p. 5. Reprinted with permission from the National Academy of Sciences License Number 3943050931661.

Notably, the field of general education struggled to measure conceptual understanding (Ernest, 2002; Rittle-Johnson, Schneider & Star, 2015). There is broad recognition that procedural fluency and conceptual understanding are intertwined; however, the methods for measuring conceptual unerstanding remain elusive. Ernest (2002) noted that most attempts to measure conceptual understanding relyed on participants explaining or defining a procedure, a rationale, or a concept during the mathematical process; the process of measuring understanding required the participant to communicate with language. Similarly, Rittle-Johnson et al. (2015) described several types of studies conducted in the past twenty years that measured conceptual understanding with language. Participants evaluated concepts, explained steps, or justified decisions. However, understanding can procede the development or use of language, and individuals can and do understand the underlying concepts without being able to express the concepts with language (Ernest, 2002). Regardless of the challenges related to measuring conceptual understanding, the general education curriculum and the general education assessments have changed, and attempts to challenge students to demonstrate conceptual understanding have been integrated into the *Common Core* assessments (Rittle-Johnson et al., 2015; Kendall, 2011). The changes to the general education math curriculum required a change in special education practices to enable students with disabilities to make progress according to the academic standards (Courtade et al., 2012; Creech-Galloway, Collins, Knight, & Bausch, 2013; ESSA, 2016; Wehmeyer, 2006).

However, few studies have explored algebra for students with ID (Browder et al., 2008, Jimenez et al., 2008; Göransson et al., 2016). To help students to access the general education curriculum, the field of special education needs to expand the research of mathematics instruction beyond procedural fluency (Göransson, et al., 2016). Traditionally, the behaviorists within the field of special education define three stages for learning new skills: acquisition, in which the individual is developing the skills; fluency, in which the individual performs with proficiency; and generalization, in which the individual applies or extends a skill (Deshler, Alley, Warner, & Shumaker, 1981; Snell & Brown, 2014; Stokes and Baer, 1977). Operationally, behaviorists would define many of strands found in the Kilpatrick et al. (2001) model as generalizations of the procedural fluency skill. For instance, Cease-Cook (2013) and Root (2016) demonstrated a method of inferring conceptual understanding using procedural fluency. They directly measured procedural fluency using a task analysis and they demonstrated conceptual understanding by changing from concrete forms of the task to abstract forms of the task; the participants generalized procedural fluency behaviors thereby demonstrating understanding; Stokes and Baer (1977) proposed that any generalization of a behavior implies some level of understanding, so using behavior to infer conceptual understanding could address the Rittle-Johnson et al. (2015) and Ernest et al. (2002) concern that current methods for measuring understanding in mathematics (including algebra) implies understanding by measuring language usage which could under identify an individual's level understanding.

Statement of the Problem

Expansion of the algebra curriculum in the general education community created a divide within the special education community. Algebra participation for students with ID has remained under-researched, and some in the field of special education continue to believe that teaching these academic skills is impractical (Ayres et al., 2011). Demonstrating that individuals with ID can understand algebra will help to address the concerns of impracticality (Göransson et al., 2016; Monari Martinez & Pelligrini, 2010). Special education teachers felt ill-prepared to teach academic algebra to students with ID because it is still relatively new (Creech-Galloway et al., 2013); and teachers have reported they lacked the tools for teaching algebra to students with disabilities (Johnson et al., 2013). Yet, accountability systems and general education policies have maintained that students with ID should participate in algebra (Courtade et al., 2012; Creech-Galloway et al., 2013, ESSA, 2016; Johnson et al., 2013; Kilpatrick et al., 2001). Compounding the frustration of special education teachers, the laws (ESSA, 2016; IDEA, 2004; NCLB, 2001) required the use of evidence-based practices, but few studies have explored algebra instruction for high school students with ID, so few evidence-based practices have been documented.

The laws (ESSA, 2016; IDEA, 2004; NCLB, 2001) promoting academic inclusion at the high school level have lacked the scholarship needed to support the practice, and the laws have created friction among the practitioners (Ayres et al., 2011; & Ayres et al., 2012; Courtade et al., 2012). For example, Monari Martinez and Pelligrini (2010) described teachers as resistant to the idea of including students with Down Syndrome in Algebra I classes. Some scholars have alluded to teacher unfamiliarity with specific teaching practices (Browder, Jimenez, & Trela, 2012; Browder, Trela et al., 2012; Creech-Galloway et al., 2013), and others described previous experiences with algebra as an obstacle. Lee, Mims, and Jimenez (2016, April) cited their own personal, negative experiences with high school algebra combined with doubts about student abilities as the source of this obstacle.

Research continues to be conducted in the area, and the adoption of *errorless learning* techniques as a teaching methodology shows promise. *Errorless learning* is a modified form of behaviorism designed to minimize the use of negative feedback (Mueller, Palkovic & Maynard, 2007; Touchette, 1971; Touchette & Howard, 1984). Although traditionally used to teach functional or life skills, Browder et al. (2008) found the practices associated with *errorless* *learning* (e.g. time delay prompting and simultaneous prompting) were commonly and effectively employed to teach mathematics to elementary and middle school students.

At the high school level, much of the research conducted to date has simplified the typical high school algebra tasks or focused on algebra skill development for students in middle school. More critically, the research still has not documented the full academic potential of students with ID (Ayres et al., 2012; Courtade et al., 2012). To date, few researchers have explored algebra for students with ID because the group was traditionally excluded from general education (Connolly, 1973; Trent, 1994). Similarly,an *a priori* assumption led researchers to believe that algebra instruction for students with ID was impractical (Ayres et al., 2011; Connolly, 1973; Courtade et al., 2012; Kirk, 1955; Kirk & Johnson, 1951; Lee et al., 2016, April, April; Trent, 1994).

Statement of Purpose

Research regularly suggests students with ID exceed assumptions (Browder, 2015; Creech-Galloway, 2013; Connolly, 1973; Courtade et al., 2012). However, research documenting the high school achievement of students with ID has been limited to individuals with Down Syndrome or limited to skills found in the elementary and middle school curricula. The purpose of this study is to demonstrate that individuals with ID can learn grade-level algebra skills using *errorless learning*, commonly practiced in special education instruction.

Rationale for the Study

Researchers have initiated an exploration of the development of algebra skills for students with ID, challenging historical assumptions associated with the disability. Originally, the line of research was abandoned in the late 1890's because psychologists believed that students with ID were cognitively unable to learn algebra and the skill of reading was more important (Binet & Simon, 1914; Connolly, 1973; Kirk & Johnson, 1951; Trent, 1994). However, the conditions supporting the historical assumptions changed.

Changes in the underlying assumptions. More recent research suggests the historical assumption that students with ID were unable to learn math, was flawed (Agran, 2014; Browder, 2015; Hord & Bouck, 2012; Lee, et al., 2016, April). Access to the curriculum, technologies, and attitudes of the scholars have changed.

Access to the curriculum. Today's discussion parallels the scholarly dialogue of the 1970's when researchers assumed students with ID were unable to learn arithmetic. As noted by Connolly (1973), the mathematics research contained an underlying the assumption that students with ID could not learn arithmetic; the historical assumption was developed using a population of participants living in institutions without the benefits of an academic education. Similarly, today, researchers use assumptions derived from research conducted with cohorts of students who did not participate in grade-level academic standards. The right to participate in grade level standards was solidified in the policy after the passage of NCLB (2001) and IDEA (2004), so prior cohorts were not exposed to the increased the academic expectations (Browder, Jimenez et al., 2012; Browder, Trela et al., 2012; Creech-Galloway et al., 2013; Johnson et al., 2013). Increased academic standards have likely influenced the prior knowledge of the students entering the secondary schools.

Changes in technology. New cohorts of students with ID have been found to demonstrate the ability to complete more algebra skills because technologies have changed. First, researchers have applied behavioral technology to teaching algebra skills to individuals with ID (Browder et al., 2008; Creech-Galloway et al., 2013; Jimenez et al., 2008). Second, technology has changed how students engage with algebra. Graphing calculators can help students with ID access a broader selection of the general education curriculum (Yakubova & Bouck, 2014). Technology has also changed the algebra curriculum. Algebra curriculum now focuses on a wide array of thinking skills designed to help students to use math more in daily life. The definition of algebra has been expanded to include broader thinking skills, and the algebra skills learned today are different than the algebra skills developed two decades ago (Kendall, 2011; Kilpatrick et al., 2001). Technology has also provided teachers with new tools to help students with disabilities to learn (Browder et al., 2008; Cooper, Heron, & Heward, 2007; Creech-Galloway et al., 2013).

***Changes in the attitude of scholars*.** Additionally, the attitudes of scholars are changing. Ayres et al., (2011) described the historical rationale for providing individuals with ID the traditional functional curricula. They noted that scholars believed the functional curricula provided individuals with ID with the skills necessary for life in work and the community. The traditional method for instructing students with disabilities deemphasized academics. Courtade et al. (2012) described the following reasons for students with ID to participate in academic curriculum: (a) IDEA (2004) and NCLB (2001) give students the right to take part in the curriculum; (b) academic skills are important and necessary for post-secondary life; (c) the full potential of students remains unknown; (d) students can learn academics and functional skills simultaneously, and (e) academic participation helps to change the individual student's view of self (p. 3). Similarly, Browder (2015) noted her views of student academic changed when she began to see students with ID demonstrating new skills.

Ineffective long-term outcomes. The change in attitudes among scholars stemmed from the ineffective long-term outcomes that were a product of traditional practices; for example, disability education policies failed to deliver improvements in employment outcomes (Bouck, 2012). The US Bureau of Labor Statistics (2015) reported that only 20% of adults with disabilities are part of the labor force, and the unemployment rate among the group remains around 12.5%. Because traditional educational practices focused on functional skills development, participation in algebra skill development constitutes a change in practice. All students including those with ID must pass algebra to access high school math and science (ESSA,2016; Kilpatrick et al., 2001; Kress, 2005; Moses et al., 1989). In some states, passing algebra is required to earn a general education diploma (ESSA,2016; Kilpatrick et al., 2001; Kress, 2005; Moses et al., 1989, VDOE, 2009), and earning a diploma can create more post-secondary transitional options. For example, entry-level employment requires algebra skills (Rosenbaum & Binder; 1997), and algebra is a prerequisite to participate in college math and science classes (Moses et al., 1989). Individuals with ID who know and can apply algebra concepts can obtain employment in higher paying jobs (Moneri-Martinez & Benedetti, 2011; Rodriguez, 2016). Individuals with ID who have these pre-requisite academic skills can also attend college programs (Thoma et al., 2011). Finally, students with ID need algebra to conduct the basic financial transaction in daily life (Rodriguez, 2016).

Overview of the Literature

Two previous literature reviews have been published. Browder et al. (2008) examined math instruction for students with severe disabilities, and Hord and Bouck (2012) examined secondary school math instruction for students with ID. Neither literature review focused on algebra; however, the number of publications exploring algebra instruction for students with ID has expanded over the past decade. Browder et al. (2008) was only able to locate one algebra study, and Hord and Bouck (2012) only located seven studies. A broad search of literature published after 2001, shows at least 12 studies directly related to algebra, with three additional studies exploring related math concepts. Table 2 presents the identified literature. Two-thirds of the studies were published within the past six years (Browder, Jimenez et al., 2012; Browder, Jimenez et al., 2012; Brown, Ley, Evvett, & Standen, 2011; Cease-Cook, 2013; Creech-Galloway et al., 2013; Göransson, et al., 2016; Hammond, Hirt, & Hall, 2012; Monari Martinez & Benedetti, 2011; Monari Martinez & Pellegrini, 2010; Rodriquez, 2016; Root, 2016).

A series of studies from Italy demonstrated that individuals with ID could learn Algebra I skills when they were provided with supported instruction in the general education environment. The studies documented the development and application of a broad range of algebra skills after Italian legislation mandated individuals with Down Syndrome to participate in general education classes (Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005). Additionally, in one case, Monari Martinez and Benedetti (2011) observed individuals with ID obtaining employment after learning the algebra skills.

Allsopp, van Ingen, Simsek, and Haley (2016) identified three *big ideas* for algebra knowledge: (a) number patterns (e.g. fractions and counting), (b) variables, and (c) the concept of equality, and another group of studies explored or observed students with ID as they developed the *big ideas* fundamental to algebra (Allsopp et al., 2016, Kilpatrick et al., 2001). According to the literature, conceptual understanding of fractions was developed using computer-aided instruction (Brown et al. 2011; Hall, DeBernardis, & Reiss, 2006; Hammond et al., 2012). Göransson et al. (2016) observed students with ID using inquiry-based social learning methods to develop the concept of equality, and Cease-Cook (2013) examined how a *concrete-representational-abstract* intervention could help students with ID to learn how to simplify algebraic expressions or to solve algebraic equations.

Another group of studies focused on developing discrete algebra problem-solving skills with individuals with ID. Using *errorless learning* strategies, three participants solved one-step equations (Jimenez et al., 2008). In a separate study, four participants acquired the procedural fluency needed to solve geometry problems, then generalized the skill to applying the Pythagorean Theorem to solve different problems (Creech-Galloway et al., 2013). Hord and Xin (2014) illustrated how three participants adapted geometric formulas (Hord & Xin, 2014). Two studies demonstrated a method for participants to plot points on a coordinate plane (Browder, Jimenez et al., 2012; Browder, Trela et al., 2012). Root (2016) expanded basic equation solving, by having four participants solve word problems. Similarly, Rodriguez (2016) observed ten participants generalizing algebra to financial literacy problems found in daily life.

Research Questions

The current literature shows that individuals with ID can learn more algebra, but the studies focusing on high school level algebra skills are limited. Collectively, a selection of Italian studies (Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005) demonstrated that a narrow group of students with Down Syndrome could learn a broad range of Algebra I skills, but the studies lacked clear descriptions of the interventions. Conversely, a separate group of studies (Browder, Jimenez, & Trela, 2012; Browder, Trela et al., 2012, Cease-Cook, 2013; Creech-Galloway et al., 2013; Jimenez et al., 2008; Root, 2016) demonstrated that a wide range of students with ID can learn a narrow selection of middle-school level algebra skills. The current literature does not address algebra skills instruction, at the high-school level, for a wide variety of students with ID, and it is unclear if the students are learning skills beyond procedural fluency (e.g. conceptual understanding). Therefore, the following research questions were explored:

1. When provided with evidence-based behavioral intervention and adapted materials, will participants with ID acquire:

a. procedural fluency in solving a linear equation to create a line?

b. procedural fluency in creating an equation from a picture of a line?

2. Does the acquisition of procedural fluency for creating a linear equation or creating a line affect conceptual understanding?

a) does acquisition of procedural fluency generalize to the inverse skill (Creating a line is the inverse skill for creating an equation and creating an equation is the inverse skill for creating a line)?

b) does acquisition of conceptual understanding require less time in instruction than with procedural fluency of the inverse skill?

Description of the Methodology

A single-case experimental multiple-baseline across participants design was used to monitor the performance of six participants with ID from a central Virginia public high school. Participants were randomly assigned to one of two experimental paths. In Experiment 1, three participants received an intervention for a target skill (create-an-equation), and generalization to the inverse skill (create-a-line) was monitored for evidence of conceptual understanding. In Experment 2, three different participants recevied intervention for a different target skill (create-a-line), and generalization to the inverse skill (create-an-equation) was monitored for evidence of conceptual understanding. The procedural fluency and the generalization of conceptual understanding was measured using a task analysis for each skill. The *errorless learning* intervention package included practice, systematic feedback, self-monitoring, a modeling prompt, and formula templates.

Definition of Terms

1. **Adaptive Behavior-** The cognitive process permitting students to independently generalize information to new environments or problems. Constructs of adaptive behavior include the individual's ability to apply skills not only to new self-care skills but also to academic situations. Normed psychometric evaluations measure adaptive behavior (American Association on Intellectual and Developmental Disabilities, AAIDD, 2010; American Psychological Association, APA, 2013).
2. **Adaptive Reasoning-** The cognitive process used in mathematics and requiring logic, reflection, explanation, and justification (Kilpatrick et al., 2001).
3. **Algebra-** Algebra is the manipulation of symbolic mathematical representations using logic and syntax. Algebra integrates five cognitive skills including (1) Adaptive Reasoning, (2) Strategic competence, (3) Conceptual Understanding, (4) Productive Disposition, (5) Procedural Fluency (Hills, 1948; Kilpatrick et al., 2001).
4. **Conceptual Understanding**- Conceptual understanding is a cognitive process in mathematics. It occurs when students build understandings about the relationships between (a) numbers and variables, (b) the operations applied to numbers and variables, and (c) *big ideas* found in mathematics. Allsopp et al. (2016) defined three categories of *big ideas*, number patterns, equality, and variables. Because conceptual understanding is intertwined and bi-directional with procedural fluency, procedural fluency can be used to infer conceptual understanding (Allsopp et al., 2016; Cease-Cook, 2013; Ernest, 2002; Kilpatrick et al., 2001; Rittle-Johnson et al., 2015; Root, 2016).
5. ***Errorless learning-*** A form of behavioral instruction that supports and encourages individuals during the learning process in a manner designed to minimize errors (Mueller et al., 2007; Touchette, 1971; Touchette & Howard, 1984)

6. **Intellectual Disability-** A condition that is comprising of less than one percent of the overall population. Significant supports are necessary to instruct the individual in intellectual functioning, learning, and problem-solving as well as adaptive behavior in every daily life skill. Intellectual functioning and adaptive behavior can be measured with normed tests with scores falling two standard deviations below the mean ($\bar{x}$ =100) (<70); however, the definition of intellectual disability assumes the assessments are free of bias (cultural or linguistic), the individual's limitations are used to determine the supports, and individuals can learn with support. (AAIDD, 2010; APA, 2013)
7. **Procedural Fluency-** Procedural fluency is the cognitive process of following procedures efficiently and appropriately. Procedural fluency includes the procedural application, generalization, of a formula to solve problems (Kilpatrick et al., 2001).
8. **Productive Disposition-** Productive disposition is a cognitive view of the world. It permits students to view algebra as a "sensible, useful, and worthwhile" tool, and it encourages students to adapt or generalize algebra functions to new situations (Kilpatrick, 2001, p. 59).
9. **Skill Acquisition-** The first stage of learning within an *errorless learning* structure. Participants demonstrate up to 60% of the steps to complete a task, (Deshler et al., 1981; Snell & Brown, 2014; Stokes & Baer, 1977).
10. **Skill Generalization-** a stage in the behavioral learning process that occurs after a student has acquired a skill. Utilization of acquired skills occurs across environments, skills, or people. (Snell & Brown, 2014; Stokes & Baer, 1977)
11. **Strategic Competency-** Strategic Competence is the cognitive process permitting individuals to choose tools to solve mathematical problems. Productive disposition can be inferred if the individual explains the steps involved in a process, creates new algorithms

based on existing knowledge, or if the student can choose a procedure from a list of procedures (Ernest, 2002; Kilpatrick et al., 2001)

12. **Task analysis-** A task analysis is sequence of discrete steps needed to describe a behavior leading to the completion of a task. A task analysis can be used to measure, monitor, and teach the acquisition of a skill by counting the number of steps completed or by calculating the percentage of steps completed (Cooper et al., 2007; Liberty, 1976; Snell & Brown, 2014)
13. ***Trial and error* Learning-** A learning approach where the individual's behavior changes after the individual attempts a behavior, receives positive or negative feedback for the behavior, and then, over time, adjusts the behavior to avoid the negative feedback and to receive the positive feedback (Skinner,1969/2013; Thorndike, 1913).

Chapter II

Review of the Literature

A Conceptual Framework for Algebra Instruction

During the latter half of the twentieth century, the general education community started shifting the focus of mathematics education from basic arithmetic towards algebra as the fields of science, technology, engineering and mathematics gained importance (Kaput, 1998; Kilpatrick et al., 2001; Kress, 2005; Moses et al., 1998). The general education curriculum requires students to master basic algebra (Kaput, 1998), and the trend continues to promote Algebra I completion in grades seven and eight (Loveless, 2008). Additionally, states are integrating algebra within the elementary general education curriculum (K-5). Elements of algebra exist within the elementary school sections in the *Common Core Curriculum* and the individual state curricula (Alaska Department of Education and Early Development, 2012; Indiana Department of Education, 2014; Kendall, 2011; Minnesota Department of Education, 2008; NGACBP & CCSSO, 2010; Nebraska Department of Education, Oklahoma State Department of Education,2016; South Carolina Department of Education, 2015; Virginia Department of Education, 2009). The general education curriculum incudes an expanded the definition of mathematics to promote a deeper understanding of algebra (Kendall, 2011: NGACBP & CCSSO, 2010).

In 2001, the National Academies of Science (Kilpatrick et al., 2001) developed a comprehensive model of mathematics. The mathematics model (Figure 1) deemphasized the view that mathematics is a process composed of procedures, saying that mathematics was a set of multiple, intertwined cognitive skills. Kilpatrick et al. (2001) defined mathematics learning as a braid of five skills. The cognitive processes associated with the model included the processes of (a)working through the steps of a problem (procedural fluency), (b) applying acquired procedures to unique situations (productive disposition), (c) creating equations or using words to explain procedures (conceptual understanding), (d) choosing tools to solve problems (strategic competence), and (e) applying logic to solve or explain problems (adaptive reasoning). The broader view of mathematics focused mathematical instruction on algebra, and the general education embedded algebra across the K-12 curriculum (Kendall, 2011; Kilpatrick et al., 2001; NGACBP & CCSSO, 2010).

The Kilpatrick et al. (2001) model helped to redefine algebra. Instead of systems of procedures, the general education curriculum defined algebra as the syntax needed to manipulate the abstract set of symbols and concepts found in math and geometry (Kilpatrick et al., 2001; Kress, 2005; NGACBP & CCSSO, 2010). Broadly, algebra was viewed as a tool to unlock the conceptual understanding associated with mathematics (Kendall, 2011; Kilpatrick et al., 2001). As such, the general education curriculum deemphasized procedural fluency when the other four strands of mathematics were added to the curriculum (Kendall, 2011; Kilpatrick et al., 2001). The new curriculum encouraged students to explore and build mathematical knowledge. Importantly, although deemphasized, procedural fluency remained an important component of algebra (Kilpatrick et al., 2001, Rittle-Johnson et al., 2015).

Defining *algebra for all* atudents. Including all students in algebra is an important issue identified with the civil rights movement. Moses et al. (1989) described algebra access as a civil right for disenfranchised students because algebra skills are necessary to access the high school and college classes needed for higher paying jobs. To increase the opportunities available to students the general education shifted towards a conceptual framework of mathematics from a sequential model (Ernest, 2002; Kilpatrick et al., 2001; Kendall, 2011), and the general education community established two *algebra for all* policies (Kilpatrick et al., 2001). First, states embedded algebra across the general education curriculum (K-12). Secondly, the states encouraged all students to complete Algebra in the eighth or ninth grades (Kendall, 2011; Kilpatrick et al., 2001; NGACBP & CCSSO, 2010). Policy makers hoped more students would complete algebra to access the high school and college classes needed for careers in science, technology, engineering, and mathematics (Kilpatrick et al., 2001; Kress, 2005; Moses et al., 1989).

Does *all* mean students with ID? General education scholars debated the merits of the *algebra for all* movement, and although concerns were raised by Loveless (2008), who argued that the *algebra for all* movement was harming elite students, the consensus in the general education community suggests that *algebra for all* students is important. The *algebra for all* movement defined disenfranchised students coming from minority backgrounds (e.g. Black or Hispanic), low economic status, and students with disabilities (Kilpatrick et al., 2001; Kress, 2005; Moses et al. 1989; NGACBP & CCSSO, 2010), and the policies never explicitly mentioned students with ID. Initially, there were indications that students with ID should be excluded from algebra. NCLB (2001) permitted one percent of students (with significant cognitive disabilities) to participate in alternative assessments using alternate curricular standards. Although the law required students with ID to make progress within the general education curriculum (Ayres et al., 2012; Courtade et al., 2012; Wehmeyer, 2006), Ayres et al. (2011) explained alternate standards excluded students with ID from academics in favor of practical educational programs aimed at improving independent living and employment. The practical curricula included basic arithmetic (adding, subtracting, multiplying and dividing) found in the elementary school curricula but not algebra (Browder et al., 2008; Bouck, 2012; Courtade et al., 2012, Jimenez et al., 2008).

The conditions supporting the exclusion of students with ID from the academic curricula changed. First, researchers began questioning the effectiveness of the traditional practices. Bouck (2012) observed that traditional approaches (basic academic, functional skills, and community based) were ineffective in leading students with ID to the transitional outcomes. Secondly, the traditional curricula set arbitrary limits on student academic potential (Courtade et al., 2012). As more students with ID were included in the algebra-infused general education curriculum in Kindergarten to Eighth grade, researchers noted students were exceeding previously assumed abilities (Browder, 2015; Courtade et al., 2012). Second, the law changed. Ayres et al. (2011) questioned the rationale for the academic shift, but when the NCLB legislation was reauthorized, ESSA (2016) reinforced the academic participation of students with ID. ESSA (2016) maintained the participation of one percent of students with significant cognitive disabilities in the alternate curriculum, but the new law increased access to the high school curriculum. Changes included the following: (a) alternate assessments must link to the grade level standards; (b) states must ensure students with ID have access to the academic classes needed for the general education diploma, and (c) a student participating in an alternate assessment in one subject (e.g. reading) cannot be forced to take part in the alternative assessment for another subject (e.g. mathematics).

The ESSA (2016) clarifications affirmed the right of students with ID to participate in the high school math curriculum which includes algebra. ESSA (2016) also stressed the importance of the Individuals with Disabilities Education Improvement Act (IDEA, 2004) to provide students with the opportunity to make progress within the general education curriculum. Because policy makers infused algebra standards across the K-12, general education curriculum (Kilpatrick et al., 2001; Kress, 2005; Moses et al., 1989; NGACBP & CCSSO, 2010), and because the students with disabilities must have access to the general education curriculum, students with ID are required to access to algebra.

Organization of the Review of Literature

This literature review explores empirical research publications related to algebra achievements for students with ID. The literature is organized thematically around the targeted algebra skills documented in the literature using the conceptual framework of Kilpatrick et al. (2001). A few studies documented broad algebra skill development. However, most studies examined one to four narrowly defined skills, each of which generally fell into one of two of the conceptual framework categories. Thus, the literature review will focus on the two categories, conceptual understanding and procedural fluency.

Methodology of the review of literature. The researcher conducted a multi-stage systematic search for literature documenting algebra interventions for students with ID. In the first stage, the researcher examined the summary research related to mathematics and algebra

Table 1

ProQuest™ Keyword Searches and Initial Results

Keyword Search	ProQuest Results
"Algebra"	61,932
"Algebra" AND "Intervention"	1,436
"Algebra" AND "Intervention" AND "Disability"	62
"Algebra" AND "Intellectual Disability"	10
"Algebra" AND "Intellectual Disability" AND "Intervention"	3
"Algebra" AND "Severe Disabilities"	5
"Algebra" AND "Severe Disabilities" AND "Intervention"	4
"Algebra" AND "Significant Cognitive Disabilities"	3
"Algebra" AND "Significant Cognitive Disabilities" AND "Intervention"	1
"Algebra" AND "Multiple Disabilities"	3
"Algebra" AND "Multiple Disabilities" AND "Intervention"	2
"Algebra" AND "Mental Retardation"	62
"Algebra" AND "Mental Retardation" AND	2
"Algebra" AND "Developmental Disability"	3
"Algebra" AND "Developmental Disability" AND "Intervention"	8
"Math"	175,569
"Math" AND "Severe Disability"	18
"Math" AND "Severe Disability"	8
"Math" AND "Intellectual Disability"	104
"Math" AND "Intellectual Disability" AND "Intervention"	54
"Math" AND "Significant Cognitive Disabilities"	17
"Math" AND "Significant Cognitive Disabilities" AND "Intervention"	6
"Math" AND "Multiple Disabilities"	20
"Math" AND "Multiple Disabilities" AND "Intervention"	10
"Math" AND "Mental Retardation"	426
"Math" AND "Mental Retardation" AND "Intervention"	246
"Math" AND "Developmental Disability" AND "Intervention"	122

interventions. The *What Works Clearinghouse* (U.S. Department of Education, Institute of Educational Sciences, 2015) maintains a database of intervention research categorized by topic and grade level. The research typically excluded students with disabilities, so in the second stage, the researcher began a more comprehensive keyword search within the *ProQuest database*. Boolean combinations of keywords were created using disability terms (e.g. "Intellectual Disability," "Mental Retardation," "Significant Cognitive Disability," "Down Syndrome," and "Fragile X"), math terms (e.g. "Math" and "Algebra"), and "Intervention." Table 1 presents the search terms and the search results. Initial keyword searches yielded fewer than 500 results per search, so the researcher read all the abstracts. If the study described a mathematics intervention (instructional strategy) for secondary students with ID, then the researcher read the methods sections. Similarly, qualitative studies that observed students with ID learning algebra were read because the qualitative observations likely contained descriptions of an intervention. A third search stage manually examined the ancestral lineage of promising studies, policy documents, related literature, and meta-analytical documents. Finally, the authors were used as keywords with *Google Scholar Alerts;* the system monitors the release of dissertations, scholarly literature, and policy documents.

Criteria for inclusion and exclusion. To be included in the analysis, the documents needed to comply with the quality standards found in January 2005, special issue of *Exceptional Children*. Qualitative and quantitative studies were included because they both contribute to the understanding of student learning Odom et al., (2005). Observational qualitative studies must have documented systematic, objectively collected, and meaningful descriptions of the participants, interventions, or work samples (Brantlinger, Jimenez, Klingner, Pugach, & Richardson, 2005). Experimental and quasi-experimental studies must have had a clear conceptualization of the ideas explored, utilized participant samples appropriate for the question, and clearly described the interventions, outcomes, and measures (Gersten et. al., 2005). Single-case studies must have described participants, settings, dependent variable, and interventions clearly; measured the dependent variable over time with clear, stable baselines, and the study was replicated across different points of time using different participants, settings, or materials (Horner et al., 2005; p. 174). As well as meeting the standards of research quality found in the *Exceptional Children* special issue (Brantlinger et al., 2005; Gersten et al., 2005; Horner et al., 2005; Odom, et. al., 2005), studies must have met additional criteria. The intervention in the study must have focused on math (all K-12 math) interventions or teaching strategies for secondary students (age 10-21) with intellectual disability.

Several types of documents were excluded from this review, including general education intervention studies, meta-analytical studies, professional development literature, and policy documents. Studies were rejected unless they targeted students with ID, and were published after the implementation of NCLB (2001) because the *algebra for all* policies were not in effect for students with disabilities (Browder et al., 2008; Johnson et al., 2013) prior to that time.

Results from the search. Initially, 19 documents met criteria; however, if a dissertation was published as a peer-reviewed article, the publication was included, and the dissertation was excluded. For example, the dissertation (Neodo, 2004) was later published as Neodo and Monari Martinez (2005), and it was the publication that was included in this review. Two dissertations and 13 peer-reviewed publications were included in this review. Table 2 presents the study, sample size, design, intervention style, and mathematical skill focus. Additional searches located related research from the general education research community and the fields of learning disabilities, and the studies were included if they met the inclusion criteria.

Table 2

List of Studies Included in the Review of the Literature

Study	Date	(n)	IQ	Framework Skills	Interventions
Browder, Jimenez, & Trela	2012	4	30-41	Procedural Fluency Multiple Skills	Behavioral (*Errorless learning*)
Browder, Trela, Courtade, Jimenez, Knight & Flowers	2012	16	30-54	Procedural Fluency Multiple Skills	Behavioral (*Errorless learning*)
Brown, Ley, Evett, & Standen	2011	16	<60	Conceptual Understanding Fractions	Behavioral (Technology Based *Trial and error*)
Cease-Cook (Dissertation)	2013	3	63-68	Adaptive Reasoning & Conceptual Understanding	Constructivist (Concrete-Abstract)
Creech-Galloway, Collins, Knight & Bausch	2013	4	41-57	Procedural Fluency to Apply Pythagorean Theorem	Behavioral (*Errorless learning* with Imitation)
Göransson, Hellblom-Thibblin & Axdorph	2016	31	Na	Conceptual Understanding *	Inquiry Based Social Learning/ Constructivist
Hall, DeBernardis, & Reiss	2006	5	40-69	Conceptual Understanding	Behavioral (Technology Based *Trial and error*)
Hammond, Hirt, & Hall	2012	22	53-90	Conceptual Understanding	Behavioral (Technology Based *Trial and error*)

Table 2 Continued

Study	Date	(n)	IQ	Framework Skills	Interventions
Hord & Xin	2014	3	63-73	Strategic Competence & Procedural Fluency	Constructivist (Concrete to Abstract)
Jimenez, Browder, & Courtade	2008	3	40-45	Procedural Fluency Solving Equations	Behavioral
Monari Martinez & Benedetti	2011	2	Down Syndrome	Algebra I	Broad Integrated Support
Monari Martinez & Pellegrini	2010	15	33-73	Algebra I	Broad Integrated Support
Neodo & Monari Martinez	2005	6	65-70	Algebra I	Broad Integrated Support
Rodriguez	2016	10	ID, Multiple Disabilities, and/or autism	Procedural Fluency: Money Skills	Social Learning
Root (Dissertation)	2016	4	50-66	Procedural Fluency: Word Problems	Constructivist

Algebra I for Students with ID

Kilpatrick et al., (2001) distinguished algebra from geometry (the study of shapes) and arithmetic (e.g. addition, subtraction, multiplication, division) because algebra requires people to use symbols and abstractions with a syntax to guide the symbolic manipulation. In general, Algebra I curricula embed clusters of skills that broadly integrate abstract concepts with procedures (Alaska Department of Education and Early Development, 2012; Indiana Department of Education, 2014; Kilpatrick et. al., 2001; Minnesota Department of Education, 2008; NGACBP & CCSSO, 2010; Nebraska Department of Education, 2015; Oklahoma State Department of Education, 2016; South Carolina Department of Education, 2015; Texas Education Association, 2016; Virginia Department of Education, 2009). Courtade et al., 2012 and Johnson et al. (2013) pointed out the potential of students with disabilities to learn the global, abstract concepts in algebra remain undocumented in America.

The Italian studies. Researchers in Italy studied the potential of students with ID to learn a broad spectrum of algebra skills while participating in Algebra I classes. In 1992, Italian legislation extended the inclusion for students with ID to the secondary school (Monari Martinez, 2002). Three quantitative studies and one qualitative study examined the potential of students with ID to succeed in a comprehensive Algebra I course.

In the first study, Neodo and Monari Martinez (2005) followed six participants with Down Syndrome as they attended Algebra I. Participant IQ's were greater than 65 and less than 70. The curriculum divided skills into the following 10 categories: (a) conducting operations with fractions with identical denominators; (b) carrying out operations with fractions with different denominators, (c) simplifying fractions, (d) using fractions to solve word problems, (e) using numbers and operations, (f) solving a variety of single-variable linear equations, (g) using a formula to calculate points for a line, (h) plotting points on a Cartesian plane, (i) utilizing the Pythagorean Theorem to calculate the distance between two points, and (j) examining a line from an equation. The study did not provide a detailed description of the intervention because teachers were instructed to provide extra support to participants based on the teachers' informal assessment of independence. As participants practiced a targeted skill, teachers ranked the participants' level of independence within each category (e.g. 1- Complete assistance, 2- Some assistance, and 3- independent). General education teachers provided extra support and instruction to students until the teachers ranked student achievement as independent. After students had demonstrated the ability to complete tasks independently, the researchers administered a test for each skill category. Scores for the six participants across the ten skills averaged 82 percent correct and independent and ranged between 53 and 88 percent. The authors concluded students with Down Syndrome could learn algebra; however, as an exploratory phenomenological study, the authors recognized the limitations of the study and recommended more research to confirm the results.

Monari Martinez and Pelligrini (2010) conducted a larger within-subjects repeated measures study with 15 randomly selected participants with Down Syndrome as they attended an Algebra I class over the course of one year. Participants engaged in a series of 33 skills involving fractions. The skills included creating basic fractions (e.g. creating fractions from pictures and simplification of fractions (e.g., $m \cdot n)/n = m$; or $n/m \cdot m/n = 1/m$), solving equations with fractions (e.g. $\frac{1}{4} \cdot a = 3$), and solving word problems from physics classes. Participants were assessed after the teacher provided instruction, and again at the end of the course. The study did not give a detailed description of the intervention exercises because teachers were given latitude and flexibility. The study tracked participant performance as they completed daily practice activities and during final assessments at the end of each unit. The results of the study showed participants performed better on the final assessments (*M=69.79)* than on daily classroom work (*M*=65.22). The results were analyzed with a within-subjects Analysis of Variance (ANOVA) test for significance. Statistically significant improvements occurred across the mean ($F_{(1,14)}$ = 10.82; $p<0.01$). The study helped to confirm the findings from Neodo and Monari Martinez (2005); however, the study did not explore the processes used by students to solve the problems.

In a qualitative testing study, Monari Martinez and Benedetti (2011) demonstrated some of the problem-solving strategies employed by participants. The study provided work samples showing the participants using coordinate planes, solving word problems, and graphing lines from equations (e.g. $y=mx+b$). In one work sample, the participant solved a word problem designed to calculate the length of a loan given a compound interest rate. The study noted the participant used the formula regularly in an employment setting (bank). To solve the problem, the participant utilized a logarithm. In another work sample, the participant created a graph of a line from an equation. The participant was provided with the formula $y=(½)\ x+1$; to solve the problem, the student substituted the values for *x* to receive the corresponding *y* values. The *x* and *y* values created points that the participant plotted on a coordinate plane before connecting to create a line. Monari Martinez and Benedetti (2011) demonstrated that students with ID used multiple steps to solve algebra problems that required the application of formulas, and extension of the algebra skills (e.g. simplifying expressions and solving equations) to geometry using the coordinate plane.

Summary of the Italian studies. Collectively, the Italian studies demonstrated that individuals with Down Syndrome could participate in Algebra I classes. Each article described providing participants with extra support, but the articles neglected to describe the additional aid in detail. Participant achievement of skills, after the intervention, exceeded 65 percent (sufficient to pass high school algebra). The studies demonstrated the abilities of students to learn a broad range of algebra I skills, incorporating all five strands of the mathematics conceptual model proposed by Kilpatrick et al., (2001).

Building Conceptual Understanding with *Big Ideas*

Kilpatrick et al. (2001) noted the practice of using verbalizations to infer conceptual understanding was common in the math literature, but they also pointed out verbalizations would likely underestimate the number of students with conceptual understanding because, "students often understand before they can verbalize that understanding" (p. 118). In the model of Kilpatrick et al. (2001), conceptual understanding occurs when students understand the underlying process of a mathematical procedure. The underlying knowledge permits students to build new mathematical knowledge. Allsopp et al., (2016) and Witzel (2016) stressed the need to develop conceptual understanding in students because the understanding provides a solid basis for future learning. Witzel (2016) recommended building the concepts of number patterns, equality, and variables; he called the concepts the *big ideas* of mathematics. Collectively the research shows secondary students with ID learning some of the foundational skills. Secondary students with ID are learning the number patterns through fractions, exploring the concept of equality with inquiry-based experiments, and building the concept of variables.

Number pattern studies. In algebra the interplay between two variables, *slope (*change in the value of *y* divided by the change in the value of *x)* is often expressed as a fraction or the decimal equivalent, and Allsopp et al., (2016) asserted that it is important for students to build a strong understanding of the ratio relationships within fractions. Three studies examined how students with ID could build an understanding of the relationship. The studies attempted to help students to recognize the equivalence of fractions in pie graphs, fractions, or decimal formats (e.g. ◕= ¾ = .75).

Hall et al. (2006) found inconsistent development of the same fraction skills across participants using a pre-test/post-test design. They encouraged five participants with Fragile-X Syndrome to match the image of a pie graph to a fraction or an image. Participants' ages ranged between 12 and 19, and the IQ scores fell between 40 to 65. Students were given a pre-test, participated in a computerized intervention program, and then received a post-test. The computerized intervention comprised the following steps: (a) present problems to the student; (b) ask the student to find an equivalent fraction, number, or symbol from an array of choices; and then (c) provide feedback to the student. To examine the effects of the intervention, Hall et al., (2006) looked at the skills under five separate matching conditions. Participants could match the fraction to the pie graph (3/4 = ◕), the fraction to the decimal (3/4 =.75), the decimal to the pie graph (.75 =◕), the pie graph to the fraction (◕=3/4), or the pie graph to the decimal (◕ =.75). "Four of the five participants successfully learned the math relations, requiring between 64 and 847 trials to complete the training" (p. 647, Hall et al. (2006) defined mastery of the skill if participants matched the items 60% of the time, and the results were inconsistent. One participant improved in all five-skill areas, one participant improved in three areas, and two participants showed improvement in two areas and skill loss in two areas. Hall et al. (2006) suspected the different performances in accuracy) correlated to IQ scores.

Hammond et al. (2012) attempted to establish the correlation between IQ and participant performance, using the intervention and pretest-posttest experimental design from Hall et al. (2006). The researchers randomly assigned 22 participants with Fragile-X syndrome (IQ's ranging from 54-90) to a computerized intervention or to a control group. Hammond et al.,

(2012) used the same computerized intervention package and targeted the same skill examined by Hall et al. (2006), and participants engaged in as many as 1500 trials. They found that the IQ score positively correlated with improvements in accuracy for the group in the intervention with correlations ranging between .32 to .70 ($p < .05$); however, no improvements were associated with the control group. The study established IQ as a potential covariate for interventions, but the authors over extended the conclusion, and the authors asserted “it is possible that specific brain abnormalities associated with FXS [Fragile X Syndrome] may hamper stimulus equivalence (or 'concept') formation" (p.8).

In a similar study, Brown et al. (2011) documented some growth in student achievement with a different computerized intervention. Instead of requiring participants to memorize associations as in the Hall et al. (2006) and Hammond et al. (2012) studies, participants played a video game to build the associations. Including 16 participants with ID (IQ's not provided) from the British school system, the researchers matched individuals using scores from the British Picture Vocabulary Scale before randomly assigning them to an intervention or control group. The authors reported improvement for students in the intervention group where median scores rose from 35 percent to 40 percent accuracy. Again, participants failed to reach 60 percent achievement criteria, but the growth suggested students could improve conceptual understandings in mathematics.

Equality study. Göransson et al. (2016) conducted a qualitative study to directly observe the math lessons taught to students with ID. The qualitative study purposively selected six classrooms from a pool of 60 compulsory Scandinavian schools. The researcher's video recorded 18 lessons and debriefed teachers with 21 semi-structured interviews. The observations included 31 students between the ages of 7 and 18 as they engaged in a variety of conceptual mathematical activities. In one lesson, the researchers observed students exploring the concept of equality using a balance beam. The teacher encouraged students to create combinations of numbers to balance the equation. For example, students might use four groups of five to equal 20, or they might choose two groups of ten to equal 20. Göransson et al. (2016) inferred that the students understood the concept of equality because the students could be observed helping each other to find solutions and because the students responded to the inquiry activity with comments like "'I thought the same way'" (p. 13). The inferences Göransson et al. (2016) attached were limited because the authors did not describe the participants in detail; without the demographic data related to the diagnosis of ID, it is difficult to say for sure if the students had ID. Additionally, the purpose of the study was to document alternative intervention techniques for students with ID, so the researchers did not attempt to quantify the effect of the intervention. Göransson et al. (2016) implied students developed conceptual understanding because students could verbalize thoughts.

Studies building the concept of variables. Algebra is a language, and students learning algebra must develop an understanding of the symbols and the grammar of algebra (Hills, 1948; Kilpatrick et al., 2001; Root, 2016; Witzel, 2016). Developing a conceptual understanding of variables can help students to develop stronger algebra skills (Kilpatrick et al., 2001; Witzel, 2016). Cease-Cook (2013) used a single-case, multiple probe design to show that students could develop the concept of variables. The study provided three middle school participants with ID (IQ scores 63 to 68) with a *concrete to representational to abstract* intervention to increase the participants' abilities to manipulate algebraic expressions and equations to find equivalent terms (e.g. $\frac{xz+yz}{z} = \frac{z(x+y)}{z} = (x+y)$). The systematic approach was consistent with the *errorless learning* models because participants were provided with support in a manner designed to minimize errors

(Mueller et al., 2007; Snell & Brown, 2014). In the concrete-representational – abstract intervention, participants were provided with the concrete objects representing variables (e.g. cups, paper clips) which were phased out as the participants developed the skills. To measure the participants' skill development, Cease-Cook (2013) used a task analysis and calculated the number of steps and problems that students completed correctly. After two types of interventions, all three students increased performance from a baseline score of zero with 80% of the steps completed correctly during the testing baseline phase. Cease-Cook (2013) inferred the development of conceptual understanding because participants performed the inverse operations after the intervention.

Summary of the studies exploring conceptual understanding. The five studies examining skills related to conceptual understanding focused on number patterns, equality, and variable manipulation. The studies exploring conceptual understanding included a total of 68 participants, and the researchers documented three distinct types of interventions. All three of the studies that examined the number patterns (fractions) used technology-aided feedback for the intervention (Brown et al., 2011; Hall et al., 2006; Hammond et al., 2012). Computers were programmed to provide feedback as the participants employed a *trial and error* approach to learning. Participants engaged in a trial; the computer checked the students work and provided encouragement for correct answers or negative feedback (e.g. "…incorrect") for incorrect answers. Göransson et al. (2016) described a community-referenced intervention to help students to develop the concept of equality, and Cease-Cook (2013) described a progressive reduction in structured concrete supports to develop the conceptual understandings of variables. The three approaches demonstrated different levels of skill improvements with the Cease-Cook (2013) study documenting the growth within the fewest trials.

Developing Procedural Fluency

` Procedural fluency is the cognitive process of solving algebra problems using a set of steps (Kilpatrick et al., 2001), and many studies indirectly examined the development of procedural fluency for participants with ID. The studies demonstrated the abilities of students to solve, apply, and generalize algebraic procedures. In general, researchers used behavioral, *errorless learning* interventions to teach participants with ID to solve problems.

Procedural fluency to solve one-step equations. A seminal study, Jimenez et al., (2008), demonstrated students with ID could solve one-step equations. The skills performed by the participants aligned best with procedural fluency because the study focused on procedures for solving the equations. Jimenez et al. (2008) used a single-case, multiple probe design across participants. The study was one of the first attempts to show that middle school participants with ID could solve abstract algebra problems. In the study, three participants (IQs between 41 and 45) learned to solve one-step linear algebra equations using addition or subtraction (e.g. $5 + x = 10$). The authors described a multi-component intervention package consisting of concrete objects (templates), a ten-step task analysis, and a systematic, Constant Time Delay (0 seconds) prompting strategy. The participants' ages ranged between 15 and 17 years old. Jimenez et al. (2008) monitored participant performance with a nine-step task analysis to document the completion of each of the observable steps required for solving the equations. Participants completed 80 to 100 percent of the steps for the skill after participating in 10 to 30 intervention sessions.

Generalization of procedural fluency. Kilpatrick et al. (2001) showed three other strands of mathematics learning -- productive disposition, adaptive reasoning, and strategic competence, but no studies explicitly explored the remaining strands. Göransson et al. (2016)

observed that much of the existing research examining math skill development for participants with ID was rooted in behaviorism, of which generalization is a stage of learning that follows the acquisition of behaviors (Snell & Brown, 2014; Stokes & Baer, 1977). Some studies showed the procedural fluency of algebra skills generalizing to a mix of competencies, that included algebra, geometry, word problems, or life skills.

Mixed skills. One study examined the application of procedural fluency generalized across four skills. Browder, Jimenez et al. (2012) reviewed the impact of an intervention using a nine- or ten-step task analysis, with graphic organizers (templates) for three students with ID (IQ's less than 41). The participants were nominated by the teacher, and the teacher implemented the intervention. The intervention targeted solving one-step equations from word problems, plotting points on a coordinate plane to create lines, and graphing data from stories. Researchers employed a multiple-probe across conditions design, and participants showed improvement in procedural fluency for each skill. The intervention ran for seven to eighteen sessions, and achievement after intervention ranged from 20% to 100%. Similar studies showed improvements in procedural fluency unique to geometry.

Using a larger group of participants, Browder, Trela et al. (2012) expanded the intervention to include 16 student participants. The study trained 10 teachers to implement an intervention package using a task analysis, a prompting strategy, and a story of the participants solving steps for solving the math problems (e.g. Simon takes the number and moves it to the square, then…). The teacher-nominated participants had ID (IQ between 30 and 54). The target math skills for the study included modified general education standards from algebra (e.g., solving one-step equations), data analysis (e.g., determining which group has more), geometry (e.g. plotting a point on a coordinate plane), and measurement. Procedural fluency was measured for each skill using a task analysis, and participant performance was measured by the percentage of correctly performed steps. The eleven students showed an improvement in mean scores from a pretest (32%) to the post-test (60.3%) with the algebra skill showing an increase in student performance from a pretest (31.9%) to the post-test (65.6%), and an increase in the geometry skill from the pretest (43.1%) to the post-test (77.8 %).

Geometry. Two studies examined how students could apply procedures to solve geometry related problems. Hord and Xin (2014) conducted a single-case design across participants for three middle school participants with IQ scores between 65 and 75. The participants were asked to apply geometric formulas to find the area or volume for different shapes. The formulas for finding the area included: rectangles (*A=lw where A= area, l= length, and w = width*); triangles (*A=1/2 bh; where A=area, b= base, and h= height*); and circles (*A=2Πr where A= area, Π ≈ 3.14, and r= radius*). The formulas for finding the volume included: rectangular, triangular, and cylindrical prisms (*V= Bh where V= volume, B= the area of the base of the shape, and h=height*). Participants also used the formulas to find missing variables. To complete the skill participants identified the shape, applied the appropriate formula, identified the missing value, and solved for the value. To assess participant performance, criterion test probes were administered during baseline, intervention, and during testing conditions. Like the Cease-Cook (2013) intervention, Hord and Xin (2014) employed a concrete-to-abstract intervention. The use of two and three-dimensional shapes was faded as participants gained experience with the skill. After seven to ten lessons of using three-dimensional blocks under the guidance of a teacher, the participants switched to using two-dimensional paper drawings of shapes with the formulas. Hord and Xin (2014) demonstrated improvement during the intervention phase for all three participants. Two participants increased the number of problems answered correctly from a baseline of 0% to 80 %, and one participant increased from a baseline of 20% to at least 60%. In the Kilpatrick et al. (2001) model adapting different formulas to the right situation was described as strategic competence, and in the Hord and Xin (2014) study, participants generalized the math procedures because the formulas used flexibly; participants adapted the different formulas to different problems. When participants demonstrate strategic competence, they are also demonstrating a level of conceptual understanding (Kilpatrick et al., 2001).

In single-case multiple-baseline probes across participant's design, Creech-Galloway et al., (2013) looked at a different geometry skill. They demonstrated that students with ID could use the Pythagorean Theorem to solve a variety of problems with triangles. The four middle school participants had IQ scores between 41 and 57. Using the Pythagorean Theorem ($a^2 + b^2 = c^2$), participants solved problems with a 32-step task analysis. As part of the intervention package, participants activated and viewed videos showing the steps of the procedure. Participants required between four and ten trials during the intervention to show improvement. In all cases, the participants solved the equations to find the value of the hypotenuse (c) of each equation. Three participants increased the percentage of steps completed from a baseline of 0% to 100% during the intervention. Although the students applied the Pythagorean Theorem to different triangles, the skill aligned with Kilpatrick et al.'s (2001) procedural fluency because the skill itself was limited. Participants were consistent in their application of the theorem, did not use the formula to find other missing sides (*a* or *b*), and solved the same problems repeatedly (Creech-Galloway et al., 2013).

Word problems. Root (2016) illustrated the ability of participants to generalize procedural fluency to solve general word problems. In her study, three participants with IQ scores between 50 and 66, applied word problem vocabulary to solve different types of algebraic word problems. Participants engaged in a series of vocabulary building activities before beginning an intervention to solve word problems. The intervention package included a 10-step task analysis, a self-monitoring checklist, and *errorless learning* feedback. The procedures encouraged students to identify vocabulary words and key information before solving the problem. For instance, when a problem used the word “total,” the operation was addition. Participants could choose from the four functions (addition, subtraction, multiplication, and division) to solve the problems, and after participating in a minimum of eight intervention sessions, all three participants demonstrated an increase in performance in the procedures from baseline (<25%) to intervention (100%).

Daily life. One qualitative study examined the role of algebra in everyday life for adult participants (ages 22-27) with ID. Rodriguez (2016) invited participants to engage in a community education program designed to teach the functional algebra skills needed for banking. Qualitative pre-intervention and post-intervention interviews and observations were collected. During the intervention, participants engaged in weekly lessons and practiced skills such as calculating hours worked in each pay period. (e.g. If James is paid $7.55 an hour the paycheck is for $241.60, how many hours should James have worked?) During the individual practice sessions, staff provided the participants with individualized attention, feedback, and instruction. Although explicit assistance was provided to everyone to develop procedural fluency, Rodriguez (2016) permitted everyone to develop personal strategies to solve the problems. The qualitative analysis examined the different strategies that participants used to generalize the skill. Rodriguez noted individuals solved multi-step equations using the correct steps, but instead of using variables (e.g., x) the individuals would use “placeholders” or blank spots in place of the variables. Instead of writing, “3x-5 =150” participants would write “3__- 5=

150). In post-observations and interviews, participants recognized the need for algebra because algebra was needed in their daily lives, so participants could generalize the procedural fluency to productive disposition (the belief that algebra is useful in everyday life). However, Rodriguez's observations also illustrated how procedural fluency interacted with conceptual understanding because the participants adapted and constructed the procedures, generalizing the practice sessions into different algorithmic procedures.

Summary of studies exploring procedural fluency. Excluding the Rodriquez (2016) study, the studies included 37 participants with documented IQ's between 33 and 73. Rodriquez (2016) provided an intervention to ten adults with a previously reported diagnosis of ID; however, IQ scores were unavailable. Again, except for Rodriguez (2016), interventions included instruction using a behaviorally based intervention strategy paired with a task analysis, and a prompting and feedback strategy consistent with the Mueller et al. (2007) definition for *errorless learning* (e.g. delayed, simultaneous, or fading prompts). In all cases, participants demonstrated improvements in procedural fluency. Rodriquez's (2016) intervention took place as part of a community-based club where individuals were provided with whole group instruction and individual practice. The procedural fluency skills targeted for intervention included solving equations and word problems, applying formulas to geometric shapes, and solving money-based problems. In some cases, the research demonstrated how procedural fluency could generalize to a mix of challenges (Browder, Jimenez et al., 2012; Browder, Trela et al., 2012), geometry (Creech-Galloway et al., 2013), word problems (Root, 2016), and daily life (Rodriguez, 2016).

Summary of Algebra Research for Students with ID

The literature showed that participants with ID engaged in academic algebra instruction, and provided evidence that students with ID can improve conceptual understanding and procedural fluency, and generalize skills to productive disposition and strategic competence. Algebra interventions for students with disabilities were explored after the passage of NCLB (2001). The reviewed studies documented three basic algebra interventions- social, constructivist, and behavioral approaches. Across the studies, 135 participants engaged in secondary mathematics, and 12 of the 16 studies related directly to algebra skills. Of the 12 studies demonstrating algebra skills for students with ID, only four studies showed students with ID learning high school level algebra.

Social interventions. Several studies demonstrated student involvement in algebra because of social intervention. Social interventions allow for the natural social environment to provide students with feedback and reinforcement from peers (Bandura, 1971). These interventions provided students with broad access to the general education environment or encouraged students to work with peers.

Access. A group of studies from Italy described the academic benefits of providing access to algebra for individuals with Down Syndrome (Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005). Noting that Italian law requires students to participate in the secondary academic curriculum, Monari Martinez and Pelligrini (2010) described the series of studies as an attempt to document the abilities of students as they participated in a comprehensive Algebra I curriculum, that monitored individual performance and provided remediation in the form of warranted, individualized attention. Although participants in the series of studies demonstrated the ability to engage in algebra (Monari

Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005), the studies lacked detailed descriptions of the intervention.

Peer-Based Interventions. Rodriguez (2016) set up a community-based club to allow participants to work with each other to solve money problems. The researchers provided the club with weekly topics to explain math applications, and then smaller groups of peers were encouraged to work together to solve problems. As needed, the additional individualized instruction was provided by staff members. Similarly, Göransson et al. (2016) observed teachers guiding groups of students as they solved puzzles or problems, and the researchers felt the key to the intervention was the verbal engagement that occurred between the students as they worked through the problem-solving process. In both the Rodriguez (2016) and the Göransson et al. (2016) studies, conceptual understanding was constructed by the participants, and the social interventions overlapped with the constructivist approaches.

Constructivist elements. Kilpatrick et al. (2001) noted that the general education math community traditionally emphasized rote learning, and as an alternative, they promoted an alternative constructivist approach to learning. In contrast, within the special education literature, there were two types of constructivist methods employed. The first used a concrete-to-abstract intervention approach (Cease-Cook, 2013; Hord & Xin, 2014), and the second attempted to boost algebra skill performance by building the language that supports the skill. The concrete-to-abstract approach was used to establish skills for students. In the concrete-to-abstract approach, instructional supports were faded from the concrete objects (e.g. popsicle sticks, paper clips, or chips) to the more symbolic or abstract representations (e.g. numbers or letter variables). Cease-Cook used this strategy to help individuals to solve equations or simplify algebraic expressions, and Hord and Xin (2016) provided participants with objects so the individual could apply geometric formulas to area or volume problems.

Behavioral interventions. Studies demonstrated two types of behavioral interventions. The first category integrated technology to provide feedback to participants practicing a skill. The second type of behavioral intervention integrated multiple interventions into a comprehensive intervention package for participants.

Trial and error. *Trial and error* learning is a traditional method for providing an intervention. Participants are provided with positive or negative feedback after completing a step of a task (Skinner,1969/2013; Thorndike, 1913). Three articles evaluated the *trial and error* method when it was embedded into computer software as participants matched fractions to decimals or shapes (Brown et al., 2011; Hall et al., 2006; Hammond et al., 2012). Participants made some progress, increasing performance to at least 60% accuracy, but participants required between 87 to 1500 trials to develop the skill.

Errorless-learning intervention packages. *Errorless learning* provided an alternative to trial-and-error learning interventions. *Errorless learning* scaffolds an environment around participants to reduce the need for negative feedback (Mueller et al., 2007; Snell & Brown, 2014; Touchette, 1971; Touchette & Howard, 1984). Spooner, Knight, Browder, and Smith (2011) called the prompting strategies associated with *errorless learning* method an evidence-based practice for students with ID. Many studies used multi-level intervention packages to support participants as they learned algebra related skills (Browder, Jimenez et al., 2012; Browder, Trela et al.,2012; Creech-Galloway et al., 2013; Jimenez et al., 2008). The packages often included a task analysis, a prompting strategy, templates, and a self-monitoring component.

Task analysis. A task analysis is a systematic sequence of discrete and observable steps required to complete a skill. Together, the steps map the algorithmic procedures needed to complete a problem. In two studies, Browder, Trela et al. (2012) and Browder, Jimenez et al. (2012) reported that task analyses were key components of the intervention package for algebra instruction. Similarly, Jimenez et al., (2008) embedded a nine-step task analysis for solving one-step equations into the intervention, Root (2016) and Creech-Galloway et al. (2013) used task analyses to aid participant self-monitoring during interventions.

Self-monitoring. Three studies, Creech-Galloway et al. (2013), Jimenez et al., (2008), and Root (2016), included self-monitoring interventions. Self-monitoring provides participants with the opportunity to practice meta-cognition to solve problems (Lee et al., 2016, April). The Jimenez et al. (2008) noted one of the three participants needed extra support during the intervention, and thus a copy of the task analysis was provided to the participant. The participant used the task analysis as a set of directions. Root (2016) modified the task analysis putting empty boxes next to each step. The participants were instructed to check steps as they progressed through the algorithm. Creech-Galloway et al. (2013) automated the self-help. The iPad videos presented a model performing each step of the task analysis. Participants self-activated the video prompts by touching the screen (Creech-Galloway et al., 2013).

Templates. Templates are like the semi-concrete objects used in Cease-Cook (2013) study. The templates support participants as they learn algebra using a semi-abstract scaffold for the participant to follow (Saunders, Bethune, Spooner, & Browder, 2013; Lee et al., 2016, April). Jimenez et al. (2008) included equation templates with spacers as part of the intervention package (e.g. x + 5 = 3 → x + 5 __ __ = 3__ ___); the three participants moved numbers and symbols into the spaces. Root (2016) described the use of templates as a support for participants solving different types of word problems. The participants would read the word problem, decide which type of algorithm was necessary, and select the template for the problem. The templates provided participants with places to write numbers and math functions (e.g. □ ○ □ = □).

Prompting strategies. Interventions included *errorless learning*, prompting strategies. Although Mueller et al. (2007) identified three prompting strategies, fading prompts, time delay prompts, and superimposition prompts, the current research used variations of the time delay approaches. Time-delay provides participants with feedback within a set amount of time. Zero-time delay, or simultaneous prompting, provides the participant with instructional feedback in the form of a prompt as the individual receives a stimulus to proceed with activity. The Constant-Time-Delay procedure is a little different. It provides the individual with feedback with preset or progressively longer times (e.g., between a half a second to five seconds) (Cooper et al., 2007). For example, Jimenez et al. (2008) increased for participants from zero seconds to four seconds and then eight seconds for the steps needed to solve one-step equations, and Creech-Galloway (2013) used simultaneous prompting to deliver the video modeling prompt for each of 32 steps involving they Pythagorean Theorem.

Skills. Legislation required access to the grade-level curriculum (IDEA, 2004; NCLB, 2001). However, participants with ID were excluded from the general education intervention research (Haas, 2005), and the skills demonstrated tend to be pre-algebraic without regard for Algebra I. The inclusion of students with ID in the Algebra I curriculum was limited to three studies (Monari Martinez & Benedetti, 2011; Monari Martinez & Pellegrini, 2010; Neodo & Monari Martinez, 2005). In each case where participants with ID completed Algebra I level activities, the participants had Down Syndrome, and it is unclear if the same level of algebra can be compared to the wider group of individuals with ID. The remaining research shows individuals with ID engaged in activities consistent with pre-algebra. For example, students with ID (a) identified equivalent fractions (Brown et al., 2011; Hall et al.; 2006; Hammond et al., 2012); (b) used formulas to find area, volume, or distance (Cease-Cook, 2013; Creech-Galloway, 2013; Hord and Xin, 2014); (c) solved one-step equations (Browder, Jimenez et al., 2012; Browder, Trela et al., 2012; Jimenez et al., 2008); (d) solved world problems or financial math problems (Rodriquez, 2016; Root, 2016), and (e) plotted points on a coordinate plane (Browder, Jimenez et al., 2012; Browder, Trela et al., 2014). Although researchers explored many skills, the evaluation of the algebra skills was conducted outside of the mathematics framework from Kilpatrick et al. (2001), and the literature tended to focus on either conceptual understanding or procedural fluency (Göransson et al., 2016). The limited focus of the researcher led to limitations in the research.

Limitations

Limitations of the Italian studies. Although comprehensively tied to algebra, the Italian studies did not explicitly categorize the skills using the model from Kilpatrick et al., (2001). The skills did appear to solicit integrated responses. For example, in Monari Martinez and Pelligrini (2010) participants solved word problems using formulas in physics where the problems required participants to identify key information, select the appropriate formula, substitute data for the variable, and solve using a complex set of cognitive processes that include the different strands from the conceptual framework model, but the studies did not describe how the various strands of mathematics were used by participants. Without clear links to the conceptual framework, the results are limited to the broad category of Algebra I as opposed to the five skills (productive disposition, conceptual understanding, procedural fluency, adaptive reasoning, and strategic competence) in the framework from Kilpatrick et al., (2001).

The Italian studies did show participants with Down Syndrome could learn algebra, but they did not compare the results to the performance of general education students or other students with Down Syndrome. Instead, the researchers compared participant results to a hypothetical and undocumented, historical assumption that students with ID were incapable of learning algebra. While the studies did show the historical assumption is false, at least for students with Down Syndrome in the Milan region of Italy, it is unclear if the individuals knew the material prior to the intervention. Without the within-subject comparisons, the research did not adequately examine the influences of prior knowledge, environment, or the interventions. Participant achievement could be related either to prior educational experiences or to the intervention.

If the performance results were related to the student intervention, it would be difficult to determine how. None of the Italian studies provided a detailed description of the intervention. Instead, vague references were made to an intervention. Neodo and Monari Martinez (2005) described the teachers providing additional practice for the student if the student was unable to performed the task independently, but the level of supports provided during the individual practice was not described. It is unclear if participants were provided with prompts, models, feedback, or just extra work. It would be difficult to replicate the intervention or to expand the intervention into different classrooms without a more detailed description.

Additionally, the ambiguous intervention would be difficult to classify. In some respects, the intervention might align with the social inclusion described by Göransson et al. (2016) because students were included in the general education environment, but the Italian studied did not describe interactions within the environment. Neodo and Monari Martinez (2005) did describe how the teachers use a ranking system to determine student independence. Ostensibly, participants were provided with additional instruction or practice if the teacher felt the student needed to be more independent.

Another limitation of the studies related to the sample which limits generalizability or results. Samples were selected from the individuals diagnosed with Down Syndrome in the Milan region of Italy. The results might not generalize to the larger population of individuals with Down Syndrome or the population of individuals with ID.

Limitations of the conceptual understanding studies. The studies examining conceptual understandings had many limitations. First, the researchers are overstating the accomplishments of the individuals with ID. For example, the studies focusing on fractions (Brown et al., 2011; Hall et al., 2006; Hammond et al., 2012) examined skills found within third, fourth, and fifth grade *Common Core Curriculum* (NGACBP & CCSSO, 2010); the skills were not the skills found in the general education middle or high school curriculums. Similarly, the concept of equality described by Göransson et al., (2016) was listed in the first grade, general education standards (NGACBP & CCSSO, 2010). The most advanced conceptual skills were explored by Cease-Cook (2013). Participants solved and simplified expressions with variables; however, the skill is listed in the general education curriculum as a sixth-grade skill (NGACBP & CCSSO, 2010). When focusing on conceptual understanding, researchers concentrated on the *big ideas* (number patterns, equality, & variables) from Allsopp et al. (2016) necessary to advance in algebra, but the studies did not attempt to demonstrate student advancement in algebra.

In general, the skills were examined outside of the general education mathematics framework of Kilpatrick et al., (2001); specifically, none of the studies focused on the deep learning emphasized described by Kilpatrick et al. (2001). Without a theoretical framework describing the cognitive processes of mathematics, the measurements used to monitor progress could lack construct validity, thus invalidating the research findings (Göransson et al., 2016; Mitchell & Jolley, 2010). Specifically, without the framework, researchers may have overstated the scope of the algebra skills learned by the participants because most of the strands of algebra (productive disposition, strategic competence, procedural fluency, conceptual understanding and adaptive reasoning) were missing from the studies.

The inverse also appears to be true. By focusing on narrow bands of conceptual understanding (e.g. equivalent fractions) researchers understated the potential of participants with ID to learn concepts. Hammond et al.'s (2012) finding of a correlation between an IQ and participant math performance did not necessarily mean individuals with low IQ's are unable to learn the concept of fractions. Instead, the results could suggest that students with lower IQ's require supports different than *trial and error* learning.

Limitations of the studies exploring procedural fluency. Similarly, the studies examining procedural fluency were limited in scope, and none of the studies examining procedural fluency linked to the mathematics model from Kilpatrick et al., (2001) making comparisons to the general education algebra skills difficult. Browder, Jimenez et al. (2012) and Browder, Trela et al. (2012) explicitly linked the target skills to North Carolina's grade-level standards, but the links were to the alternate achievement standards which are different from the actual general education standards. Additionally, studies often simplified the math activities. For example, Jimenez et al. (2008) simplified the task of solving one-step equations by eliminating equations requiring multiplication or division. To date, none of the discrete skill studies examine high school level skills.

Moving Forward

Although promising, the current body of literature is insufficient to establish a reliable pedagogy for students with ID. The published studies only demonstrate grade-level achievement for high school level algebra achievement for students with Down Syndrome (Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005) and studies focusing on broader groups of students with ID involved elementary school or middle school standards (Browder, Jimenez…et al., 2012; Browder, Trela… et al., 2012; Cease-Cook, 2013; Creech-Galloway et al., 2013; Jimenez et al., 2008; Root, 2016). Additionally, none of the special education literature used the broader definitions of mathematics described by Kilpatrick et al., (2001). Moving forward, research needs to expand the settings, student characteristics, and skills. Additional research is required to examine the application of interventions to support the achievement of high-school-aged students as they participate in grade-level algebra activities. Ideally, the research would explore many interventions; however, at this time, new research needs to establish the boundaries of student potential in Algebra I because the full potential of students with ID remains undocumented (Courtade et al., 2012). Without valid and reliable demonstration, researchers and teachers will continue to argue that full inclusion at the secondary level is infeasible, and that students with ID should continue to develop functional skills (Ayres et al., 2011; Ayres et al., 2012; Courtade et al., 2012; Johnson et al., 2013; Loveless, 2008).

Chapter III

Methodology

Limited research exists related to methods of algebra instruction for participants with ID. While a cluster of researchers demonstrated that individuals with Down Syndrome could participate in a wide variety of algebra skills using general education instructional practices (Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005), the studies did not describe the instructional methods. A second group of researchers provided detailed descriptions of intervention packages used to teach isolated algebra skills; however, this second group of investigators did not target high school level algebra skills (Browder, Jimenez et al., 2012; Browder, Trela et al., 2012; Cease-Cook, 2013; Creech-Galloway et al., 2013; Jimenez et al., 2008).

The skills targeted in published research articles have focused on procedural fluency but have not addressed the broader set of cognitive math skills according to the Kilpatrick et al. (2001) model, which includes conceptual understanding, adaptive reasoning, productive disposition, and strategic competence. Notably, Cease-Cook (2013) and Göransson et al. (2016) were the only studies explicitly examining conceptual understanding of algebra skills, though neither research study explored *errorless learning* behavioral interventions as a tool to build conceptual understanding. For non-algebraic math skills, another group of researchers linked behavioral interventions to teaching conceptual understanding of fractions, but did not target any algebra skills (Hall et al., 2006; Hammond et al., 2012; Hord & Xin, 2014).

This study examined the performance of high school participants with ID on two algebra skills: (a) creating an equation from a graph of a line and (b) creating a graph of a line from an equation. This study included the delivery of an intervention package composed of a two-dimensional template, a prompting procedure, and a self-monitoring procedural guide to facilitate the acquisition of the targeted skills. The intervention and the subsequent measurement of both procedural fluency and conceptual understanding of the skills will be guided by two task analyses (the description of steps required to complete the task). The purpose of this study was to investigate the development of procedural fluency and conceptual understanding of algebra skills by participants with ID. Specifically, the study sought to answer the following research questions:

1. When provided with evidence-based behavioral intervention and adapted materials, will participants with ID acquire procedural fluency in solving a linear equation to create a line, and in creating an equation from a picture of a line?

2. Does the acquisition of procedural fluency for creating a linear equation or creating a line affect conceptual understanding?

    a) does acquisition of procedural fluency generalize to the inverse skill (Creating a line is the inverse skill for creating an equation and creating an equation is the inverse skill for creating a line)?

    b) does acquisition of conceptual understanding require less time in instruction than with procedural fluency of the inverse skill?

Pilot Study

A pilot study was conducted in the spring of 2014 at a central Virginia high school. The pilot field tested a behavioral intervention on similar target skills, using measurement tools, and intervention procedures as currently proposed. Specifically, the pilot study explored the participants' abilities to complete the following four algebra skills (a) solving one-step equations with multiplication and (b) division, (c) modeling of equations given a line, and (d) creating a graph of a line given an equation ($y=mx+b$). To prepare for the *Virginia Alternate Assessments* (VDOE, 2014), four students with ID participated in a series of intervention activities designed to teach the procedural fluency for each skill.

Results from the pilot study helped to shape decisions for the proposed study. Pilot participants demonstrated the ability to solve one-step equations during baseline (minimum range = 40 to 60 percent correct responses before intervention), so skills related to solving one-step equations were eliminated from the proposal. Results indicated the other skills, creating a line and creating a graph, were feasible and sensitive to the intervention. All the students showed 0 percent growth during baseline for the skills involving graphing the equation and creating the equation, and the performance increased after intervention from 40 to 100 percent accuracy for all students. Field testing the intervention facilitated refinement of procedures for the intervention and helped to refine the task analysis for the target skills.

The pilot study also helped identify risks to implementation fidelity. During one pilot fidelity check (baseline), a participant was observed reading their responses as recorded by staff on the data sheet containing the task analysis. For the current study, the data sheet was shielded from participants' view.

Research Design

A single-case experimental multiple-baseline design across participants was used to determine the effectiveness of an intervention package for a minimum of six participants with ID. A task analysis of each of the two skills (i.e. creating an equation from a graph of a line and creating a line from an equation) was implemented to guide the intervention for and measurement of the participants' acquisition of procedural fluency. Indirectly, each task analysis was also used to measure the impact of participants' learning on the respective inverse skill.

The research design had multiple advantages. Single-case designs allow researchers to document the effects of an intervention on a target behavior (Horner et al., 2005). The design requires small numbers of participants (in this case, $n=6$), which allows researchers to conduct studies with small, unique populations (Johnson, Hough, King, Vos, & Jeffs, 2008; Kratochwill et al., 2010; Shadish, Hedges, Horner, & Odom, 2015). Additionally, studies using this design can contribute to a larger research program for establishing research-based evidence. Johnson et al., (2008) noted a researcher can plan, *a priori*, to replicate the design directly as part of a body of research, and after conducting a series of identical studies, the researcher can conduct a meta-analytical study. Moreover, when the studies are replicated across time with different teams of researchers, the use of single-case design can be instrumental in establishing an evidence base (Kratochwill et al., 2010; Shadish et al., 2015).

The study incorporated the quality indicators from both Horner et al. (2005) and Kratochwill et al. (2012). Horner et al., (2005) and the *What Works Clearinghouse* (Kratochwill et al., 2010) established the quality indicators for all single-case designs. Single-case study designs should have (a) clear descriptions of settings and participants, (b) measurable operational dependent variables with (c) verification methods, and (d) social validity (Horner et al., 2005). Similarly, Kratochwill et al. (2012) emphasized the need to establish experimental control with replication of the effects three times at different points throughout the study. The implementation protocol for this study built in replication by using two experiments with three participants each.

Research Protocol

Setting. This study was conducted within a central Virginia public school system. The district participated in the pilot study, and the central administration committed to the study after the University's Institutional Review Board (IRB) approval was obtained. The division ran a single school district with four high schools, located near a state university. The schools ranged in population size, the largest school serving approximately 2,100 students, and the smallest high school, a public charter school, serving 50 students. Table 3 presents the current US Census Bureau (2015) data for the locality. Only the largest school committed to participation.

Table 3

Demographic Data from the Participating District

Population Density	Income		Race			
	Median	Poverty	W	B	H	A
137.3 persons per Sq. mile	$67,725	10%	82%	10%	6%	5%

Note: From the 2010 US Census. Race was self-reported in the Census W- White; B-Black; H-Hispanic; A-Asia

The school maintained a classroom dedicated to instruction and assessment for participants involved in the *Virginia Alternate Assessment*. The classroom included workstations basic supplies, chairs for the participant and the staff member, and desktop space to work on tasks. The desktop was shielded from the view of other students. In the room, up to five students could work on separate skills individually with teachers and paraprofessionals. Typically, students rotated into the room for 15- minute instructional or assessment sessions throughout the school day, so the classroom room was kept sterile of academic materials that could provide information that would invalidate the state testing.

The school administration assigned the staff to work on the study. They provided one of five individual staff members assigned to work in the assessment room during the day. Three individuals were highly qualified special education teachers with master's degrees and endorsements in reading, mathematics, and social studies. Two individuals were paraprofessionals who were earning their special education teaching endorsements. The first individual possessed a Bachelor of Education degree, and the second individual had a bachelor's degree in psychology with a minor in special education.

Participants. Consistent with the protocol approved by the VCU IRB, a three-phase process was used for selecting and assigning participants. First, potential participants were recruited and screened to ensure individuals met the criteria for participation. Subsequently, the participants were assigned to one of two experiments.

Recruitment. After obtaining VCU IRB approval and school division permissions, school principals were invited to participate. Only one school responded to the request. The district distributed recruitment letters (Appendix A) to a pool of 18 potential participants who were identified by the school as Alternate Assessment participants. Nine parents returned the permission slips (Appendix B).

Screening. The participants were screened to verify eligibility for the study. Criteria for consideration included a primary or secondary diagnosis of ID, attendance at a public high school in the identified school district, participation in the *Virginia Alternate Assessment,* and demonstrated ability to utilize the materials without physical assistance. Individual Education

Program (IEP) documents were reviewed and compared to the definition of Intellectual Disability, whereby both intellectual, and adaptive behavior testing scores must be two or more standard deviations below the norm (<70 for both the IQ and Adaptive Behavior Score) (AAIDD, 2010). IEP accommodations were also reviewed to make sure the accommodations would not interfere with the experiment. For instance, one individual required hand-over-hand assistance to draw a line. The study included one participant who could independently communicate with a Dynamic Display Voice Output Communication Aid (DD-VOCA) (iPad™ with Proloquo2go™). The communication device was available to the student throughout all activities. The screening process excluded three participants: (a) one required physical support (hand-over-hand) to use her communication device, (b) another had a medical issue requiring a change-of-placement to the homebound setting; and (c) one individual declined to participate after all were provided with an assent form (Appendix M) prior to participation in the study.

Description of participants. Table 4 presents the basic demographic information for each participant. All the students were members of the same class, Ed, Guion, and Chiaki participated in the first experiment, and Mukai, Dwight, and Bluford participated in the second experiment.

Ed. There were three participants for Experiment 1, Ed, Guion, and Chiaki. Educational records identified Ed (pseudonym), as a 17- year- old, male, African American student in the eleventh-grade. Ed was eligible for special education services as an individual with autism, and was also identified as a student with ID. The district provided Ed with support from a speech language pathologist for 20 minutes per week, simplified reading assignments (second/third grade reading level), Tier II (small group general education) literacy classes, functional skills training, job training, and Tier III (Individualized) math instruction. Ed received additional support from his mother (an elementary school teacher) on a nightly basis. Ed's participation in

Table 4

Participant Demographics Including Pseudonym, Disability (DIS), Ethnicity (ETH), Age, Sex, Cognitive Test (IQ), Adaptive Behavior (AB), Grade, Years Enrolled in Alternate Assessment (AA), and Experiment Assignment (Ex)

Participant	DIS	ETH	Age	Sex	IQ	Adaptive	Grade	VAAP
Chiaki	VI	Asian American	15	F	TONI-3 66	Battelle 54	9	7
Ed	AUT	African American	17	M	WISC-IV 51	Vineland 52	11	7
Guion	MD	African American	16	M	DAS 64	Battelle 54	11	8
Dwight	ID	African American	17	M	WISC-IV 65	PBI "At Risk"	11	4
Mukai	AUT	Caucasian	17	M	WISC-IV 62	Not mentioned.	11	8
Bluford	ID	African American	15	M	WISC-IV 52	"Adaptive Component" 69	10	7

Note: All participants had a primary or secondary diagnosis of Intellectual Disability (ID); secondary disabilities include. Autism (AUT), Multiple Disabilities (MD), and Visual Impairment (VI) were comorbid conditions with ID. Cognitive tests included Wechsler Intelligence Scale for Children Forth Edition(WISC-IV), Differential Abilities Scales (DAS), Test of Nonverbal Intelligence Third Edition (TONI-3). Educational files did not always document the Assessments used to measure adaptive behavior; however, documented tests included the Battelle Developmental Inventory (Battelle) Vineland Adaptive Behavior Scale (Vineland), and Scales of Independent Behavior (SIB-R). Eligibility was determined by the local district. All students participated in the Virginia Alternate Assessment (VAAP) for multiple years.

the *Virginia Alternate Assessment* began in fifth-grade. His high school special education teachers reported that Ed could operate a cash register, solve one-step equations without a calculator, simplify algebraic expressions, and use formulas to solve word problems. He had experience reading line graphs, bar graphs, and circle graphs. One of his special education teachers noted, "He likes to read charts and graphs about basketball. He might know how to use a linear equation."

Guion. The second participant, Guion (Pseudonym), was in the eleventh-grade, 16- years -old and identified for special education as a student with multiple disabilities because of a orthopedic impairment (Cerebral Palsy) and ID. During screening, Guion demonstrated the ability to draw a straight line using a standard pencil and a ruler. The records showed Guion began participating in the *Virginia Alternate Assessment* in the third-grade. As a high school student, Guion participated in several extracurricular activities, and these activities interrupted his participation in the study. He was an active member of the chorus, and attended a week-long trip to Florida. He also participated in a vocational assessment conducted by a state rehabilitation agency. In high school, Guion attended separate math classes with other special education students, and the records stated that Guion was building his skills with fractions and money. His IEP mentioned math organizational issues, but the document did not elaborate. His special education teachers noted that Guion struggled in math, and they also described problem behaviors (skipping class, frequent trips to the bathroom, and frequent trips to the nurse). A functional behavioral assessment had speculated his behavior served to avoid the math activities. His teacher did not believe he had ever attempted algebra, and she stated she did not believe he could solve one-step equations because he was still learning to count money.

Chiaki. The final participant for Experiment 1, Chiaki (pseudonym), was a tenth-grade, 15-year-old student during the study. The only female participant, her primary diagnosis for eligibility was ID; however, she had a second disability of a visual impairment. School records described the visual impairment caused by muscle weakness near the eye; specifically, the school records described a sixth nerve palsy which manifested with a tendency for the eyes to turn towards the nose. A note from an optometrist stated that the condition was untreatable. School documents also stated Chiaki compensated for the vision difficulties by moving closer to objects that she needed to read, and the IEP documents also noted that she could become physically fatigued by constantly changing visual activities. Her preferred activities included drawing, cooking, and exercise. Chiaki could communicate verbally, but she preferred to communicate through writing, with American Sign Language, or with a DD-VOCA. The IEP described math goals related to counting objects, and the annual goal aimed to increase her ability to count from 15 to 21. The IEP did not list goals related to time or money, but the teachers included Chiaki in a daily fundraising activity (selling coffee). Chiaki counted the daily coffee cup inventory, and she sold coffee to other adults in the high school.

Mukai. Experiment 2 included Mukai, Dwight, and Bluford. Mukai was a 17-year old, eleventh grade student during the study. He had a primary diagnosis of autism with a secondary diagnosis of ID. Mukai used verbal communication and enjoyed talking about the Pittsburgh sports franchises. He stated he was not very good at math, that he made many mistakes, and the IEP supported his statement. Specifically, in the *present level of performance*, the teacher stated that Mukai made mistakes on multi-step word problems because “he raced through the assignments.” Mukai did participate regularly in the class fundraising activity (selling coffee), and the IEP described Mukai’s ability to accurately make change. However, Mukai’s annual IEP goal was to “Improve functional math skills.” Mukai’s math teacher described activities designed around financial literacy math problems, but the teacher emphasized that Mukai did not use variables or equations to solve the problems. Mukai demonstrated the prerequisite counting and drawing skills during screening.

Dwight. Dwight was a 17-year -old eleventh-grade student. His IEP documents ID as the only disability, and it describes a young man interested in sports and poetry. Dwight was also described by the IEP as a peer leader. However, in mathematics, the IEP noted that Dwight was struggling to learn how to solve money related word problems requiring addition or subtraction. The math teacher noted that Dwight could identify points on a coordinate plane, but he sometimes inverted point *x* with point *y;* she also noted that Dwight had not learned about one-step equations. During the screening process, Dwight communicated verbally, and he self-described his top interest as basketball. He also stated he, “Really, really,” wanted to learn algebra. Dwight demonstrated how to count forward and backwards without an issue, but he struggled drawing a line in free form. During the screening process, the teacher needed to show teach him how to lay a ruler across two points; however, after the demonstration, Dwight repeated the task independently.

Bluford. During the study, Bluford was a 16-year-old, tenth-grader. He expressed an interest in music, and he regularly participated in an audio production class. Bluford was verbal, but he preferred to answer “yes” or “no” questions. His only listed disability was ID. His IEP described partial participation in math activities, specifically noting that Bluford often failed to complete assignments. In mathematics, the *Present Level of Performance* described difficulties solving problems involving money, and solving word problems involving multiplication and division. The IEP also noted that Bluford preferred activities where he worked alone. Bluford did indicate a desire to participate in the study. When responding to the assent form he was heard whispering, “like everyone else.”

Dependent Variable. The dependent variable was defined as the percentage of steps of the target skill completed correctly and independently (without prompting). The task analyses presented in Appendices C and D show the eleven steps needed to create a line, and the ten eps required to create an equation, respectively. The percentage of correctly performed steps were calculated by dividing the number of steps completed correctly by the total number of steps required. Initial skill acquisition was said to have occurred when the participants met the criteria level of 60%. (Snell & Brown, 2014, p. 125). Snell and Brown (2014) suggested an instructional level of 60% correct responding during the acquisition stage of learning as the minimum performance criteria needed for students to move from the acquisition stage of learning to the fluency, maintenance, or generalization stages. The selection of the 60% criteria does not necessarily indicate skill mastery or proficiency, but is minimal criteria needed to initiate instruction on related skills or generalization.

Target Skills. Two skills were targeted for the intervention. Participants were asked to create an equation when given a graph of a line Experiment 1), and to create a graph when given an equation of a line (Experiment 2). The skills were selected from the Virginia high school *Standards of Learning* curriculum (VDOE, 2009) based on state standards, and task analyses were developed for both skills, creating an equation from a graph and creating a chart from the equation. Snell and Brown (2014) recommended a seven-step approach for teachers to use to develop task analyses: (a) select a skill, (b) define the skill, (c) perform the task and observe others performing the task, (d) adapt the task to accommodate for a student’s disabilities, (e) validate the task analysis with students by practicing with students, (f) revise the sequence, and

Table 5

Alignment of Target Skills to State Curricula

State	Standards	Creating an Equation	Graphing a Line
Common Core	CCSS.MATH.CONTENT.HSA.CED.A.2 “Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.”	✓	✓
Alaska	“A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales” (Alaska Department of Education & Early Development, 2012, p. 184).	✓	✓
Indiana	“AI.L.6: Translate among equivalent forms of equations for linear functions, including slope-intercept, point-slope, and standard. Recognize that different forms reveal more or less information about a given situation… AI.L.4: Represent linear functions as graphs from equations (with and without technology), equations from graphs, and equations from tables and other given information (e.g., from a given point on a line and the slope of the line) “(Indiana Department of Education, 2014, p. 10)	✓	✓
Minnesota	“9.2.1.6 Identify intercepts, zeros, maxima, minima and intervals of increase and decrease from the graph of a function.” (Minnesota Department of Education, 2008, p. 34). “9.2.2.3 Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables, and symbolic representations. Know how to use graphing technology to graph these functions” (Minnesota Department of Education, 2008, p. 36).	✓	✓
Nebraska	“MA 11.2.1.e Analyze and graph linear functions and inequalities (point-slope form, slope-intercept form, standard form, intercepts, rate of change, parallel and perpendicular lines, vertical and horizontal lines, and inequalities)” (Nebraska Department of Education, 2015, p. 28).	X	✓

Table 5
Continued

State	Standards	Creating an Equation	Graphing a Line
Oklahoma	"A1. A.4 Analyze mathematical change involving linear equations in real-world and mathematical problems. A1. A.4.1 Calculate and interpret slope and the x- and y-intercepts of a line using a graph, an equation, two points, or a set of data points to solve real-world and mathematical problems" (Oklahoma State Department of Education, 2016, p. B-20).	✓	✓
South Carolina	"FA.ACE.2* Create equations in two or more variables to represent relationships between quantities. Graph the equations on coordinate axes using appropriate labels, units, and scales. (Limit to linear; quadratic; exponential with integer exponents; direct and indirect variation.)" (SCDOE, 2015, p. 83)	✓	✓
Texas	"The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to...(B) write linear equations in two variables in various forms, including y = mx + b… given one point and the slope and given two points…" (Texas Educational Agency, 2016, p.2).	✓	X
Virginia	"A.6 The student will graph linear equations and linear inequalities in two variables, including a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line. VDOE, 2009, p. 2)."	✓	✓

Note: ✓ denotes the standard relates to the skill. X denotes the standard does not relate to the skill.

(g) develop a data collection form. The Snell and Brown process was used to develop the task-analyses.

A general education math teacher (from the participating district) helped to interpret the standards and provided examples of similar skills performed by general education participants. Although the skills were initially selected from the Virginia state curriculum, both skills were present in other curricula. Table 5 presents the academic standards related to the target skills for the *Common Core Curriculum* and the eight states who used state-generated math standards. For the study, the two target skills were aligned with standards from 49 out of the 50 states.

Create- an-equation. In Algebra I, participants were presented with a graph of a line on a coordinate plane, and they are asked to create an equation (Kilpatrick et al., 2001; Powell & Witzel, 2016). The related standard stated, "A.6 The student will graph linear equations …in two variables…[and]…b) writ[e]… the equation of a line when given the graph of the line.... "(VDOE, 2009, p. 2). To write an equation of a line (in *slope-intercept form)*, the participants needed to use parts of the graph to create the equation. First, students identified two points. The easiest points to identify were the *x-intercept* and the *y-intercept (b)*. Then participants needed to use the two points to calculate *slope (m=rise/run).* Once the participant calculated *slope*, they constructed and placed the identified elements into the correct coordinates within the equation (*y=mx+b)*, where *m* equals the *slope (rise/run=-y-intercept/x*-intercept when the *x-intercept* and the *y-intercept* ≠ *0*[1] *)*, and *b* equals the *y-intercept.* Appendix C presents the skill and the steps needed to create an equation of the line given a graph of the equation.

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[1] Given that the y-intercept ≠0 and the x-intercept ≠ 0. Point 1 is the *x-intercept* (X,0) and Point 2 is the *y-intercept* (0,Y); therefore when using the standard formula for slope $m=(Y_2-Y_1)/(X_2-X_1)$, $m= (0-Y_1)/(X_2-0)$. Simplified the formula becomes $m=-Y_1/X_2$ or *m=-(y-intercept/x-intercept)*

Creating-an-equation. The second skill selected was the inverse skill, that is, creating a line from an equation. The Virginia (2009) standard states, "A.4 The student will solve multistep linear … in two variables, including …d) solving multistep linear equations … graphically" (p. 1). To solve equations graphically, participants needed to identify key parts of the *slope-intercept formula* (*y=mx+b),* where *y* and *x* represent the infinite combination of solutions for the equation, *m (m=rise ÷ run=-y-intercept ÷ x-intercept*) accounts for the slope of the equation, and *b* accounts for the *y-intercept.* To complete the problem, participants must identify the number that represents *slope (m=rise ÷ run),* the *rise* (numerator of the *slope* and the change in *y*), and the *run* (the denominator of the *slope* and the change in *x*). Appendix D presents a sample with the steps needed to create a line from an equation. In general, participants needed to take the formula (identify the *rise* and the *run* of the slope and the *y-intercept)*, and then plot two points before connecting the points to form a line graph.

Data collection tools. Data collection sheets using the task analyses served as a reminder for staff to deliver prompts consistently and was also used in the review process to code data collection. Appendix E shows the data collection sheet for the 10-step task analysis for creating a line from an equation. Appendix F presents the data collection sheet with the embedded 11-step task analysis for creating an equation from a line. All sessions were video recorded and graduate students enrolled in a Ph.D. program reviewed and coded the student performance. For all phases, the graduate students recorded a participant's performance using the following codes: "1" for a step independently completed by a participant, "0" for a step incorrectly completed by participants. As the videos were screened, the graduate students recorded responses on the task-analysis (Appendices E and F). During the intervention sessions, the graduate students recorded 'P' if intervention staff delivered a prompt (for both correct and incorrect prompted responses), and the graduate students recorded an "N" if the participant did not respond. A "1" was recorded for correct independent responses. Camera errors (e.g. blurry images, blocked images, or battery failures) and experimental fidelity errors were recorded with "e." Data coding deviate from the traditional "+" and "-" codes found in most of the behavioral literature because the percentage of steps completed correctly were calculated using Microsoft Excel, and Excel viewed math symbols ("+/-") as spreadsheet codes for functions.

Two experiments. The dependent variables were used differently in the two different experiments. In the first experiment, participants received an intervention for the target skill create-an-equation, and the inverse skill, create-a-line, was monitored for evidence of conceptual understanding. Similarly, for Experiment 2, participants received an intervention for the target skill Create-a-Line and the inverse skill Create-an-equation. A suplimental intervention was provided for the inverse skill if participants did not reach the 60% criterian. Similarly, adding the inverse skill to the experiment allowed the researcher to infer conceptual understanding in the event the participant would be able to construct the sequence of steps without intervention supports. Providing the supplemental intervention for the inverse skill helped to confirm the participants could reach criterion for the inverse skill.

Assignment to experiments. The proposed research protocol planned for more participants than were available, so it was unnecessary to random select participants; however, groups of participants were randomly assigned to experiments. *A*fter three participants were available for the study, a coin was flipped to determine the target and inverse skill for the first group (Experiment 1). By default, the next three participants recruited and screened were assigned to Experiment 2.

Independent variable. The intervention package consisted of randomly generated equations (Table 6) printed on a template (Appendices G & H) containing within-stimulus prompts (salient cues that facilitate correct responding before external prompting), a visual prompting procedure, a self-monitoring list of steps based on the task analysis that presented the steps for a generic math problem (not the assigned problem), and a verbal feedback procedure. The visual prompting procedure consisted of a five-second time delay procedure followed by a modeling (verbal and visual demonstration of the correct action) prompt, and verbal feedback.

Staff training. Five staff were trained and monitored to deliver the intervention accurately and consistently. Three of the staff members were the special education teachers and two individuals were paraprofessionals who each had bachelor's degrees and were working to earn their teacher certification. Before initiating baseline or intervention procedures, the staff practiced the procedures for the two skills (solving one-step equations with multiplication and division, creating a line from an equation, and creating an equation of a line). The staff rehearsed the intervention with role-playing to practice administering prompts. The practice continued until all staff members could (a) complete the math algorithms with 100 percent accuracy using the templates, (b) deliver verbal/modeling prompts to participants within five seconds of presenting a stimulus, and (c) provide verbal feedback to the participant.

Table 6

Sample of Randomly Generated Linear Equations

Session/Trial	*x-Intercept*	*y-Intercept*	*Slope*	Equation
1/A	*9*	*4*	*-4/9*	*y=-4/9 x + 4*
1 B	*-8*	*5*	*-5/-8*	*y=-5/-8 x + 5*
1 C	*-4*	*-2*	*2/-4*	*y= 2/-4 x + -2*
1 D	*5*	*-10*	*10/5*	*y=10/5 x +-10*
2 A	*7*	*8*	*-8/7*	*y=-8/7 x + 8*
2 B	*-1*	*4*	*-4/-1*	*y=-4/-1 x + 4*
2 C	*-2*	*-3*	*3/-2*	*y= 3/-2 x + -3*
2 D	*8*	*-7*	*7/8*	*y=7/8 x +-7*
3 A	*1*	*3*	*-3/1*	*y=-3/1 x + 3*
3 B	*-6*	*3*	*-3/-6*	*y=-3/-6 x + 3*
3 C	*-7*	*-10*	*10/-7*	*y= 10/-7 x + -10*
3 D	*7*	*-7*	*7/7*	*y=7/7 x +-7*
4 A	*2*	*7*	*-7/2*	*y=-7/2 x + 7*
4 B	*-9*	*2*	*-2/-9*	*y=-2/-9 x + 2*
4 C	*-7*	*-10*	*10/-7*	*y= 10/-7x + -10*
4 D	*3*	*-10*	*10/3*	*y=10/3 x +-10*

Note: Variables were randomly generated within parameters using Microsoft Excel.

Implementation Protocol. The two experiments (a) assessed participants' current knowledge of the procedures for both skills and (b) monitored participants' development of procedural fluency of one skill and its impact on the second skill. In Experiment 1, three participants received intervention on one skill (create-an-equation) while monitoring the impact on the second skill (create-a-line). In Experiment 2, three different participants received intervention on the second skill (create-a-line) and monitored its impact on the first (create-an-equation).

Random assignment to experiment. Kratochwill et al. (2012) recommended introducing random assignment to single-case designs. Although the impact of random assignment for a small-*n* design is likely to be negligible, the practice may contribute to later meta-analytical study (Johnson et al., 2008).

After recruiting and screening three individuals, participants were randomly assigned to one of two experimental conditions by coin toss. The first three participants were assigned to Experiment 1, learning how to create an equation from a graph (create-an-equation). Recruitment continued until a second group of participants could be assembled; the second group was assigned to Experiment 2, learning how to create a graph from an equation (create-a-Line).

Baseline protocol. At least five baseline sessions, consisting of four trials each, were conducted before the intervention phase. After providing participants with the initial cue, participants were expected to complete a step of the algorithm. If they did not complete the step, staff performed the step out of the participant's view to provide the natural stimulus for the desired response. To minimize the effects of deferred imitation, a curtain shielded the participant from work as the staff member performed the step. During baseline for both skills, the staff used the following semi-structured script.

- Set up the session with four randomly generated equations or four randomly generated graphs (e.g. Appendices G & H).
- Set up the curtain and the data collection sheets.
- Invite the participant to the work area.
    - Point to the work and say, “Create-an-equation.” or “Create-a-line.”
    - Wait five seconds (tap fingers on the table or leg behind the curtain e.g. count silently, one thousand…).

If the participant completes the step correctly,

- Record a “1” on the data sheet, and say, “What’s next?”

If the participant does not complete the step correctly (no response or incorrect response),

- Record a “0” for an incorrect response or an “N” for a non-response on the data sheet, and
- Shield the work behind the curtain, and complete the step for the participant.
- Present the revised problem to the participant, and say, “What’s next?”

Repeat until all steps are completed (create- a- line had 11 steps; Create-an-equation has 10 steps).

Transitioning between baseline and intervention phases. Experiment 1 began before Experiment 2, and within each experiment, all participants started in the baseline condition. Everyone moved from baseline to intervention individually in a staggered fashion. Participant 1 started intervention, and Participant 2 and Participant 3 remained in baseline to ensure the intervention from Participant 1 was not impacting Participants 2 or Participant 3. When baseline was stable for Participant 2, intervention began, while Participant 3 remained in baseline. To avoid constant exposure to baseline (i.e., non-instructional, non-reinforced, testing) trials, periodic baseline probes were used instead of continuous baseline sessions.

During the target skill intervention, participants continued baseline probes for the second, inverse skill until they had reached Snell and Brown's (2014) acquisition stage of learning (60% criterion) on the target skill, and they had completed a minimum of five intervention sessions. The baseline probes on the inverse skill were used to determine whether a participant required direct intervention to improve performance. Therefore, if the participant performed below the 60% criteria for the inverse skill during the target intervention condition, the participant transitioned into a supplemental intervention for the inverse skill.

Intervention protocol. The intervention phases included verbal feedback, modeling prompts, and participant access to the list of steps. The intervention also included a template to work on the problems and a flip book with the task-analysis and a model picture of a similar problem being solved. The same intervention protocol was used for both skills.

- Set up the session with four randomly generated equations or four randomly generated graphs.
- Set up data collection sheets.
- Invite the participant to the work area.
- Point to the work and say, "Create-an-equation." or "Create-a-line."
- Wait five seconds (tap fingers on the table or on leg behind the curtain).

If the participant completed the step correctly,

- Provide verbal feedback (e.g., "Good Job! I see that you used the directions to ________").
- Say, "What's next?"

If the participant did not complete the step correctly (no response or incorrect response) in five seconds,

- Say, “That is not correct. Let me show you the step.”
- Model and verbally describe executing the step. (e.g., say, “I am looking at the directions. I see I should _____, so I am going to ________.”
- Undo the step, and present it to the student. “Now it is your turn; you are going to look at the directions and try to follow the steps.”

If the student missed the step again,

- Say “I am sorry, that isn’t correct.” I am going to reset this, and we are going to move forward”
- Correct the work, and (behind the curtain)
- Present the completed step to the participant, and say, “What’s next?”

If the student completed the step say, “What’s next?”

- Repeat until all steps are completed (creating-a-line has 10-steps; creating-an-equation has 11 -steps).

Booster protocol. If a student did not reach criterion (60%) after five to seven intervention sessions, the participant engaged in at least three booster sessions. The protocol for the first booster phase modified the intervention protocol by limiting the formulas and the graphs to the first quadrant on the coordinate plane and by adding color prompts to the templates (see Appendix I). If the participant participant’s performance stabilized below the criterion. If the participant’s performance stabilized below the criterion after three sessions with no growth, or after the participant engaged in seven sessions, the protocol was adjusted again. During the second booster phase, the booster protocol was adjusted, and the templates were adjusted. Blank spaces in the template were replaced with color coded Velcro™, and key parts of the formula were replaced with laminated cards that could be affixed to the Velcro™ spots. If the participant failed to achieve criterion after participating in the second booster session, the protocol was simplified, to permit modeling of the steps in front of the participant (instead of behind the curtain).

Testing conditions protocol. Participants in the study were also participating in the Virginia Alternate Assessment Program, and if the skill was performed under testing conditions, the work samples could be included in the participant's alternate assessment portfolio. In the baseline protocol adjusted the template (behind a curtain) to permit the participant to perform the next step correctly; testing conditions were different because the protocol did not permit staff to interact with the template. In the testing conditions phase, participants were provided with the materials (template, math problem, ruler, and writing items), and staff provided the initial stimuli (e.g. "Create-a-line"). Participants were observed, and the session ended when the participant stopped working, more than five minutes had elapsed, or the student indicated they had completed the task. When the participant indicated they were finished with the problem, the staff collected the work.

Threats to validity. This study controlled for both internal and external threats to validity. Threats to internal validity that obscure valid conclusions about the data within the study were controlled by ensuring that data collected on the dependent variable were reliable (through interobserver agreement) and that the treatment phase of one participant did not affect the performance of another (Gast & Ledford, 2014, Kennedy, 2005). Threats to external validity that prevent generalizing results to a broader population of individuals or conditions were controlled by replicating treatment with multiple participants (Gast & Ledford, 2014, Horner et al., 2005; Kratochwill et al., 2012).

***Internal validity and experimental control*.** This study maintained experimental control for the target skills in Experiment 1 and Experiment 2 by using a multiple-baseline across participants design. Experimental control was established when the researcher staggered the introduction of the intervention for individual participants by conditionally restricting the intervention. Only one participant moved from the baseline to intervention procedure at a time, and the move only occurred after that participant had demonstrated a stable baseline concurrent with the previous participant's intervention (Horner et al., 2005; Kratochwill et al., 2012). Additionally, the use of multiple participants and multiple experiments embedded replication within the study. Replication increases the internal and external validity respectively (Horner et al., 2005).

***External validity*.** Horner et al. (2005) asserted that external validity could be established in single-case study designs with replication of the treatment phase across participants, materials, or settings, in this case, across participants. Johnson et al. (2008) noted that replications of single-case designs assists in meta-analytical studies, and stressed the importance of providing detailed descriptions of the experiment and the participants to permit such statistical comparisons across studies. This study included detailed descriptions of the intervention package (materials protocol) to facilitate replication.

Interobserver reliability. All the experimental sessions were video recorded, assigned a session number, digitally encrypted, and saved onto an external hard drive. The videos were then viewed and coded by the researcher using the task analysis as a guide. Additionally, a computer randomly selected just over 20% of the videos to be coded for a second time by one of three doctoral level graduate students at the Virginia Commonwealth University. The data collected by the graduate students via video, were compared to the data collected by the staff during the sessions. Interobserver reliability was calculated using a total agreement formula (Interobserver Reliability = Total Agreements / Total Observations X 100) (Hartman, 1977; Kennedy, 2005).

Fidelity monitoring. Appendix J includes the fidelity monitoring checklist used for each session. The researcher used the check-list during each session, and monitored the intervention daily. On two occasions, the researcher noted the intervention staff made a mistake providing feedback to the participant. Specifically, the errors were mathematical in nature. The staff incorrectly identified the *x-intercept as the y-intercept,* and then the staff provided incorrect feedback to the participants (inverted the *x-intercept* and *y-intercept).* In both cases, the errors were coded as "e" on the task analysis, and the researcher paused the sessions for one day to recalibrate the intervention staff to the task. In addition to the daily fidelity checks, a random sample of 20% of the trials were reviewed by the researcher and VCU graduate students.

Social validity**.** Social validity is established when the activities for the participants facilitate their meaningful engagement in the larger community (Kazdin, 2011). Wolf (1978) stated that research should meet the goals for, "…what society really wants" (p.207). Congressional legislation continues to require schools to include students with ID in the general education curriculum, including access to algebra instruction. Access to algebra instruction remains a civil rights issue (Moses et al., 1989, Kress, 2005), and ESSA (2016) retains language requiring students with ID to access the academic high school curriculum. Research that explores algebra instruction for students with ID is part of the *AAIDD National Goals for Research* (Thoma et al., 2015). Courtade et al. (2012) noted academic achievement is a goal of society and Jorgenson (2005) stressed the importance of maintaining high academic standards.

The participants and a panel of experts was utilized to establish social validity. The panel included: (a) three general education, high school mathematics teachers, (b) three special education teachers, (c) two general education students who completed algebra I, and (d) two parents of a high school student with ID. Each member of the panel completed a continuously ranked scale to respond to questions about the interventions and the skills (Disagree to Agree). Appendix K includes the statements from the panel of experts, and a second continuous scale, Appendix L, presents the statements for the participants.

Analysis

Data collection and security. Participants were assigned a pseudonym which was the only direct identifier attached to the two forms of data collected. First, participants were videotaped during all sessions as they complete the tasks. Second, electronic spreadsheets (Microsoft Excel, 2016) were used to record performance data, interobserver reliability, fidelity of implementation, and social validity documented by the staff and observers on the appropriate data sheets.

The following security protocols were observed. First, the electronic videos were maintained on an encrypted hard drive. Only the researcher, primary investigator, dissertation committee members, and specified graduate students associated with the project could access to the files. Excel spreadsheets were created without direct identifiers and/or VCU's secure online data management system, REDCAP™ (Harris et al., 2009) was used for the social validity surveys.

Data analysis. The raw data were converted to percentages and then graphed for visual analysis using Microsoft Excel. The percentage of correct responses was calculated for each trial in each session for all phases of each experiment (baseline, target skill intervention, inverse skill, and testing conditions). Line graphs were created to show individual participant's results across time. The graphs presented a single line per participant across all the phases. Session percentages for participants' target skill are presented in Figure 2 (Experiment 1) and Figure 3 (Experiment 2). The graphs present the dependent variable (percentage of steps completed) for the target skill (create-an-equation or create-a-line) on the vertical axis, and the independent variable (conditions) will be presented across time on the horizontal axis. Vertical condition lines will demarcate the different phases of the trial (baseline, target Intervention, & testing conditions). In one case, booster interventions were also presented.

Additionally, a second series of graphs are presented in Figures 4 to Figure 9. Each figure presents a graph of the percentage of steps completed for the target skill above a graph of the percentage of steps completed for the inverse skill. Vertical condition lines separate the baseline, target intervention, inverse intervention, booster phases (if applicable) and testing conditions phases. The graphs were used for visual analysis.

Using visual analysis, the researcher examined each individual participant's performance within each phase of the trial. Visual patterns of change between phases due to treatment effects included observations within-conditions and across conditions. Changes in the percentage of steps completed (level), the trend (slope), range in performance between trials and sessions (variability) were assessed; and post-intervention observations examined the immediate change (immediacy of effect), and any changes to other participants' performances (overlap) (Gast & Ledford, 2014), as well as the speed of change (slope).

Although not common in single-case designs, statistical analysis can be performed when enough data are present (Kratochwill et al., 2010; Shadish et al., 2015). Horner et al., (2005) noted the statistical analysis can be used, but it should be used to complement not supplant the analysis. In this study, statistical analysis helped to determine if findings were due to random chance (Howell, 2010). The interpretation of the data depended on the research question. Specifically, the researcher examined the data to compare the individual participant's performance between the baseline and intervention phases. To supplement the visual analysis, the researcher used descriptive statistics to compare the within-condition means, medians, maximum and minimum numbers; to ensure that random chance did not contribute to the change in performance across three conditions a Repeated Measures Freidman's Analysis of Variance (RM-Friedman's ANOVA) supplemented the analysis.

Delimitations

Several delimitations helped to eliminate threats to internal validity. First, the study excluded participants who required direct staff support for communication (interpretation) or physical movements (hand-over-hand assistance). Sign language interpretation and/or hand-over-hand supports could inadvertently function as visual and physical prompts. Failing to control for these inadvertent prompts would have compromised experimental control. Second, the study intentionally focused on two entry-level algebra skills. This decision reduced the number of algorithmic derivations participants needed to employ to solve problems during the study. For example, participants were not required to reduce fractions because the skill would have required a separate task analysis as well as introduce possible threats to construct validity. Third, the study only focused on two of the mathematics cognition strands described by Kilpatrick et al. (2001) to allow for close monitoring of the relationship between procedural fluency and conceptual understanding. Finally, the study took place in a single school. Limiting the study to one location reduced the number of staff delivering the content, thus minimizing variability with delivering the intervention (Snell & Brown, 2014; Stokes & Baer, 1977).

Summary

This study explored the potential of students with ID to participate in the academic, standards-based curriculum for high school algebra. Using a single-case, multiple-baseline across participants, experimental design, the study determined that participants with ID could improve both procedural fluency and conceptual understanding in algebra. The study focused on two algebra skills, (a) create a linear equation from a graph (create-an-equation) and (b) create a graph from an equation of a line (create-a-line). The researcher provided six participants with ID an intervention package that included a task analysis, a self-monitoring strategy, a time-delay procedure, a template, and verbal feedback. Measurement of participant performance was documented using the task analysis of each skill, and a visual analysis of data was used to determine the participants’ progress.

Chapter IV

Results

Overview

This study explored the effects of an evidence-based intervention on the ability of six participants with ID to acquire linear algebra I skills as they prepared for the *Virginia Alternate Assessment* (VDOE, 2016). The study also measured the impact of skill acquisition and procedural fluency on the related inverse of the skill (the other target skill). A multiple-baseline design across participants was utilized. Two groups of three participants each were randomly assigned to either Experiment 1 where the target skill was to *create-an-equation* or Experiment 2 where the target skill was to *create-a-line*. In both experiments, the inverse skill (i.e., the other target skill) was monitored while the target skill was developed with the intervention protocol.

Reliability

A procedural checklist was used by the school's staff, and 100% video recordings of the session were reviewed by the researcher. Additionally, a random sample of 20% of the videos was reviewed by three doctoral level students. During the review process, the procedural reliability (fidelity) was established and the interobserver agreement was calculated.

Procedural Reliability. Procedural fidelity was measured using a fidelity checklist (Appendix J) across a random sample of 20.15% of the sessions and trials. Overall, fidelity to the intervention was 93.01%. Most of the error (4.54%) was due to issues related to the implementation of the Constant Time Delay procedure; staff tended to offer more time to the participant than the prescribed five seconds. A smaller number of errors were related to staff errors in mathematics (1.63%) with the remainder of the errors (0.82%) related to camera issues or template misprints, in which a response could not be reliably observed. In all cases, the procedural reliability was above the recommended 80% (Gast & Ledford, 2014; Kennedy, 2005).

Interobserver Agreement. Each video session was reviewed and coded by the researcher, and a group of doctoral level students reviewed and coded a random sample of 20.15% of the sessions. The coding results were compared, and the total percentage of agreement between the observers was 94.70%. In single-case designs, interobserver reliability must be greater than 80% (Gast & Ledford, 2014; Kennedy, 2005).

Question 1

When provided with evidence-based behavioral intervention and adapted materials, will participants with ID acquire:

- procedural fluency in creating an equation from a picture of a line?
- procedural fluency in solving a linear equation to create a line?

To answer the research question, two separate experiments were conducted. In Experiment 1, instructors provided a picture of a line, and asked each participant to, “Create the equation.” In Experiment 2, staff provided an equation and asked each participant to, “Create a line.”

Experiment 1. Experiment 1 addressed the first part of the research question, i.e., to determine if procedural fluency improved when individuals were asked to create an equation from a picture of a line. In Experiment 1, three participants were trained to create-an-equation across three conditions (baseline, intervention, & testing) for a minimum of three sessions for each phase. The data were processed, coded, and line graphs were created in Microsoft Excel™ for visual analysis with line graphs presenting the percentage of steps completed during a session across time (weeks).

The visual analysis was conducted to look for within and across condition changes. Across participants and conditions, experimental control was maintained. In single-case designs, comparisons across conditions are essential, because the change occurring from one condition (independent variable) to another helps to establish a causal link between the dependent variable and the independent variable (Gast & Ledford, 2014; Kennedy, 2005). Figure 2 presents the line graphs used for the visual analysis in Experiment 1. A separate graph (Figure 3) presents the trend lines; to allow a standard visual analysis comparison between participants, the trend lines were calculated for the percentage of correct steps across sessions. Trend lines were calculated using the Least-Squares Regression Method described by Kennedy (2005, pp 99-100) because the method only requires three points of data, and the visual analysis compared the direction of the slope (positive, neutral, or negative).

Ed. Participant 1, Ed engaged in with 15-minute-long sessions for a maximum of three days a week for eleven weeks. Ed was unavailable to participate in the study on three occasions. In week three, Ed missed a session due to a suspension, and in week 7 the study was paused because the program was conducting community based vocational assessments. Ed completed his participation with the target skill before the spring break (week 11).

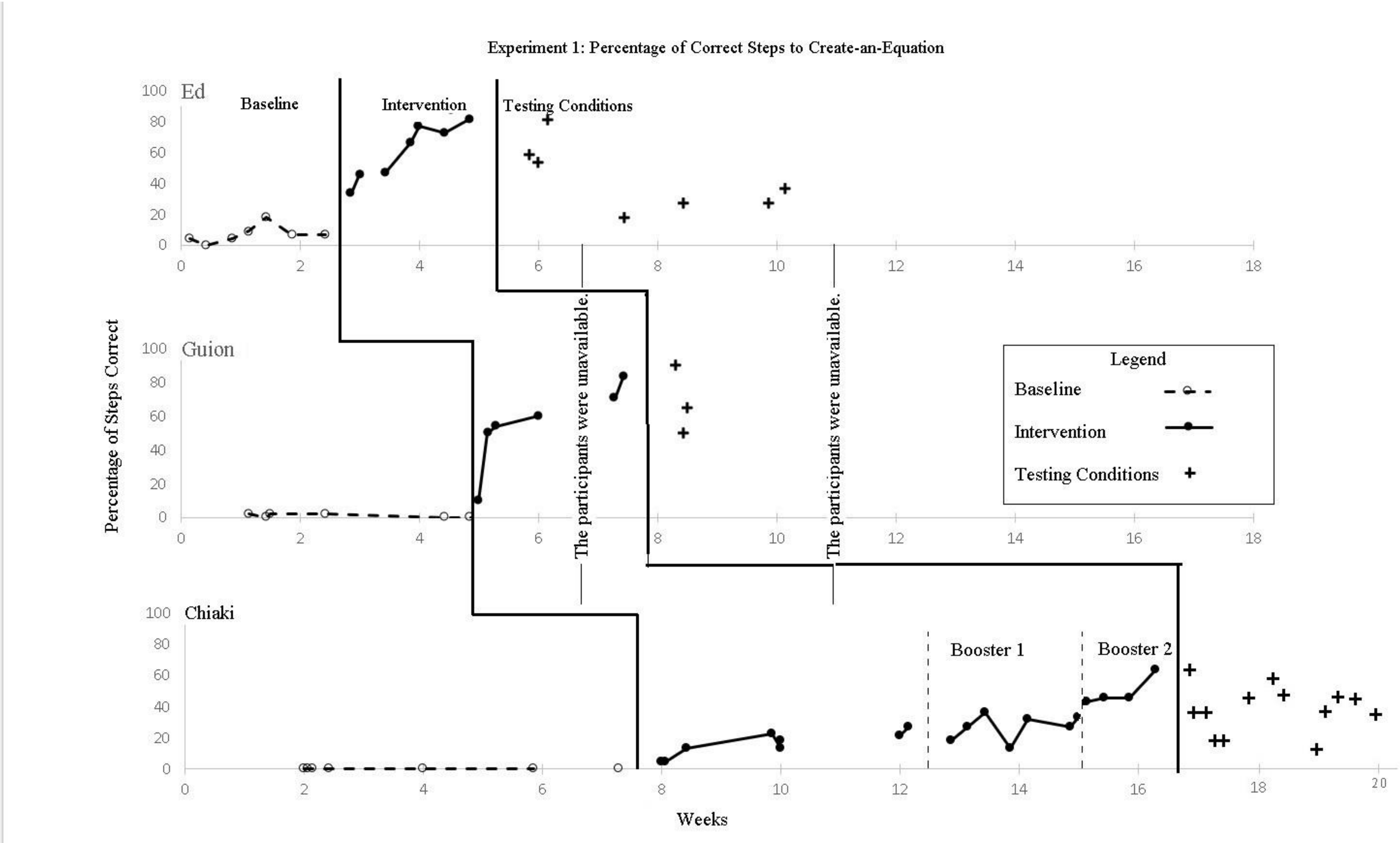

Figure 2. Experiment 1 graph of participant performance (percentage of correct steps) across time (weeks).

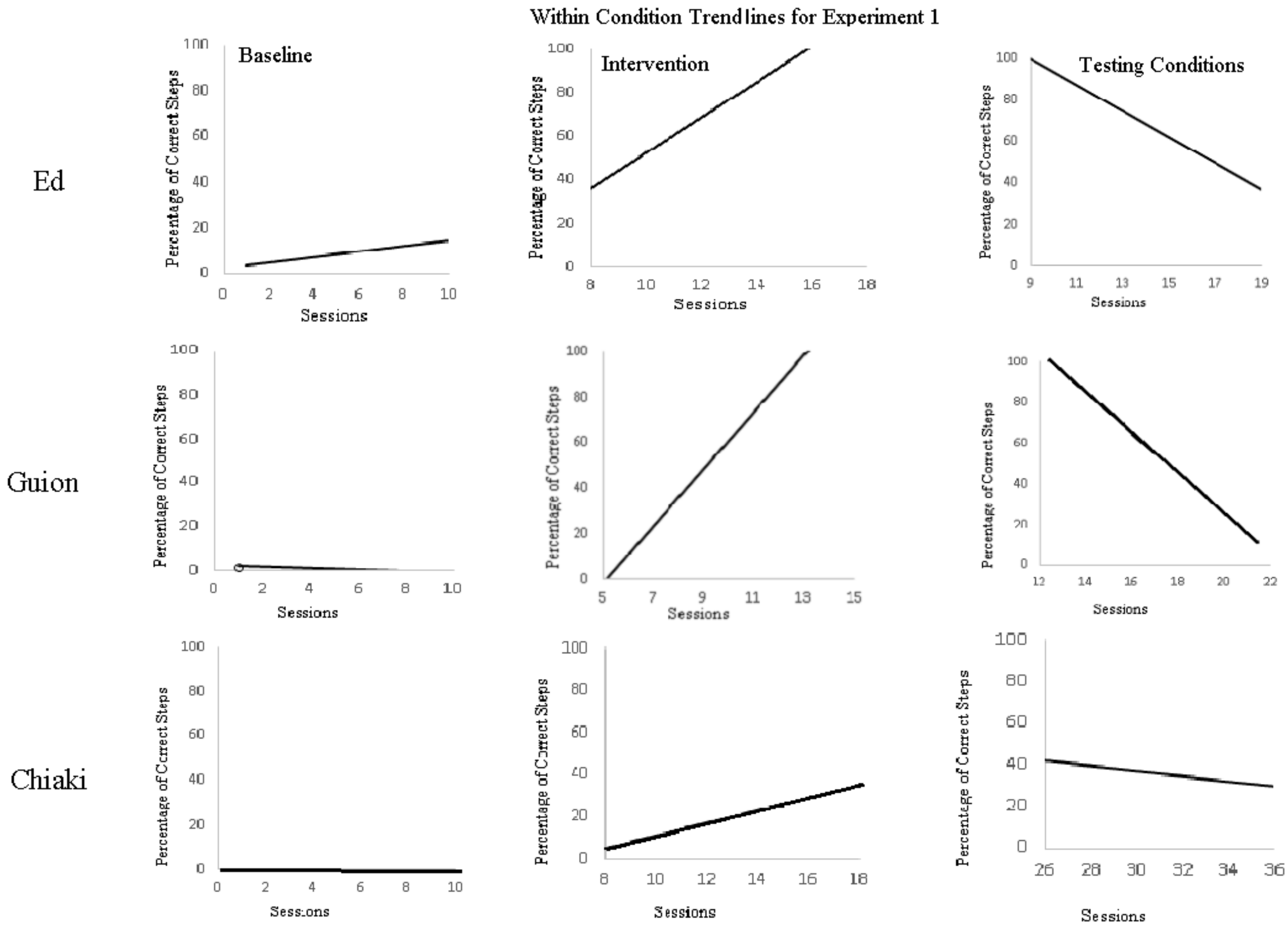

Figure 3. The within-conditions (baseline, intervention, and testing conditions) trendlines for Experiment 1 (Create-an-Equation). Trendlines were calculated using the Least-Squares method across sessions (instead of weeks) using MS Excel.

Visual Analysis. Ed participated in seven baseline sessions. Figure 2 presents Ed's performance data. A stable baseline was achieved after seven sessions with an overall positive slope. Although Ed initially demonstrated a negligible percentage of steps (<20%), he finished the baseline sessions with two consecutive low scores (<7%). Immediately after starting the intervention protocol, Ed's performance increased to 34%, and continued to rise steadily over the next four sessions (45%, 47%, 67%, & 77%). Growth continued a positive trend with Ed completing the intervention condition with 82% of the steps completed accurately. The intervention protocol was not in effect during the testing condition phase. Ed's initial performance declined from the 82% on during the last intervention session to 59%, 54%, and 81% during the first three sessions of the testing conditions. After a pause in the study (due to the unavailability of the participant), Ed's performance continued to decline (18%, 27%, 27%, & 36%). A visual analysis of the within-condition trends (Figure 3) confirmed the visual analysis of the performance data. In baseline Ed's performance had a slight positive trend; however, the rate of changed increased during intervention, and Ed's performance switched to a negative trend during the testing conditions phase.

Descriptive statistics. The descriptive statistics supported the visual analysis. The within-condition for the percentage of correct steps with means(M), medians (Med), minimums (min) and maximum (max) are presented in Table 7; the table also presents the trend line formula, with the Pearson product-moment correlation (*r*), and the formula accuracy (r^2). During baseline, the percentage of correct steps for Ed ranged between 0% and 18% for the percentage of correct steps. The slope during the baseline condition was positive but shallow (0.5). The median (6.81%) matched the two final data points for the condition, and the within-baseline-condition mean was 7.13%. The was higher in the intervention condition (67%). Ed increased performance during the intervention condition from a minimum of 34% to a maximum of 81%. The within-intervention trend line slope was high (8.1); however, Ed's performance started to plateau after five intervention sessions. During the intervention, the median score for Ed was 67%, and the mean score was 60. Within the testing condition, the intervention was withdrawn for the primary skill (create-an-equation), and the performance decreased *(*M=43%; Mdn= 36). Ed's performance in the testing condition had an overall negative slope (-6.2) with a maximum of 85% and a minimum of 26% of the steps completed independently.

Changes occurred between conditions. His performance increased from a mean of 7% during baseline to a mean of 61% during intervention. Similarly, his accuracy in performance decreased as Ed moved from intervention (M=61%) to the testing conditions (43%). The slopes for each trend line also changed across conditions. The trendline slope increased from baseline (1.2) to intervention (8.4) and testing conditions (-6.26). The variability or accuracy of performance also changed across conditions with greater variability in the baseline condition (r=0.47) and the testing condition (r=-0.61) compared to the intervention condition (r =0.95).

Guion. Participant 2, Guion, engaged with the first part of the experiment for nine weeks. He was unavailable for large blocks of time because he participated in a week long vocational assessment with a regional residential program (week 7). Additionally, Guion participated in a week-long out-of-state chorus trip during week 10, and spring break during week 11. Guion's fine motor skills interfered with his ability to manipulate the self-monitoring notebook consistently; he sometimes requested help to turn the pages of the self-monitoring notebook.

Table 7

Experiment 1: Descriptive Statistics & Within-condition Trendline Formulas

	M	*Min*	*Mdn*	*Max*	*n*	*Trendline Formula*	*r*	r^2
Ed								
Baseline	7.12	0.00	6.81	18.18	7	y= 2.24 + 1.22x	0.47	0.22
Intervention	60.71	34.09	66.67	81.80	7	y= -28.83 + 8.14x	0.95	0.91
Testing	43.31	18.18	36.40	81.00	7	y= 156.01 - 6.26x	0.61	0.37
Guion								
Baseline	1.13	0.00	1.10	2.30	6	y= 2.23 - 0.31x	0.47	0.22
Intervention	54.75	9.98	57.00	84.09	6	y= -64.07 + 12.51x	0.93	0.87
Testing	69.95	50.00	67.70	90.20	3	y=226.19 - 10.07x	0.75	0.57
Chiaki								
Baseline	0.00	0.00	0.00	0.00	7	y=0	na	na
Intervention	15.58	4.40	15.80	27.00	8	y= -18.10 + 2.93x	0.87	0.75
Booster 1	26.80	13.60	27.20	36.60	7	y= - 0.08 + 1.42x	0.37	0.14
Booster 2	49.43	43.10	45.50	63.60	4	y=-101.25 + 6.15x	0.84	0.7
Testing	31.07	9.90	36.00	45.40	11	y= 74.36 - 1.25x	0.37	0.14

Note: Descriptive statistics includes the Means (M), Median (Mdn), Minimum (Min), Maximum (Max), and number of samples *(n)*. Trendlines were calculated using the Least Squares Method *(y=a + bx)* across sessions, and *Pearson* product-moment correlations *(r)* were reported as not applicable (na) for horizontal lines. Trendline accuracy was reported as the coefficient of determination *(r^2)*.

Visual Analysis. Figure 2 presents Guion's performance during Experiment 1 for the create-an-equation skill. Probes were conducted during the baseline phase, and his performance was stable with all six probes showing less than 2.5% of the steps completed for each session. Guion's stable baseline continued throughout Ed's intervention, and did not change until Guion began intervention. During the first intervention session Guion's performance increased to 9.8%, and the performance increased sharply to 50% for the second session. Guion continued to demonstrate an increase in performance (54%,60%, 70%, & 80%). Notably, the increase continued during the seventh week when Guion was participating in a vocational assessment; algebra instruction did not take place during the vocational assessment. The deadline for Guion's state assessment limited his ability to participate in the testing session to three sessions. Guion's increase in performance continued for the first testing session (90%) before decreasing to 50% and 60%. Figure 3 presents the within-condition trendline changes for Guion. The slope changed from a negative during baseline, to a positive during intervention, and a negative during the testing conditions.

Descriptive statistics. The visual analysis observations for Guion were confirmed by the with the descriptive statistics (Table 7). The first condition, baseline, included six sessions with a small negative trend line slope (-0.31). Scores ranged from a minimum of 0% to a maximum 2.3%. The mean score for Guion during the intervention was 1.13%, and the median was 1.1%. Guion participated in six intervention sessions. Within the intervention condition, Guion increased independent performance from a minimum of 10% to a maximum of 84% during the final session. The mean for the intervention condition was 54.75%, and the median was 57%. The trend-line slope for within-the-intervention conditions was 12.5. Guion's testing period was limited to three sessions. The maximum score was the first score of 90.2%, the minimum score was 50%. Within-the -testing condition, the trend-line was negative (-10.1). Overall, Guion's performance during the testing conditions remained higher than the criterion (60%). The mean during the testing condition was 69.95%, and the median was 67.7%.

The trendlines confirmed changes in Guion's performance changes within-conditions. The trendlines changed from a negative slope during baseline to a positive during the intervention, and back to a negative across conditions. The rate of change increase from the baseline condition (-0.31) to the intervention (12.51); the change decreased as Guion moved from intervention (12.51) to the testing conditions (-10.07). The variability was larger during baseline (r = 0.47) compared to the intervention (r=0.93), and variability increased as Guion moved from intervention to the testing conditions (r=0.75).

Chiaki. Participant 3, Chiaki, required accommodations to participate in the experiment. She required more time to complete each task, so the number of problems presented in each session were reduced (two items for the target skill and one item for the inverse skill), additionally, Chiaki required two booster phases to achieve criterion. Chiaki's intervention was interrupted in week 11 (spring break). The graph of Chiaki's performance is available in Figure 2.

Visual analysis. Chiaki maintained a flat performance with 0% of the steps completed across the seven baseline sessions. This level of performance continued during Ed's and Guion's intervention phases. When intervention began for Chiaki. there was an increase in performance with a steady but positive trend upward; however, after eight sessions Chiaki had not achieved criterion, so a booster session was added. Color prompts were added to the template for the booster session, and Chiaki's scores initially showed an increase; however, the overall trend after seven additional sessions showed that her performance stabilized around 30% (below criterion).

A second booster session was added which included a template with the color prompts and concrete manipulatives. There was an increase in performance and on the fourth session Chiaki's performance had increased to criterion. Criterion was maintained for one session during the testing condition; however, Chiaki's performance decreased. Her lowest scores during the testing condition were higher than her preintervention scores in baseline. Visual analysis of the trendlines confirmed changes between conditions. During baseline the slope was 0, in intervention the slope was positive, and during the testing condition the slope was negative.

Descriptive Statistics. Table 7 presents the descriptive statistics. Across all seven of the baseline measurements, Chiaki performed 0% of the steps; the median, minimum, maximum, and slope were all zero. During the eight intervention sessions, Chiaki's scores ranged from a minimum of 4.4 % to a maximum of 27%; the median score was 15.8%, and the mean was 15.6%. The within-conditions trend line showed a positive slope of 2.93 ($r=0.87$). During the first booster condition the minimum score was 13.6% and a maximum of 36.6 %. The median for the condition was 27. 2%, and mean was 26.80; however, the rate of changed decreased and the slope was 1.42 ($r=.37$). Within the second booster condition the descriptive statistics increased. The mean increased to 49.4 %; the median increased to 45.5%, and the minimum (43.1%) and maximum (64.6%)scores also increased. The rate of change was greater for the second booster condition with a slope of 6.15 ($r= .87$) compared to the intervention condition where the slope was 2.93($r=.87$).

Between participants. Importantly, there was no evidence of between-participant effects. Each participant remained stable during baseline conditions as other individuals were under intervention conditions. Specifically, Guion's baseline performance remained at 0% during the two sessions of intervention that occurred after Ed began intervention. Similarly, Chiaki's

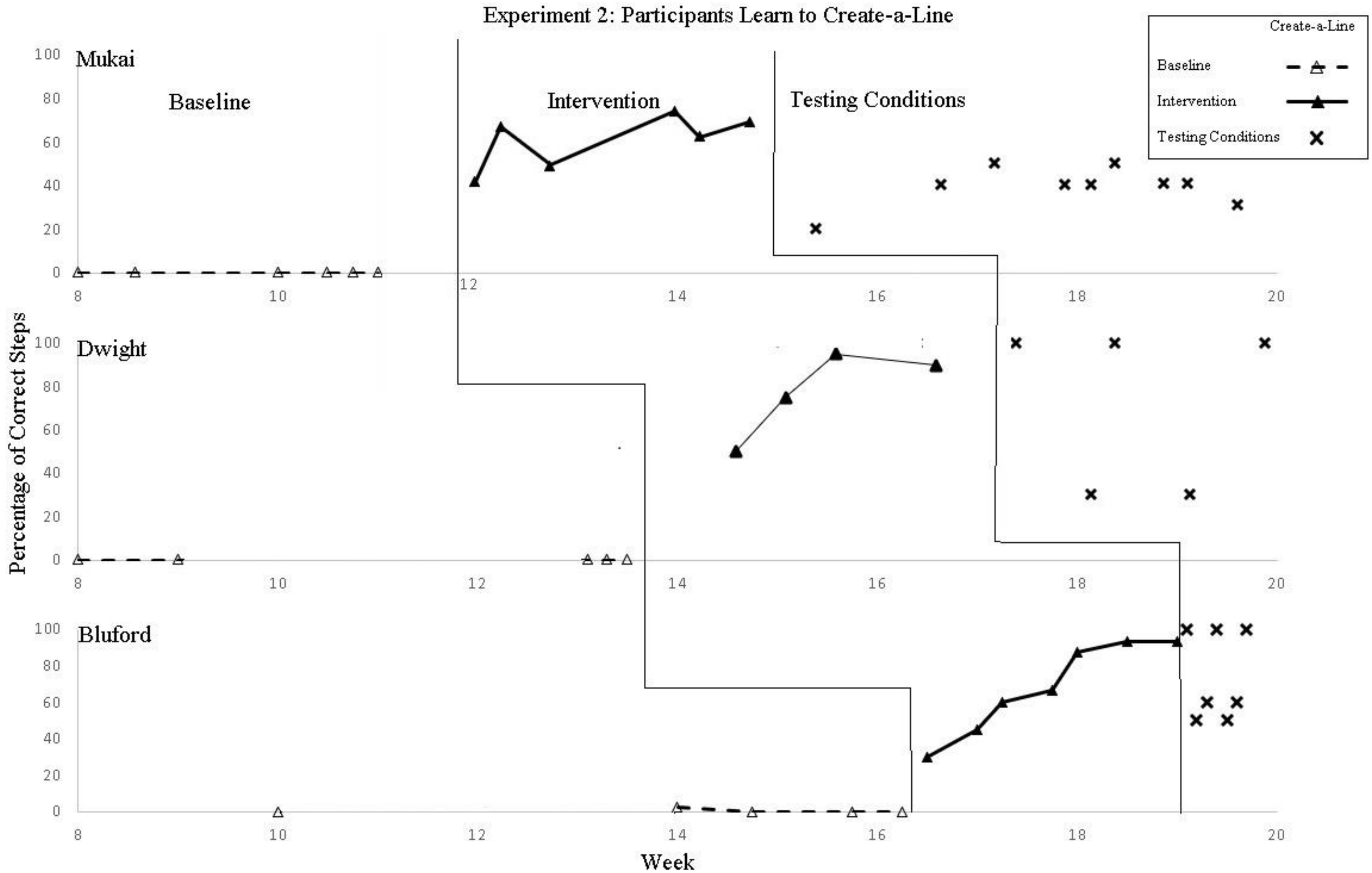

Figure 4. Percentage of correct steps presented for participants as they learned to create-a-line from a formula.

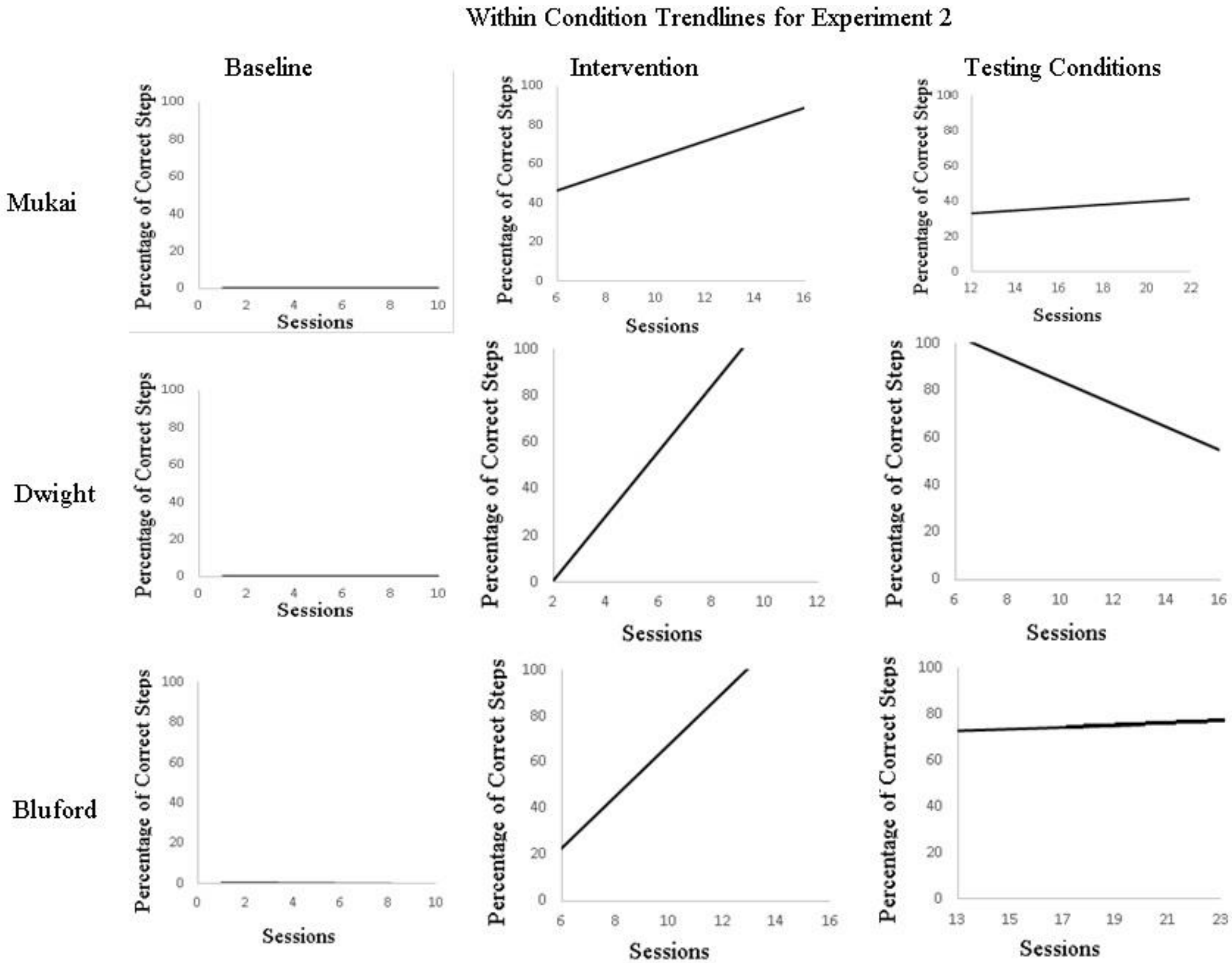

Figure 5. Within-condition Trendlines for Experiment 2. Trendlines were calculated with the Least Squares Method across session.

baseline performance remained at 0% throughout Ed's intervention and during the overlap with Guion's intervention. The effects of intervention were demonstrated for all three individuals.

Experiment 2. Participant were assigned to the skill (create-an-equation) in Experiment 1 by a coin flip, so by default, the next three participants were assigned to the second skill (create-a-line) for Experiment 2. The second experiment used the same intervention, the same procedure, and the same measurement tools; however, Experiment 2 targeted at the skill of creating-a-line for intervention, and the create-an-equation skill was monitored indirectly. Again, the three participants were observed learning the target skill create-a-line across three conditions (baseline, intervention, & follow-up). Again, each participant engaged in a minimum of three sessions per condition. The same methods for coding, and creating graphs for visual analysis were utilized, and Figure 4 presents the percentage of steps completed by each participant for each session, and the graph was used for the visual analysis, and Figure 5 presents the trend lines. Table 8 presents the descriptive statistics and trendline formulas.

Mukai. The fourth participant in the study, Mukai, started baseline measurements during week 8, but he did not begin intervention until after the spring break (week 11). His performance is presented in Figure 4, and the trendlines used for the visual analysis are presented in Figure 5. The descriptive statistics are presented in Table 8.

Visual analysis. Across the baseline condition, Mukai completed zero steps, and he showed no improvement. When intervention began he demonstrated an increase in performance, and his growth continued with some variability throughout the intervention condition. When the testing condition began, Mukai's performance decreased immediately, but his performance remained relatively stable within the condition. His testing condition scores were higher than the

Table 8

Experiment 2: Descriptive Statistics and Trendlines

	M	*Min*	*Mdn*	*Max*	*n*	*Trendline Formula*	*r*	*r2*
Mukai								
Baseline	0.00	0.00	0.00	0.00	6	y=0	na	na
Intervention	61.33	42.50	65.25	75.00	6	y=20.89 + 4.27x	0.40	0.28
Testing		20	40	50.00	9	y=22.89 + 0.85x	0.28	0.08
Dwight								
Baseline	0.00	0.00	0.00	0.00	5	y=0	na	
Intervention	77.50	50.00	82.50	90.00	4	y= -27.5 + 14x	*0.94*	0.88
Testing	72.00	30.00	100.00	100.00	7	y=132.56 - 4.89x	-0.26	<0.00
Bluford								
Baseline	0.42	0.00	0.00	2.50	5	y= 0.1 + 0.67 x	-0.13	0.2
Intervention	67.98	30.00	66.70	93.33	7	Y=-44.1 + 11.28x	0.98	0.95
Testing	74.29	50.00	60.00	100.00	7	y= 68.21 + 0.36x	0.03	<0.01

Note: Descriptive statistics includes the Means (M), Median (Mdn), Minimum (Min), Maximum (Max), and number of samples *(n)*. Trendlines were calculated using the Least Squares Method *(y=a + bx)* across sessions, and *Pearson* product-moment correlations *(r)* were reported as not applicable (na) for horizontal lines. Trendline accuracy was reported as the coefficient of determination (r^2).

baseline scores. A visual analysis of the trendlines (Figure 5) shows a horizontal line for the bassline condition, with an increasing slope during intervention. The trendline slope during the testing conditions was also positive; however, the rate of change was less than the rate of change during the intervention session.

Descriptive statistics. Table 8 presents the descriptive statistics of Mukai's performance. Descriptive statistics for Mukai's performance within each condition is presented in Table 8. Mukai completed 0 % of the steps during each of the six baseline sessions. Changes occurred Mukai's performance improved over baseline immediately after the intervention began changing from 0% to 42.5%). During intervention, Mukai's scores ranged from a minimum of 42.5% to a maximum of 75% with a mean score of 61.3% and a median score of 62.5%. The rate of change during the intervention as indicated by the trendline slope was 4.26. During the testing sessions, Mukai's scores remained higher than the baseline scores but lower than the intervention scores. The statistics ranged between a minimum of 20% to a maximum of 50% with a median of 40% and mean of 38.9%. Mukai's trendline during the testing sessions was positive with a slope of 0.85. The trendline statistics helped to quantify the variability within Mukai's performance. During the intervention Mukai was less variable (*r=.40)* than during the testing condition (*r=.28).*

Dwight. The fifth participant in the study, Dwight, engaged with the study for 12-weeks in the spring semester. The graph of his performance is presented in Figure 4. Dwight's within-condition trendlines are presented in Figure 5.

Visual analysis. Dwight completed 0% of the steps for each session of the baseline condition was 0%. His performance remained stable with a score of 0 % after Mukai started the intervention protocol. During the intervention condition, Dwight's performance increased steadily across the four sessions until 90% of the steps were mastered. When the intervention ceased, Dwight's performance continued to improve, and he achieved 100% accuracy in the first session; however, within the condition his fluctuated between 30% accuracy and 100% accuracy. A visual analysis with the trendlines (Figure 5) confirmed differences between conditions. The trendline during the baseline condition was horizontal with 0% of the steps completed; however, during intervention, the rate of change was positive. During the testing condition, Dwight continued to demonstrate improvements; however, the rate of change decreased, and the trendline was nearly horizontal.

Descriptive statistics. Descriptive statistics of Dwight's performance also confirmed the changes. Table 8 presents the descriptive statistics. Dwight participated in five baseline sessions, and in all sessions, he completed 0% of the steps correctly. During the intervention condition, which included four sessions, Dwight's performance ranged from 50% to 90% accuracy with a mean score of 77.5% and a median of 82.5%. Across the intervention condition, the change in performance showed a slope of 14 with little variability (r=0.94). In the testing condition, Dwight's performance across seven sessions varied between a maximum of 100% and a minimum of 30% (r=-0.04) with a within-conditions mean score of 72% and a median score of 100%. The trendline slope (-4.89) decreased from the rate of change during intervention (14).

Bluford. The final participant, Bluford was also the most communicative of the six participants. The video recordings show that he frequently asked questions during the intervention. For example, during the second intervention session, he asked the staff, "Does the rise always go up and down?" The graph of his performance is presented in Figure 4, and the within-condition trendlines are presented in Figure 5.

Visual analysis. In baseline, Bluford demonstrated a stable baseline with near 0% of the steps completed, and his performance remained constant during Mukai's intervention, and Dwight's intervention. When Bluford started intervention, his performance increased immediately, and the increase continued steadily before reaching 93%. During the testing condition, Bluford's performance became more variable with some scores at 90% and others falling to 50%. A visual analysis of the trendlines (Figure 5) shows a nearly flat slope during baseline that changes to a positive slope during the intervention. The slope of the trendline during the intervention was steeper than the positive slope within the testing condition.

Descriptive statistics. Table 8 presents the descriptive statistics of Bluford's performance. Bluford participated in six baseline sessions. His scores in baseline ranged from a maximum score of 2.5% to minimum score of 0%. Within-condition for baseline, the mean score was 0.42%, and the median score was 0.0%. Bluford's performance was slightly positive but mostly flat (slop*e=0.67*). Within-condition for intervention, Bluford's performance ranged from a minimum of 30% and a maximum of 93.33 % with a mean score of 67.98% and a median score of 66.70%. The slope within-condition for intervention was positive (slope=11.28). However, the shape of the performance was less linear and more logistic shape with the slope decreasing as the performance approached 100% accuracy. Bluford's improvements in performance continued during the testing condition, and the within-condition trendline slope was a moderate 0.36; however, the variability increased as Bluford moved from intervention ($r=0.98$) to testing ($r=-0.03$).

Statistical analysis. For analysis, the individual participants' mean within-condition performance data for the baseline, intervention, and testing conditions were compared. The codes were entered IBM's SPSS Version 24, assumptions were checked, and because the nonparametric repeated measures data included three groups (baseline, intervention, and testing conditions), a one-way Repeated Measures Freidman's Analysis (Friedman's RM) looked for differences between conditions (Friedman's RM). Friedman's RM compares the rankings of related samples across time (Howell, 2010). For analysis, the *p-value* was set at a 0.05, and the null hypothesis assumed the performance would be the same across the conditions. However, the null hypothesis was rejected because the analysis revealed statistically significant results ($X^2_{Friedman}$ (2) = 9.33; $p<.01$). To determine where the differences existed, a *post-hoc* Wilcox Signed Ranked test was used to test across conditions. A Bonferroni adjustment was not made to the *p-value* because the risk of type-II errors would be increased with the low participant numbers. The test confirmed significant differences between the baseline and intervention (Z=-2.2, $p<.03$) and the testing to baseline conditions (Z=-2.2, $p<.03$).

Question 2

Does the acquisition of procedural fluency for creating a linear equation or creating a line affect conceptual understanding?

a) does acquisition of procedural fluency generalize to the inverse skill (creating a line is the inverse skill for creating an equation and creating an equation is the inverse skill for creating a line)?

b) does acquisition of conceptual understanding require less time in instruction than with procedural fluency of the inverse skill?

Importantly, all six individuals were eventually able to reach criterion (60%). One of the six individuals reached criterion without supplemental intervention; one individual required booster sessions, and four individuals could reach criterion with supplemental intervention. For each individual participant, the inverse skill was monitored for evidence of generalization while the participant completed the intervention from Experiment 1 and Experiment 2. Inverse skill development was monitored using a procedural fluency task analysis, and the percentage of steps completed during each session were recorded and presented in graphs.

Experiment 1. Three participants, Ed, Guion, and Mukai, participated in the first experiment. Each participant was provided with a picture of an equation and a template to create-an-equation. After solving all the create-an-equation problems for a session, the participants were provided with a coordinate plane, a ruler, and an equation, and the participants were asked to problem-solve to create-a-line. After participating in five intervention sessions focused on the target skill, the participants received supplemental intervention for the inverse. The inverse skill intervention followed the same intervention protocol. Participants received a task-analysis based self-monitoring tool, modeling prompts and supports from the staff members delivered using a semi-structured prompt, and a template for solving the problem. If a student struggled to show significant growth after three consecutive sessions, then booster sessions were used. Booster session 1 added color prompts to the template, and reduced the number of problems per session; booster session 2 added concrete manipulative objects, and booster session 3 reduced the number of concrete manipulatives that were deemed distracting by a severe disabilities expert.

Ed. The graph used for the visual analysis is presented in Figure 6. Ed's performance for the inverse skill showed variability during the baseline condition. His scores fluctuated between 0 and 40% before achieving as stable baseline between 0 and 10%. The graph shows the largest increase in performance during baseline occurred immediately after Ed began the intervention for the target skills. The graph also shows that Ed's performance for the inverse skill reached criterion sooner for the inverse skill than for the target skill.

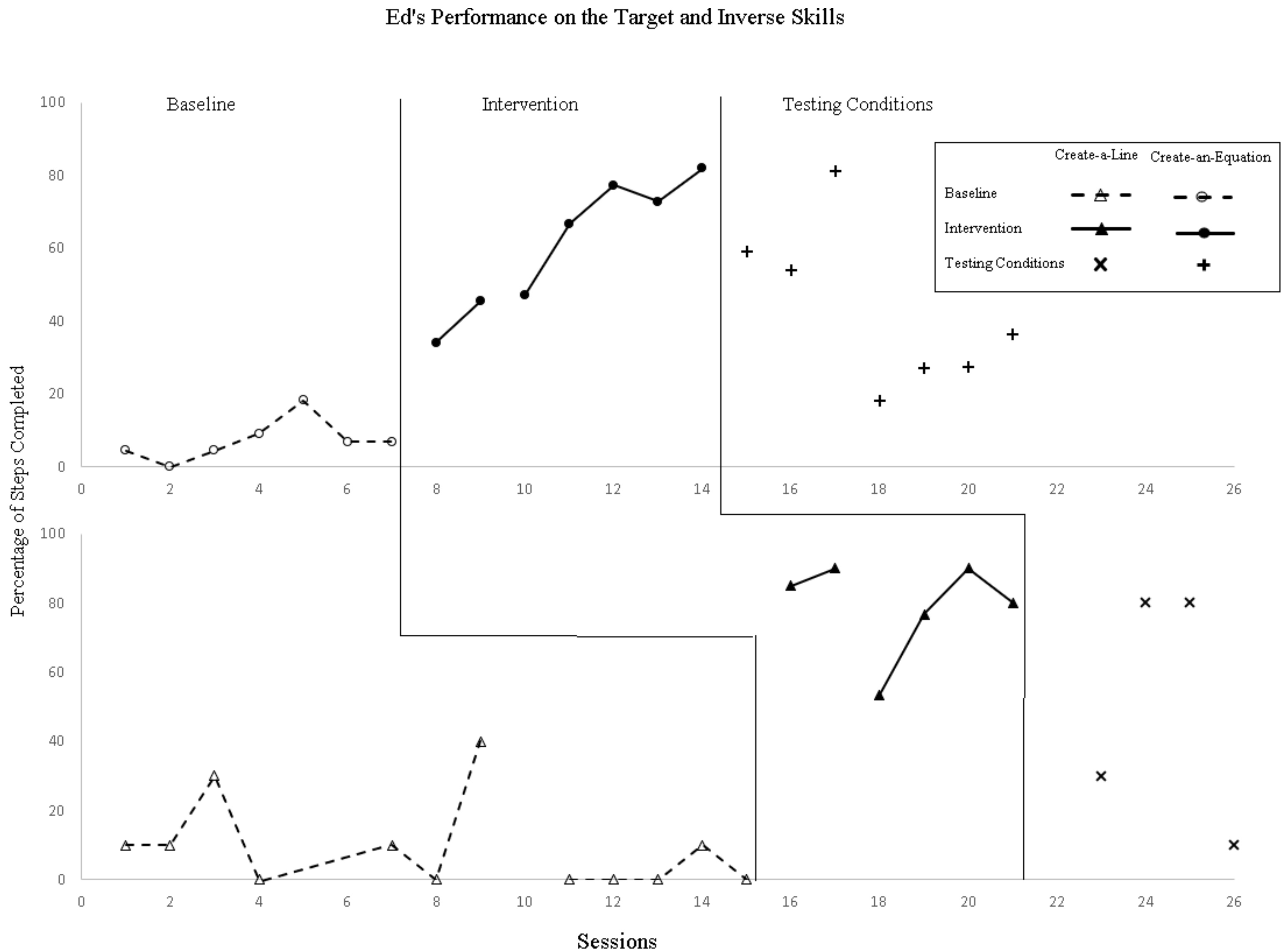

Figure 6. The graph shows Ed’s performance across sessions (not weeks) for the target skill (create-a-line) and the inverse skill (create-an-equation).

Descriptive statistics confirmed Ed's performance. For the inverse skill, Ed's baseline performance was inconsistent. He fluctuated between a minimum of 0% to a maximum of 30% of the steps completed correctly. After intervention for the target skill began, Ed's performance became less stable and the range between the minimum (0%) and the maximum (40%) increased. The graph shows a spike in Ed's performance after intervention started for the target skill; however, this performance was followed by a decrease in performance. His score fell below his initial performance of 11% to 0%. However, when the supplemental intervention began, Ed's performance increased sharply from 0% to 85%, and during the supplemental intervention period, Ed's performance was relatively stable with scores ranging from 53% to 90% accuracy.

Ed's performance did reach criterion faster for the inverse skill than for the target skill. During baseline with intervention for the target skill, Ed did not achieve criterion with the inverse skill, and a supplemental intervention protocol was used to teach the inverse skill. The supplemental intervention protocol was identical to the procedure used during the intervention for the target skill. Ed reached criterion with the first supplemental intervention with a score of 85%. In contrast, Ed required four intervention sessions for the target skill before he reach criterion.

Guion. Guion's performance for both the target and the inverse skill are presented in Figure 7. A visual analysis of the graph shows that Guion's performance completed 0% of the steps during baseline for the inverse skill until the intervention for the target skill began. Two moments of instability occurred; at session 8 and session 13, Guion's performance increased to 20% and 20% respectively. Additionally, his performance for the inverse skill reached criterion (>60%) immediately after starting intervention. In contrast, Guion required four sessions of

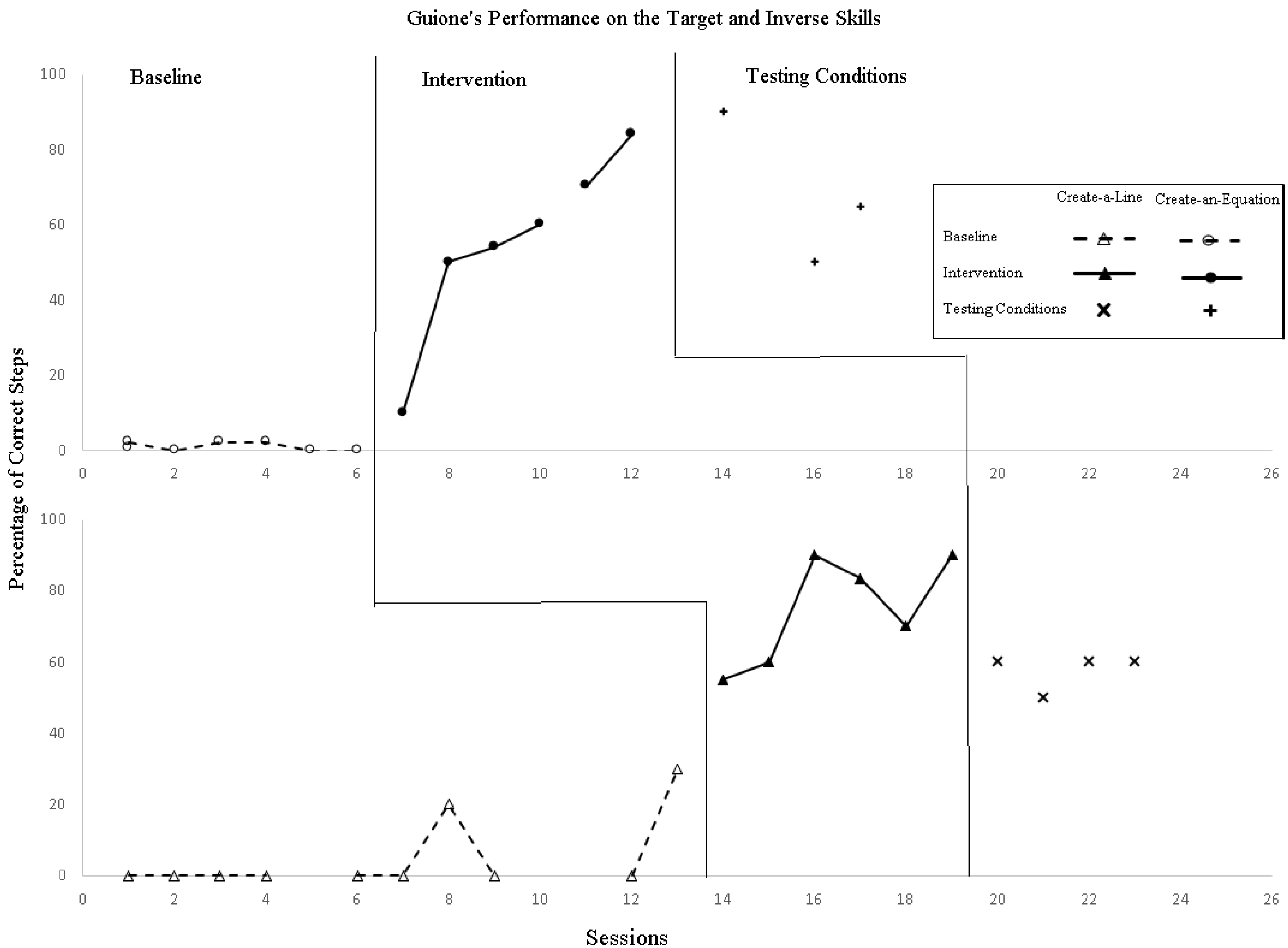

Figure 7. Guion's performance across sessions for the target (create-an-equation) and inverse (create-a-line) skills.

intervention for the target skill to reach criterion. Also, Guion's performance on the inverse skill remained stable and near criterion during the testing sessions.

Descriptive statistics confirmed the visual analysis. Guion generalized the inverse skill before supplemental intervention began. Importantly, Guion's performance within baseline was stable with 0% of the steps completed for the inverse skill. However, as soon as the intervention for the target skill began, Guion's performance for the inverse skill became more variable. For the inverse skill, his scores fluctuated between a minimum score of zero to a maximum of 30%. Similarly, the mean scores for the inverse skill changed. Before the target intervention began the mean was 0%; however, after the target intervention began the mean for the final five baseline sessions was higher (M=10%). Guion finished the baseline session for the inverse skill with an upward trend and the maximum within-condition score (30%), and the positive trend continued when supplemental intervention began for the inverse skill. Guion's scores during the inverse skill intervention condition ranged between a minimum of 55% and a maximum of 90%. Within the inverse intervention condition, Guion's mean performance increased to 74.72 with a median score of 76.67. During the testing condition, the median score was 60%, and the mean score was 57.5%.

Guion reached criterion more quickly for the inverse skill than the target skill. For the target skill Guion crossed the 60% criterion level during the fourth intervention session (70%), but he reached criterion for the inverse skill after participating in two supplemental intervention sessions (60%). Additionally, Guion's performance during the testing phases was less variable and more stable for the inverse skill than the target skill. For the target skill, Guion demonstrated a negative trend within the testing condition, but for the inverse skill, the trend was horizontal.

Chiaki. A graph of Chiaki's performance for the target and inverse skills is presented in Figure 8. The visual analysis shows that Chiaki demonstrated 0% of the steps for the inverse skill throughout the intervention for the target skill, and when the intervention for the inverse skill began the increase in performance was negligible. Similarly, the inverse booster intervention sessions also showed negligible increases in performance. After starting the second booster intervention condition for the inverse skill, Chiaki's performance did increase; however, her scores remained below criterion after four sessions, so a third booster session began. During the third booster condition Chiaki's performance increased to the criterion level. The school year ended before Chiaki could participate in the testing condition.

Descriptive statistics confirmed the visual analysis. Chiaki's performance on the inverse skill remained flat with 0% of the steps completed correctly throughout the baseline and during the intervention and booster interventions for the target skill. After the inverse intervention had begun, she performed 10% of the steps for all three sessions. This increased to 15% accuracy for the first inverse booster session which added color prompts to the template before returning to 10% accuracy for the final two sessions within the condition. During the second inverse booster session (n=4), which included concrete objects and color prompts, the performance score ranged between 23.3% accuracy and 40% accuracy. Chiaki's performance within the third inverse booster sessions ranged from a minimum of 30 to a maximum of 65 (criterion=60). Chiaki reached criterion for the target skill after 19 sessions, and she did reach criterion faster for the inverse skill with 15 sessions. However, the intervention package was different than the booster intervention package used for the target skill, so the results are not comparable.

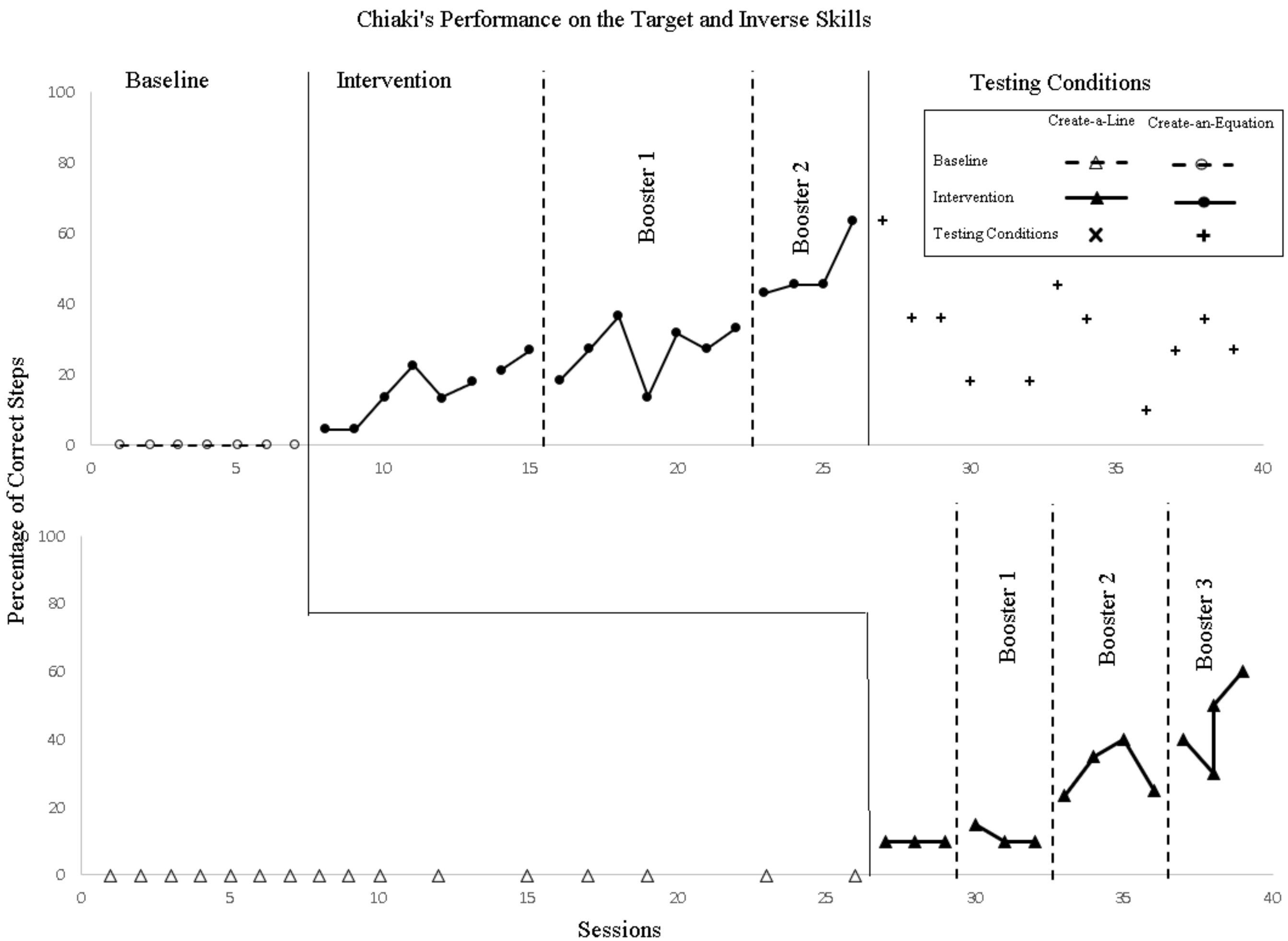

Figure 8. The graph shows Chiaki's performance as the percentage of correct steps for the target (create-an-equation) and inverse (create-a-line) skills.

Experiment 2. The three remaining participants were assigned to Experiment 2 where the individuals learned the target skill to create-a-line, and the inverse skill create-an-equation was monitored. The intervention package in Experiment 2 was identical to the intervention package provided in Experiment 1. After results were coded, a graph was created to permit visual analysis of the data. Overall only one participant, Bluford, demonstrated generalization to the inverse skill without intervention, and the remaining two individuals did not reach criterion faster for the inverse skill when compared to the target skill.

Mukai. Mukai's results are presented in Figure 9. Mukai maintained the same percentage of correct steps through the baseline condition for the inverse skill. This included the three sessions that overlapped with his target skill intervention condition. Mukai's performance did increase for the inverse skill after the intervention for the inverse skill began, and his performance graph shows a rapid increase in performance for the first three intervention sessions; the graph also shows that Mukai's performance plateaued and remained stable for the remaining four intervention sessions. During the testing conditions Mukai's performance decreased to the baseline performance level for the first session.

Descriptive statistics confirmed the visual analysis. Mukai's performance with the inverse skill remained stable with 9% of the steps completed independently. The means increased from 9% within the baseline condition to 63% in the intervention period; similarly, the median scores increased from 9% during baseline to 72.7% during the intervention condition. Mukai's performance decreased when the testing condition began, and the mean during the testing condition was 34%, and the median decreased to 36%. Notably the rate change was higher for the inverse skill. The within-trend slope for the target skill during intervention was

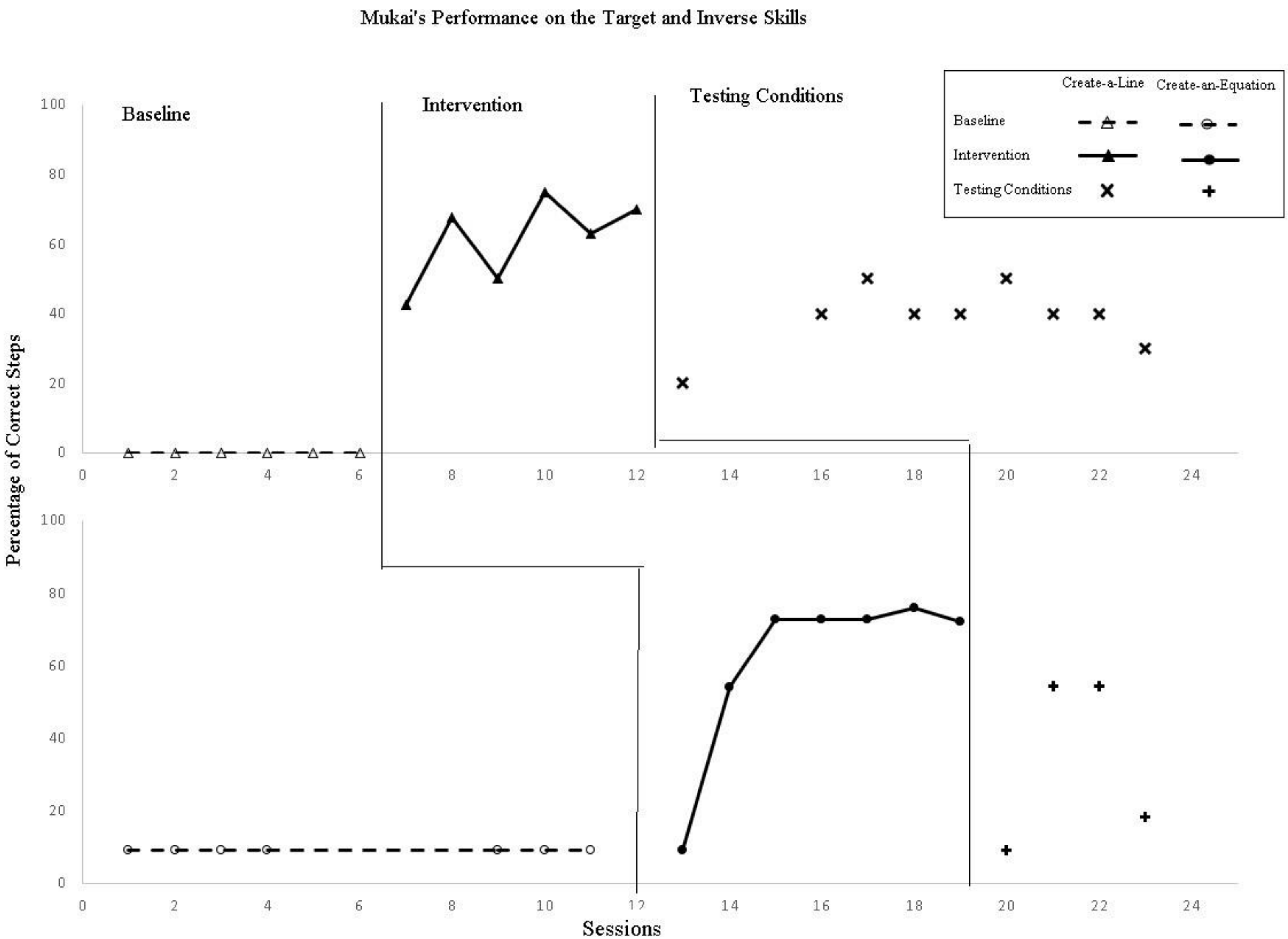

Figure 9. The percentage of correct steps across sessions for the target skill (create-a-line) and the inverse skill (create-an-equation).

4.27(*r=0.4; r^2=0.28),* and the within-trend slope for the inverse skill intervention was 8.32 (*r=.74; r^2=0.55).*

Dwight. Figure 10 presents a graph of Dwight's performance for both skills. For the inverse skill, Dwight's performance remained unchanged with 0% of the steps completed. This score was maintained through the target skill intervention period until the inverse skill intervention began. The graph displays a sharp increase in performance during intervention, and the increase continued with some variability across the six sessions. During the testing condition, the performance decreased and ranged more widely, but all three scores from the testing condition were higher than the pre-intervention baseline scores.

The descriptive statistics confirmed the visual observations. The 0% performance for the inverse skill continued during the intervention for the target skill, and there was an increase in the intervention. During the six supplemental intervention sessions, Dwight's scores ranged from a minimum of 45.5% and a maximum of 100%. The mean was 72.7% with a median of 82.5%. The descriptive statistics decreased within the testing condition. Dwight's mean decreased to 48.4% with scores ranging between 18.1% and 72.73%; the median during follow up.

Differences existed between Dwight's performance during the intervention for the target skill to the supplemental intervention for the inverse skill. Dwight's performance slope was smaller for the inverse skill (6.5) than for the target skill (14); however, some of the difference can be attributed to the higher initial condition value for the inverse skill (57%) compared to the initial condition value of the target skill (50%). Variability for the target skill during intervention (*r*=0.94*;* r^2=0.88 was consistent with the variability for the inverse skill (*r=0.93*) than for the target skill(r^2*=0.86*).

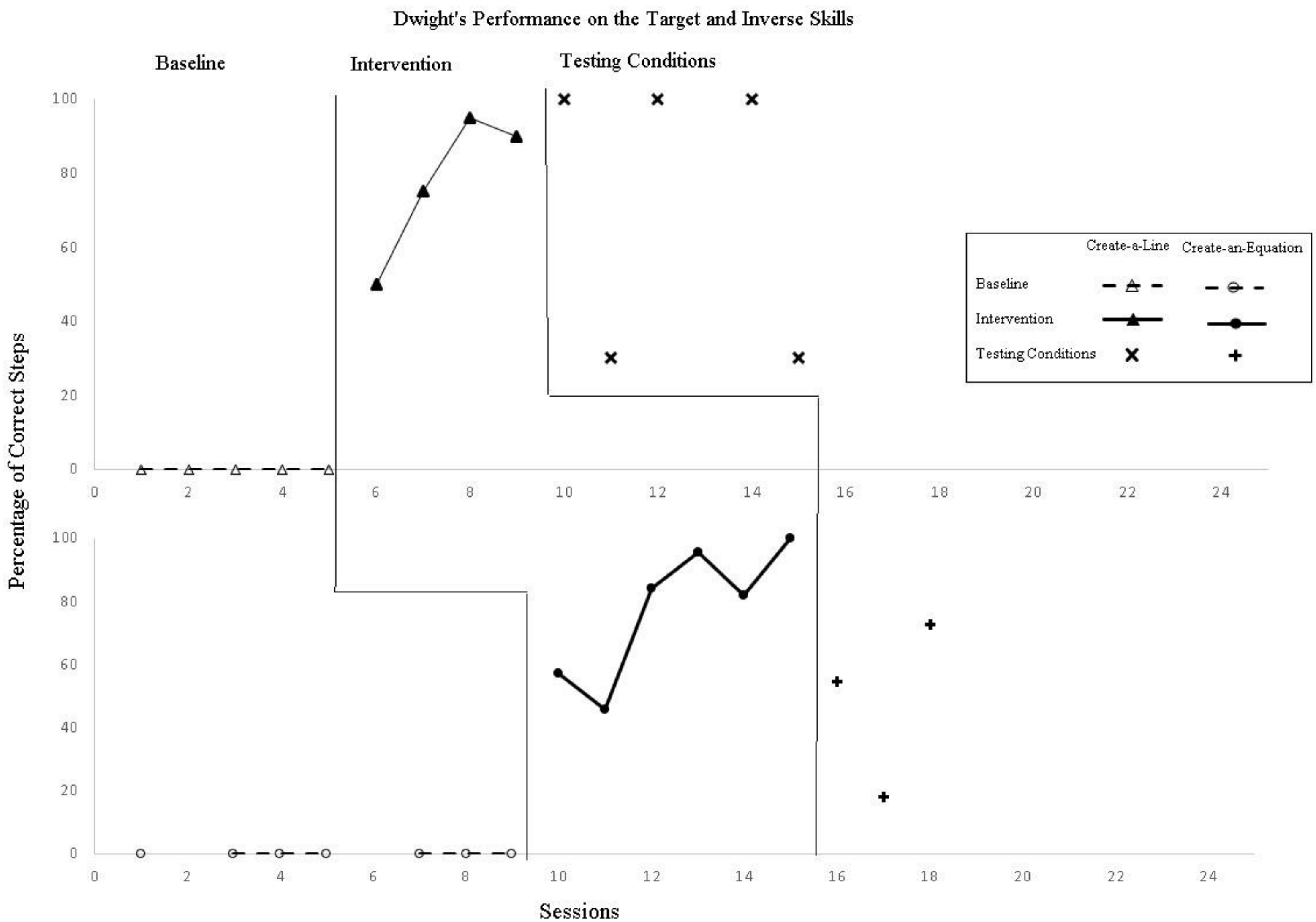

Figure 10. Dwight's performance for the target (create-a-line) and inverse skill (create-an-equation).

Bluford. A graph of Bluford's performance is presented in Figure 11. Bluford was the only one of the six participants to reach criterion with the inverse skill without supplemental instruction. Prior to the start of the intervention for the target skill, the baseline performance for the inverse skill was stable with 0% of the steps completed for four out of the first five sessions. There was an anomalous performance for the second step when Bluford completed 9 % of the steps correctly. However, after the intervention for the target skill began, performance for the inverse skill increased to criterion for three of the next four sessions with a decrease in performance on one occasion. Bluford's scores were determined to be high enough by the teachers to proceed to the testing condition, and within the testing condition, Bluford's scores fluctuated between 0% and 60%. Although Bluford's performance during the testing condition for the inverse skill was like the performance during the testing condition for the target skill, in general, Bluford's scores appeared higher for the target skill.

The descriptive statistics verified the visual analysis. During baseline for the inverse skill, a change in performance occurred after the target intervention began. Before the target intervention began, scores ranged between 0% and 9% with a mean of 1.8% and a median of 0%. After the intervention for the target skill began the scores for the inverse skill increased. The mean for the inverse skill mean increased to 36 % and the median increased to 54%. Bluford's performance for the inverse the skill was stronger than the score would suggest. He was getting the algebra correct, but he was often skipping over the orientation steps associated with the problems. For instance, Bluford did identify the y-intercept value (algebraically correct), but he did not "Touch the y-axis."

Anecdotally, teachers reported that Bluford was the "chattiest" participant, and the tapes confirmed. He asked questions during the intervention period. For example, on one occasion, Bluford asked, "Where does the rise come from?" and on another occasion, he can be heard saying, "Rise is Y."

Statistical Analysis

Comparing within-condition means. Visual analysis suggested three out of the six individuals showed some improvement in the inverse skill while receiving intervention for the target skill, so a statistical analysis of the data was performed to rule out random chance as an explanation for the results. For everyone the mean within-condition scores (baseline, target intervention, & inverse intervention) were computed. The three groups of data were screened, and the distribution looked non-parametric, so a non-parametric Friedman's RM was selected. The critical significance value of .05 was set as the significance level. The null hypothesis assumed there would be no differences between the conditions; however, the null hypothesis could be rejected because the difference was statistically significant ($X^2_{Friedman}$ (2) =10.38; $p<0.01$). Post-hoc tests were conducted using the Wilcoxon Signed Ranks Test. Differences did exist between the baseline condition and the supplemental condition ($Z=2.02$; $p<0.05$) and target intervention and supplemental conditions ($Z=2.02$; $p<0.05$). However, the difference between the baseline and target intervention conditions was not found to be significant ($Z=1.07$; $p>0.2$).

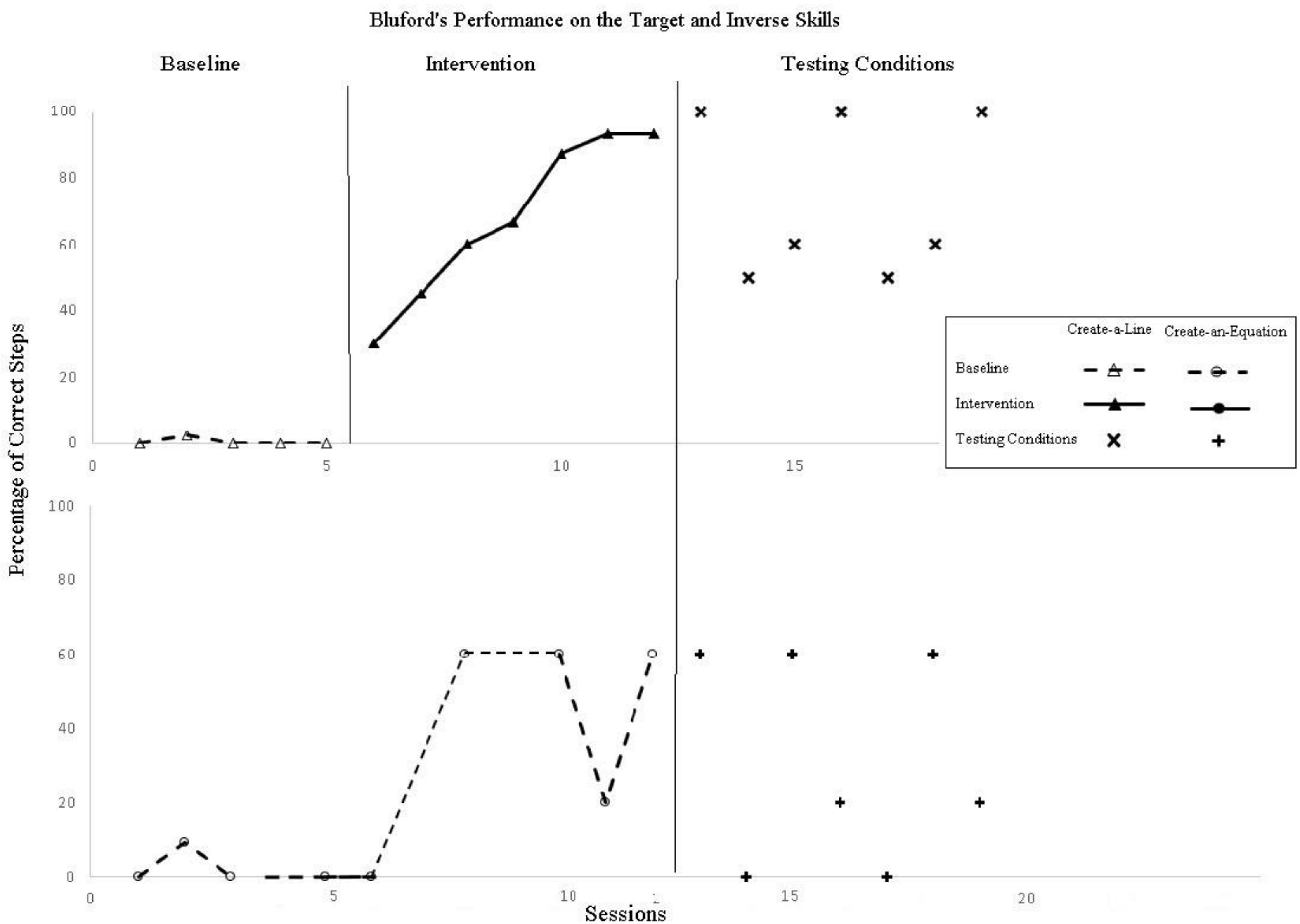

Figure 11. Bluford's percentage of correct steps for the target skill (create-a-line) and the inverse skill (create-an-equation).

Changes in baseline. *A priori*, the researchers hypothesized that the generalization to the inverse skill during baseline or the rate of learning would be faster for the inverse skill. To test the first part of the hypothesis, the researcher calculated the means during baseline for the target and inverse skill during as well as the mean during baseline for the inverse skill that occurred during the target intervention. The repeated measures data set was nonparametric with three groups (baseline,intervention, & testing conditions), so a Friedman's RM test was selected. The critical value (*p-value)* was set to 0.05, and the test was conducted with a null hypothesis stating there would be no difference between the baseline conditions. The null hypothesis was rejected, as the data was statistically significant ($X^{2}_{Friedman}(2)$ =6.53; $p>0.04$). Post-hoc tests were conducted using the Wilcoxon Signed Ranked tests. The means for the inverse baseline compared to target baseline (Z=-1.826; $p<0.064$) and the inverse baseline compared to the inverse baseline during intervention were (Z=-1.826; p<0.064).

Trend changes across conditions. The second half of *a priori* hypothesis suggest that participants, during the supplemental intervention, would learn the inverse skill at a faster rate than the target skill. To compare the data across conditions, the slope was calculated using the final three consecutive points from the baseline condition and the first three points from the intervention condition. One participant, Bluford was eliminated from the analysis because he did not require supplemental intervention for the inverse skill. The data were (separated into two groups. The rate change for the target skill and the rate change for the inverse skill. With two groups (change for target skill & change for inverse skill) of five participants with non-

Table 9

Participant Responses to Statements on the Social Validity Questionnaire.

	Ed	Guion	Chiaki	Mukai	Dwight	Bluford	M
1. I need algebra to graduate.	60	97	81	51	91	48	71
2. I need algebra to attend college.	60	100	78	30	50	48	61
3. I need algebra to be in other science and math classes.	96	95	47	28	18	89	62
4. The staff helped me learn algebra.	100	100	84	87	90	90	92
5. I want to learn algebra.	60	100	91	38	82	89	77
6. I should learn algebra like other students in the school.	100	99	97	35	87	87	85
M	79	99	80	45	70	75	

Note: Participants responded with scores between 0 *(strongly disagree)* to 100 *(strongly agree).*

parametric data, the Wilcoxon Signed Ranks test was selected. The null hypothesis stated there would be no difference between the two groups. The difference between groups was not found to be statistically significant ($Z=-.67$; $p > 0.62$), and the null hypothesis was upheld. However, power was relative low ($ß<.35$).

Social Validity Panel of Experts

To measure the social validity surveys (Appendices J & K) were distributed to a panel of experts. The panel included all six of the study participants, the four special education staff who administered the intervention, two parents of the participants, two high-school general education math students, and three general education high school math teachers. Results from the participant responses are presented in Table 9, and Table 10 presents summary results from the Panel of Experts. In addition, the researcher recorded the unsolicited comments of the participants, staff, and parents.

Participants. Two forms of social validity data were gathered from the participants. First, a social validity survey with six Likert statements (Appendix K) was administered to the participants. Second, anecdotal observations included teacher reports, behavioral observations, and student comments.

Survey. Each participant was provided with a six statement Likert survey in REDCap™ (Harris et al., 2009), and instructed to indicate their level of agreement by sliding an electronic indicator towards “0-Strongly Disagree” or “100-Strongly Agree.” Appendix L presents the survey, and Table 9 presents the responses for each participant. The mean scores for statements related to the purpose of algebra were lower than scores related to inclusion. In response to the statement,” I need algebra to graduate.” the mean score was 71, and the mean response score was 61 for “I need algebra to attend college,” and “I need algebra to be in other science and math classes." In contrast, participants responded with higher mean scores for statements related to staff, "The staff helped me to learn algebra," (*M=92*) and the statement, "I should learn algebra like other students in the school" (*M=85*). Five students responded with some agreement to the statement, "I want to learn algebra" (*M=85*); however, one participant, Mukai, responded that he disagreed with the statement (35).

Mukai also had the lowest responses to the statements with a mean score of 45 across statements. Mukai also expressed eagerness to participate in the actives, and on the tapes, he can be heard asking, "Is it my turn yet?" However, in general, the remaining five individual participants responded with scores between neutral (50) and strongly agree (100). Guion (participant 2) responded with the highest agreement (99).

Anecdotal observations. The special education teachers reported impacts of the study on participants. For example, they reported an increase in counting accuracy for Chiaki (Participant 3). They also noted that Chiaki increased the use of math related sign-language. Specifically, she started signing *positive* and *negative* when working as a cashier.

The teachers also reported comments made by students outside of the study. For example, Guion (Participant 2) and Mukai (Participant 4) would ask daily, "Is today an Algebra day?"

Behavioral observations during the study netted the only negative incident occurring during. During session 49, Chiaki (Participant 3) wrote the word "hell" at the top of the page. Staff reported the student was having a "bad day" and they described an incident from the morning involving a negative interaction with another teacher. Consistent with the IRB approved procedures, the researcher confirmed the participant's willingness to continue with the study. She agreed, so it is not entirely clear if the interpretation of the comment was associated with the math activity.

In contrast, most of the comments made by the participants during the study indicated a desire to learn the algebra material. Dwight (Participant 5) and Bluford (Participant 6) would ask for the intervention to begin. Dwight said, “I really want to learn this.”

After starting the intervention, Bluford (Participant 6) made an excited comment. He said, “This is pretty tight! I like this [algebra].”

More powerfully, at the end of one session, an interaction was observed between the teachers and Guion (Participant 2). After the teacher had noted that Guion was working harder on the algebra than his IEP related math counting activities, Guion commented, “Do you know why I am so good at this? It is because this isn’t the baby stuff that I am normally asked to do. I like this.”

Students without disabilities. Table 10 presents the social validity results from the two students without disabilities. Responses from the participants were higher than the responses from one the general education students. Two students without disabilities (one male and one female) also responded to the panel of experts’ survey. As panel members, both general education students completed Algebra I, and both had experience volunteering in classrooms with students with severe disabilities. The male student responded with high scores for the statements “It is important to lean algebra” (91); “Algebra skills are needed to graduate with a general education diploma” (100), and “In high school, students are asked to create graphs of lines from linear equations” (100). The male student responded with a lower score (76) in response to the statement, “In high school, students are asked to create formulas in the slope-intercept form,” noting verbally that the skill was “really from middle school.”

The female general education student responded with the lowest scores from all the panel members. She moderately agreed with the statemen that algebra was a requirement for high school or math classes (69), and she responded with total agreement (100) to the statement, "In high school, students are asked to create graphs of lines from linear equations in the slope-intercept form (y=mx+b)." However, she disagreed with the statement about the inverse skill (39). She responded with a neutral score (50) to two statements, "It is important for students with ID to participate in algebra," and "algebra is a prerequisite for college math or science classes." She also responded, "strongly disagree" (0) to the statements, "Algebra is a prerequisite for college admissions" and "Algebra is required for graduation with a general education diploma." Concerned that she might have misread questions on the survey, the research followed up with a conversation, and the female student laughed and said, "nobody really needs algebra after high school."

General Education Teachers. Results from the general education teachers are presented in Table 10. Three Algebra I teachers responded to the survey with scores on the agree side of the scale. With two exceptions, the general education teachers responded with scores consistent with strongly agree (score > 80). They responded with variability (Agree to Strongly Agree) to the statement "It is important for students with ID to learn algebra;" one general education teacher responded with a score of 75, another 90, and a third with a score of 82. For the second statement, "Algebra skills are required for graduation with a general education diploma," the general education teachers responded with scores of 100, 100, and 80 respectively.

Special Education Staff. The special education staff included three special education teachers and one paraprofessional. Results are presented in Table 10. They responded consistently with scores of 97 or higher for five of the statements (a) "Algebra skills are required for graduation with a general education diploma;" (b) "In high school, students are asked to create graphs of lines…;" (c) "In high school, students are asked to create formulas…;" (d)

"Algebra is a prerequisite for college admissions…," and (e) "…college math or science classes." The special education staff responded positively to the statement "It is important for students with ID to participate in algebra" with scores of 75, 97, 100, and 100. The staff that responded with 75 noted, "I agree, but I really don't think everyone uses algebra every day." However, the special education agreed the least with the statement, "Algebra skills are needed to participate in high school science or math classes." The scores were 26, 51, 70, and 100. One teacher commented, "Really? Then what are we doing *every day?"*

Parents. Table 10 includes the responses from parents. The parents who responded to the statements also strongly agreed with most statements (score >80) with one exception. The parents rated the statement, "Algebra is a prerequisite for college admissions" with a score of 75. One of the parent explained, "There are college programs for students with ID. Algebra helps, but it isn't required."

Summary

This study examined the effects of a multi-faceted treatment package on the six participants' procedural fluency abilities. The study also observed the generalizability of the participants' procedural fluency to inverse skills. All six participants showed improvement in procedural fluency scores after direct intervention began for the target skill. One of the six participants generalized to the inverse skill without direct supplemental intervention; two additional individuals showed some signs of generalization without supplemental intervention. For one participant, it was necessary to intensify the intervention supports using concrete manipulatives and color prompts so that the participant could reach criterion. Experimental control was maintained, and the effects of the intervention package on the development procedural fluency for the target skill were strong. Additionally, the supplemental intervention package appeared to be effective to help the students to generalize the target skill to the inverse skill in four out of six cases.

Table 10. Mean Responses from the Subgroups on the Panel of Experts

Statement	GE Teachers	SE Staff	Male GE Student	Female GE Student	Parents
(1) It is important for participants to participate in algebra.	82	93	91	50	90
(2) Algebra skills are required for graduation with a general education diploma.	93	100	100	0	95
(3) In high school, students are asked to create graphs of lines from linear equations in the slope-intercept format (y=mx+b).	99	100	100	100	100
(4) In high school, students are asked to create formulas in slope-intercept form (y=mx+b).	100	99	76	30	100
(5) Algebra skills are needed to participate in high school science or math classes.	96	62	NR	69	95
(6) Algebra is a prerequisite for college admissions.	97	99	NR	0	75
(7) Algebra is a prerequisite for college math or science classes.	100	100	NR	50	100

Note: General Education (GE), Special Education (SE), No-Response (NR). Participants responded to statements with scores ranging between 0 (*strongly disagree*) to 100 (*strongly agree*).

Chapter V

Conclusion

Endrew v. Douglas County School District (2017) underscored the importance of academic activities for individuals with disabilities and called on teachers and schools to recognize the increased academic requirements in IDEA (2004) and ESSA (2016). The new court standard affirmed the individual students' civil right to access the grade-level curriculum (Endrew v. Douglas School District, 2017), and further supported the belief that access to algebra, an academic skill integrated across grades the general education curriculum, is a civil right (Kendall, 2011; Kress, 2005; Moses, et al., 1989). At present, the special education community is just beginning to include students with Intellectual Disability (ID) in grade level academic standards at the high school level (Ayers et al., 2011; Ayers et al., 2012; Creech-Galloway et al., 2015; Courtade et al, 2012; Kleinert et al., 2015), and the shift to grade-level academic skills has exposed deficits in both the general and special education pedagogy. The general education community has redesigned the general education curriculum; algebra was integrated across *all* grades, and algorithmic processes were deemphasized and replaced with problem solving for five strands of learning (a) productive disposition, (b) procedural fluency, (c) adaptive reasoning (d) strategic competence, and (e) conceptual understanding (Kendall, 2011; Kilpatrick et al., 2001; NGACBP & CCSSO, 2010). However, the changes have been met with skepticism from educators of at-risk students (students with disabilities, different ethnic backgrounds, and low socio-economic status) because teachers are not sure how to teach algebra to the new groups of students (Creech-Galloway et al., 2013; Ernest, 2002; Haas, 2005; Loveless, 2008).

In contrast, the special education community developed *errorless learning* practices in mathematics that focus on functional skills or procedural fluency (Browder et al., 2008; Browder, Jimenez, et. al., 2012; Browder, Trela et al., 2012; Creech-Galloway et al., 2015; Courtade et al., 2012; Göransson, et al., 2016; Jimenez et al., 2008). Typically, the method is applied to discrete skills involving the procedural fluency being developed in higher level mathematics (Browder et al., 2008; Browder, Jimenez, et. al., 2012; Browder, Jimenez et al., 2008). However, more recent research applied the methodology to introduce procedures conceptually. Creech Galloway et al. (2015) showed students solving problems with the Pythagorean Theorem and Root (2016) demonstrated students could solve word problems. Browder et al. (2017) and Root et al. (2017) replicated the Root (2016) study showing the technique could be used to teach word problems. *Errorless learning* stems from the behaviorist tradition (Mueller et al., 2007), and researchers from the constructivist perspective have criticized the method. Göransson, et al. (2016) noted that the existing studies were designed to meet the procedural skills of the old math curriculum, and only one study, Göransson, et al. (2016) attempted to explicitly document conceptual understanding in algebra and beyond for individuals with ID.

Without a bridge between the general and special education approaches, students with ID will continue to be educated in segregated environments. Overwhelmingly, 93% of students with ID receive academic instruction in separate, self-contained special education classrooms taught by special education teachers who do not have mathematical backgrounds or by math teachers who do not have experience teaching students with disabilities (Courtade et al., 2012; Creech-Galloway et al., 2015; Johnson et al., 2013; Kleinert et al., 2015; Lee et al., 2016, April). Historically, special educators assumed students with ID were incapable of learning algebra (Ayres et al., 2011; Browder, 2015; Connolly, 1973; Courtade et al., 2012; Kirk, 1955; Kirk & Johnson, 1951; Lee et al., 2016, April; Monari Martinez & Pelligrini, 2010), and more reseach needs to be conducted to challenge the historical assumption (Browder et al., 2008; Browder, Jimenez, et. al., 2012; Browder, Trela et al., 2012; Courtade et al., 2012; Creech-Galloway et al., 2015; Göransson, et al., 2016; Jimenez et al., 2008). Additionally, the skills need to move beyond rote learning to demonstrate conceptual understanding (Göransson, et al., 2016, Spooner, Saunders, Root et al., 2017).

This study used a single-case multiple baseline design across participants to observe six high school-aged participants before, during, and after receiving instruction to improve procedural fluency with one of two algebra skills (create-an-equation or create-a-line). Instruction was provided to everyone in a public-school setting for a target skill (e.g. create-an-equation) while the inverse skill (e.g. create-a-line) continued to be observed. The researcher hoped to find evidence of some impact on the inverse skill that could signal that generalization of the procedure took place without supplemental instruction. This is important because generalization of procedure to the inverse skill could imply some level of understanding (Rodriguez, 2016; Snell & Brown, 2014; Stokes and Baer, 1977).

Summary of the findings

Results from the study demonstrated that high school age participants with ID can improve procedural fluency to create-a-line (from an equation) and to create-an-equation (from a line). The *errorless learning* intervention package included a semi-structured script, a time-delay procedure, a modeling prompting strategy, participant self-monitoring, and a task-analysis. All six participants reached criterion of 60% correct; however, Participant 3, Chiaki, needed two more intensive booster sessions to reach the 60% criterion specified in the protocol. The more intensive interventions used concrete objects. Results also indicated that students with ID will sometimes generalize learning for the target skill to impact performance on the inverse skill. Half of the participants (n=3) completed a greater percentage of steps in baseline performance for the inverse skill before receiving direct instruction, and one of the participants achieved criterion for the inverse skill without receiving the instruction intervention.

Implications

Academic expectations for students with ID are increasing. The Every Student Succeeds Act (ESSA, 2016) increased the requirements for individuals with significant cognitive disabilities to participate in the grade-level academic standards for mathematics. Under No Child Left Behind (NCLB, 2001), accountability increased for each state, locality, and classroom; however, teachers struggled and continue to struggle to meet the new academic standards (Creech-Galloway et al., 2015; Loveless, 2008). The requirements are compounded by the *Endrew v. Douglas County School District* which mandates schools to maintain *adequately ambitious* expectations for students with ID. The results of this study have implications for research, practice, and policy.

Research. The research implications cross two fields of education research. First, the results of this study relate to the field's efforts to clarify the boundaries of potential for individuals with ID, and help to validate *errorless learning* as a method to support students in algebra. Second, for the field of mathematics education, the results help to clarify the struggle to measure conceptual understanding. This study highlights the need to develop new measurement strategies to accommodate participants whose use of language may be too limited to describe mathematics to an observer (e.g. students with disabilities or students learning English).

Special Education. This study fits incrementally within the existing body of special education research exploring procedural fluency of math skills. It used a single-case design across participants with a multi-component *errorless learning* intervention. One of the purposes of the body of research is to clarify the boundaries of individual potential (Ayers et al, 2012; Courtade et al., 2012) because high school is the *frontier* for inclusion for students with ID (Jacobs & Saperstein, 2017, June). Direct replication of the study will be needed to validate the results and to establish the *errorless learning* methodology as an evidence based practice for algebra instruction, and indirect replication will help to answer more detailed questions about the intervention. The study adds to a growing amount of research documenting the capabilities of students with ID to complete algebra problems; the research community should begin to explore *why* the change is occurring.

Potential. Courtade et al. (2012) noted that the full potential of individuals with ID remains undiscovered, and Courtade et al. (2012) and Ayres et al. (2012) described the need for researchers to explore the outer limits of individual abilities. Previous research established the abilities of individuals with ID to solve equations (Browder, Jimenez et al., 2012; Browder, Trela et al., 2012; Jimenez et al., 2008; Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2011; Rodriquez, 2016), to solve equations with the Pythagorean Theorem (Creech-Galloway et al., 2012), and to apply formulas to a variety of real-world financial problems (Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2011; Rodiguez, 2016). Similarly, Browder, Trela et al. (2012) documented the ability of students with ID to use a coordinate plan, and to create a data related graph.

This study builds on all the existing research. Individuals with ID can use bivariate equations to create linear graphs, and inversely, individuals with ID can use linear equations to create formulas. Simply, this study supports and adds to the findings in existing research. Six out of the six individuals with mild ID (55<IQ<70) could complete 60% of the steps needed to create-a-line or create-an-equation, and they retained higher than baseline scores (means, medians) after intervention sessions ended.

The findings of this study extend the feasibility of Algebra I level skills to *some* students with autism, multiple disabilities, and ID. The skills in this study were high school level skills found in 49 out of the 50 state algebra curricula. The skills were verified by general education teachers, special education teachers, parents, and high school students as high school level math activities. Six out of six individuals reached criterion for the procedural fluency for the algebra skills. Specifically, the results expand on the Italian studies (Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005) demonstrating that more students with ID can participate in high school level algebra. Of course, replication needs to be completed. Replications should include students with different comorbid diagnosis (e.g. autism, Fragile X Syndrome), different geographic locations, and more schools.

Best way to teach? How to teach algebra was one focus of this study, and should continue to be explored with future research. Indirect replication of the study would also be helpful to explore the effectiveness of the teaching methodology; indirect replication changes a minor item in the intervention (e.g. changing the prompting strategy). Johnson et al. (2008) described a program of research where indirect replications of single-case studies were chosen *a priori* with the intention of answering larger questions in a meta-analysis. The question of how best to teach higher math to individuals with ID still requires documentation (Browder, Jimenez et al., 2012;

Browder, Trela et al., 2012; Creech-Galloway et al., 2012; Göransson, et al., 2016; Jimenez et al., 2008; Root, 2016; Rodriquez, 2016), and the Johnson et al. (2008) provides a research solution to answer the question. The multi-faceted intervention package could be systematically adjusted to permit a meta-analysis the opportunity to compare the effects of intervention components.

For example, in this study, all six individuals showed improved performance over baseline, and there is strong experimental evidence showing that the intervention package was key to improving the performance in six out of six cases. However, at this point, it is difficult to identify the key components of the intervention package. The template, the time delay strategy, the modeling prompt, the self-monitoring strategy, or some combination of the intervention components could be the key intervention strategy. By itself, the *time delay* strategy is the only strategy that has been declared an evidence based strategy for teaching mathematics to individuals with ID (Browder et al., 2008); a future meta-analytical study like what Johnson et al. (2008) described could look at the other intervention components across studies to see if they now meet the criteria of an evidence based strategy.

Interventions in algebra depend on context (Haas, 2005). The group of students, the skill focus, the teacher's skill level, and the intervention all interact within the classroom (Darling-Hammond & Bransford, 2005; Haas, 2005). The indirect replication of studies could help to clarify the conditions in which constructivist conditions work and the conditions in which behavioral conditions work. Specifically, future studies could document under *what* conditions the intervention components were effective. This study employed a low-tech personnel-intensive modeling prompt. The strategy differed from the high-tech video modeling prompts employed by Creech-Galloway et al. (2012) and Kellum et al., (2016). In this study, a low-tech flip book with directions and diagrams was used instead of the high-tech model; however, the results of this study were similar to both the Creech-Galloway et al. (2012) and Kellum et al., (2016). This study did not directly compare high tech verses low tech approaches, and a future study could compare the use of high-tech prompting strategies to low tech strategies. Similarly, indirect replications of the experiment could be conducted with and without the self-monitoring supports, the templates, or the prompting. Again, using the Johnson et al. (2008) model for a meta-analysis, across time, researchers would be able to determine which parts of the intervention package are having the greatest impact on participant performance, and they should be able to tease out the interactions between intervention components.

In a similar fashion, researchers could explore *when* different interventions are effective. Recently, the field of special education has started to shift focus from procedural fluency to conceptual understanding, and many new publications explored the use of a Concrete-Representational-Abstract method paired with the errorless/structured teaching method (Bouck et al., 2017; Cease-Cook, 2013; Root, 2016, Root et al., 2017). Chiaki's results in the study were anomalous. Although she did show improvements during baseline that were above the intervention, she was the only participant who was unable to reach criterion without additional interventions. The adjustments to the intervention included adding concrete objects, reducing the complexity of the math problems, (Quadrant I only), and adding color prompts. The final booster intervention was like the intervention described by Root (2016) and Root et al. (2017) where an intervention package using a task-analysis, self-monitoring, and a phased Concrete-Representational-Abstract procedure to teach participants how to solve word problems. Hord and Xin (2012) used a similar strategy to teach participants with ID how to solve geometry formula problems. In this study, Chiaki required concrete objects to solve the math problems; however, the other five individuals did not require concrete objects. It appears that *some* students might require the use of concrete objects to solve math problems, but the technique is not needed for *all* students. An indirect replication of this study could be conducted by adding the Concrete-Representational-Abstract system to explore the schema based approach defined by Root, Browder, and Saunders (2016); in time, a comparison between the two methods will be easier.

Why is change occurring? At some point, the field needs to acknowledge Browder's (2015) observation that something is happening in the field of special education; individuals with intellectual disabilities can, do, and have performed mathematics skills previously assumed to be impossible. In this study, the participants with ID were preforming algebra skills common in the Algebra I curriculum (Alaska Department of Education and Early Development, 2012; Indiana Department of Education, 2014; Kendall, 2011; Minnesota Department of Education, 2008; NGACBP & CCSSO, 2010; Nebraska Department of Education, Oklahoma State Department of Education,2016; South Carolina Department of Education, 2015; VDOE, 2009); the tasks were once viewed as impossible for individuals with ID to complete (Browder, 2015; Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005). At some point, the field needs to ask *why* students with ID are demonstrating skills previously believed impossible. There are many possible explanations.

The change could be explained by the *Flynn Effect.* Across generational cohorts, and across time there is a tendency for cohorts from the current generation to outperform the previous generation (AAIDD, 2010; APA, 2013; Flynn,1984; Flynn, 1987). The results of this study showed that six students with ID could demonstrate high school level algebra, a skill that Browder (2015) noted was not considered feasible a few decades ago. The Flynn Effect could explain the results because the meaning of an IQ score would be contextual to the time. For an instance, an IQ of 70 measured in 1975 meant something different than an IQ of 70 measured in 2017 because the normed groups were different (AAIDD, 2010; APA, 2013). Although the effect could influence the performance of students with ID in algebra, it would be difficult to document. Students with ID have historically been excluded from national mathematics assessments. For instance, individuals participating in the alternate assessment are excluded from the National Assessment of Educational Progress (NAEP, IES, 2017), and the alternate assessment program has only been in existence since the establishment of the NCLB act of 2001. However, a qualitative historical analysis of the math studies and educational materials published over the past 100 years might prove useful. An historical study would document the changes in practitioner training, researcher focus, and outcomes.

An historical analysis would also likely support the Connolly (1973) hypothesis as a second explanation for why students with ID are learning more math today than previously thought possible. Connolly argued that the inclusion of elementary school students in arithmetic classes, based on the hypothesis that the previous research (from the 1950's and 1960's) sampled participants who lived without access to education. He predicted that individuals with ID (at the time it was called Mental Retardation) would demonstrate greater mathematical potential after being included in academic mathematical instruction. In the current study, six out of six individuals demonstrated the ability to learn algebra skills previously thought not possible for individuals with ID, and an application of the Connolly (1973) hypothesis could explain the results because the participants in this study are part of the first generation of students that were included in and assessed in academic mathematics through middle school (NCLB, 2001; IDEA, 2004); the public publication of test results focused attention on school performance (Kendall, 2011). However, annual curriculum testing was not part of the Rodriguez (2016) study, nor the

European studies (Göransson, et al., 2016; Monari Martinez & Benedetti, 2011; Monari Martinez & Pelligrini, 2010; Neodo & Monari Martinez, 2005). Connolly (1973) predicted improved mathematical abilities of cohorts of students with ID if the students were included in academics. However, the impact of inclusion might not fully explain the gains of individuals with ID.

The systematic measurement of the intervention might have led to the gains. In this study, a targeted intervention with systematic measurement was used, and six participants with ID demonstrated improvements in algebra skills. As part of the intervention the participant performance was monitored and in one of the six cases, the intervention was adjusted because the measurement showed the participant (Chiaki) did not make the expected progress. Using data to monitor and adjust interventions for individuals with disabilities is an old practice, and in mathematics there can be greater gains from the systematic intervention than the practice of inclusion. For instance, in a recent analysis of fraction instruction for students with disabilities, Fuchs et al. (2015) found inclusion had a smaller impact on student performance than targeted intervention; targeted intervention provided direct support to students.

A targeted and adjustable intervention might explain the gains of individuals with ID in other studies. Martinez and Benedetti (2011) and Monari Martinez and Pelligrini (2010) did have a nebulous description of individualized supplemental math support. Similarly, Rodriguez (2016) described supplemental support to the adults with ID who participated in the *money club*. A metanalytical study might tease out the differences. In any case, the inclusion of students with ID in algebra with behavioral interventions or targeted intervention support constitutes a change in practice from the traditional functional skills focus (Ayres et al., 2011; Ayres et al., 2012; Courtade et al., 2012; Kleinert et al., 2015).

More generally, the gains in student performance might be explained by changes in educational practices. Within this study, six participants showed improved procedural fluency for an algebra task after receiving an *errorless learning* intervention. *Errorless learning* strategies attempt to speed up the natural trial-error-learning process with the strategic adjustments in tasks designed to reduce the errors during trials (Mueller et al., 2007; Touchette, 1971; Touchette & Howard, 1984). *Errorless learning* has been applied to most of the interventions provided to participants in the US, engaged in algebra tasks (Browder, Trela et al., 2012; Browder, Jimenez et al., 2012; Cease-Cook, 2013; Creech-Galloway et al., 2012; Root, 2016; Root et al., 2017; Jimenez et al., 2007).

Systematic vs. natural generalization. As part of this study, the apparent impact of participants' behavior on inverse skills was observed before and during intervention. One participant, Bluford generalized the procedure (create-a-line) to the inverse skill (create-an-equation) shifting from no accuracy to 55% accuracy. Guione only received intervention for the target skill. Two additional participants (Ed & Guione) showed brief improvements in the percentage of steps completed for the inverse skill (create-a-line) while receiving intervention for the target skill (create-an-equation). However, to reach criterion, Ed and Guione required supplemental intervention support for the inverse skill. Similarly, the remaining participants (Chiaki, Mukai, & Dwight), showed no skill carry-over until direct intervention for the inverse skill began.

In the existing literature, there are three intervention techniques related to generalization. First, there are examples of spontaneous generalization of math skills. Rodriquez (2016) demonstrated that individuals with ID were understanding the procedures for solving algebraic equations because the participants self-scaffolded the algorithm. His participants were generalizing skills without explicit instruction. Specifically, the participants used blank spaces to represent variables. Similarly, a second method intervention promoted generalization of knowledge without explicit instruction. Göransson, et al. (2016) described teachers using inquiry based activities paired with strategic questioning to develop concepts; the participants generalized observations to language. A third, more structured approach blended constructivist methods with behavioral methods (Concrete-Representational-Abstract) procedure paired with self-monitoring and a behavioral prompting procedure to develop abstract concepts to generalize procedures across representations (Root, 2016; Root et al., 2017; Spooner et al., 2017).

In the current study, participants were monitored to see if they could generalize an algebra procedure to the inverse skill. Although only half of the participants, demonstrated impact, construed as generalization to the inverse skill. The fact that the participants did so when using a structured behavioral intervention suggests that, in some cases, behavioral approaches can develop some level of conceptual understanding without direct intervention support. The idea would support the Spooner et al. (2017) hypothesis that students with ID are generalizing more skills because they have been exposed and trained to problem solve with more complex math curricula. However, if only 50% of the participants are naturally generalizing knowledge when using behavioral, the approach does not appear to be reliable enough to rely on natural generalization, implying that more research is needed.

Noting a similar issue when teaching functional skills to individuals with ID, Stokes and Baer (1977) argued that explicit planning needed to take place to ensure reliable generalization; they specifically recommended developing staged interventions that generalized to other people, places, or skills. Stokes and Baer (1977) did not seem to differ from the approach advocated by Spooner et al. (2017) who mapped out an explicit behavioral instruction to ensure generalization of conceptual skills, and in this study, the other half of the participants generalized the inverse skill with more explicit behavioral interventions. Saunders et al. (2013) described generalization as an essential component of the process used to adapt algebra standards for individuals with ID. Determining the effectiveness of behavioral or constructivist approaches to generalize knowledge to conceptual understanding was not a focus of this study, but could be an avenue of research for future researchers. At a minimum, as researchers move forward, the rate of generalization to conceptual knowledge should be monitored.

General Education. Including students with ID in algebra benefits *all* students, and the results of this study help to clarify how all individuals learn algebra. Butterworth and Kovas (2013) stressed the importance of studying how individuals with intellectual disability learn algebra because individuals with disabilities help to define the neurocognitive processes used to solve mathematical problems. The current study contributes to the larger general education literature related to mathematics because the results identified issues associated with the definitions and process for measuring conceptual understanding. Additionally, the study's results help to clarify the role of language in the process of learning algebra.

Defining and measuring conceptual understanding. In this study, six participants with ID developed procedural fluency for a target algebra skill (create-a-line or create-an-equation), and half of the participants demonstrated that the intervention also had an impact on the inverse skill. It appears that those participants generalized the procedural knowledge to the inverse skill without supplemental intervention. Generalization of a skill denotes understanding (Stokes & Baer, 1977); therefore, the researcher is inferring that the generalization of knowledge to the inverse skill denotes development of conceptual understanding. However, demonstrating conceptual understanding as an outcome of procedural fluency is unlikely to convince critics of the *algebra for all* movement that individuals with ID are truly understanding algebra, and without clear demonstrations of conceptual understanding, understood to be manifested through verbal explanation, and who will continue to argue that the exclusion of individuals with disabilities from academic instruction is justified. For instance, Ayres, et al. (2011) argued against the adoption of academic standards for students with ID in part because there remains a belief that individuals with ID are unable to understand the academics enough to generalize them. However, the risk of exclusion is the same for general education students without disabilities. In 2007, Loveless argued to exclude all at-risk youth, including students with disabilities, from algebra instruction because he believed the students were not capable of understanding or benefiting from algebra. In both cases, the risk of exclusion results from the existing definition of understanding, which only accepts understanding as occurring when the individual can communicate understanding.

Currently, the construct of understanding relies on language to demonstrate comprehension (Ernest, 2002), and researchers within the general education research community struggle to measure conceptual understanding without the language (Ernest, 2002; Kilpatrick et al., 2001; Kendall, 2011; Rittle-Johnson, Schneider, & Star, 2015). Rittle-Johnson et al. (2015) examined the existing literature exploring conceptual understanding, and they found that language was being used as a proxy for understanding. However, Ernest (2002) theorized that understanding of a procedure occurs before the individual can explain the process, and there are no methods for measuring conceptual understanding without relying on the individual to communicate their thoughts (Ernest, 2002; Kilpatrick et al., 2001; Kendall, 2011, Rittle-Johnson & Schneider, 2015). In fact, there are "Currently, no standardized approaches for assessing conceptual understanding and procedural knowledge with proven validity, reliability, and objectivity….” (Rittle-Johnson & Schneider, 2015, p. 14). This study provides an alternate way to measure understanding and, by using the generalization of procedural fluency as a proxy for conceptual understanding, could increase the accessibility of measurements for individuals with communication complications (non-English speakers, individuals with communication disorders, and/or individuals with Autism or ID).

The body of special education literatureregarding algebra learning in students with ID and the results from this study, could assist the general education community. The generalization of any behavior indicates a level of understanding (Stokes and Baer, 1977; Thorndike, 1917), in this case, the generalization of skills from the target to the inverse. The idea of using generalization of procedural behaviors would be compatible with the Rittle-Johnson & Schneider (2015) framework for conceptual understanding. In the Rittle-Johnson & Schneider model, conceptual understanding and procedural fluency exist within working memory, and modifications to the working memory within long-term memory would indicate changes to both conceptual understanding and procedural fluency. The modifications to long-term memory occur when the individual changes behavior. Improving procedural fluency for an inverse skill would indicate a change in behavior and therefore would demonstrate conceptual understanding without the need to communicate with language.

Language & mathematics. Although it is desirable to develop a measurement of conceptual understanding separate from language and communication, language might be a separate strand of mathematical learning. In this study, Chiaki who required supplemental boosters to learn the algebra skills, was the most challenged with communication, and some of her difficulties appeared to be related to the language of the problem. For instance, when asked to find the “y-intercept” Chiaki would often point to the letter “y” in the formula. Chiaki’s confusion is understandable because in algebra multiple symbols often represent the same concept. The y-intercept is represented by the letter "b," but in some situations, in algebra, the intercept could be represented by the letter "a." Regardless of the algebraic symbolic representation, the y-intercept is the point where a line crosses the vertical axis. The language becomes more complicated when looking at the symbols and language of *slope.* Slope can be represented by the letters "b," "k," or "m," and linguistically, the terms *constant, rate-of-change, rise-over-run, slope,* and *modulation* may be used to describe the algebraic concept. The idea that linguistic inconsistencies can hinder the development of mathematical skills is not new to the general education literature. Miller, Smith, Zhu, and Zhang (1995) attributed longitudinal differences in arithmetic abilities between English speaking and Chinese speaking math students to the linguistic inconsistencies of the English counting system. More directly, Morin and Franks (2009) noted that issues with language complicated mathematical instruction for students with disabilities, and they recommended that teachers intentionally examine the impact of language in the mathematics classroom. Although speculation of the role of language in mathematics learning is not new, the field is still developing the tools needed for a complete investigation. The first tool needed will be a corpus (comprehensive list of words and symbols used in an area), and researchers are just starting to create the corpus for algebra words and symbols. Leibowitz (2016) compiled a list of symbols and language used in a single algebra classroom, and Alcock et al. (2017) attempted to compile a list of words and symbols used in Algebra by scanned an electronic library of algebra textbooks. After a more comprehensive list of the actual language used in the classroom is compiled, researchers could test to see if explicit instruction in terminology impacts the performance of students.

Practice. In the meantime, all students will continue to participate and be evaluated in the grade-level standards (ESSA, 2015; IDEA, 2004), and for high school students, the standards will continue to include algebra (Alaska Department of Education and Early Development, 2012; Indiana Department of Education, 2014; Kendall, 2011; Minnesota Department of Education, 2008; NGACBP & CCSSO, 2010; Nebraska Department of Education, Oklahoma State Department of Education,2016; South Carolina Department of Education, 2015; Virginia Department of Education, 2009). This study demonstrated that an *errorless learning* intervention package helps individuals with ID to improve procedural fluency with some generalization to indicate conceptual understanding. At a minimum, the study confirms that the *errorless learning* method is a flexible tool for teachers to use in the day-to-day classroom; however, the tool can be adapted to new situations.

Flexibility across students. In this study, all six participants showed growth with as they learned grade-level algebra skills. The intervention package was consistent with the *errorless learning* method. Also described as the structured teaching process, the *errorless learning* method provides teachers with a tool to teach students with ID. The process is not new to the world of mathematics or even algebra. Saunders et al. (2013) took the previous research (Browder et al.,2008; Browder, Jimenez et al., 2012; Browder, Trela et al., 2012; Jimenez et al., 2007) and mapped out a step-by-step guide for teachers to develop an errorless-based instructional program to teach algebra procedures. The steps included (a) selecting a skill from the grade-level curriculum, (b) identifying a real-life activity that required the skill, (c) find an evidence based instructional strategy, (d) include instructional supports, (e) measure and monitor the students' progress, and (f) plan for generalization.

This study demonstrated that the strategy is more flexible than described by Saunders et al. (2013). First educators had assumed it would be necessary to find a real-world application to teach skills to individuals with ID. Although practical applications of algebra exist, sometimes the applications are taught in separate, higher level mathematics or science classes (Kendall et al., 2011; Kress, 2005; Moses, 1989). The results from this study showed that students could learn the abstract skill without the real-world application, so the *errorless learning* tool could help teachers to provide access to grade-level algebra standards taught within the general education curriculum without real-world applications. Second, the results of this study indicated that the use of instructional supports (e.g. integrated color prompts and concrete objects) will depend on the individual's need. Five out of the six participants reached criterion without the additional supports, and the one participant who required additional supports did not benefit from a color prompting strategy. Finally, teachers should plan for generalization, but teachers should also be prepared for some students to generalize without direct instruction. The results of the study showed one out of the six participants generalized to the inverse skill without direct instruction, and two others showed indications that generalization was occurring. However, showing signs of generalization does not indicate that the individuals reached criterion, and in total, five out of the six participants required supplemental instruction to generalize to the inverse skill.

Adapting the technique. As an experiment, this study demonstrated the *errorless learning* technique with limited intervention variables. For instance, the experiment used a low-tech form of self-monitoring with abstract materials coupled with staff support. It may be likely that a classroom teacher would be able to adapt and expand the technique. Using self-monitoring, staff supports, big idea skills focuses, and generalization supports may lend themselves to be more flexible than the experimental controls would suggest.

Self-monitoring. Flexibility with the self-monitoring process is likely to be essential. In this study, the self-monitoring occurred using a low-tech flip book. The book contained text of the step prompt and a picture of a similar problem being solved. After completing a step in the procedure, the student turned the page to the next step. A teacher might choose another form of self-monitoring support. Examples of different approaches are in the existing literature. In previous research studies, self-monitoring occurred when the individual checked boxes next to steps of a task-analysis (Creech-Galloway et al., 2013; Jimenez et al., 2008; Root, 2016). Teachers could also embed the self-monitoring within a video modeling structure. Kellems et al. (2016) integrated the self-monitoring routine within a video modeling structure. Participants needed to swipe the screen after completing a step. The screen would then present a video of how to perform the next step of a math problem. The results from the current study and the Kellems et al. (2016) study suggest that the self-monitoring method is effective regardless of the method of delivery.

Alternative prompting support. In this study, teachers and paraprofessionals provided prompting supports for experimental consistency. This capital-intensive model might not be necessary. The video modeling structure demonstrated by Creech-Galloway et al. (2012) and Kellems et al. (2016) suggested that prompts can be delivered effectively with technological devices and without out staff. It might also be possible to replace staff with peers. In other studies, not related to mathematics, peers have delivered the prompts. Watkins et al. (2015) describes several peer-mediated instruction practices where prompting, reinforcement, redirection, and supplemental instruction helps the individual to achieve in the general education environment.

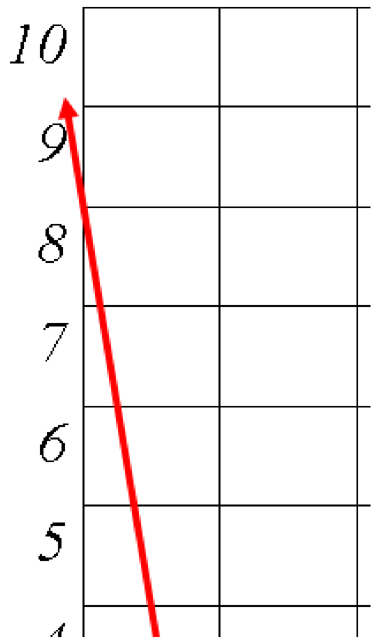

Figure 12. Example of a Common Mistake: Students would often misread the y-intercept for problems with steep slopes. In the example above the correct is 8; however, students would incorrectly identify the y-axis as 9 or 10.

Other Big Ideas. A teacher could also have flexibility breaking down the task into smaller components to focus on numeracy stressed by Witzel (2016), numeracy includes reading numbers on rulers, graphs, gauges, and dials. Again, for experimental consistency, the current study did not deviate instruction from the target or inverse skills, and in some cases, the participants demonstrated a need for numeracy instruction. For instance, error analysis and teacher reports noted that participants were incorrectly reading the x-intercept and y-intercept passed through a point but over another number. Figure 10 presents an example. Outside of an experiment, classroom teachers would be able to pause instruction to build the skill of reading graphs, rulers, and dials. Similarly, students would invert the x-intercept and the y-intercept, and teachers, outside of an experiment, would be able to provide supplemental instruction for the component skill.

Generalization to life. From the pragmatic perspective, teachers of students with ID could modify and adjust the generalization of the skill for direct employment applications. Algebra skills like the skills targeted in this study are useful in entry-level employment settings. For instance, the ability to create formulas is a fundamental skill used in data entry where the content of a spreadsheet, and the ability to create a graph from data or a formula is a skill that relates to reading a map, a newspaper, and as part of a self-improvement graph (Browder & Spooner, 2014; Browder, Trela et al., 2012; Fuchs, Deno & Mirkin, 1984; Rodriguez, 2016; Rosenbaum & Binder, 1997). Similarly, being able to find, translate, or read slopes is essential in carpentry and construction (Rosenbaum & Binder, 1997).

Policy. The results of this study impact two strands of policy. First the results reinforce the need to have robust professional development and teacher development programs to provide teachers with the tools for including students with ID into algebra activities. Second, the results reinforce the need to maintain policies that include students with ID in the state assessment programs.

Professional development. This study demonstrated one method for providing academic algebra instruction to students with ID. However, it will be necessary to train teachers to use the technique. Professional development for teachers is available. ESSA (2016) describes dedicating funding to train educators in the uses of state-wide alternate assessment programs. The professional development should include instruction in *errorless learning* techniques, and instruction in the moral and legal obligations requiring students with ID to participate in grade-level standards.

Errorless learning. This study demonstrated the effectiveness of an *errorless learning* method to improve the procedural fluency of an algebra skill for individuals with ID. The method should be included in professional development activities related to the alternate assessment. Without training, the status quo will be maintained: most students with ID are being educated in segregated environments (Kleinert et al., 2015), and in the segregated environments well-meaning advocates for students with ID prioritize functional skills instruction over academic instruction (Ayers et al. 2011, Ayers et al., 2012; Courtade et al., 2012). Integrating the method of instruction into teacher training programs or as part of the alternate assessment training conducted by states will likely improve student performance (Courtade et al., 2012). Spooner, Baker, Harris, Ahlgrim-Delzell & Browder (2007) conducted a professional development study where teachers were provided with one hour of professional development training in the *errorless learning* methodology. They measured improved academic performances for students of the teachers who participated the training. Updated professional development should include the concrete-representational-abstract methods described by Bouck et al. (2017).

Inclusion. The results of this study demonstrated that individuals with ID can develop algebra skills. Participants improved procedural fluency and demonstrated the ability to generalize the skills. However, the techniques will not be adopted, and students will not be included if parents, teachers, and administrators dismiss academic instruction as a novelty. To help mitigate the resistance to academic instruction for individuals with ID, it will be helpful to develop professional development materials that focus on the legal and ethical rationale for including students in grade-level activities.

Current policy regarding academic inclusion is based on legal requirements. The requirements have been considered and clarified by the courts, and the courts are supporting access to the general education curriculum. The *Endrew v. Douglas County School District* (2017) recognized that individuals with ID not only have a right to *access* the general education curriculum, they have a right to *make progress* in the general education curriculum. School districts are no longer able to justify *de minimums* benefit for the individual; instead, districts must be *adequately ambitious.* To be adequately ambitious, districts need to encourage teachers, parents, and administrators to shift from the assumptions about student abilities from *cannot* to *can* learn (Courtade et al., 2012). Professional development should include instruction on evidence based practices, including the Italian studies (Monari Martinez & Benedetti, 2011; Monari Martinez & Pellegrini, 2010; Neodo & Monari Martinez, 2005), the American studies (Bouck et al., 2017; Browder, Jimenez et al., 2012; Browder, Trela et al., 2012; Courtade et al., 2012; Creech-Galloway et al., 2013; Jimenez et al., 2007; Rodriguez, 2016;Root, 2016; Root et al., 2017), and the conceptual studies (Göransson et al., 2016).

Limitations

The results of the study are tempered by the limitations. The study was predicated on the assumption that the local school district accurately identified the participants as having an Intellectual Disability. In two cases, the adaptive behavior scores were questionable. Adaptive behavior scores were unavailable for Mukai who was identified by the school district as a student with autism. In another case, the records showed outdated adaptive behavior scores; Chiaki's scores were from a test (Scales of Independent Behavior- Rating). The test was administered when she was in preschool with triennial updates using "anecdotal records."

Additionally, the study's small sample size prevented a complete exploration of the data. The researcher was unable to determine if the order of skill development was important. It might be easier to learn how to create-a-line before learning how to create-an-equation, or vice versa. Order effects could explain some of the differences found in the results. Specifically, participants tended to obtain higher scores on the skill to "create-a-line" as both the target and the inverse skill. However, the difference could also be due to the inequality inherent in the skill. To "create-a-line" required only ten steps to complete, but to "create-an-equation" individuals needed to complete eleven steps.

The site used for the study maintained a robust extracurricular program for participants, and in some cases the time devoted to academic mathematics instruction was limited to as little as 30 minutes twice a week. In addition, the limited amount of time was sometime interrupted by other issues. Ed's intervention was stopped abruptly during the follow-up period after disciplinary issue (suspension) unrelated to the study, and Guione's intervention was interrupted for two, week-long absences. The first interruption occurred when Guione participated in a state sponsored vocational evaluation conducted at an out-of-town location, and the second interruption occurred when Guione attended a week-long chorus event in another state. In at least one case, the interruption in the intervention was followed by a decrease in the participant's performance.

Additionally, the participants might have been unique. Each participant had engaged with grade level standards as part of their participation with the Virginia Alternate Assessment Program. As such they had received instruction with a robust set of mathematical standards in third through sixth grade, and some of the participants had experience solving one-step equations using addition or subtraction. The familiarity with formulas, variables, and prealgebra concepts could have provided essential background knowledge.

Although enough participants were recruited for the study, not enough participants were recruited to conduct a true random sample, and because individuals opted in to the activity self-selection might have positively contributed to the participant's positive performance gains. Additionally, Chiaki's co-occurring disabilities likely interfered with her participation in the study. The severe disabilities expert who recommended adjustments to the booster sessions noted the intervention modeling protocol required Chiaki to shift visual fields too frequently. It is likely that Chiaki's performance scores were depressed due to accessibility issues.

Conclusion

Federal policy has changed instructional practices, and students with ID are required to participate in grade level academic skills (ESSA, 2015; IDEA, 2005; NCLB, 2001). The changes sparked an interest in new math skills instruction for individuals with ID (Bouck et al., 2017; Browder, Jimenez et al., 2012; Browder, Trela et al., 2012; Courtade et al., 2012; Creech-Galloway et al., 2013; Jimenez et al., 2007; Rodriguez, 2016; Root, 2016; Root et al., 2017). However, teachers, faced with the new accountability requirements, questioned the feasibility of teaching students with ID higher math like algebra. Limited research examined grade-level algebra instruction for students with ID; some studies suggested that students with ID could learn procedural math skills (Bouck et al., 2017; Browder, Jimenez et al., 2012; Browder, Trela et al., 2012; Courtade et al., 2012; Creech-Galloway et al., 2013; Jimenez et al., 2007; Rodriguez, 2016; Root, 2016; Root et al., 2017), and most existing studies did not examine if students were developing conceptual understanding (Göransson, et al., 2016).

This study established that students with ID could learn grade-level procedural fluency skills in t algebra using an *errorless learning* intervention package. Participants in this study included six students with ID. Three individuals had comorbid conditions (physical impairments, visual impairments, or autism). Their IQ scores ranged from 51 to 66. Participants were assigned to one of two experiments. In Experiment 1, the participants received intervention for the target skill (create-an-equation) while being monitored for an inverse skill (create-a-line). In Experiment 2, the other three participants received an intervention for a different inverse skill (create-a-line), and the inverse skill (create-an-equation) was monitored. In this study, five of the six individuals with ID could reach criterion (60% accuracy) with an *errorless learning* intervention package; however, a sixth individual required a modified intervention (concrete manipulatives) to reach the same criterion level. Three of the six individuals demonstrated generalization to the inverse skill without direct supplemental intervention.

The results supported research suggesting that students with ID can learn algebra skills, and in some cases, the individuals are clearly demonstrating some level of conceptual understanding . Future research should explore more grade-level algebra skills for students with ID, and researchers should work to develop a better method for measuring conceptual understanding in algebra. In high school algebra classrooms, the *errorless learning* teaching methods should be used to support students with ID, and policies should be developed to promote professional development to train teachers in the methodology.

However, the take-away from this study is larger than the establishment of new potential in algebra.. In general, researchers, practitioners, care-givers, and policy makers should question the historical assumptions for students with ID. Historically, assumptions related to the mathematical abilities of students with ID have been proven false (Browder, 2015; Connolly, 1973; Courtade, 2012; Jimenez et al., 2008; Rodriquez, 2016). This study, in concert with the other recently published studies, has proven the historical assumptions false. Students with ID can learn algebra skills. Students with ID can understand algebra, and students with ID can benefit from algebra instruction. In the long-run, the inclusion of students with ID in the grade-level algebra curriculum likely benefits the students and helps them to naturally generalize more functional life skills (Rodriquez, 2016; Spooner et al., 2017).

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APPENDIX A

SAMPLE RECRUITMENT COVER LETTER

Dear Parents,

I am a graduate participant at the Virginia Commonwealth University, and I am interested in teaching techniques. Specifically, I am interested in the techniques that would help students with special needs to access the high school, algebra curriculum. I would like to include your child in this study.

Attached is a permission slip for you to review. The permission slip describes the activities in which your child will be asked to participate. Contact information is also available.

Please carefully review the permission slip. If you are willing to let your child participate in the study, sign and return the document to your child's teacher.

Sincerely,

Andrew Wojcik M.Ed.

Virginia Commonwealth University

APPENDIX B

This Box for IRB Office Use Only – Do Not Delete or Revise
Template Rev Date: 12-10-15

RESEARCH SUBJECT INFORMATION AND PERMISSION FORM

TITLE: Developing Conceptual Understanding and Procedural Fluency in Algebra for High School Students with Intellectual Disability

VCU IRB NO.: HM20008463

INVESTIGATOR: Andrew Wojcik

PURPOSE OF THE STUDY
The purpose of this research study is to find out how students with Intellectual Disability learn algebra.

You are being asked to provide permission for your child to participate in this study because your child is preparing for the Virginia Alternate Assessment. Findings from the study will help other students with Intellectual Disability to learn algebra.

DESCRIPTION OF THE STUDY AND YOUR CHILD'S INVOLVEMENT

The study will be described, and your questions will be answered. If you decide to allow your child to participate in this study, you will be asked to sign at the bottom of this permission slip.

In this study, your child will be asked to participate in short math lessons twice a day for 3 weeks. The sessions will last less than 15 minutes, and your child will be asked to solve 5 algebra problems. Sample problems are attached. Your child might know how to solve the problems already, so for the first 5 sessions, your child will be observed as they attempt to solve the problems. Then for the remaining sessions, the teacher will model the process needed to solve the problems and provide feedback. Additionally, your child will be asked to respond to 5 statements about algebra. To make sure we are recording the steps your child uses to solve the problems, the sessions will be video recorded. To protect your child's identity, we will avoid speaking your child's name during the recordings and will not use your child's name when we same or store the recording.

RISKS AND DISCOMFORTS
Many children feel uncomfortable when they learn algebra, and the intervention is designed to encourage your child. The teacher will model how to solve the problem and they will provide positive feedback.

BENEFITS TO YOU AND OTHERS

Because your child is participating in the Virginia Alternate Assessment the teacher working with your child will use high quality instruction, and the work completed and collected will be counted as part of your child's assessment. Results from the study will help teachers design better programs for other students.

COSTS

If you choose to participate in this study, the time your child spends in the math sessions is the only thing that is considered a cost. **There is no monetary cost associated with this study.**

ALTERNATIVES

If you choose not to let your child participate in this study, he or she will still have the opportunity to learn the same math skills. Your child will still participate in the Virginia Alternate Assessment.

CONFIDENTIALITY

If you provide permission for your child to participate in the study, the researchers will access your child's educational records. Potential information that is being collected by the researcher that could identify your child would be the video tapes, notes about your child (grade level, age, race, IQ score, Adaptive Behavior score), and performance on the two algebra skills. Data is only being collected for research purposes. Your child's name will not be used on any of the information collected.

Instead of your child's name, a participant number will be used to record the information. The information/data files will be stored electronically on the researcher's computer, and within the VCU file lockbox. The files will be password protected, and only the VCU researcher will be able to access the files. The files will be destroyed 5 years after the study ends.

The videos of your child will also be stored electronically and with password protection. At the beginning of the study, all school personnel will be asked not to use your child's name during the recordings. The video files will be stored in the secure VCU file lockbox. After the data is recorded from the videos, the files will be destroyed.

VOLUNTARY PARTICIPATION AND WITHDRAWAL

Your child's participation in this study is voluntary. You may choose not to let your child participate. Your choice will not involve a penalty, and your child will still participate in their normal school day. If your child begins to participate in the study, you will be allowed to remove she or he from the study at any time. There are no penalties if you choose to remove your child from the study. Your and your child's rights and benefits will remain.

Your child's participation in this study may be stopped at any time by the researcher if it is deemed to be in your child's best interest.

QUESTIONS
If you have any questions, complaints, or concerns about your child's participation in this research, contact:

Donna Gilles Ed.D.
VCU- Associate Professor
Executive Director for the *Partnership for People with Disabilities*
(804) 828-8244

and/or

Andrew Wojcik M. Ed.
Doctoral Student
VCU School of Education
wojcika@k12albemarle.org
804-212-7060

The researcher/study staff named above is the best person(s) to call for questions about your child's participation in this study.

If you have any general questions about your rights as the parent of the participant child in this or any other research, you may contact:

Office of Research
Virginia Commonwealth University
800 East Leigh Street, Suite 3000
P.O. Box 980568
Richmond, VA 23298
Telephone: (804) 827-2157

Contact this number to ask general questions, to obtain information or offer input, and to express concerns or complaints about research. You may also call this number if you cannot reach the research team or if you wish to talk with someone else. General information about participation in research studies can also be found at http://www.research.vcu.edu/human_research/volunteers.htm.

PERMISSION
I have been given the chance to read this consent form. I understand the information about this study. Questions that I wanted to ask about the study have been answered.

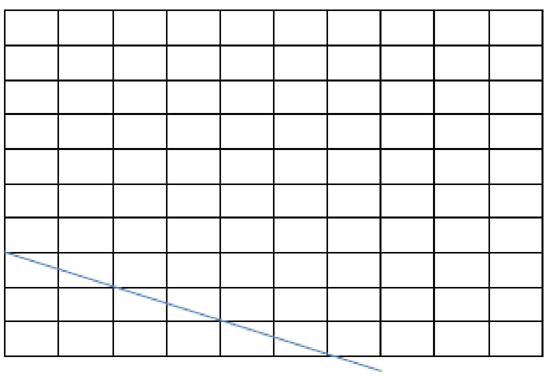
My signature says that I am willing to let my child participate in this study. I will receive a copy of the consent form once I have agreed to participate.

____
Name of Child

____
Participant name printed   Participant signature   Date

Sample Problem 1.

Directions: Use the graph of the line to create an equation.

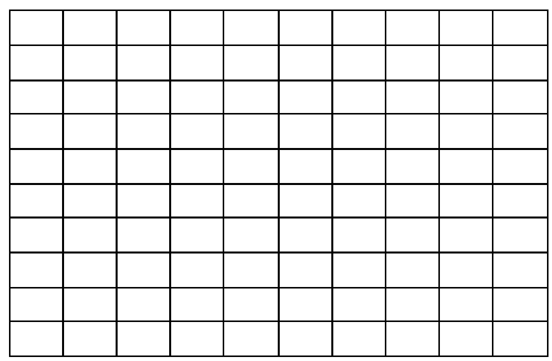


y= _________*x* + __________

Sample Problem 2.

Directions: Use the formula to draw a picture of the line.

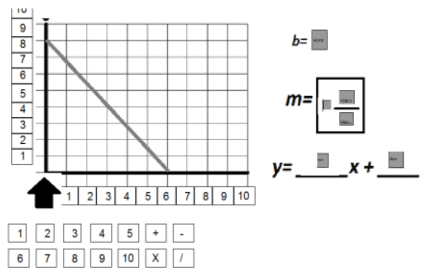
y= 1/3 x + 1



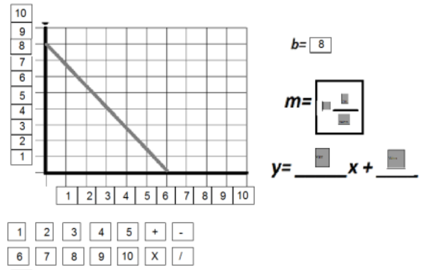
Appendix C

EXAMPLE OF STEPS FOR CREATING A LINEAR EQUATION FROM A GRAPH

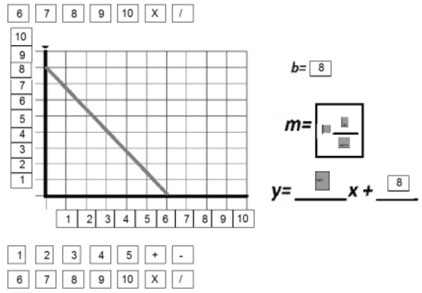
1. Identify the *y-axis.* (e.g. participant touches the *y-axis* or a number on the axis.)



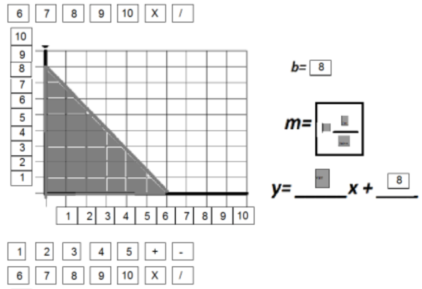
2. Identify the *y-intercept (*Participant places the *y-intercept* in the *"b" part of the template*.)



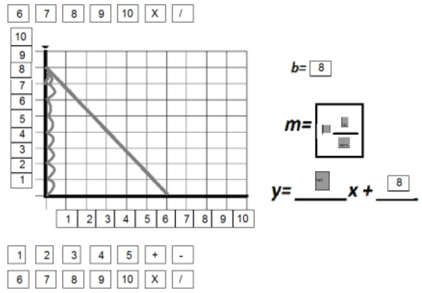
3. Place the *y-intercept* into the formula.



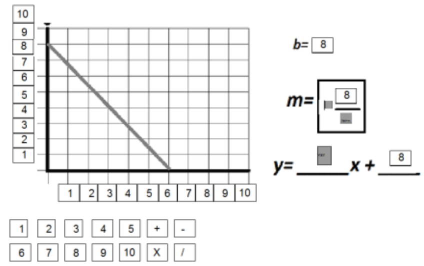
4. Trace the triangle. (Participant touches the *y-intercept*, moves across to the *x-intercept*, then to the origin, and back to the *y-intercept*)



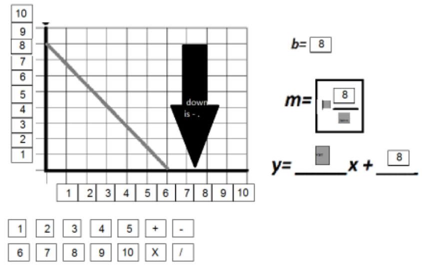
5. Count *Rise*. (from 0)



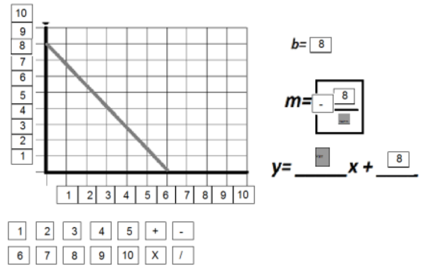
6. Put *Rise* into the *slope* formula



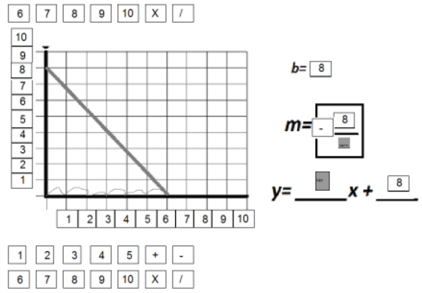
7. Negative or positive *slope?*



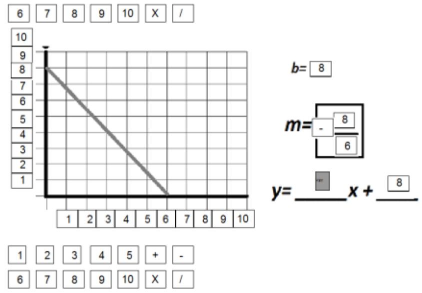
8. Place negative or positive into the formula.



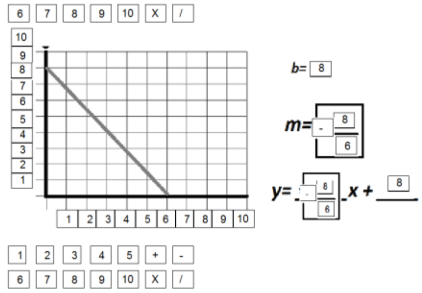
9. Count *run.* (from 0)



10. Place *run* into the formula.



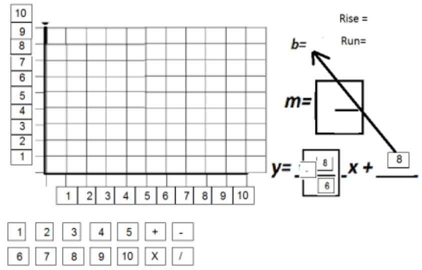
11. Place *Slope* into the line formula.



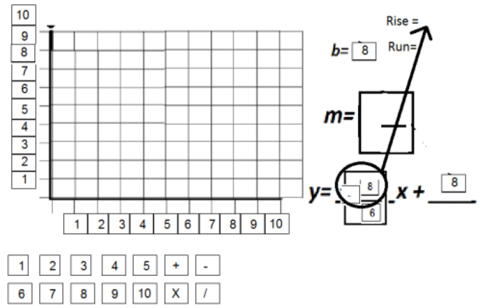
APPENDIX D

TASK ANALYSIS FOR CREATING A LINE FROM AN EQUATION

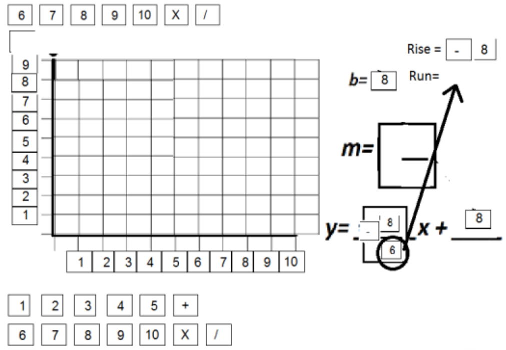
1. Find Begin (*y-intercept)*



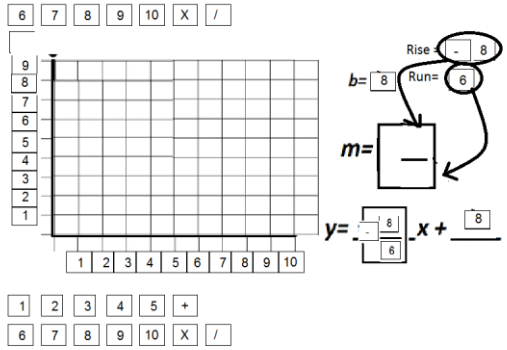
2. Identify *rise.*



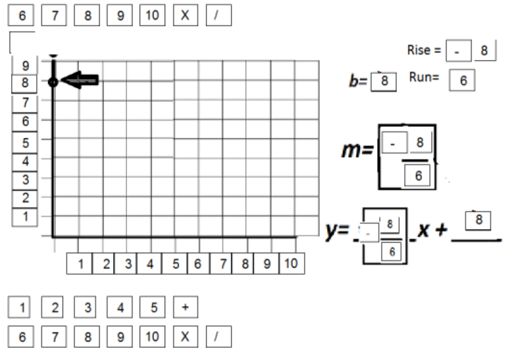
3. Identify *run.*



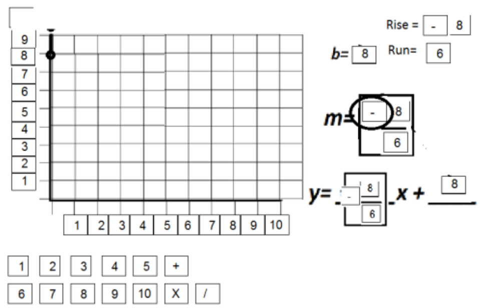
4. Create *slope (m).*



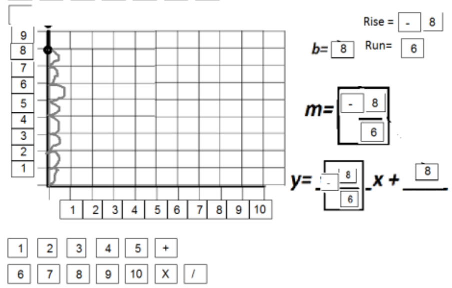
5. Place a dot on the *y-intercept*



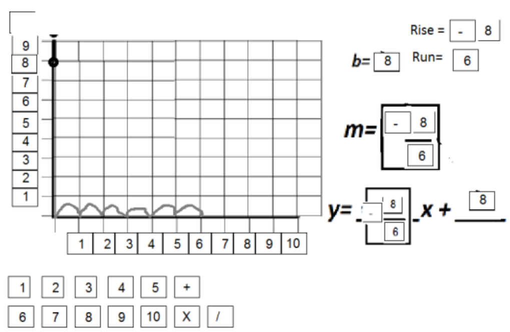
6. Decide Count Up or Down.



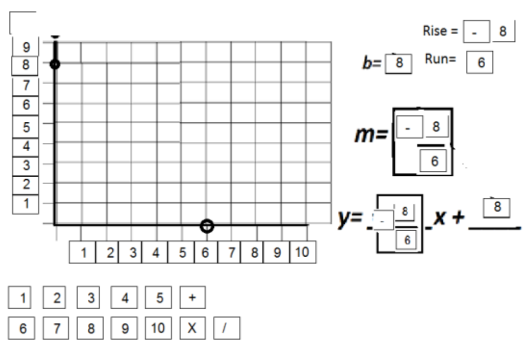
7. Count rise.



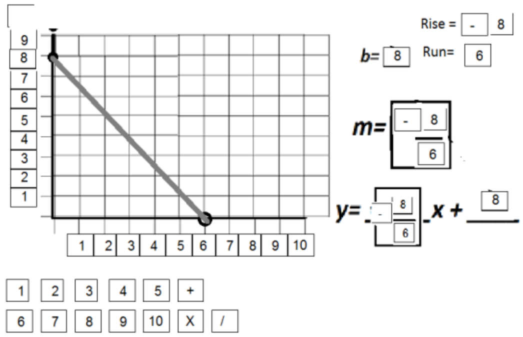
8. Count run.



9. Place 2nd Point.



10. Draw a line to connect the points



APPENDIX E

DATA SHEET FOR CREATING A LINE FROM AN EQUATION

RECORDTIMESTAMP ____________________

Session Number ____________________

Participant number

- ○ 1
- ○ 2
- ○ 3
- ○ 4
- ○ 5
- ○ 6

Phase of the Trial

- ○ Baseline
- ○ Intervention 1
- ○ Intervention 2
- ○ Follow-up

Randomly generated x-intercept

- ○ 10
- ○ 9
- ○ 8
- ○ 7
- ○ 6
- ○ 5
- ○ 4
- ○ 3
- ○ 2
- ○ 1
- ○ -1
- ○ -2
- ○ -3
- ○ -4
- ○ -5
- ○ -6
- ○ -7
- ○ -8
- ○ -9
- ○ -10

Randomly generated y-intercept

- ○ 10
- ○ 9
- ○ 8
- ○ 7
- ○ 6
- ○ 5
- ○ 4
- ○ 3
- ○ 2
- ○ 1
- ○ -1
- ○ -2
- ○ -3
- ○ -4
- ○ -5
- ○ -6
- ○ -7
- ○ -8
- ○ -9
- ○ -10

1. Find Begin (y-intercept)

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

2. Identify Rise

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

3. Identify run.

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

4. Create Slope

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

5. Place a dot on the y-intercept

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

6. Decide count up or down.

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

7. Count rise.

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

8. Count run.

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

9. Place second point.

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

10. Draw line.

- ◯ 0
- ◯ Independent, unprompted, and correct
- ◯ PROMPT
- ◯ NO RESPONSE

Record 1 for an independent, unprompted, and correct answer. During the baseline and follow-up phases, avoid prompts, feedback, and reinforcement.
Record 1 for an independent, unprompted, and correct answer.
Record P if a prompt was provided.
Record N for a non-response.
Record 0 for an incorrect answer

APPENDIX F
DATA SHEET FOR CREATING AN EQUATION FROM A LINE

Session Number	____________________
Participant number	◯ 1 ◯ 2 ◯ 3 ◯ 4 ◯ 5 ◯ 6
Phase of the Trial	◯ Baseline ◯ Intervention 1 ◯ Intervention 2 ◯ Follow-up
Randomly RISE	◯ 5 ◯ 4 ◯ 3 ◯ 2 ◯ 1 ◯ -1 ◯ -2 ◯ -3 ◯ -4 ◯ -5
Randomly generated RUN	◯ 5 ◯ 4 ◯ 3 ◯ 2 ◯ 1
Randomly Y-intercept	◯ 5 ◯ 4 ◯ 3 ◯ 2 ◯ 1 ◯ -1 ◯ -2 ◯ -3 ◯ -4 ◯ -5
1. Identify y-axis.	◯ 0 ◯ Independent, unprompted, and correct ◯ PROMPT ◯ NO RESPONSE
2. Identify the y-intercept (participant places the y-intercept in the "b" part of the template.	◯ 0 ◯ Independent, unprompted, and correct ◯ PROMPT ◯ NO RESPONSE
3. Place the y-intercept into the formula.	◯ 0 ◯ Independent, unprompted, and correct ◯ PROMPT ◯ NO RESPONSE

4.Trace the triangle (Participant touches the y-intercept, moves across to the x-intercept, then to the origin, and back to the y-intercept.	○ 0 ○ Independent, unprompted, and correct ○ PROMPT ○ NO RESPONSE
5. Count rise (from 0).	○ 0 ○ Independent, unprompted, and correct ○ PROMPT ○ NO RESPONSE
6. Put RISE into the slope formula (student places the y-intercept value into the numerator of the fraction)	○ 0 ○ Independent, unprompted, and correct ○ PROMPT ○ NO RESPONSE
7, Negative or positive slope?	○ 0 ○ Independent, unprompted, and correct ○ PROMPT ○ NO RESPONSE
8. Place negative or positive in the formula.	○ 0 ○ Independent, unprompted, and correct ○ PROMPT ○ NO RESPONSE
9. Count run. (from 0)	○ 0 ○ Independent, unprompted, and correct ○ PROMPT ○ NO RESPONSE
10. Place run in the formula.	○ 0 ○ Independent, unprompted, and correct ○ PROMPT ○ NO RESPONSE
11. Place SLOPE into the line formula.	○ 0 ○ Independent, unprompted, and correct ○ PROMPT ○ NO RESPONSE

APPENDIX G

SAMPLE TEMPLATE FOR CREATING A LINE

Create the graph from the equation.

$$y = 1/4\, x - 1$$

b=______

rise=______

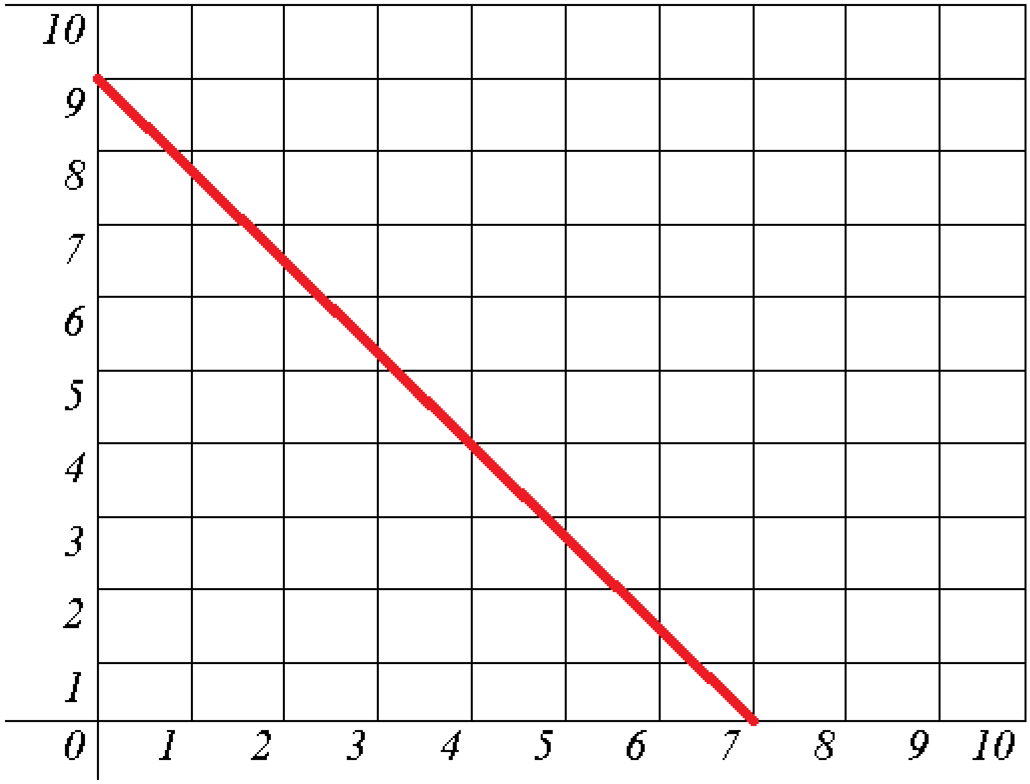
m= ______

run=______

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
-1										
-2										
-3										
-4										
-5										
-6										
-7										
-8										
-9										
-10										

APPENDIX H
SAMPLE TEMPLATE FOR CREATING AN EQUATION

Directions: Create an equation of the line.



X-intercept=

+ - SLOPE (M) =

Y-intercept=

Y= ______x + ______

APPENDIX I
SAMPLE OF COLOR PROMPTS

Create the graph from the equation.

$$y = 1/4\,x + 1$$

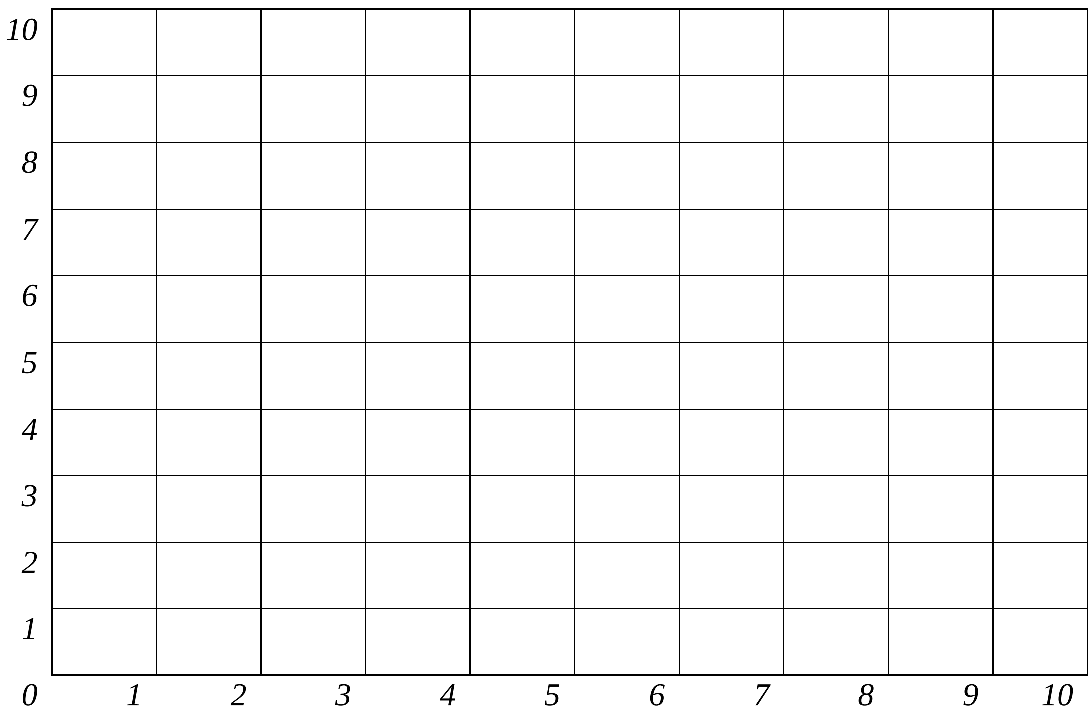
b= ______

rise=+/- ______

m=

______
______

run= ______



APPENDIX J

FIDELITY CHECKLIST

Directions: For each trial within the study, please indicate you completed the following.

	Date:	Date:	Date:	Date:	Date:
The participant matches the Pseudonym.					
The task analysis matches the skill taught.					
Participants were provided with the following materials: (a) a coordinate plane, (c) a formula template, (d) a ruler, and (d) a pencil.					
The task analysis was shielded from the participant.					
The answer key was hidden from the participant.					
The participant worked separately (from other participants).					
Prompts were delivered using the script.					
Prompts were delivered after a 5-second constant time delay.					
The data collection accurately portrays participant performance.					

Log of unusual events:

Date	Description of the event.

APPENDIX K
SOCIAL VALIDITY QUESTIONAIRE FOR THE PANEL OF EXPERTS

RECORDTIMESTAMP ______________________________

(1) It is important for participants to participate in algebra.

Strongly Disagree   Neutral   Strongly Agree

(Place a mark on the scale above)

(2) Algebra skills are required for graduation with a general education diploma.

Strongly Disagree   Neutral   Strongly Agree

(Place a mark on the scale above)

(3) In high school, students are asked to create graphs of lines from linear equations in the slope-intercept format (y=mx+b).

Strongly Disagree   Neutral   Strongly Agree

(Place a mark on the scale above)

(4) In high school, students are asked to create formulas in slope-intercept form (y=mx+b).

Strongly Disagree   Neutral   Strongly Agree

(Place a mark on the scale above)

(5) Algebra skills are needed to participate in high school science or math classes.

Strongly Disagree   Neutral   Strongly Agree

(Place a mark on the scale above)

(6) Algebra is a prerequisite for college admissions.

Strongly Disagree   Neutral   Strongly Agree

(Place a mark on the scale above)

(7) Algebra is a prerequisite for college math or science classes.

Strongly Disagree   Neutral   Strongly Agree

(Place a mark on the scale above)

APPENDIX L
SOCIAL VALIDITY QUESTIONAIRE FOR THE PARTICIPANTS

Participant Number

I need algebra to graduate.	Strongly Disagree — Neutral — Strongly Agree (Place a mark on the scale above)
I need algebra to attend college.	Strongly Disagree — Neutral — Strongly Agree (Place a mark on the scale above)
I need algebra to be in other science and math classes.	Strongly Disagree — Neutral — Strongly Agree (Place a mark on the scale above)
The staff helped me learn algebra.	Strongly Disagree — Neutral — Strongly Agree (Place a mark on the scale above)
I want to learn algebra.	Strongly Disagree — Neutral — Strongly Agree (Place a mark on the scale above)
I should learn algebra like other students in the school.	Strongly Disagree — Neutral — Strongly Agree (Place a mark on the scale above)

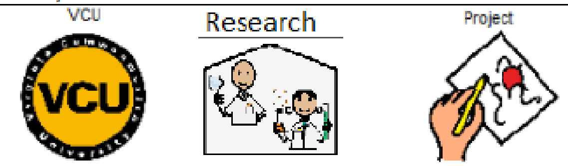
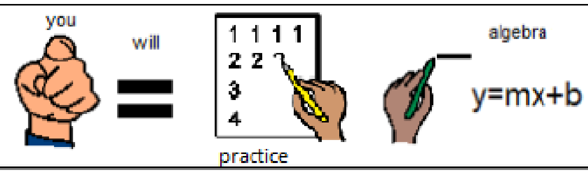
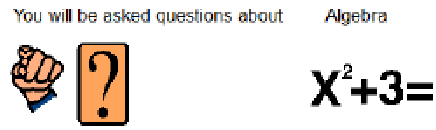
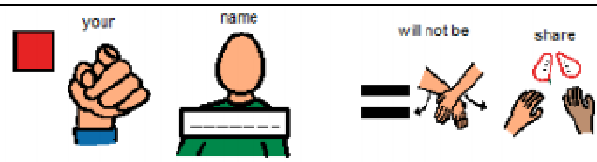
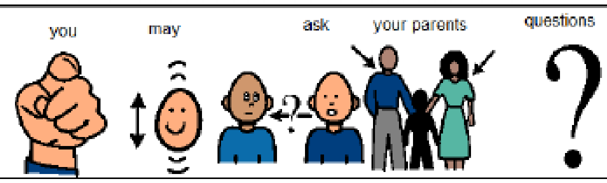
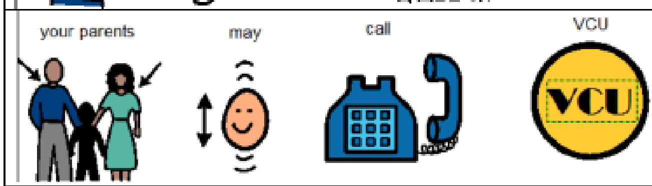
APPENDIX M

PARTICIPANT ASSENT FORM

Participant Assent Form

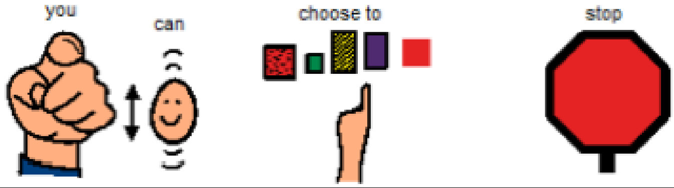
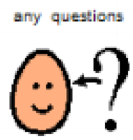
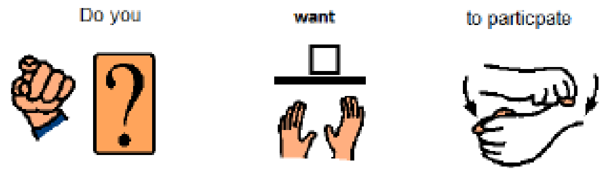
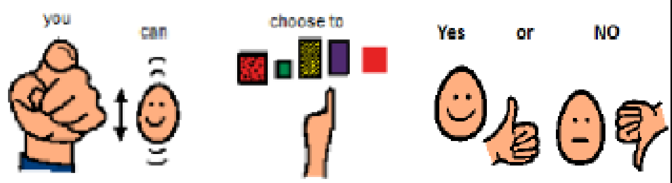
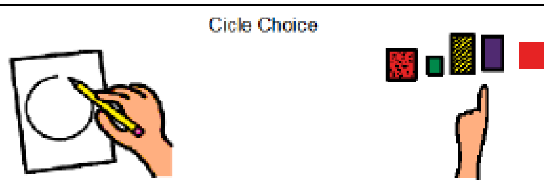
Study Title: Developing Conceptual Understanding and Procedural Fluency in Algebra For High School Students with Intellectual Disability

VCU IRB Study: **HM20008463**

 VCU / Research / Project	This is a research project being conducted by students at VCU.
 you / will / practice / algebra	You will practice algebra.
 You will be asked questions about / Algebra	You will be asked questions about algebra.
 you / will be / video / recorded	You will be video recorded.
 your / name / will not be / share	Your name will not be shared.
 you / may / ask / your parents / questions	You may ask your parents for advice.
 your parents / may / call / VCU	Your parents may call VCU.

Give a Copy of the Form to the Participant to keep.

Participant Assent Form
Study Title: Developing Conceptual Understanding and Procedural Fluency in Algebra For High School Students with Intellectual Disability
VCU IRB Study: **HM20008463**

	You may choose to stop at any time.
	Do you have any questions?
	Would you like to participate?
	You can choose to participate or not.
	Circle your choice.
	NO YES

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Fax: 1 (866) 585-6260
Email: Mjg@tobiidynavox.com
Web site: www.mayer-johnson.com

QUESTIONS

If you have any questions, complaints, or concerns about your participation in this research, contact:

Donna Gilles Ed.D.
VCU- Associate Professor
Executive Director for the *Partnership for People with Disabilities*
(804) 828-8244

and/or

Andrew Wojcik M. Ed.
Doctoral Student
VCU School of Education
wojcika@k12albemarle.org
804-212-7060

The researcher/study staff named above is the best person(s) to call for questions about your child's participation in this study.

If you have any general questions about your rights as the parent of the participant child in this or any other research, you may contact:

Office of Research
Virginia Commonwealth University
800 East Leigh Street, Suite 3000
P.O. Box 980568
Richmond, VA 23298
Telephone: (804) 827-2157

Contact this number to ask general questions, to obtain information or offer input, and to express concerns or complaints about research. You may also call this number if you cannot reach the research team or if you wish to talk with someone else. General information about participation in research studies can also be found at

http://www.research.vcu.edu/human_research/volunteers.htm.

Vitae

Andrew Wojcik was born in Great Lakes, Illinois on December 25, 1972. He graduated from Menchville High School in 1991, and obtained his Bachelor of Science in Education from Pennsylvania State University in 1995. He obtained a Master of Education from James Madison University in Virginia in 2003. He taught for two years for a private company called VisionQuest in Philadelphia, Pennsylvania and Osceola County, Florida. Then he worked for 19 years as a special education teacher in Albemarle County Virginia. Andrew continues to work as an administrator in the field of Special Education for Albemarle County Public Schools.