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Academic Rigor for All: A Review of Literature

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M E T R O P O L I T A N E D U C A T I O N A L R E S E A R C H C O N S O R T I U M
Background

Virginia Commonwealth University and the school divisions of Chesterfield, Colonial Heights, Hanover, Henrico, Hopewell and Richmond established the Metropolitan Educational Research Consortium (MERC) on August 29, 1991. The founding members created MERC to provide timely information to help resolve education problems identified by practicing professional educations. MERC membership is open to all metropolitan-type school divisions. It currently provides services to over 12,000 teachers and 152,000 students. MERC has based funding from its membership. Its study teams are composed of university investigators and practitioners from the membership.

MERC is organized to serve the interests of its members by providing tangible material support to enhance the practice of educational leadership and the improvement of teaching and learning in metropolitan educational settings. MERC’s research and development agenda is built around four goals:

- To improve educational decision-making through joint development of practice-driven research questions, design and dissemination,
- To anticipate important educational issues and provide leadership in school improvement,
- To identify proven strategies for resolving instruction, management, policy and planning issues facing public education, and
- To enhance the dissemination of effective school practices.

In addition to conducting research as described above, MERC conducts technical and educational seminars, program evaluations, and publishes reports and brief on a variety of educational issues.
# Academic Rigor for All: A Review of Literature

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**EXECUTIVE SUMMARY**

Rigor can be defined in any number of ways. We found an imbalance between the ways in which rigor has been defined by the Virginia Department of Education, and how education scholars define rigor in the respective academic disciplines.

- *The Commonwealth of Virginia* defines rigor as college and career readiness as measured by attendance in post-secondary educational institution, achievement of high Standards of Learning (SOL) test scores, as well as participation in Advanced Placement and International Baccalaureate programs.

- *Educational Psychologists* tend to define rigor in ways that are generalizable across contexts. Researchers in this tradition focus on *academic press*, or the extent to which educational stakeholders, including students, are oriented towards demanding coursework. Studies in this tradition have also found that student motivation is crucial, and that this motivation is mediated by the extent to which tasks are challenging, related to the world outside of school, and provide opportunities for students to collaborate when problem solving.

- *Discipline-Based Scholars* of Teaching and Learning define rigor in ways that reflect the core concepts of their discipline. Thus, a rigorous math class is one where students are encouraged to think mathematically, i.e. to use mathematical approaches to solve problems. Although specific pedagogical styles are discussed in this literature, the overall emphasis is on depth rather than breadth, with curricula being designed around building understanding of key concepts rather than covering (or efficiently delivering) factual information and procedural steps. In other words, a US history course might be organized around the way that the idea of freedom developed over the course of US history.

**BACKGROUND: ACCOUNTABILITY AS A POLICY CONTEXT**

Since the publication of *A Nation at Risk* in 1983, policy makers at the state and federal level have sought to improve the rigor of instruction and achievement of American K-12 students (Hamilton, 2003; Hess, 2003; Ravitch, 2010). The most popular of the reforms that has emerged since *A Nation at Risk* (1983) is what scholars call *high-stakes*, or *test-based* accountability. Over the past three decades, policy makers have coalesced around the idea that the root cause of our nation’s (supposed) educational decline is a lack of accountability (Hess, 2003; Loveless, 2005). As Loveless (2005) articulates it,

> The standards and accountability movement is based on the theory that a sequence of three activities will improve education: first, defining what students should learn (setting standards); second, testing to see what students have learned (measuring achievement); third, making the results count (holding educators and students accountable). (p. 7)

Education historian Larry Cuban (2005) listed a set of assumptions that underlie the theory articulated by Loveless (2005). Cuban (2005) writes that these assumptions include that:

- Strong economic growth, high productivity, long-term prosperity, including a higher standard of living, and increased global competitiveness depend upon a highly skilled workforce.
♦ Public schools are responsible for equipping students with the necessary knowledge and skills to compete in an information-based workplace.

♦ Public schools are doing a poor job of preparing high school graduates for college and the workplace, with urban schools doing the worst job of all.

♦ Schools are just like businesses. The principles that have made businesses successful can be applied to schools to produce structural changes that will improve academic achievement as measured by standardized tests, end the skills mismatch, and increase public confidence in schools.

♦ Higher test scores in school mean future employees will perform better in college and in the workplace. (pp. 39-40)

The assumptions and theory of action laid out by Loveless (2005), a supporter of these policies, and Cuban (2005), a critic, have proved enduringly popular with law makers. Politicians and policy makers are responding to what they perceive is the public’s demand for improved educational rigor. Studies touting evidence of the successes and failures of accountability policies have filled the pages of a wide range of education journals, and it is difficult to distill a conclusion regarding their overall effects. One recent meta-analysis of the research on the effects of test-based accountability policy over the past two decades concluded that

> since 1992, the era of test-based accountability has been associated with increasing student achievement, but improvements have not been as clear-cut or dramatic as had been hoped and cannot be attributed solely to accountability policies. Although the trend continues to be positive, the intensification of pressures since NCLB has not produced commensurately higher gains. (Shepard, Hanaway, & Baker, 2009, p. 2)

Although it is possible that the pressure produced by NCLB has not produced the desired gains, education administrators are faced with important decisions, often prescribed by law, and have little time to use research as an aid in decision making. District administrators are accountable to the public they serve, members of which may share the assumptions about education articulated by Cuban (2005). Chief among these assumptions is one that emerged in 1983 in *A Nation at Risk*, the fear that America is losing ground to economic competitors, and that public schools are responsible for this shift. Policy makers have tried to address this fear by enacting accountability policies that are designed to enhance the value of educational credentials. Writing standards and measuring achievement with tests is supposed to signal to employers that a high-school graduate has a set of cognitive skills that he or she can put to use as a member of the workforce. Doubts, however, remain in the business community about the extent to which students graduate from high school with the skills that they need to be successful (Hess, 2008). Colleges also continue to have to invest in re-educating freshmen and sophomores so that they have the academic skills to be successful (Nguyen, Bibo, & Engle, 2012).

After a substantial investment of time, effort, and money in testing systems (Chingos, 2012) administrators are asking whether classroom teaching and learning has the rigor to support the building of valued academic skills. A recent report by the Scholastic and the Bill and Melinda Gates Foundation (2012) indicates that teachers across the country do not believe that testing systems have increased academic rigor. The study found that teacher support for standardized testing is very low across the country. For example, “only 26% of teachers say that the results of standardized tests are an accurate reflection of student achievement” (Gates Foundation, 2012, p. 29). Almost half of the teachers surveyed (45%) reported that students do not take standardized tests seriously, nor do they perform to the best of their ability on them (Gates Foundation, 2012). In addition, only 20% of high school
teachers surveyed believed that district-level tests were “absolutely essential or very important in measuring student achievement” (Gates Foundation, 2012, p. 27). In contrast, 92% of the teachers who participated in this study reported that measures of achievement, such as formative and ongoing classroom assessments are “absolutely essential” or “very important” (Gates Foundation, 2012, p. 26). What is not known, however, is the extent to which these reported classroom practices support the research community’s definition of rigorous instruction.

There have been attempts at meta-analytical studies of the effects of high-stakes testing policies that rely on various kinds of evidence. For example, two meta-analyses of research on the effects of high-stakes assessment on rigorous practices were produced in the area of social-studies education (Au, 2007; Grant & Salinas, 2008). These meta-analyses, however, highlight the difficulty of making a definitive statement about the effects of these policies on classroom rigor. Nevertheless, both analyses agree that the evidence from a wide variety of research reports suggests that accountability has not delivered on its promise of greater rigor in history/social studies classes (see also, Grant, 2006; 2003). Au (2007) interprets the overall effect as one in which teaching is more narrowly focused on exam achievement leading to an “increase in teacher-centered instruction associated with lecture and the direct transmission of test-related facts” (p. 263) rather than a more rigorous approach. Grant and Salinas (2008) were more circumspect in their conclusions about the effects of current accountability policies on the climate of rigor in schools, emphasizing the great variability in how district leaders, administrators and teachers have interpreted and acted upon these policies.

**Academic Rigor**

In this literature review, we will attempt to address the issue of academic rigor in several ways. First, we will review federal and Virginia policy documents that discuss academic rigor, and the research reports that influenced these documents. We will then attempt to articulate a clear definition of academic rigor that applies across academic contexts. This definition draws on the work of educational psychologists, sociologists, and scholars of teaching and learning. Finally we will review the research literature on rigorous classroom practice in two disciplines, history and mathematics.

**Virginia’s Department of Education Defines Rigor**

*Rigorous instruction* is the term used frequently to describe the goals for teachers and students in documents which seek to influence educational policy in Virginia. The state has defined and discussed the issue of academic rigor in several official state documents (Commonwealth of Virginia Board of Education [CVBE], 2011; Virginia Department of Education [VDOE] 2011; 2010a; 2010b). These documents, in turn, cite two key reports as sources for how rigor is defined and framed (ACT, 2007; International Center for Leadership in Education [ICLE], 2011). These sources define rigor as the quality of the high school curriculum (ACT, 2007), and specify that a rigorous curriculum promotes in-depth learning and the use of cognitive skills similar to those found in the higher-order thinking levels of Bloom’s Taxonomy (e.g. application, evaluation, synthesis) (ICLE, 2011). Although the VDOE’s definition of rigor is drawn from these reports, state documents rely on measures, such as exam scores and advanced courses taken, as well as participation in post-secondary education as indicators of the existence of rigorous instruction (VDOE, 2011; 2010a: 2010b).
In a number of Virginia Department of Education documents, measures of student achievement are used as evidence to indicate the existence of instructional rigor in schools (VDOE 2010a, 2010b, 2011). These measures include student attainment of advanced proficient level—defined as achievement above a particular cut-score on a Standards of Learning (SOL) exam, attainment of college-ready SAT or ACT scores, participation in Advanced Placement, International Baccalaureate, dual-enrollment courses, and participation in the Virginia Early College Scholars program (Virginia Department of Education 2010a, 2010b, 2011). Virginia policy also refers to achievement on NAEP assessments as an indicator of rigor (VDOE, 2011). Virginia students’ NAEP scores have remained slightly higher than the national average, but have not risen or dropped significantly since 1998 (National Center for Educational Statistics [NCES], 2011; Schmidt, 2012). The VDOE has not based claims on the existence of rigor only on the measures mentioned above. The VDOE has publicly committed itself to the preparation of young Virginians for post-secondary education and the world of work (VDOE, 2010a; 2010b). In both the VDOE’s “College and Career Readiness Initiative” (2010a) and “Summary of Virginia’s Race to the Top Competitive Application” (2010b), rigor is defined in relation to students’ post-secondary success. By these measures, Virginia students are succeeding. In the latest report by the Federal Graduation Indicator (FGI), which followed Virginia’s graduating class of 2011, 62% of graduates who held standard or advanced diplomas were enrolled in post-secondary education within sixteen months of graduation (VDOE, 2012).

While the number of students enrolling in post-secondary education after high school graduation may serve as an indicator of rigor of the Virginia public school curriculum, questions have emerged about the extent to which high-school graduates are prepared for college-level work. For example, Virginia Commonwealth University’s University College was founded in 2006 after administrators realized that incoming freshmen, particularly minority students, needed greater academic support in order to succeed during the first years of college (Nguyen et al., 2012; VCU University College, 2012). Since then, Virginia Commonwealth University has seen graduation rates of African American and Hispanic students rise to approximately the same rate of Caucasian students, around 50% (Nguyen et al, 2012).

The necessity of programs like VCU’s University College suggests a need for a greater understanding and push for academic rigor in the PreK-12 curriculum in order to provide students with a stronger foundation of academic skills prior to enrollment in college.

Rigor appears to be a major concern for Virginia’s educational policy-makers, as exhibited by the frequency of the term in policy documents (Commonwealth of Virginia Board of Education, 2011; Virginia Department of Education 2010a, 2010b, 2011). However, its summative definition does little to aid administrators, teachers, parents, and students as they attempt to determine how rigor is manifested in schools. The development of formative definitions for rigor, in conjunction with the existing summative definitions provided by Virginia educational policy, may be useful for educators as they work to increase rigor in Virginia’s schools.

**Studying Rigor**

Academic rigor has been studied both quantitatively (e.g., Burris, Wiley, Welner, & Murphy, 2008; Matsumura, Slater, & Crosson, 2008) and qualitatively (e.g., Boston & Wolf, 2006; Bower & Powers, 2009), though most studies of rigor employ post-positivistic methods. Studies of rigor have been conducted with gifted and regular education students, but often focus on schools with low socio-economic status (Burris et al., 2008; Cohen & Poon, 2011; Harris & Harington, 2006; Lee & Smith, 1999). Typically, studies of rigor have been conducted in middle and high schools (Hoy & Hannum, 1997; Lee, Smith, Perry, & Smylie, 1999; MDRC, 2008; Stein, Grover, &
Quantitative studies of rigor often employ teacher or student surveys designed to assess the perceived level of rigor in lessons or the school climate as a whole (Matsumura et al., 2008; Phan, 2009; Shouse, 1996; Sweetland & Hoy, 2000). In such studies, student scores on tests of achievement serve as the chief proxy for rigor (Lee & Smith, 1999; Matsumura et al., 2006; Newmann, 1991). Quantitative studies of rigor often seek to reveal a causal relationship between rigor and student achievement (Burris et al., 2008). These methods favor the descriptions and guidelines for rigor presented by ACT (2007) and the Virginia Department of Education (2010 a, b; 2011).

Qualitative studies of rigor typically employ either classroom observation, sometimes combined with interviews (Boston & Wolf, 2006; Cohen & Poon, 2011; Wasley, Hampel, & Clark, 1997), or document analysis in which teacher lesson plans are analyzed for indicators of rigor (Henningsen & Stein, 1997; Hess, Carlock, Jones, & Walkup, 2009; Wolf et al., 2004). Such studies often seek to describe rigorous school and classroom climates, identifying proxies for rigor and how rigor is perceived by teachers and students. These studies often utilize or help to develop criterion-referenced rubrics for rigor which allow researchers and administrators to determine the level of rigor present in lessons or the school climate (Boston & Wolf, 2006; Matusecich, O’Connor, & Hargett, 2009; Mitchell et al., 2005). Proxies for rigor in these studies include high-level classroom discourse and questioning (Bower & Powers, 2009; Matuswievich et al., 2009; Wehlage, Newmann, & Secada, 1996; and lessons which require students to solve problems and make connections (Henningsen & Stein, 1997; Matuswievich et al., 2009; Wehlage et al., 1996).

### Defining Rigor Across the Disciplines

Rigorous teaching. Academic rigor typically describes curriculum or instruction which holds students to high standards, includes opportunities for the development of connections and deep knowledge, and fosters application of knowledge to real-world problems (Darling-Hammond, 1995; ICLE, 2011; Newmann, 1996). Rigorous teachers exhibit a disposition towards teaching that stresses the demand for great effort or commitment on the part of students to reach a certain standard (Blackburn, 2008; Darling-Hammond, 1995; Newmann, 1996). Teachers with this disposition are primarily concerned with student learning, teach within their students’ zone of proximal development, teach their students to think and work in disciplined ways, and provide opportunities for students to connect in-school knowledge to out-of-school knowledge (Newmann, 1996).

Holding high expectations for student learning is at the heart of academic rigor (Bower & Powers, 2009; Darling-Hammond, 1995; Mitchell, Shkolnik, Song, Uekawa, Murphy, Garet, & Means, 2005; Newmann, 1996; 1991). For example, Hoy and Hannum (1997) found that teachers and administrators in over eighty middle schools described academic emphasis as

*the extent to which a school is driven by academic excellence. High but achievable goals are set for students, the learning environment is orderly and serious, teachers believe in their students’ ability to achieve, and students work hard and respect those who do well academically.* (p. 294)

These findings relate to what Hoy and Hannum (1997) describe as academic press, a term which was used in many psychological studies in the 1990’s and is now synonymous with rigor amongst educational psychologists (Hoy & Hannum, 1997; Lee & Smith, 1999; Lee, Smith, Perry, & Smylie, 1999; Shouse, 1996). Academic press often refers specifically to aspects of the
educational or school climate that work in concert to foster high expectations and achievement (Murphy, Weil, Hallinger, & Mitman, 1982). Studies framed by the academic press construct investigate the relationship between academic press and student achievement (Hoy & Hannum, 1997; Lee & Smith, 1999; Lee, Smith, Perry, & Smylie, 1999; McDill, Natriello, & Palas, 1986; Murphy, Weil, Hallinger, & Mitman, 1982; Shouse, 1996), and have been conducted using both quantitative and qualitative methods. Studies such as that of McDill and colleagues (1986) found that student achievement varied systematically with levels of academic press, indicating that academic press and achievement were related.

Similar to Vygotsky’s idea of the zone of proximal development (Vygotsky, 1978), rigorous teaching assumes all students can learn if they experience educational activity that is set at an appropriately challenging level and provides time for mastery of new concepts (Blackburn, 2008; Bower & Powers, 2009; Brimfield, 1988; Common Core, 2012; Olvera & Walkup, 2010). Many studies of academic rigor suggest that systems of stratification typically found in secondary education contribute to the deterioration of rigorous education for students tracked in classes deemed to have lower-ability students (Bower & Powers, 2009; Burris, Wiley, Welner, & Murphy, 2008; Reed, 2008; Resnick, 1995, 2001, 2006). Resnick (1995) suggests that this may be due to the prevailing view of intelligence as a fixed property, meaning that students’ possess a level of aptitude that does not change over time (see also Dweck, 2000). This leads educators to modify the pedagogical approach, academic press, and cognitive complexity of instruction (i.e. rigor) for their students who have been placed in non-college tracks (Resnick, 1995). For example, when comparing the expectations and practice of two mathematics teachers who taught both honors and regular pre-calculus classes, Reed (2008) found that “tasks become less demanding for the regular students as they are not required to do the same amount of mathematical activity as the honors students” (p. 57).

This dilemma can be remedied by differentiating instruction in order to challenge students at appropriate levels (Blackburn, 2008). Education is still considered rigorous if students are held to expectations that are considered high for the individual. Challenging discourse, connections between prior knowledge and new concepts, and real-world applications help to foster high expectations for all students (Matsumura, Slater, & Crosson, 2008; Newmann, 1991; Newmann, 1996; Stein, Grover, & Henningsen, 1996; Wehlage, Newmann, & Secada, 1996). Building these concepts into instruction allows teachers to address definitions of rigor identified in policy documents as well as the definition of rigor developed by scholars. However, teachers are not solely responsible for increasing the level of rigor in education. Students also play a role in determining the level of rigor of their education.

**Students and Rigor**

Engagement is central to students’ participation in a rigorous education (Blackburn, 2008; Brimfield, 1988; Kilpatrick, Swafford, & Findell, 2001; Matsumura, Slater, & Crosson, 2008; MDRC, 2008; Stein & Lane, 1996), and is also increased when students are intrinsically motivated to learn. In their evaluation of educational reform in underprivileged schools in which increased academic emphasis (i.e. rigor) was a central focus of reform, Stein and colleagues (1996) found that student products that reflected high levels of academic rigor were related to students’ self-reports of intrinsic motivation to learn. Teachers fostered such motivation by increasing the complexity and real-world relevance of tasks, encouraging students, for example, to develop their own solutions to mathematical problems. Teachers can foster engagement by careful task selection, including tasks that have relevance to students’ interests and real-world applications (Blackburn, 2008; Kilpatrick,
Swafford, & Findell, 2001; NCTM, 2000; Williamson & Blackburn, 2010). High-level questioning and discourse, along with mixed-ability cooperative groups, can also help to increase student engagement (Matsumura, Slater, & Crosson, 2008; Newmann, 1991; Stein, Grover, & Henningsen, 1996).

In recent years, cognitive psychologists have studied higher-order thinking or critical-thinking skills as related to student achievement (Barak, Ben-Chaim, & Zoller, 2007; Franke, Webb, Chan, Battey, Ing, Freund, & De, 2007; Phan, 2009). Promotion of critical thinking has been linked to academic rigor and includes skills indicative of academic rigor, such as high-level discourse and the application of classroom knowledge to real-world problems (Wolf, Crosson, & Resnick, 2004). Critical or higher-order thinking is defined in these studies as the process of using prior knowledge, reflection, analysis, and synthesis to address new and perplexing, often real-world, problems (Phan, 2009; Seixas, 2006; Waring & Robinson, 2010). Such studies have been conducted both quantitatively and qualitatively, and often cite classroom discourse and questioning as indicators of critical thinking (Barak et al., 2007; Franke et al., 2007; Kracle, 2012; Waring & Robinson, 2010; Wolf et al., 2004). Education scholars who specialize in particular disciplines, however, tend to have more elaborate definitions of critical thinking that are closely related to the kinds of thinking necessary for that particular discipline. This review investigates how scholars conceptualize rigor more specifically in the math and history disciplines below.

The following two sections take up the issue of rigor as it relates to the specific disciplines of mathematics and history/social studies respectively. In these sections we highlight the findings of scholars from a variety of backgrounds whose studies of academic rigor are framed by the big ideas of a particular discipline. Although some findings are congruent, the studies discussed below differ from those mentioned above. Rather than endeavoring to make universal statements about academic rigor, the scholars discussed below are interested in studying the pedagogical practices that lead students to adopt disciplinary modes of thinking, e.g. thinking mathematically, or historically. These modes of thinking involve understanding key concepts and solving authentic problems.

**RIGOR IN MATHEMATICS**

In 2001, the National Research Council's (NRC) Mathematics Learning Study Committee, under the sponsorship of the National Science Foundation and the U.S. Department of Education, published a report synthesizing research on mathematics learning. The committee consisted of individuals with diverse backgrounds ranging from school teachers to principals, business executives and university professors. The report, published under the title *Adding It Up* (Kilpatrick et al., 2001), plainly portrayed the changing nature of the meaning of successful mathematics learning in school, and in society at large, throughout the twentieth century. As described in the report, in the first half of the past century, successful mathematics learning primarily meant gaining facility in using computational procedures within the discipline. Starting from the late 1950s, until the end of the 70s, success in mathematics meant gaining understanding of the structure of the unifying ideas of the discipline. Acquiring the necessary mathematical problem solving skills were also part of the espoused successful learning criteria of these decades. This era, also referred to as the new math, came to an end with 1980s emphasis back to accuracy and speed in carrying out the computational procedures in mathematics.

In 1989, the leading national professional organization in mathematics education, the National Council of Teachers of Mathematics (NCTM), published its first of a series of "standards" documents, which started the contemporary reform movement in mathematics education. Combining and synthesizing the goals of the past century, these
standards documents (NCTM, 1989, 1991, 1995, 2000) gradually characterized successful mathematics learning as the development of 'mathematical power,' which includes correct and sophisticated mathematical reasoning and communication skills, conceptual understanding of the big ideas of the discipline, knowledge of the necessary procedures and computations, as well as the ability to solve mathematical problems (Kilpatrick et al., 2001; Stanic, 2003). Thus, with the publication of these NCTM standards documents, a growing consensus has been built among mathematics education leaders about the need to define successful mathematics learning to include a wide range of knowledge, understanding, skills and dispositions, rather than a focus on one particular proficiency as was done in the past century.

To help clarify the goals of NCTM’s reform movement, NRC’s Mathematics Learning Study Committee also offered, in their report Adding It Up (Kilpatrick et al., 2001), five proficiency strands in mathematics that all students should attain at all levels. These proficiency strands are viewed as the pillars of successful mathematics learning today. The five strands are coined as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

Conceptual understanding refers to a student’s grasp of the underlying mathematical concepts and relations. Achievement of understanding involves meaningful activity on the part of the learner, who develops deep and relational understanding of central mathematical concepts. Procedural fluency is similar to the goal of facility in quickly and accurately carrying out the computational procedures in mathematics that was espoused in the past. The current conception, however, adds flexibility to efficiency and accuracy, which includes flexibly choosing and using procedures in particular situations based on an understanding of how and why the procedures work. Strategic competence involves being able to approach problem situations in a variety of ways and planning and carrying out effective mathematical strategies to solve problems. Adaptive reasoning is defined as the ability to persuasively explain one’s reasoning while mathematically justifying the solution steps used to arrive at the correct answers. Finally, productive disposition is the ability to perceive and appreciate mathematics as sensible, useful, worthwhile and relevant. There is a strong consensus in the field of mathematics education that all of these five proficiencies should be at the center of rigorous teaching in all mathematics classroom across the country. Thus, it is widely agreed that successful learners of mathematics demonstrate strength and power in all of these proficiencies.

As evident in the currently espoused mathematical proficiencies, rigorous mathematics instruction demands that students engage in meaningful mathematical activities that involve disciplinary reasoning, effective communication, strategic problem solving and fluent computation, and that result in the growth of conceptual understanding (Kilpatrick et al., 2001). To elucidate the basis for each of these tenets of a rigorous mathematics lesson, mathematics education scholars extensively discussed the major theoretical perspectives that guide the recommended pedagogical practices. Cobb’s (2007) account of the current major theoretical perspectives in the field of mathematics education, which was published in the National Council of Teachers of Mathematics (NCTM)’s second handbook for research in mathematics education, serves as a useful overview. According to Cobb (2007), four major theoretical perspectives underlie current research and practice in mathematics education: Experimental psychology, cognitive psychology, socio-cultural theory, and distributed cognition theory. In-depth historical origins and more detailed accounts of each perspective can be found in Cobb (2007) and elsewhere.

Similar to research in other disciplines, researches on how mathematics is taught and learned, and suggestions for its improvement, are based on findings from a
number of different research communities, including experimental psychology, cognitive psychology. These studies are framed by theories of mind that tend to emphasize the development of individual constructions of mathematical knowledge, or the development of social constructions of mathematical knowledge (Cobb, 2007). As Simon (2009) contends, these different theoretical perspectives should be viewed as complementary sources for educational scholarship and practice. This balance and harmony of theoretical bases in mathematics education underlie the NCTM’s widely embraced five process standards: problem solving, reasoning and proof, communication, connections, and representation, which summarize the research-based practices that help increase the aforementioned proficiencies. It is recommended that these five processes are implemented in every mathematics lesson and become an integral part of mathematical practice in school. Mathematics educators commonly believe that rigorous mathematics instruction that combines challenging content with these mathematical processes on a daily basis has the highest potential to increase the aforementioned proficiencies in all students and thus bring about successful learning for all. The NCTM’s (2000) latest standards document describes the five processes as follows:

− **Problem solving:** Instructional programs should enable all students to build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.

− **Reasoning and proof:** Instructional programs should enable all students to recognize reasoning and proof as fundamental aspects of mathematics; make and investigate mathematical conjectures; develop and evaluate mathematical arguments and proofs; select and use various types of reasoning and methods of proof.

− **Communication:** Instructional programs should enable all students to organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; use the language of mathematics to express mathematical ideas precisely.

− **Connections:** Instructional programs should enable all students to recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics.

− **Representation:** Instructional programs should enable all students to create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; use representations to model and interpret physical, social, and mathematical phenomena (NCTM, 2000).

Although today’s mathematics classrooms are changing to include these processes, if we look at a typical mathematics classroom across the country, it is still likely to observe a teacher mostly trying to help his or her students carry out a certain solution method or algorithm correctly (Kilpatrick et al., 2001). This method or algorithm is likely to be presented in its entirety at once and demonstrated several times until most students seem to have mastered its correct execution. Similar to the mathematics education practices of the previous century, computation is likely to be the overarching mathematical process, and obtaining right answers to the computations is likely to be considered the manifestation of successful learning. Vis a vis these typical practices, Ball (1991) writes: “When we hear right
answers simply as representing understanding, we miss opportunities to gain insight into students’ thinking” (p. 45). The ways in which students reason and think about a given mathematical situation are crucial for teachers to know because, based on the theories of mathematical learning outlined earlier, students’ existing knowledge and ways of thinking shape their current learning. Even if a student gives a correct answer to a question, the meanings and understandings that the student holds should be known to teachers in order to promote conceptual development (Ball, 1991). To achieve such conceptual development in their students, teachers should design effective learning environments with carefully chosen tasks and activities, facilitate students’ learning by providing suggestions, listening and posing questions, interacting, explaining, telling, showing, demonstrating, and establishing effective norms for discussion and communication. In these learning environments, teachers should also monitor the setting for doing mathematics in which the students are making sense of their experiences and growing understandings, they have autonomy with respect to the methods they use to solve the problems and they themselves decide whether an idea or solution is correct or reasonable, and the classroom culture exhibits an appreciation for mistakes as opportunities to learn (Hiebert, Carpenter, Fennema, et al., 1997). Furthermore, in effective mathematics classrooms that integrate the recommended processes and practices, students actively and fully participate in the carefully designed learning activities and continually reflect on their activity as well as other students’ comments and ideas.

According to NCTM’s problem solving standard, students should solve mathematical problems “for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific ‘correct’ solution method” (Hiebert et al., 1997). Students should also discuss and explicate their reasoning while explaining to each other the steps of their solution strategies.

One of the most important studies that provide details into how mathematics is taught in the United States is the Third International Mathematics and Science (TIMMS) video study conducted in 1995. National samples of teaching were collected in three countries from 81 U.S., 100 German and 50 Japanese eight-grade mathematics classrooms. This video study was a small part of the larger TIMMS study with 41 countries and three different grade levels. With the goal of investigating how eight-grade mathematics was taught in the U.S. and in Germany and Japan, the researchers videotaped one lesson in each classroom (Stigler & Hiebert, 1997). The following quote describes the nature of most common mathematics teaching practice observed:

“The typical eight-grade mathematics lesson in the U.S. is organized around two phases: an acquisition phase and an application phase. In the acquisition phase, the teacher demonstrates or leads a discussion on how to solve a sample problem. The aim is to clarify the steps in the procedure so that students will be able to execute the same procedure on their own. In the application phase, students practice using the procedure by solving problems similar to the sample problem. (Stigler & Hiebert, 1997, p. 18)

Besides this common teaching practice in the U.S., there are two important findings of this study that provide significant insight into how mathematical rigor might look like in the classroom. First, mathematical concepts and procedures can be either simply stated or developed through examples, demonstrations and discussions. When a procedure is developed, students investigate why the procedure works and go beyond its accurate execution. While the average percentage of topics containing concepts that were developed was around 80 percent in both Germany and Japan, it was 20 percent in the U.S. Likewise, while the average percentage of topics containing concepts that were simply stated was around 20 percent in both Germany and Japan, it was 80 percent
in the U.S. This finding gives us a good sense of what American students and teachers are not doing in the mathematics classroom.

Second, the nature of work students do in the mathematics classroom can be grouped into three categories: practicing routine procedures, applying concepts in new situations, and inventing new procedures. According to the TIMMS video study, average percentage of seatwork time spent in these three kinds of tasks in Germany and the U.S. was very close, but, was significantly different in Japan. In both Germany and the U.S., between 90 and 95 percent of seatwork time was spent practicing procedures. Time spent applying concepts and inventing procedures were less than 5 percent each. Time spent in these two rigorous and conceptually demanding tasks were slightly less in the U.S. than in Germany. In contrast with Germany and the U.S., Japanese students' average percentage of seatwork time spent in these three kinds of tasks were: 40 percent practicing procedures, 40 percent applying concepts, and 20 percent inventing strategies.

Vis a vis these findings, Stigler and Hiebert (1997) write: "But to assume that Japanese teachers are less active or directive than German or U.S. teachers would be a mistake. Although it is true that Japanese teachers give students time to struggle with challenging problems, they often follow this up with direct explanations and summaries of what the students have learned. This is why Japanese teachers were coded as engaging in more direct lecturing than either German or U.S. teachers. Although the time devoted to lecturing was minimal in all three countries, 71 percent of Japanese lessons contained at least some lecturing, compared with only about 15 percent of German and U.S. lessons" (Stigler and Hiebert, 1997, p. 18). Thus, this study has significantly contributed to our understanding of the nature of mathematical problems and activities that American students engage in mathematics classrooms.

**Rigorous in History**

This review is focused on the research tradition that emphasizes the importance of helping students to adapt more disciplinary modes of reasoning. Thus, a discussion of the teaching literature in history education must begin with a subject specific definition of rigor. This definition will enumerate the habits of mind that are valued in the history community. History and social studies are subjects that straddle both the humanities and the social sciences. Historians offer theories or reasoned arguments about change and continuity over time, usually in the form of a narrative account of the past based on the careful consideration of available evidence (Seixas, 1996; Wineburg, 2001). Historical narratives are rarely, if ever, evident from piecing together the evidence, however. Historians must use a number of tools and theories to interrogate and interpret the evidence (Wineburg, 1991). Many of these tools, such as econometrics, anthropology, forensics, archaeology, statistics and social theories have emerged from the social and natural sciences, others, such as textual analysis emerge from the humanities.

Ideally, history/social studies teachers should provide experiences that strengthen their students’ abilities to use factual knowledge, historical concepts, and interpretation techniques to make sense of the past. Teaching students to make sense of the past, however, is not the only goal of history/social studies teaching. The reason that the subject is included in the school curricula is to prepare the next generation for democratic citizenship (Hess, 2009; Reuben, 2005; Westheimer, 2004). This includes knowledge about government, as well as the origins and development of the United States and the rest of the world. It also includes a set of dispositions, such as considering evidence before making a decision, empathizing with people whose life circumstances are different than one’s own, and playing an active, positive role in one’s community. These dispositions, or habits of mind, are more difficult to
measure using standard behavioral objectives and measurement techniques, such as multiple-choice tests (Reich, 2009). There is, nevertheless, a broad consensus that these civic purposes of history/social studies instruction are crucial aspects of democratic citizenship that schools should help foster (Barton & Levstik, 2004; Wineburg, 2001).

To provide a general overview of research on rigorous history teaching, it is useful to identify a few key features of rigorous practice. What emerges from the literature is not so much a particular style of teaching (Barton & Levstik, 2004), for example student centered or teacher centered (Wilson and Wineburg, 1988; Grant, 2003), but rather ambitious goal setting based on the conceptual, knowledge and academic skill needs of students (Grant, 2003; Grant & Gradwell, 2010). It is difficult to narrow such broad ideas into a set of behavioral categories. Nevertheless, a few areas emerge as particularly important: conceptual focus, historical literacy (including writing), conceptual explanations, and classroom discussion.

**Conceptual Focus**

Beginning in the late 1980s, a number of researchers responded to the call made by Shulman (1988) for in depth studies of teaching that focused on the *pedagogical content knowledge* of teachers. Researchers at this time made important findings in regards to the way in which conceptual focus supports rigorous pedagogical practice in history/social studies classes (Wilson, 2001). Onosko (1990; 1989) found that more successfully rigorous teachers were those who placed "thinking as the central focus with content understanding a valued outcome" (Onosko, 1989, p. 191). Like other scholars (e.g. Barton & Levstik, 2004; Grant, 2005; Wilson & Wineburg, 1988; Wineburg, 2001), Onosko (1990; 1989) found that rigorous teachers prized depth over breadth, and had more well-thought-out and elaborate definitions of thinking than less rigorous teachers did. Onosko also found that these teachers framed thinking as dispositions, or habits of mind, such as: skepticism of historical claims, looking for evidence to support arguments, suspending judgment before coming to a conclusion, willingness to entertain other perspectives (see also Barton & Levstik, 2004; Grant, 2003; Levesque. 2008; Wineburg, 2001). Similarly, when studying the extent to which teachers were effective at teaching their students to use higher-order-thinking, Onosko (1990) found that the more rigorous teachers’ lessons were more focused, coherent, included more opportunities for students to explain their answers, and to have their reasoning—rather than their answer—critiqued and challenged (see section on discussion below).

Many of the studies inspired by Shulman’s (1988) call for research has been focused on teacher content knowledge and understanding (e.g. Wilson & Wineburg, 1989??). In their review of research on history/social studies teaching, Barton and Levstik (2004) were critical of the narrow focus on teacher knowledge, citing studies that indicate that teacher instructional goals are a more salient factor in regards to actual pedagogical practice (e.g. Grant, 2003).

Scholars such as Au (2007), have raised the concern that the current focus on preparing students to perform well on high-stakes exams has altered the pedagogical focus of teachers away from disciplinary rigor. Recently, a group of scholars have attempted to study the extent to which teacher practice in 6 states, including Virginia, with high-stakes history exams are focused on student conceptual growth in history/social studies. Called the Social Science Inquiry Research Consortium (SSIRC), the group studied the relationship between classroom instruction and student achievement on standardized history/social studies tests (SSIRC, 2011). SSIRC researchers observed 52 teachers at 17 school sites in the six participating states. The researchers used a protocol developed by Newmann and associates (1996)
for assessing the extent to which classroom teaching exhibits four key elements of rigor:

- Higher-order thinking
- Deep knowledge
- Substantive conversation
- Connection to the real world

**Higher-order thinking** was operationally defined as activities in which students are engaged in problem solving and are expected to be producers of knowledge who manipulate facts and ideas in order to arrive at a conclusion through some form of synthesis, generalization, or explanation. **Deep knowledge** was operationalized as the organization of instruction around the central concepts of a discipline (see also Wiggins & McTighe, 2006). For history/social studies these include the idea that history is an attempt to explain change over time through a rigorous, but fallible, analysis of the evidence, and synthesis of that evidence into a plausible narrative (Lee, 2005; Levesque, 2008; Seixas, 1996; Wineburg, 2001). **Substantive conversation** was a measure of the extent to which there was sustained back and forth among teachers and students focused on the deep knowledge of the lesson that is not controlled entirely by the teacher. Finally, the researchers measured the level of **connectedness to the real world**, or the extent to which classroom learning is connected to the lives students lead outside of schools and to persistent public issues.

The first report from this study (SSIRC, 2011) found that 78.9% of the teacher participants were not teaching in a way that would be regarded as focused on student understanding of history/social studies concepts. This finding is consistent with research on history/social studies teaching conducted over the past 40 years (for reviews see Barton & Levstik, 2004; Seixas, 2001; Wilson, 2001). The researchers found some evidence that students whose teachers were more rigorous outperformed the students whose teachers were less rigorous on standardized tests, but the correlation between test scores and rigor were not statistically significant. This finding furthers the argument that the tests being used to measure achievement of history standards to not accurately measure student understanding of disciplinary concepts (see also Reich, 2009).

**Historical Literacy**

The study of history pedagogy has benefitted over the past few decades from cognitive studies of reading and literacy. Researchers have taken studies of reading in history (Beck, McKeown, & Gromoll, 1989) and coupled them with a deeper understanding of historical thinking (Wineburg, 2001), creating new frameworks for **historical literacy** (Wineburg 2009; Reisman, 2012; Monte-Sano, 2011). With historical literacy, scholars have been able to build on earlier studies of historical thinking (e.g. Seixas, 1993; Wineburg, 1991) and operationalized some specific skills and strategies that help students read and make sense of historical texts. This research has informed effective curricular reform efforts (see Reisman, 2012 below).

Kucan and Beck (2003) found that students understand texts, and remember more information from them, when they conduct a mental conversation with the author. Reading research has shown that this is what competent readers do with all kinds of texts that they read. Researchers in historical thinking, however, point out that reading is not a set of universal skills that transfer from one domain to another (Moje, 2008). Different genres require different conversations between reader and author. In history, the two major genres of writing include textbooks and historical documents. Textbooks, as Beck and McKeown (1988; Beck, McKeown & Gromoll, 1989) found, pose some serious hurdles for a struggling reader. Understanding them, even at the elementary level, requires more background knowledge than most children have (Beck & McKeown, 1988). In addition, the
lack of a personal authorial voice in textbook writing (Paxton, 2002), a specialized academic vocabulary (Hinchman & Zalewski, 2001) and an omniscient voice that suggests that there are no controversies or unsolved mysteries in history (Paxton, 2002; Wineburg, 2001) all serve to make textbooks a hurdle, rather than an aid for many students. A number of studies have shown that when texts are written in more reader friendly ways, such as making fewer assumptions about background knowledge (Beck and McKeown, 1988), and writing in a personal rather than impersonal voice (Paxton, 2002), more students are more able to remember more information.

The other genre of history writing that has become more popular among classroom teachers are historical documents. The reading of such documents poses different problems than do textbooks. For example, Wineburg (1991) compared how Advanced Placement history students in an elite high school approached the reading of historical documents with the approach of a group of professional historians. He found that the high school students had learned to read for information, but not how to read historically. As a result, they were unable to draw a conclusion from the texts that they read, or to construct an accurate depiction of an event. Evidence has emerged in both the UK and in the US that approaching the difficulty of teaching and learning history by focusing on disciplinary literacy can be effective (Lee & Ashby, 2000; Reisman, 2012). Recently, the importance of student writing, particularly the opportunity to write multiple drafts, has come into focus as a major area in which rigorous history teachers can engender higher-order-thinking as well as higher order academic skills (Monte-Sano, 2011; 2008).

Reisman (2012) explored the results of a quasi-experimental treatment-intervention study in an urban California district. The study was designed to measure the extent to which a more rigorous approach to historical study would affect “(a) students’ historical thinking; (b) their ability to transfer historical thinking strategies to contemporary issues; (c) their mastery of factual knowledge; and (d) their growth in general reading comprehension” (p. 86). Teachers who were in the treatment group received extensive professional development and fully developed unit and lesson plans that covered U.S. history from early European settlement to the Vietnam War. The PD and materials inverted the traditional approach to teaching history. Rather than being told a story and asked to memorize details of it for an exam, students were asked to read historical documents and to come to reasoned conclusions of their own. In these document based lessons (Reisman, 2012), students were guided by their teacher through “four distinct lesson segments: (1) Background knowledge; (2) Central historical question; (3) Historical documents; and (4) Discussion” (p. 89).” The study found that students in the treatment group had statistically significant improved general reading skills, historical thinking skills, and factual recall (i.e. standardized test performance) compared to non-treatment students. The latter finding replicates that of Nokes, Dole and Hacker (2007) who also found that an approach to history that focuses on students’ ability to read, interpret and synthesize an historical argument from documents increases factual recall. Perhaps most significantly, the Reisman (2012) study found that treatment effects were more pronounced among the sub-group of struggling readers. In other words, struggling readers in the treatment group improved significantly more than their non-treatment counterparts on tests of historical thinking and factual knowledge.

Historical Explanation

Effective history teachers are able to provide students with powerful explanations of historical events and phenomena (Leinhardt, 2001; Paxton & Wineburg, 2000). Leinhardt (2001) explains that in history, explanations are designed to help students understand historical events (e.g. the signing of the Emancipation
teaching tools when the teacher is clear about what idea, structure or theme they are trying to exemplify. Leinhardt (2001) cites research that shows that it is usually a good idea to use multiple examples in an explanation. To be useful, representations should “connect in relevant and explicit ways to the explanation being developed” (Leinhardt, 2001, p. 348). That said, the danger of using representations in an explanations is that they can confuse students as well oversimplify and otherwise distort the explanation of an idea, event or theme.

**Classroom Discussion**

In history/social studies instruction, discussion is highly valued, but rarely attempted and often poorly executed (Hess, 2004). Hess (2004) defines discussion in the following way:

*First, discussion is dialogue between or among people. It involves, at a mini-mum, the exchange of information about a topic (a controversy, a problem, an event, a person, etc.). Second, it is a particular approach to constructing knowledge. The approach is based most fundamentally on the idea that something positive can occur when people are expressing their ideas on a topic and listening to others express theirs. ... it takes many forms and is used for many purposes. (p. 152)*

When orchestrated well by a teacher, discussion can be a key tool for raising academic rigor in the classroom (Hess, 2009; Kucan & Beck, 2003; Onosko, 1990; Rossi, 1995). As Hess (2004) points out, the interplay of diverse ideas and information in good discussions provides a crucial opportunity for students to practice the skills of critical thinking, including the complex forms of historical thinking mentioned above. Perhaps more importantly, it is through deliberative discussion that young people learn the skills of democratic citizenship (Hess, 2009; 2004; Westheimer, 2004; Westheimer & Kahne, 2004).
Unfortunately, discussions such as those outlined above appear to be rare events (Nystrand, Gamoran, Carbonary, 1998; SSIRC, 2010). One study (Nystrand, et al., 1998) found that 90 percent of social studies instruction in 106 middle and high schools contained no discussion at all. What discussion did exist consisted of exchanges lasting less than one minute. Wilen (2004) calls these short interchanges “a quasi-discussion form called recitation” (p. 33) that is primarily aimed at assessing student attention to teacher talk.

In a study of 58 teachers in 8 states, the SSIRC (2010) found that authentic pedagogy, including discussion, was more likely to occur in classrooms that were predominantly white and female, and less likely to occur in classrooms that were predominantly made up of students of color. Overall, this study (SSIRC, 2010) found that 42 of the 58 teachers observed exhibited minimal or limited authenticity, a judgment that includes an assessment of the level of classroom discussion. Teachers exhibiting minimal authenticity primarily lectured with power point and recitation rather than discussion. Even when they engaged students in project based work, there was little opportunity for students to discuss their work with their peers.

Good discussions begin with questions for which there is no one obvious correct answer (Bain, 2006; Hess, 2004; 2009; Newmann, 1996; Wiggins & McTighe, 2006). In her review of the relevant literature on discussion in social studies classes, Hess (2004) concludes that

_Virtually all of the case studies of high-quality discussions in the literature share as their central feature a problem, text, topic, question, or issue that provokes multiple interpretations._ (p. 154).

Good discussions are more likely to be the result of careful teacher planning and orchestration (Hess, 2009), from the pre-planned questions to the attention given to preparing students with the information and preparation needed to make meaningful contributions.

Another crucial factor in the existence of good discussion is the classroom culture that the teacher co-creates with his or her students (e.g. Bain, 2006). Classrooms that value habits of mind such as listening, respect for differing opinions, the use of evidence to support claims and a cooperative rather than competitive ethos help students feel comfortable sharing their views and taking risks (Hess & Poselt, 2002).

**Conclusion**

Policy makers and members of the general public have been anxious about the state of education in this country since the early 1980s. Chief among these concerns has been the fear that high school graduates will not have the academic skills and knowledge needed to be successful in post-secondary educational institutions and to compete in the global economy. Policy makers have chosen to address this situation with a system of accountability that relies heavily on standardized tests. Critics of these policies have pointed out that these tests measure the attainment of only a fraction of the academic skills and conceptual knowledge that students need to be successful.

Scholars who study academic rigor have found that it is more likely to exist in schools with cultures that foster high expectations of all students and that have an overall focus on providing students with educational experiences that challenge them. Education scholars who focus on particular disciplines have added much to these findings. These scholars have enumerated specific academic and pedagogical skills that are crucial to rigorous instruction in a particular discipline. This literature is particularly useful for helping stakeholders to make sense of what rigorous instruction looks like in a math, history, or science class. We suggest that the scholarship on disciplinary learning contains key insights into how more rigorous pedagogical approaches might be developed.
REFERENCES


